Weather derivatives as an instrument to hedge against the risk of high energy cost in greenhouse production

by

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1 Introduction

While weather derivatives become more and more important in the risk management of energy companies, this innovative financial instrument is hardly used in agriculture. They serve as a financial coverage of weather related business risks which play an important role in agriculture. These risks relate to the revenues as well as to the costs of livestock, crop and horticultural production.

Depending on the production programme, greenhouse production is characterised by high heating energy demand especially in wintertime. During this period greenhouses have to be heated at low outdoor temperatures to keep a constant temperature inside. Thus, changing temperatures lead to a significant variability of costs which, in turn, cause reverse fluctuations of returns on the part of the energy supplier.

Thus, it is worth to analyse the effect of weather derivatives in greenhouse production. Therefore we explain the characteristics of weather derivatives in a first step. Based on a greenhouse model we analyse the energy cost variations. Finally we check whether the financial risk can be reduced with a weather derivative based on a temperature index.

2 Characteristics and construction of weather derivatives

Generally speaking, a weather derivative is “a derivative whose payoff is based on a specified weather event, for example the average temperature in Chicago in January” (CFTC, 2005). Other underlyings like precipitation, snowfall, wind velocity or solar radiation are possible as well. Thus, weather derivatives have no tradable asset as underlying. Their main objective is to convey a compensation payment if the underlying develops unfavourable. Most of the weather derivatives are traded “Over the counter” (OTC). Contrary to financial exchange trading the OTC market is less formalised and is characterised by individual agreements between contract partners.
In addition to the underlying, other contract parameters like duration (monthly or seasonal) have to be determined. The *strike* is the index value where a payment is exercised. The payment is determined by the *tick size*. This is the amount to be paid per index point. The underlying fixed in the contract is measured at a predefined weather station (Cao and Wei, 2003).

**Figure 1: payoff structure of different option types and positions**

Although different kinds of future transactions are possible, options dominate the market as they are particularly appropriate to reduce downside risk (Becker and Bracht, 1999). Generally, the buyer and seller of an option are in the following situation: the buyer (long position) purchases a right and pays a premium for it. The seller (short position) accepts an obligation and receives the premium. Depending on the content of the right, options are distinguished into two basic types: in the case of a *call option* the buyer purchases the right to buy an underlying at a certain price and at a certain time. Contrary the buyer of a *put option* purchases a right to sell the underlying at a certain price and time.

The buyer of a call option hedges against increasing market prices of the underlying that he wants to buy in the future. If the market price exceeds the strike price, the option will be exercised. If the market price falls underneath the strike the buyer loses the premium. In both cases the difference between market price and strike price determines the payoff of the option.

The payoff structure of the respective option at maturity is depicted in figure 1. The positions “long” and “short” represent the buyer and seller. K marks the strike level that corresponds to
the strike price of a traditional underlying (e.g. stock prices). P is the premium or price of the option to be paid. The payoff is determined by the positive difference between the observed index value at maturity and the strike level K. The difference is multiplied by the tick size V that corresponds to the payment per index point. Deducting the premium P from the payoff leads to the profit or loss of the option. The buyer’s profit (long position) for a call option is:

\[ G^L_c(x, K) = V \cdot \text{Max}[0, (x - K)] - P_c \]  

(1)

The buyer’s profit for a put option is:

\[ G^L_p(x, K) = V \cdot \text{Max}[0, (K - x)] - P_p \]  

(2)

Symmetrical to the long position the profit of the seller is:

\[ G^S_c(x, K) = -V \cdot \text{Max}[0, (x - K)] + P_c \]  

(3)

\[ G^S_p(x, K) = -V \cdot \text{Max}[0, (K - x)] + P_p \]  

(4)

The fair premium of the option corresponds to the discounted expected value of the payoff. The fair premium is defined in the way that the expected profit of both parties is zero and no transaction costs accrue. It is calculated by multiplying the tick size V by the negative deviation of the index x from the strike level K and the probability \( \omega \) that the index x is underneath the strike level. Finally the factor \( e^{-rT} \) discounts the payment over the duration T using the interest rate r.

\[ P_p = \left[ (K - E(x \mid x < K)) \omega (x < K) V \right] e^{-rT} \]  

(5)

The fair premium of a call option is calculated analogously:

\[ P_c = \left[ (E(x \mid x > K) - K) \omega (x > K) V \right] e^{-rT} \]  

(6)
3 Definition and distribution of the underlying

In this section we describe the basis variable first and then we analyse its stochastic characteristics.

3.1 Definition of the temperature index

Since up to date weather derivatives are mainly used in the energy sector where revenues heavily depend on the temperature during a season, temperature based weather contracts have the highest market share with 89 %. They are mostly based on the concept of so called degree days (Deutsche Bank Research, 2003). This base variable is calculated according to how many degrees a daily average temperature varies from a reference value. The day’s average temperature is based on the maximum and minimum temperature from midnight to midnight (Ellithorpe and Putnam, 2003). Because there is a loss of information in the concept of degree days, derivatives whose underlying is based on the average temperature of a period, become more and more important. This parameter can be picked from the homepage of guaranteedweather.com for different locations all over the world. Most of the heating energy in a greenhouse is needed during the winter season from November 1\textsuperscript{st} (i=1) till March 31\textsuperscript{st} (i=N). Thus we have chosen this period as duration for the weather index \( x \).

\[
x = \frac{1}{N} \sum_{t=1}^{N} \frac{T_{\text{max}, t} + T_{\text{min}, t}}{2}
\]

Temperature indices have some particularities that we explain in the next section.

3.2 Nature of the seasonal temperature index

Generally speaking there are two ways to obtain the distribution of the base variable or the index derived from it. On the one hand the index can be calculated from the historical values for each year (Brody, 2002, p. 198; Turvey, 2001, p. 5; Dornier and Queruel, 2000, p. 1). Since the index is usually a continuous random variable we can estimate a distribution from
historical data. This approach is very simple but one precondition is that the contract partners fix the price at the beginning of the contract and that they do not trade the derivative before maturity. With regard to the problem described in this paper this limitation seems to be acceptable because as a result of the high specificity there is no liquid market expected. However, a large number of years is needed to estimate the probability density function properly and we only obtain one index value per year. From figure 2 that depicts the frequency of the average temperature from November till March for the years 1980 to 2003 we cannot be sure that the normality assumption is correct. The distribution seems to be left skewed. More data are needed to arrive at a certain conclusion.

Figure 2: frequency distribution of the average temperature in Berlin from November till March from 1980 till 2003; source: http://www.dwd.de

The second approach is to simulate the development of the daily temperature during the season to obtain the probability distribution. The average temperature is the computed over the simulation time. Repeating the simulation many times finally yields the distribution of the average temperature. For the simulation we need a stochastic model of the seasonal temperature. The stochastic process reflecting the temperature dynamics of the period has the following characteristics:
• Daily temperatures fluctuate seasonally. A positive linear trend is usually added to the seasonal figure.

• Daily temperatures correlate with temperatures of the days before.

• Daily temperatures follow a mean reversion process. They can only deviate from the long time seasonal average for a short time and move back again then.

• The standard deviation of the temperature time series changes over the season.

Generally temperature is a continuous time value. Thus, the process in principle should be modelled as a continuous time diffusion process. However, since only daily averages of temperature are available the parameter estimation has to be done with discrete variables.

Following the approach used by Cao and Wei (2000), Alaton et al. (2001), Tigler and Butte (2001) and Schirm (2001) a time series analysis was carried out using the model below:

\[ Y_t = \bar{Y}_t + U_t + \sigma_t \varepsilon_t \quad (8) \]

with

\[ \bar{Y}_t = a_0 + a_1 t + a_2 \sin(\omega t) + a_3 \cos(\omega t) \quad (9) \]

\[ U_t = Y_t - \bar{Y}_t = \phi_1 U_{t-1} + \phi_2 U_{t-2} + \phi_3 U_{t-3} \quad \text{and} \quad (10) \]

\[ \sigma_t = \sigma_0 + b_1 \sin(\omega t) + b_2 \cos(\omega t) \quad (11) \]

\( \bar{Y}_t \) is composed of a time dependent trend and a seasonal component. \( U_t \) is a third order autoregressive process for the detrended and seasonally adjusted values. \( t \) is the time variable starting with 1 on January 1st 1980. \( \varepsilon_t \) is a white noise process whose standard deviation \( \sigma_t \) is a sine function of the season. The frequency is 365 days (leap years remain unaccounted). Thus \( \omega \) is \( 2\pi/365 \). The coefficients are estimated using the least squares method. They are listed in table 1.
Table 1: estimated values and standard error for the coefficients of the regression equation

<table>
<thead>
<tr>
<th>coefficient</th>
<th>$a_0$</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$a_3$</th>
<th>$\phi_1$</th>
<th>$\phi_2$</th>
<th>$\phi_3$</th>
<th>$\sigma_0$</th>
<th>$b_1$</th>
<th>$b_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>estimated value</td>
<td>9.25</td>
<td>0.0001</td>
<td>3.34</td>
<td>-9.07</td>
<td>0.966</td>
<td>-.251</td>
<td>0.095</td>
<td>2.132</td>
<td>0.125</td>
<td>0.112</td>
</tr>
<tr>
<td>standard error</td>
<td>0.08</td>
<td>0.00002</td>
<td>0.06</td>
<td>0.06</td>
<td>0.011</td>
<td>0.015</td>
<td>0.011</td>
<td>0.019</td>
<td>0.026</td>
<td>0.026</td>
</tr>
</tbody>
</table>

The residuals are approximately standard normally distributed. Because of the third order autoregressive process there is no autocorrelation of the residuals.

Figure 3: frequency distribution of the residual error

We used equation 8 to simulate 10000 temperature paths from November 1st 2005 till March 31st 2006. The start value is the trend value. In figure 4 we compare the distribution of the model’s average temperature with the normal distribution of the historical data. The mean of the empirical data (2.65°C) is 0.59°C lower than the mean of the model (3.24°C). Also the standard deviation is about 0.6°C higher (1.54°C empirical and 0.95°C simulated).
The lower mean can be explained by the trend increase of the temperature. It is also possible that extreme values distort the lower tail. We could also doubt the symmetry of the empirical distribution. The deviations could, however, also be caused by specification errors of the temperature model. Since both distributions contain information we base the following calculations on a normal distribution whose mean and standard deviation are formed as the average of both distributions. Thus the mean is $\mu = 2.95^\circ C$ and the standard deviation is $\sigma = 1.29^\circ C$.

![Figure 4: comparison of the probability density function estimated from historical data and the temperature model](image)

4 Model calculations

4.1 Calculations of the heat energy demand

The greenhouse operation used for the calculations of the heating energy demand has 10 compartments of 1000 m² each. The side height is 4 m. The outer walls consist of insulated glass and the roofage is of single glass. An energy screen is installed that is closed at night. Oil is used as energy source to heat the greenhouse.
The main crops are begonias and poinsettias. They need a temperature of 18°C during the day and 16°C at night (Lange et al., 2002, p. 29). The weather data required for the energy model are provided by the Institute of Vegetable and Fruit Science of the University of Hanover. Thus, the greenhouse model farm is situated near Hanover.

We use Hortex-Light, a computer based decision support system for the design and operation of greenhouse heating systems, to estimate the heating energy consumption. The thermal demand is calculated using arithmetic calculations. The calculations are based on a surface related k’-model. The fuel consumption results from the energy that is needed to raise the inside temperature of the greenhouse to the designated target value (Rath, 1992). We assume a constant fuel price.

\[
Q_{Seg} = \sum_{n=1}^{8760} \left( (i_{n} - v_{1,Hn} - \Delta v_{Sp,n}) \cdot k'_{a} \cdot \sum_{r=HFa} A_{r} \cdot (1 - EE_{ES,n}) - \dot{Q}_{IW,n} \right) t_{Si}
\]

(12)

where

\( Q_{Seg} \) = annual heat energy consumption of a greenhouse section [Wh]
\( n \) = hour of a year
\( i_{n} \) = actual inner temperature of a greenhouse section at hour n [°C]
\( v_{1,Hn} \) = notional inner temperature of a greenhouse section at hour n without heating [°C]
\( \Delta v_{Sp,n} \) = inner temperature increase caused by the heat accumulation in a heated greenhouse [°C]

\( k'_{a} \) = heat consumption coefficient of the outer surface \([ \frac{W}{m^2K} ]\)
\( HFa \) = outlying surfaces [m²]
\( EE_{ES} \) = economy of energy by use of an energy screen [-]

\( \dot{Q}_{IW} \) = heat energy supplied / conducted by inner walls [W]
\( t_{Si} \) = time increment
Hortex-Light needs hourly outside temperature and solar radiation values as input. The values enter formula (12) via the fictitious inside temperature of the greenhouse division without heating. The output of the software is the hourly fuel demand given by the quotient of the heating energy demand and the heating value of heating fuel. One litre of heating oil provides about 10 kWh. The estimation error of the model revolves around 5% compared to effectively measured consumption.

In figure 5 the annual heat oil consumption from November to March from 1987 until 2003 is depicted along with the index. The negative correlation of -0.983 between both values is evident. Since we keep other influence factors constant the outside temperature is the only explanatory variable.

![Figure 5: heat oil consumption and temperature index](image)

With the calculated heating oil demand we use linear regression analysis to explain the heating oil demand \( H \) by the index:

\[
H = m_0 + m_1 x
\]

(13)

In the above equation \( m_0 \) is the intercept, \( m_1 \) is the slope of the straight line and \( x \) is the index distribution. The result of the regression is \( m_0 = -2,830,124 \) and \( m_1 = 24,654 \). That means that
if the index declines by 1 point 24,654 additional litres of heating oil are needed. This context is depicted in figure 6.

![Figure 6: regression of the heat oil consumption depending on the temperature index](image)

The calculations of the heating energy demand for the model greenhouse do not deliver enough values to estimate the demand distribution. We simulate the heating oil demand using:

$$H = m_0 + m_1 x + \sigma \varepsilon_2$$  \hspace{1cm} (14)

where $m_0$ is the absolute term and $m_1$ the slope of the heat oil demand estimated by $x$. $\sigma_1$ is the standard deviation of the regression and $\varepsilon_2$ is a normally distributed random variable. The index $x$ is the distribution of the average temperature. We also account for the fact that the energy demand model does not precisely estimate the energy demand.

From the comparison between measured and simulated energy demands of four farms we derive a standard deviation $\sigma_2$ of 5 % of the relative estimation error. Assuming that the price for 1 litre heating oil is $k$ the following energy cost distribution can be derived:

$$C = (1 + \sigma_2 \varepsilon_3) H k \left(1 - \frac{r}{2}ight)$$  \hspace{1cm} (15)
We assume that heating costs accrue continuously. Thus we discount the average capital to the starting date. The frequency distribution resulting from (15) is compared with the cost distribution after having signed a put option. According to (2) and (15) the following formula results:

$$C_{Put} = C - G_p^t = (1 + \sigma_2 e) H \left(1 - \frac{rT}{2}\right) - V \cdot \text{Max}[0, (K - x)] e^{-rT} + P_p$$

(16)

The payoff of the put option is also discounted to the starting date. The distributions are derived from a sample of 10000 random simulation runs.

### 4.2 Simulation results

We do not have any restrictions for the strike level or the tick size. Thus we keep them variable and optimize them for the greenhouse model. Knowing the regression coefficients for the heat oil demand and the assumed price $k = 0.35 \text{ €}$ we calculate the optimal tick size $V$:

$$V = -m_1 k = 8629 \text{ €}$$

(17)

The optimal strike level depends on the farmer’s attitude towards heating energy cost risk. Thus we calculated different risk measures at varying strike levels (Martin et al., 2001). In addition to the variance and standard deviation describing the volatility, we used risk measures describing the downside risk. They provide better results if the distribution is asymmetric.

In a final step we still have to determine the premium of the put option. Therefore we need the conditional expected value and the probability $\omega$. Since the index $x$ is normal distributed we calculate the premium as follows:

$$E(x \mid x < K) = E(x) + \sigma \left[ \phi \left( \frac{K - E(x)}{\sigma} \right) - \omega \left( x < K \right) \right]$$

(19)
\[ P(x < K) = \Phi \left( \frac{K - E(x)}{\sigma} \right) \] (20)

where \( \Phi(\cdot) \) is the standard normal distribution, \( \phi(\cdot) \) its density function and \( \sigma \) the standard deviation of the index \( x \).

Table 2 depicts statistics of the heating oil cost distribution reduced by the payoffs from the put option plus the premium. It is evident that the probability of a payoff increases if the strike level increases. At the same time the premium rises because it corresponds to the expected payoff of the option. If we choose a strike level of 6, the probability that a payment is exercised is 99%. Therefore we pay the relative high premium of 25842 €. With an increasing strike level the standard deviation diminishes. The downside risk measures like semi standard deviation and the percentiles also illustrate a risk reduction whose effect decreases with increasing strike level. Nevertheless a complete coverage against the energy cost risk is not even possible at the maximum strike. Another idea is that the greenhouse farm should not exceed a certain cost threshold. If we assume a value of -110000 € for such a threshold, the probability without option is 23% whereas the probability of an option with a strike of 4 is 8%.
Table 2: effects of different strike levels on the distribution of the heat oil costs of the modelled greenhouse

<table>
<thead>
<tr>
<th>Strike Level</th>
<th>mean</th>
<th>Prob. of payment</th>
<th>premium</th>
<th>standard deviation</th>
<th>semi std. deviation</th>
<th>5% percentile</th>
<th>10% percentile</th>
<th>threshold</th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td>-101167</td>
<td>-</td>
<td>-</td>
<td>12000</td>
<td>8759</td>
<td>-121513</td>
<td>-116560</td>
<td>23%</td>
</tr>
<tr>
<td>0</td>
<td>-101167</td>
<td>1%</td>
<td>32</td>
<td>11925</td>
<td>8663</td>
<td>-121324</td>
<td>-116587</td>
<td>23%</td>
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<tr>
<td>1</td>
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<td>6%</td>
<td>267</td>
<td>11532</td>
<td>8221</td>
<td>-120050</td>
<td>-116109</td>
<td>23%</td>
</tr>
<tr>
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<td>22%</td>
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<td>7042</td>
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<tr>
<td>6</td>
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<td>99%</td>
<td>25842</td>
<td>5648</td>
<td>4062</td>
<td>-110609</td>
<td>-108427</td>
<td>6%</td>
</tr>
</tbody>
</table>

5 Conclusions

The energy cost risk of a greenhouse farm can be reduced with the weather derivative developed in this study. However a complete coverage against high energy consumption is not possible. An important precondition for an advantageous application of weather derivatives is a high correlation between the weather index and the operating profit. Hence a weather derivative can be constructed that reduces the profit risk. The flexibility of the derivative type and the weather index on the OTC market can develop a broad field of applications especially in agriculture. The main problem of a case adapted construction is the knowledge of the relation between the probability density function of the parameter determining the profit and the weather index. Afterwards the probability density function of the weather index has to be determined to determine the fair premium. The optimal hedge position depends on the risk perception of the decision maker.
6 References