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This on-line version differs from the printed Proceedings 2004.
Ragnar Jonsson's paper is included in this version, but is missing from the paper copy.
Sustainability and Long-term Dynamics of Forests: Methods and Metrics for Detection of Convergence and Stationarity

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Abstract
Sustainability has several definitions within natural resource economics. Sustainable states may be economically or biologically determined. Age and age classes add complexity to any definition of forest sustainability. Here we focus on selected metrics that might be used to define forest sustainability, convergence to a sustainable state, and methods that allow convergence detection and classification of forest dynamics. In a deterministic world sustainable states of forests may be cyclic or fixed. If random perturbations are included, sustainability is associated with convergence of distribution parameters. Pattern recognition procedures are presented that detect whether a forest converges to a cyclic, a fixed or a stationary state and whether convergence is fast, slow or cannot be observed within the time horizon considered.

Keywords: Faustmann normal forest, forest cycles, convergence criteria, convergence detection

1. Introduction
An inherent characteristic of forest management is the need to make decisions regarding harvesting, regeneration, thinning and other silvicultural activities. These human interventions influence the current state and long-term development of the forest and its productivity, which determine economically feasible options. Some types of interventions are crucial to the state and long-term dynamics of the forest whereas other interventions have more limited impacts. Forest economics has widely focused on determining the optimal rotation age or harvest time for a stand of trees. However, long-term forest dynamics including the evolution of forest age class structure over many rotations has not received much scrutiny (Newman 2002).

Since the 1950’s, standard answers regarding optimal harvesting provided by forest economists have been based on the Faustmann Model of forest management (see Newman 1988 and Newman 2002 for comprehensive reviews). In the Faustmann Model the optimal harvest age that maximizes the net present value of bare land is determined. In most studies the age class structure of the forest is assumed to follow directly from the Faustmann harvest age, and take the form of a Normal Forest, i.e. if \( a' \) is the Faustmann harvest age, then a forest can be divided into \( a' \) compartments, each with \( 1/a' \)th of the forest area. All \( a' \) age classes within the interval [0; \( a' \)], or [1; \( a' \)] depending on the season, occupy one compartment and every year one compartment is harvested.

There are several reasons why a normal forest may be desirable, including such advantages as sustainable harvest levels and the absence of adjustment costs from changing harvest levels. However, the idea of a normal forest and particularly the Faustmann normal forest has acquired
almost mythical, unquestioned status as an inherently desirable goal. Only a limited number of studies have addressed the questions of under what circumstances is a Faustmann normal forest desirable, under what conditions will optimal harvesting generate a Faustmann normal forest, and what types of dynamic/static age class structures will otherwise arise (Mitra & Wan 1985, 1986, Salo & Tahvonen 2002a,b, 2003, Wan 1985, Wan & Anderson 1983, Wan 1994).

A re-examination of the desirability and a more extensive analysis of the conditions that generate Faustmann normal forests are timely due to the emergence of sustainability as an increasingly important natural resource management goal, and the simultaneous proliferation of optimal harvesting rules that either extend the Faustmann rule or offer alternative rules, e.g. the ‘reservation price approach’ (Brazee & Mendelsohn 1988, Brazee & Newman 1999, Lohmander 1987) and the ‘real-options approach’ (Thorsen & Malchow-Moller 2003). The development of such alternative harvest rules, incorporating risk and uncertainty or in other ways extending the Faustmann Model, have dominated the forest economics literature since the late 1980’s (Newman 2002). The focus of these alternative rules has been to incorporate omitted factors into optimal harvest rules. Thus, for example the analyses have focused on optimal response to changing prices, but have not analyzed changes of age class structures over time and the associated effects on managerial flexibility and sustainability.

Although much has been written on sustainability, little has been done to quantify the impacts of sustainability and its implications for forest management. The popularity of sustainability as a goal of forest and natural resource management suggests a concern with future harvests, future forests, and managerial flexibility. How the age class structure of a forest changes over time and how this influences the availability of mature stands for possible harvest are essential components of both determining forest sustainability and managerial flexibility.

The aim of this paper is to provide some of the tools needed to analyze age class structure and long-term forest dynamics. Metrics are a primary tool to describe forest dynamics and to distinguish between desirable and less desirable dynamics. The metrics are intended to serve as a basis for future studies that attempt to combine theoretical and applied approaches by focusing on the development and simulation of very simple systems with some standard features of standing forests. These studies are intended to supplement theoretical works with more detailed results providing a broader and clearer picture of the implications of various decision rules and management strategies.

The outline of the paper is as follows. In Section 2 general metrics are presented. In Section 3 relevant forest state variables are defined and identified. In Section 4 we present methods for convergence detection for different types of systems. In Section 5 we illustrate the characteristics of selected metrics when applied to a simple, deterministic example. Section 6 is a brief conclusion.

2. General metrics

By ‘general metrics’ we refer to parameters or statistics that are applicable to and could measure the distributional properties of almost any variable. These general metrics are intended to characterize dynamics after the time when the forest appears to have converged to some sort of dynamically stable state, or after the time when it has been decided that such a state cannot be identified within the time frame considered.

The graph in Figure 1 shows a single cycle of a forest state variable in deterministic
systems or a forest state distribution parameter in stochastic systems. Figure 1 illustrates some standard metrics within a forest state cycle. The variations of the state variable influence the level and stability of the owner’s utility as well as the risk implied by unexpected external events. The forest value is above the cycle mean most of the time but has a large negative spike. Within the context of risk aversion a large negative spike may have a greater impact than indicated by the standard deviation or the amplitude. Similarly, a large positive spike may have a smaller impact than indicated by the standard deviation or the amplitude.

![Figure 1](image-url)

**Figure 1.** Characterization of the dynamics of a cycling forest state variable or distribution parameter.

When the forest has reached a static or dynamic equilibrium it will be characterized by constancy or cycles. To characterize cycles by periodicity let the *period length*, \( p \geq 1 \) be the time between identical forest states or identical distributions of forest states.

*Dynamics type* is a basic classification metric. We distinguish three different types: *convergent*, *cyclic* and *unresolved* dynamics. For *asymptotically convergent* dynamics the period length is \( p = 1 \), for *cyclic* dynamics \( p > 1 \), and for *unresolved* dynamics, \( p \) is not identified within \( T \) years. Formal definitions are presented in Section 4. Unresolved dynamics may indicate a chaotic process but – more likely – it means that the system has not been monitored for a sufficiently long period of time (\( T \)). To identify truly chaotic dynamics is demanding, both in terms of required number of iterations and required numerical precision. Stochastic state variables are considerably more difficult to characterize than are deterministic state variables. However, deterministic state variables are often difficult enough to type. For example, in one of the cases in Meilby (2002) a forest was observed to enter a 3,500-year cycle after roughly 12,000 years. Given the large, unknown number of iterations that maybe required to identify chaotic dynamics we prefer using the term ‘*unresolved*’.

A variable that is strongly related to the identification of the dynamics type and the associated period \( p \) is the *time to convergence*. The time to convergence, \( t_c \), is defined as the
time from the start of the process to the time of the first occurrence of a repeated forest state or forest state distribution and, therefore, for unresolved dynamics the value of \( t_\ell \) is not defined.

Consider a forest state variable, \( X(t) \) at time \( t, \ t \in [0; T] \). As all cycles of a fully converged deterministic forest are identical within some numerical tolerance, for a forest that has converged we only need to consider the period \( t_0 = t \) to \( t_\ell = t_\ell + p \) i.e. it is only necessary to define the metrics for one single period of duration \( p \). However, in situations where the dynamics remain unresolved, and situations where state variables are stochastic, we may need to consider the period \( t_0 = T - p \) to \( t_\ell = T \). Since examining an entire time horizon is effort consuming, it may be desirable to examine a subperiod. For unresolved deterministic dynamics one may choose to examine a given percentage of the latest simulated states. For many series, we found examining the last 10-20% of the series to work well. For a single convergent stochastic variable a reasonable choice seems to be to evaluate all states observed after the time when convergence was detected. For a single stochastic variable that does not appear to converge autocorrelation is likely to occur, and it may be appropriate to evaluate a systematic or random sample of, e.g., 10-20% of the forest states observed within the last 50% of the simulation.

Standard descriptive statics may be used to characterize both the trajectory of \( X(t) \) in the interval \( t \in [0; T] \), or within a cycle of duration \( p \geq 1 \). For unresolved cases and for stochastic forests we operate on a sample. For deterministic cases with a fixed cycle, population formulas are used. Useful descriptive statistics include the mean, the standard deviation, the minimum, the maximum and the amplitude.

Compared to small and frequent negative deviations, rare but large negative deviations are often highly influential, when evaluating the utility of NPV or cash flow. Hence, it may be relevant to consider the skewness of the distribution of \( X(t) \). Again we need to distinguish between situations where we operate on a sample or on a population. A simple skewness measure is:

When we consider a sample:

\[
(X) = \frac{\sum_{t=t_0}^{t=t_\ell} (X(t) - \bar{X})^3}{(S-1) \cdot H(X)}
\]

When the complete cycle is known:

\[
(X) = \frac{\sum_{t=t_0}^{t=t_\ell} (X(t) - \bar{X})^3}{S \cdot H(X)^3}
\]

An alternative way to measure the asymmetry of the distribution is to calculate the proportion of the sum of squared deviations caused by negative deviations from the mean:

\[
G(X) = \frac{\sum_{t=t_0}^{t=t_\ell} (t)^2}{\sum_{t=t_0}^{t=t_\ell} (X(t) - \bar{X})^2} \quad \text{where} \quad H(t) = \begin{cases} 
0 & \text{if } X(t) \geq \bar{X} \\
X(t) - \bar{X} & \text{if } X(t) < \bar{X}
\end{cases}
\]

By applying the above metrics to the relevant forest state variables a description of the
long-term dynamics should be obtained for almost any forest system. However, as mentioned above the exact use of general metrics depends on the specific characteristics of the system under study.

3. State variables

The state of the forest may be described by a set of variables, which express the factors that have the greatest influence on economic sustainability, i.e. variables that describe the owner’s current choices, long-term flexibility and the potential for dynamic stability. No single variable can describe all factors but by combining a few variables it may be possible to construct a useful description.

An obvious measure to partially describe an owner’s current choices is the disposal or liquidation value of the forest (not including the sales value of bare land). Suppose that \( a \) is age, \( a_{\text{max}} \) is the maximum biological age of the forest or the maximum age that would ever occur in the forest, \( P(a) \) is the stumpage value at age \( a \) on a per hectare basis, and \( A(a,t) \) is the area in age class \( a \) at time \( t \). The liquidation value at time \( t \) is:

\[
L(t) = \sum_{a=1}^{a_{\text{max}}} P(a) A(a,t) ,
\]

The liquidation value provides an indication of immediate flexibility but does not characterize the range of options available to the owner in the long term. Therefore, we include the net present value of the forest at time \( t \):

\[
W(t) = \lim_{\tau \to \infty} \sum_{a=1}^{a_{\text{max}}} \sum_{\tau=1}^{\tau_{\text{max}}} A_{\tau} \cdot (P(a) - C) \left[1 + r \right]^{(\tau-1)} ,
\]

where \( A_{\tau} \) is the area harvested in age class \( a \) at time \( \tau \), \( r \) is discount rate and \( C \) is regeneration costs. In practice \( W(t) \) will be estimated on the basis of revenues from a finite period of time. However, with a positive discount rate net values in the distant future will be close to zero, and the approximation of \( W(t) \) will be close to the actual value. With the inclusion of state probabilities the extension to an expected net present value is immediate.

In the calculation of net present value future revenues and costs are weighted according to the owner’s own time preference, and thus the net present value yields an exact description of the forest value perceived by a particular owner. However, with a positive discount rate the net present value puts the most weight on revenues of the near future. NPV does not provide a good description of the long-term stability of income from the forest. To adequately describe long-term stability we introduce a metric describing the age structure’s current deviation from a normal forest. The goal of this metric is to determine the degree of disagreement between the actual land area distribution at time \( t \) and the normal forest rectangle (width \( a' \) years; height \( 1/a' \)):

\[
N(t) = \sum_{a=1}^{a_{\text{max}}} \left| A(a,t) / A_{\text{tot}}(t) - 1 / a' \right| + \sum_{a=a'+1}^{a_{\text{max}}} A(a,t) / A_{\text{tot}}(t) ,
\]

where \( A_{\text{tot}}(t) \) is the total current area of the forest.

In a sense each deviation is counted twice, i.e. where it is “missing” and where it ends up. The highest value of this metric, \( N = 2 \), would only arise in a forest that is all older than \( a' \). When a forest is all younger than \( a' \) and all land is in a single class, \( N \) converges towards 2 as \( a' \) approaches infinity.
4. Convergence

To develop methods for pattern identification and classification of dynamic state variables, we need to distinguish between strictly deterministic systems, and processes that are known to be influenced by stochastic disturbances or are likely to endogenously generate perturbations. Truly deterministic systems are unlikely to exist in reality, but serve as useful benchmarks because results for deterministic systems are usually more detailed and complete than results for systems that include random effects. In addition, the dynamics of simple deterministic systems can be evaluated using only a single simulation, while it will usually be necessary to undertake Monte-Carlo simulations to evaluate stochastic systems.

In deterministic cases dynamics can be classified by examining whether a parameter converges towards an asymptotic value, a fixed cycle or remains unresolved at the end of the time period considered. In stochastic cases classification implies recognizing if and when a distribution parameter converges in probability to a limit. Here we will both consider situations where distribution parameters converge asymptotically ($p = 1$) and situations where they converge to a cycle ($p > 1$). In all cases we will use the term ‘stationarity’ (sensu lato) to indicate a situation where convergence has been detected, no matter the particular type of pattern identified.

4.1 Convergence of deterministic systems

At time $t$ the state of the forest system is given by $X(t)$. The system is monitored for $T$ years/iterations, and the sequence is examined for occurrence of periodic/cyclic behavior. For any (small) tolerance $\delta$ we may generally state that a cycle with period $p \geq 1$ occurs for $X$ from time $t_c$ and onwards if:

$$\forall k \in N^+, \forall \tau \in [0...p-1]: \left| X_i(t_c + \tau + k \cdot p) - X_i(t_c + \tau) \right| < \varepsilon .$$

As we only monitor the development of the system for $T$ years a cycle with period $p$ is in practice observed if

$$\forall k \in [1...\text{int}((T-t_c)/p)], \forall \tau \in [0...p-1]: \left| X_i(t_c + \tau + k \cdot p) - X_i(t_c + \tau) \right| < \varepsilon .$$

If the observed period is $p = 1$, we will describe the convergence of the trajectory as asymptotical, otherwise it converges to a cycle. If $t_c$ and $p$ are not identified within $T$ years the dynamics of the system is considered unresolved.

4.2 Convergence detection for replicated stochastic series

To detect dynamics types for stochastic series, it is necessary to define convergence of a stochastic series. There are several definitions of stochastic convergence. We adopt a simple definition based on Billingsley (1995, p. 70): $\lim_{n \to \infty} P[|Y(t)-i| > \varepsilon] = 0$, where $i$ is the convergence level of the parameter $Y$ and $\varepsilon$ is an arbitrary tolerance.

Similar to deterministic systems the first task is to examine whether a process stabilizes over time or not. If the process converges key questions are: (1) how are the dynamics characterized, (2) at what level does the process stabilize and (3) how does the process vary over time. Here we attempt to categorize the stationary state by the first two moments of the distribution of outcomes. To do this it must first be determined whether the first two moments converge asymptotically or exhibit periodic behavior (cycles). In case that neither of these apply, we will consider the dynamics unresolved.

As above the system is monitored for $T$ years. The simulation is repeated $M$ times.
Accordingly, for \( X(t) \) we have \( M \) different outcomes, \( X_1(t) \ldots X_M(t) \). We define a level \( \alpha \) at which we test the value of the mean and variance of \( X \) at one point in time, \( t_c \), against the mean and variance of \( X \) at another point in time, \( t_c \). We then propose a time of convergence \( t_c \geq 0; T \geq 1 \), and repeat the test \( \forall N \geq \left[ 1 + IQ(T - t_c) / S \right] \) and \( \forall \tau \in [0 \ldots p - 1] \) with \( t_1 = t_c + \tau \) and \( t_2 = t_c + \tau + NS \). Based on the null hypothesis \( H_0 \):

\[
s^2(X(t_1)) = s^2(X(t_2)),
\]

we test the equality of the variances using the \( F \) statistic:

\[
F = \max \left\{ \frac{s^2(X(t_c + \tau)), s^2(X(t_c + \tau + kp))}{\min \{s^2(X(t_c + \tau)), s^2(X(t_c + \tau + kp))\}} \right\},
\]

which is \( F \)-distributed with \( (M-1,M-1) \) degrees of freedom under \( H_0 \).

For all combinations of \( t_c \) and \( p \) the number of times that \( H_0 \) is rejected is tallied, and if the proportion of rejections does not exceed \( \alpha \) or some other chosen threshold, we conclude that the variance of \( X \) is likely to exhibit periodic dynamics with period \( p \) from time \( t_c \) and onwards.

If it is established that periodicity with period \( p \) cannot be rejected from time \( t_c \) and onwards, we may proceed testing whether the apparent periodicity applies to the mean values too. The null hypothesis is \( H_0 : \mu(X(t_1)) = \mu(X(t_2)) \). For all combinations of \( k \) and \( \tau \) the following \( t \) statistic is used to test the null hypothesis:

\[
t = \frac{\overline{X}(t_c + \tau) - \overline{X}(t_c + \tau + kp)}{\sqrt{s^2(X(t_c + \tau)) + s^2(X(t_c + \tau + kp))}}/M.
\]

The number of degrees of freedom in each individual test can be estimated using Satterthwaite’s approximation. When the number of replications is equal in the two samples Satterthwaite’s approximation can be expressed as:

\[
df = (M - 1) \left[ 1 + \frac{2s^2(X(t_c + \tau))s^2(X(t_c + \tau + kp))}{s^4(X(t_c + \tau)) + s^4(X(t_c + \tau + kp))} \right],
\]

i.e. \( df \) approximately equals \( 2(M-1) \).

Again the number of times that \( H_0 \) is rejected is tallied. If the proportion of rejections does not exceed the chosen threshold, we conclude that the mean of \( X \) is likely to exhibit periodic dynamics with period \( p \) from time \( t_c \) and onwards.

Since the null hypothesis for mean values is only tested if the variance appears to be periodic, then it is assumed that \( \sigma^2(X(t_1)) = \sigma^2(X(t_2)) \), and we may use the pooled-variance version of the \( t \) statistic:

\[
t = \frac{\overline{X}(t_c + \tau) - \overline{X}(t_c + \tau + kp)}{\sqrt{2s^2/M}},
\]

where \( s^2 \) is the pooled variance:

\[
s^2 = (M - 1) \times \left( s^2(X(t_c + \tau)) + s^2(X(t_c + \tau + kp)) \right)/(2M - 2)
\]

and the number of degrees of freedom in each individual test is \( 2M-2 \).

It should be noted that the outcomes of the tests described inevitably depend on the number of replications \( M \) and the chosen significance level \( \alpha \). Moreover, it may happen ...
that ambiguous results are obtained, i.e. that periodicity is accepted for a number of different values of $p$ (that are not multiples of each other). To choose the most likely set of $t$ and $p$ the value of $m$ may be increased, leading to more frequent rejections of the null hypotheses, until only one candidate remains.

4.3 Convergence detection for a single stochastic trajectory

Although convergence of the mean and the variance indicate the existence of a comparatively stable region, they do not describe, neither address the convergence path. Paths may converge at different levels or exhibit fluctuations with different amplitudes, and some paths may become stationary before others. Therefore the distribution of times to stationarity ($t'$) of individual paths as well as the mean and variance of individual stationary paths with regard to a specific state variable, $X$ may be of interest. Here we define a convergence criterion for an individual series in addition to the convergence criteria defined above for the two first moments of the distribution of outcomes.

A single simulation of the system is not sufficient to enable estimation of the moments of the distribution at a given point in time. On the other hand, it appears to be a sufficient basis of examining whether the individual $X$ series converges towards a limiting distribution. Suppose that the system is monitored for $T$ years, with $T$ being so large that the system can be expected to converge within this time if it converges at all. Suppose also that the last $u$ years of the simulation ($t = T-u+1 \ldots T$) are used as a basis of estimating the first two moments of the distribution. Next, using appropriate assumptions regarding the type of limiting distribution, outliers can be detected by introducing a threshold probability of more extreme outcomes ($P_{ex}$) and by estimating the corresponding quantiles, $Q_{lo}(u)$ and $Q_{up}(u)$, of the distribution. Next, the number of outliers, i.e. values below $Q_{lo}(u)$ or above $Q_{up}(u)$, may be summed from $t = T$ to $t = 0$. The time of convergence $t$ may then be defined as the time when the calculated proportion of outliers, $R_{\text{outlier}}$, first exceeds a threshold value, i.e., $R_{\text{outlier}} = \frac{\# t \geq T | X(t) < Q_{lo}(u) \lor X(t) > Q_{up}(u)}{T+1-t}$.

A key choice is the value of $u$, i.e. the period used to estimate the moments of the limiting distribution. Clearly, the greater the value of $T$ the better, not only because it improves the chances of actually observing values representing the limiting distribution but also because it allows for greater values of $u$, improving the estimates of the moments of the limiting distribution. If autocorrelation of the series arises, it may be necessary to examine the first-order differences of state variables, $\Delta X(t) = X(t) - X(t-1)$, instead of the current state variables, $X(t)$.

A general concern with the analysis of single stochastic time series is that, not even a long relatively stable series can guarantee that the system will not at some point diverge. Not even a Monte Carlo simulation will be able to demonstrate that a given system can never exhibit divergent behavior. Of course, if divergent patterns occur with not too small probabilities, we would expect to observe them if we execute a reasonable number of simulations. However, the possibility remains for stochastic series that rare circumstances may cause divergent behavior.
5. Examples

To illustrate and compare the properties of the proposed metrics, we examine a simple deterministic forest system with four age classes. The forest is assumed to have converged, meaning that the age class structure is repeated every four periods, and that all state variables fluctuate with $p = 4$. The age classes may be thought of as 10-year classes. Using a discount rate of 0.2 therefore corresponds to roughly 0.02 on an annual basis.

The assumed harvest revenues and liquidation values are given in Table 1. The values are stated in hypothetical monetary units per ha (MU ha⁻¹) but to give the values some real meaning one may think of 1 MU as, e.g., 1000 € or 1000 $.  

First, we apply the metrics. Figure 2 presents eight age class structures, four of which (A…D) are designed to illustrate extremes, and four of which (E…H) are ‘random’ age class structures. As the forest has reached a cyclic equilibrium, and all metrics are calculated for the 4-year cycle the mean of liquidation and present values do not vary between age class structures. The mean liquidation value is 6.00 and mean present value is 25.50. Results for standard deviation, skewness and amplitude are presented in Table 2. The variation within a cycle obviously depends on the age class structure, the normal forest (A) yields no variation at all and the single age class forest (D) yields a standard deviation for liquidation value that is similar to its mean liquidation value, and a standard deviation of the present value, that is roughly 20% of the mean present value. With only a single age class (D) has a large amplitude.

Regarding the deviation from a normal forest, it should be noted that due to the fact that this metric attaches the same weight to all age classes, it does not vary between the four periods of the cycle. Consequently Table 2 only includes the ‘mean’. The maximum value of 1.5 is reached for age class structure (D). Since age classes 5 and over are not allowed in this example, 1.5 is the maximum possible value.

The skewness metrics cannot be calculated for the normal forest (A) as the standard deviations of liquidation and present values are both zero. For the forest with three age classes (B) the skewness of the liquidation value is negative because it has a marked negative spike due to the missing age class. By contrast, for this age class structure the skewness of the present value is positive because an upward spike generated when age class 1 is empty dominates. Note that for the forest with one age class (D) the results are exactly the opposite, and for the forest with two age classes (C) the distributions of liquidation and present values are symmetric so both skewness metrics are zero.

<table>
<thead>
<tr>
<th>Table 1. Harvest revenues and liquidation values used in the example.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Age class</strong></td>
</tr>
<tr>
<td>Time of payment</td>
</tr>
<tr>
<td>Harvest revenue (MU ha⁻¹)</td>
</tr>
<tr>
<td>Liquidation value (MU ha⁻¹)</td>
</tr>
</tbody>
</table>

¹ MU ha⁻¹: Monetary Units per ha
Figure 2. Age class structures applied in Table 2.

Table 2. Metrics for the eight age class stuctures A...H in Figure 2.

<table>
<thead>
<tr>
<th></th>
<th>Liquidation value, L (MU ha⁻¹)</th>
<th>Present value of forest, W (MU ha⁻¹)</th>
<th>Dev. from normal, N (Mean)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Std.</td>
<td>Skewness</td>
</tr>
<tr>
<td>A</td>
<td>6.00</td>
<td>0.00</td>
<td>-</td>
</tr>
<tr>
<td>B</td>
<td>6.00</td>
<td>2.01</td>
<td>-0.45</td>
</tr>
<tr>
<td>C</td>
<td>6.00</td>
<td>4.03</td>
<td>0.00</td>
</tr>
<tr>
<td>D</td>
<td>6.00</td>
<td>6.04</td>
<td>0.45</td>
</tr>
<tr>
<td>E</td>
<td>6.00</td>
<td>2.19</td>
<td>-0.18</td>
</tr>
<tr>
<td>F</td>
<td>6.00</td>
<td>1.44</td>
<td>-0.12</td>
</tr>
<tr>
<td>G</td>
<td>6.00</td>
<td>0.49</td>
<td>0.77</td>
</tr>
<tr>
<td>H</td>
<td>6.00</td>
<td>1.72</td>
<td>0.12</td>
</tr>
</tbody>
</table>

* MU ha⁻¹: Monetary Units per ha

Apart from ‘period length’ and the ‘time to convergence’, we have suggested 4-7 different general metrics and 3 different variables that may be used to describe the state of the forest. This implies that there are a total of 12-21 metrics that need to be considered when describing the dynamics of a deterministic forest. However, some of the metrics are likely to be highly correlated. For example it is obvious that the standard deviation and amplitude of a variable are highly correlated. Therefore, in Table 3 we have calculated Pearson coefficients of correlation for seven metrics based on a sample of 2000 random forests, each with four age classes and a cycle with p = 4, similar to the examples (E…H) in Fig. 2. As expected, the table shows that one single metric describing the variation, be it a standard deviation or an amplitude, is sufficient. On the other hand, the skewness of liquidation value and present value are quite uncorrelated, and the correlation between the skewness metrics and any of the variation metrics are low. The deviation from a normal forest is, not surprisingly, highly correlated with the variation metrics but the correlation with the skewness metrics is low. Therefore it appears that to describe a deterministic forest one metric describing the mean of present value (or liquidation value), one metric describing the variation, either standard deviation or amplitude, and two skewness
metrics are needed. In addition it may be desirable to add the deviation from a normal forest.

6. Conclusion

In this paper we have outlined a number of general metrics and state variables that, in suitable combinations, may be used to describe the long-term dynamics and economic sustainability of forests in both deterministic and stochastic settings and regardless of whether convergence is detected or not. We note that if convergence is not detected it cannot be known whether the calculated metrics are representative of the long-term dynamics or simply describe part of the path towards a long-term equilibrium, provided such equilibrium exists. Obviously, detection of convergence is much easier for deterministic than for stochastic systems. In fact, for stochastic systems there is always a possibility, however small it may be, that random events may provoke the system to leave the equilibrium that it once converged towards. Nevertheless, provided that a sufficient number of iterations and replications are included in simulations the methods presented in Section 4 should enable identification of at least temporarily convergent dynamics.

Clearly, the metrics and principles described here are of limited value when examined in isolation. Thus, the raison d’être of this paper is to provide a foundation for simulation-based studies focusing on long-term forest dynamics that arise under a specific set of decision rules, initial conditions and externalities. Together the metrics and procedures in this paper form a partial toolbox. Although not likely to be complete we still expect the paper to introduce several of the tools that will be necessary for ongoing and planned studies on the role of constraints and taxation in the convergence of forests, dynamics resulting from the reservation price approach, and dynamics arising at the forest sector level as a consequence of owner’s decision rules.

References


