Regulation of Nitrogen Pollution: Taxes versus Quotas

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This paper investigates the effects of first-best policies to regulate nitrogen application. Some nitrogen fertilizer is applied ex ante before a random rainfall, but sidedressed nitrogen may be applied ex post. First-best policy is a tax or a quota on ex ante application, because sidedressed nitrogen is not leached. Since a risk-averse farmer uses more nitrogen ex ante than a risk-neutral farmer, a higher tax must be imposed on the former. Action equivalent first-best taxes and quotas are also welfare equivalent. An empirical model for wheat in Israel was used to demonstrate the analytical findings.

Key words: first-best policy, quotas, taxes

Introduction

Recent years have witnessed a growing concern about the environmental cost of nitrogen fertilizer, which escapes to surface and groundwater supplies and is a potential source of ozone layer destruction (Swanson; National Research Council). Since the 1950s, total nitrogen fertilizer use in the U.S. has increased approximately 400%. Similarly, nitrogen use in Israel during the last two decades more than doubled.

Nitrogen is used as an essential plant nutrient, but only about 50–70% is actually taken up by crops and the remainder is mineralized, incorporated in the soil's organic matter, and lost by denitrification and volatilization and by leaching (Keeney). While pesticides and herbicides are regulated, at present, nitrogen is not regulated in many regions of the U.S. and Israel. In the absence of direct government regulation, farmers have no incentives to take into account the negative environmental externalities arising from nitrogen use. Input decisions of farmers are based solely on private interests and the potential social costs or health risks are completely ignored. In this situation, nitrogen applications are excessive when compared to the situation where the external costs are internalized into the decision-maker's objective function. Excessive production of commodities which use nitrogen intensively has adverse consequences on the surface and groundwater.[1]

Although the literature has extensively analyzed the fertilizer decisions under uncertainty (e.g., Ryan and Perrin), impacts of fertilizer regulation have received scant attention. The impact of risk-aversion on the ex ante level of fertilizer application is not obvious a priori. Anderson, Dillon, and Hardaker argued that in the absence of leaching, risk-averse farmers use less nitrogen than risk-neutral farmers and suggested that a subsidy (negative tax) be used to encourage risk-averse farmers to use more nitrogen (p. 182). Others have argued that risk-averse farmers use more nitrogen than risk-neutral ones (e.g., Ryan and Perrin). In these

[1] Water polluted with nitrogen possesses health risks. Specifically, nitrites (NO_2) in drinking water oxidizes the hemoglobin in the blood and forms methemoglobin which is lethal in small doses for embryos and babies (Lee). Second, production uncertainty causes risk-averse farmers to use nitrogen as insurance against potential yield losses.

studies, producers are assumed to make production decisions ex ante, before uncertainty is resolved.

In light of increased nitrogen use and its adverse environmental consequences, there is a need to develop an analytical model to investigate the impacts of policies to regulate farm-level nitrogen use, taking farmers' attitudes toward risk into account. This paper borrows the conceptual framework of Yohe (1976; 1978) and Weitzman comparing taxes and quotas under uncertainty and investigates the effects of taxes and quotas on nitrogen application for risk-neutral and risk-averse farmers. We assume that some nitrogen is applied ex ante before a random event (rainfall) is observed, and sidedressed nitrogen may be applied ex post. Thus, risk-averse farmers retain flexibility to adjust inputs after the random event is observed. While split nitrogen application decision has been investigated (Feinerman, Choi, and Johnson), the impacts of regulation on nitrogen use in agriculture have not received much attention.

With the advanced irrigation technologies (e.g., drip irrigation), most of the irrigated water is taken up by plant roots, and hence, leaching of nitrogen applied via the irrigation system is negligible. However, random rainfall can increase soil moisture over field capacity and cause leaching of nitrogen below the root zone to the groundwater aquifer. Since sidedressed nitrogen is applied after the rainy season, we assume that it is not subject to leaching. Thus, the first-best policy is to control ex ante nitrogen application only.

Yield and Nitrogen Leaching

Consider a representative farmer who grows an irrigated crop using water, nitrogen, and other inputs. In order to focus on nitrogen application, nonnitrogen inputs including land, machinery, and labor are assumed to be fixed.

Nitrogen is applied continuously via the irrigation system throughout the growing season. However, to facilitate the analysis, we divide the growing season into two periods and assume that nitrogen is applied only twice, one application in each period. A random event, rainfall, occurs only during period one, affecting both soil moisture content and soil nitrogen available to the crop. While some nitrogen is applied ex ante (before rainfall) at the beginning of period one, that is, the beginning of the growing season, sidedressed nitrogen also can be applied ex post after rainfall is observed.

With the advanced irrigation system (drip irrigation), the farmer is able to maintain any desired level of soil moisture, W^*. Obviously, the level of W^* is dependent on the cost of irrigation. Since some irrigation is always done, irrigation set-up costs are irrelevant for our analysis, and only variable costs (hereafter price per unit of water) should be considered. In order to focus on nitrogen use, it is assumed that at the relevant range of water and nitrogen prices, the optimal level of soil moisture content W^* is such that water stress to plants is eliminated or negligible. Maintaining soil moisture at this level is a commonly used agricultural practice in many regions of Israel. Obviously, the amount of irrigation water required to support the level W^* depends on the rainfall. Specifically, the amount of supplemental irrigation water required is $W^* - Z$, where Z is the random quantity of rainfall during the growing season. If the rainfall exceeds the threshold level W^*, no irrigation water is applied. Thus, the random cost of irrigation water is max $[w(W^* - Z), 0]$, where w is the price per unit of irrigation water.

In addition to soil moisture W, the amount of soil nitrogen available to plants is an important determinant of yield. Let A be the initial stock of plant-available nitrogen in the

soil at the beginning of period one, which includes nitrogen carryover from the previous period and can be observed by soil tests. Let X and Y be the nitrogen applied at the beginning of period one and the quantity of nitrogen applied in period two (after rainfall), respectively. Random rainfall can increase soil moisture over field capacity and cause leaching of nitrogen below the root zone to the groundwater aquifer. The total amount of nitrogen in the soil at the beginning of the season is $(A + X)$. Obviously, the more nitrogen carryover from the previous period, the less application X is required to maintain a given level of nitrogen in the soil at the beginning of period one. Nitrogen uptake by the plant at the beginning of period one (before rainfall) is $(A + X)\delta$, where δ is the uptake rate, $0 \le \delta \le 1$. The total amount of plant available nitrogen subject to leaching is thus $(A + X)(1 - \delta)$.

In period two, after rainfall is observed, some nitrogen can be applied as sidedressing. Since it is assumed that no rainfall occurs between the end of the current growing season and the beginning of the next (the off-season period), nitrogen applied ex post, Y, is not subject to leaching. This assumption applies to the situation presented in the empirical analysis (wheat grown in Israel). However, it eliminates interseasonal dynamics and, therefore, limits the generality of the analysis. The amount of nitrogen leached, L, depends on the random rainfall and is equal to $\theta Z(1 - \delta)(A + X)$, where θ is the rate of nitrogen leached beyond the root zone, $0 \le \delta \le 1$. More simply,

$$(1) \qquad\qquad\qquad L = \beta Z(A + X),$$

where $\beta = \theta(1 - \delta) > 0$. For simplicity of exposition, the stock of nitrogen at the beginning of period one, the term A is suppressed in the mathematical analysis,[2] but it is explicitly treated in the empirical application.

Yield is assumed to depend on the soil moisture W, the nitrogen applied at the beginning of the growing season (i.e., beginning of period one), and the plant-available nitrogen in period two after the application of sidedressed nitrogen, $N = (\gamma - \beta Z)X + Y$, where $\gamma = 1 - \delta$. Note that this expression describes the within-season dynamics of nitrogen carryover. Thus, yield can be represented by a production function $q = f(X, N, W)$. Recall that the soil moisture is maintained at the threshold level W^*. Suppressing W^* as an argument, the production can be written

$$(2) \qquad\qquad q = F(X, N) = F[X, (\gamma - \beta Z)X + Y].$$

The function $F(\cdot)$ is assumed to be monotone increasing and concave in both arguments, X and N. Adequate quantities of X at the beginning of the season help the plant develop an efficient root system and, as a result, increase the efficiency of nitrogen uptake during the growing season. Thus, X and N may be complements ($F_{NX} > 0$), substitutes ($F_{NX} < 0$), or independent ($F_{NX} = 0$). In the empirical analysis, F_{NX} was not found to be significantly different from zero.

Production Decisions under First-Best Taxes

To control nitrogen leaching a policy maker may employ either taxes or quotas on nitrogen application. Since the actual level of rainfall and the associated nitrogen leaching can be

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[2] It can be revived by placing X by $(A + X)$ henceforth.

observed, state-contingent taxes or quotas on nitrogen can be employed. In this section we investigate production decisions under first-best tax and quota and demonstrate welfare equivalence of the two policies. We assume that the policy maker observes the ex ante application X and knows the values of β and $\bar{Z} \equiv EZ$. With a target level of expected leaching $\bar{L} = \beta EZX$, the policy maker can control ex ante application directly by imposing a quota $X = \bar{L}\beta EZ$, or indirectly by imposing a tax t on X.

Consider the production decision problem when a first-best tax t is levied on ex ante application but not on ex post application. To provide a common reference point to compare production decisions under a quota and a tax, we assume that the producer receives a lump-sum tax rebate, R, which does not distort production decisions. This rebate scheme is consistent with the literature that compares taxes/tariffs and quotas on production/imports (e.g., Weitzman; Yohe 1976; Choi and Lapan; Choi and Johnson).

Ex Post Nitrogen Application

The farmer observes Z and then chooses Y in period two to maximize the profit:

$$(3) \qquad \pi' = pF[X,(\gamma - \beta Z)X + Y] - s(Y + X) - tX - w(W^* - Z) + R,$$

where p is output price, s is per unit cost of nitrogen, t is per unit nitrogen tax, w is the price of irrigation water, and R is a lump-sum tax rebate.[3] The first-order condition is

$$(4) \qquad pF_N[X,(\gamma - \beta Z)X + Y] - s = 0,$$

where a subscript denotes a partial derivative. Observe that the level of nitrogen applied ex ante, X, is fixed in period two. The ex post nitrogen demand, which solves (4), can be written as $Y = Y(s, p, X, Z)$. Differentiating (4) with respect to the decision variables Y and the parameters t, s, p, X, and Z gives

$$(5a) \qquad Y_t = 0,$$

$$(5b) \qquad Y_S = 1 / pF_{NN} < 0,$$

$$(5c) \qquad Y_P = -s / p^2 F_{NN} > 0,$$

$$(5d) \qquad Y_X = -F_{NX} / F_{NN} - (\gamma - \beta Z), \text{ and}$$

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[3]Obviously, some fraction of nitrogen is carried over to the next period. According to an accounting method, such nitrogen carryover is an investment in a sense, and an argument can be made to include its discounted value in the current profit π. However, in this case the value of nitrogen carryover from the previous growing season should also be deducted from current profit. While this method may also be adequate, it would complicate the present economic analysis unnecessarily without yielding additional insights. To simplify the economic analysis, nitrogen carryover from the previous growing season, A, rather than the carryover to the next, is included in the profit function. In a stationary situation, such nitrogen carryover is constant across the years, and the two methods are identical, except the discount factor. It can be shown easily that under the specific assumptions considered here, the level of A is unchanged over time.

(5e) $$Y_Z = \beta X > 0.$$

In the derivation of (5a) – (5c), X was treated as a predetermined fixed parameter. Since the tax is imposed on ex ante fertilization only, it does not affect the ex post application *directly* and hence, t is absent as an argument in $Y(s, p, X, Z)$ and $Y_t = 0$ in (5a).[4] The ex post nitrogen demand $Y(s, p, X, Z)$ is incorporated into the ex ante application problems in (6) and (8) presented below. Differentiating $N = (\gamma - \beta Z)X + Y(s, p, X, Z)$ with respect to Z and using (5e) yields

(5f) $$dN / dZ = -\beta X + Y_Z = 0, \text{ and}$$

(5g) $$dN / dX = (\gamma - \beta Z) + Y_X = -F_{NX} / F_{NN}.$$

Equation (5f) implies that plant-available nitrogen in period two after rainfall is observed, N, is independent of the random rainfall Z. An economic interpretation for this result is straightforward. Some portion of nitrogen applied early will be leached below the root zone because of rainfall. After observing the rainfall, the farmer adds sidedressed nitrogen to insure a constant level of plant-available nitrogen N required to maximize profit ex post. Because of split nitrogen application, farmers can make up the loss of nitrogen through leaching and hence retain input flexibility to respond to random weather.

Note from (5g) that the sign of dN/dX is identical to that of F_{NX}. Assuming that X declines with Y yields sign $\{dN/dt\} = -$sign $\{F_{NX}\}$. In other words, the optimal level of plant-available nitrogen in period two decreases (increases) with tax if N and X are complements (substitutes). If, however, N and X are neutral or independent ($F_{NX} = 0$), as is the case in the empirical example in the next section, the level of N is independent of t.

Ex Ante Nitrogen Application

Sidedressing of nitrogen is made in period two after the random rainfall Z is observed and, hence, is not directly affected by risk attitudes. However, the application decision of nitrogen applied ex ante, X, crucially depends on risk preference of the producer. We first consider ex ante nitrogen application decisions under a tax scheme for a risk-neutral and a risk-averse farmer, and compare them with those obtained under a nitrogen quota scheme.

Risk Neutrality. A risk-neutral farmer's problem at the beginning of period one is to choose X to maximize the expected profit:

(6) $$E\pi^{nt} = pEF[X,(\gamma - \beta Z)X + Y(s, t, X, Z)] - s(EY + X) - tX - w(W^* - EZ) + R,$$

where E is the expectation operator. Recall that although Z is random, the farmer insures a constant level of plant-available nitrogen in period two, $N = (\gamma - \beta Z)X + Y(s, p, X, Z)$.

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[4]However, t (as well as p and s) affects Y *indirectly* through a change in X. The total effect of a tax change is $dY/dt = Y_t + Y_x(dX/dt)$. Assuming that X declines as the tax increases, the indirect effect of t on Y depends on the sign of Y_x. If, for example, the inputs N and X are either substitutes or independent ($F_{NX} \leq 0$), Y_x is negative (5d), and hence, Y increases as t increases. If $F_{NX} > 0$, Y may increase with X and decreases with t. The indrect effect of t (as well as those of s and p) through a change in X depends on risk aversion. Specifically, from equations (7) and (10) below, it can be shown that $dX/dt = -E\pi_{X_t}/E\pi_{XX}$ for a risk-neutral farmer, whereas $dX/dt = -EU_{X_t}/EU_{XX}$ for a risk-averse farmer.

Moreover, observe that since N is independent of Z, yield $F(X, N)$ is not random. The first-order condition is

$$E\pi_X \equiv pEF_X + pF_N E(\gamma - \beta Z + Y_X) - sE(Y_X + 1) - t = 0.$$

Since N and F are independent of Z and $pF_N = s$ in (4), we get

$$(7) \qquad E\pi_X = pF_X[X, (\gamma - \beta Z)X + Y] - s(\delta + \beta EZ) - t = 0.$$

Thus, the derived demand for ex ante nitrogen of a risk-neutral farmer is written as $X^{nt} = X(p, s, t, \beta, EZ)$. Note that pF_X is the value of marginal product of X, and βEZ is the expected leaching rate. In the absence of tax, the equilibrium condition is $pF_X/(\delta + \beta EZ) = s = pF_N$. Observe that $pF_X/(\delta + \beta EZ)$ is the value of marginal product of X, adjusted for leaching, and must be equal to the value of marginal product of Y, which is not subject to leaching. Thus, $s(\delta + \beta EZ)$ is the opportunity cost of nitrogen applied ex ante in the absence of tax.

Risk Aversion. At the beginning of period one, a risk-averse farmer chooses X to maximize the expected utility:

$$(8) \qquad EU[\pi^{at}] = EU\{pF[X, (\gamma - \beta Z)X + Y] - s(Y + X) - tX - w(W^* - Z) + R\},$$

where $U(\cdot)$ is a monotone increasing and concave von Neumann-Morgenstern utility function, $U'(\cdot) > 0$, $U''(\cdot) < 0$. The first-order condition is

$$(9) \qquad E[U'\pi_X] = EU'E\pi_X + \text{cov}(U', \pi_X) = 0,$$

where $\pi_X = pF_X[X, (\gamma - \beta Z)X + Y] - s(\delta + \beta Z) - t$. Note that since N is independent of Z, $d\pi_X/dz = \pi_{XZ} = -s\beta < 0$, and $dU'/dZ = U''\pi_Z$, where $\pi_Z = w - pF_N \beta X = w - s\beta X$. Thus, the sign of π_Z depends on the per unit price of irrigation water w, nitrogen fertilizer s, and the leaching rate β. In places where water is scarce (e.g., Israel), the price of irrigation water is much higher than nitrogen fertilizer and π_Z is positive, that is, an increase in rainfall increases profits. We assume that $\pi_Z > 0$ because of scarcity of water.[5]

Let X^{at} denote the optimal ex ante nitrogen application of a risk-averse farmer. Assuming $\pi_Z > 0$ implies that $\text{cov}(U', \pi_X) > 0$ in (9), and hence, $E\pi_X < 0$ at X^{at}. Since $E\pi_X = 0$ at X^{nt} in (7), and $E\pi$ is concave in X by second-order condition, a risk-averse farmer uses more X than a risk-neutral farmer, that is, $X^{at} > X^{nt}$.

Proposition 1: Assume that a first-best tax is imposed on ex ante application only and that $\pi_Z > 0$. Then a risk-averse farmer applies more nitrogen ex ante than a risk-neutral farmer, that is, $X^{at} > X^{nt}$.

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[5]Note that $\pi_Z > 0$ implies an upper bound for the level of ex ante application, that is, $X < w/s\beta$. Based on the data used in the empirical example presented in the next section, the value of this upper bound is 186.9 kg/ha, whereas the highest level of optimal ex ante application (for a risk-averse farmer when $t = 0$) is less than 130 kg/ha (see table 1).

Action Equivalent First-Best Taxes under Risk-Neutrality and Risk-Aversion

Suppose the policy maker wishes to limit the expected nitrogen leaching below a target level $\bar{L}$. Then the ex ante application X must be limited to $X^* = \bar{L}/\beta EZ$. The value of tax to achieve the desired level of ex ante application, X^*, will depend on the risk attitude of the producer. Let t^n denote the tax rate which yields an ex ante application rate, X^*, for a risk-neutral farmer, and let t^a be similarly defined for a risk-averse farmer. We now compare the action equivalent tax rates for a risk-averse and a risk-neutral farmer. Differentiating (9) with respect to t gives

$$\text{(10)} \qquad \partial X / \partial t = -EU_{Xt} / EU_{XX},$$

where $EU_{XX} < 0$ by second-order condition, and

$$\text{(11)} \qquad \begin{aligned} EU_{Xt} &= EU''\pi_t\pi_X + EU'\pi_{Xt}, \\ \pi_{Xt} &= pF_{NX}\,Y_t - 1 = -1 < 0, \text{ and} \\ \pi_t &= (pF_N - s)Y_t - X = -X < 0. \end{aligned}$$

Since π_t is independent of Z,

$$\text{(12a)} \qquad EU''\pi_X\pi_t = \pi_t[EU''\pi_X] = -X[EU''\pi_X], \text{ and}$$

$$\text{(12b)} \qquad EU'\pi_{Xt} = -EU' < 0.$$

Assuming diminishing absolute risk-aversion (DARA), it can be shown that $EU''\pi_X > 0$. Let Z^* be the value of Z for which $\pi_X = 0$. Recall that $\pi_{XZ} = -s\beta < 0$, that is, π_X is decreasing in Z. Thus,

$$\pi_X < (>)\,0, \text{ for } Z > (<)\,Z^*.$$

Moreover, $\pi(\cdot)$ is monotone increasing ($\partial\pi/\partial Z > 0$). Thus, given DARA,

$$R[\pi(Z)] < R[\pi(Z^*)], \text{ if } Z > Z^*.$$

Multiply both sides by $U'[\pi]\pi_X$, which is negative for $Z > Z^*$.

$$\text{(13)} \qquad -U''[\pi]\pi_X > R^*U'\pi_X.$$

For $Z < Z^*$, $R[\pi(Z)] > R^*$ and $U'[\pi]\pi_X > 0$. Hence, the inequality (13) holds for $Z \neq Z^*$. Integrate (13) over the entire range to get

$$-EU''\pi_X < R^*EU'\pi_X = 0,$$

where the right side vanishes by the first-order condition (9). Thus, given DARA, $EU''\pi_X > 0$. Therefore, $EU''\pi_X\pi_t$ in (12a) is negative. Equations (12a) and (12b) imply that EU_{Xt} in (11) is negative, and hence, $\partial X/\partial t$ in (10) is negative (given DARA). That is,

an increase in tax reduces the level of ex ante application X of a risk-averse farmer. This result, together with that of Proposition 1, implies the following result:

> *Proposition 2*: Assume that a first-best tax is imposed on ex ante application only and that farmers exhibit DARA. Then in order to attain a target level of X^*, a higher tax must be imposed on a risk-averse farmer than on a risk-neutral farmer, that is, $t^a > t^n$.

It should be noted that profits without rebates will depend on risk attitudes. We show, however, that *profit after the tax is rebated is independent of risk attitudes*. Noting that t^a and t^n are action equivalent (they both yield the same levels of ex ante application, X^*) and substituting $R = t^n X^*$ into (6), we obtain the profit of a risk-neutral farmer:

$$(6')\qquad \pi^{nt} = pF[X^*, (\gamma - \beta Z)X^* + Y(s, \ p, \ X^*, \ Z)] - s(Y + X^*) - w(W^* - Z).$$

Similarly, substituting $R = t^a X^*$ into (6) for a risk-averse farmer, the random profits of the risk-neutral and the risk-averse producers are the same for each value of Z, as in (6'), that is, $\pi^{at} = \pi^{nt}$. In other words, with tax rebate, profit is independent of risk attitude under the first-best tax scheme.

First-Best Quotas

For an expected leaching target $\bar{L}$, the policy maker can directly control the ex ante application by imposing a quota $X^* = \bar{L} / \beta EZ$. In this case, there is no ex ante optimization problem. In period two after Z is observed, the producer chooses Y to maximize the profit:

$$(14)\qquad \pi^X = pF[X^*, \ (\gamma - \beta Z)X^* + Y] - s(Y + X^*) - w(W^* - Z).$$

Consider an ex ante quota X^* and a tax t^n on ex ante application that are action equivalent for a risk-neutral farmer, that is, $X^{nt} = X^*$. Then the ex post problem with an ex ante quota X^* is exactly the same as that under the equivalent ex ante tax t^n. Since no tax is imposed on ex post nitrogen application, the ex post application problem with an ex ante quota X^* is exactly the same as that under the action equivalent tax t^n which yields $X^{nt} = X^*$. Thus, the ex ante nitrogen tax t^n and the ex ante nitrogen quota $X^* = X^{nt}$ yield the same profit after rebate at all states, that is, $\pi^{nt} = \pi^X$ for the risk-neutral farmer, and hence, $E\pi^{nt} = E\pi^X$. Moreover, for a given ex ante quota X^*, the ex post application Y is determined after Z is realized. Thus, the ex post application Y and profit are independent of risk attitudes, and hence π^X in (13) is the same for risk-neutral and risk-averse farmers.

Assume that X^* and t^a are action equivalent for a risk-averse farmer. Then the same argument can be used to show that $X^{at} = X^*$, and hence, the ex post application problem is the same, regardless of risk attitudes, that is, $\pi^{at} = \pi^X$ for the risk-averse farmer. Thus, $EU[\pi^{at}] = EU[\pi^X]$.

> *Proposition 3*: Let t^n and X^* be a first-best tax and a first-best quota that are action equivalent on ex ante application for a risk-neutral farmer, and let t^a and X^* be action equivalent for a risk-averse farmer. Then the first-best tax and the first-best quota are also welfare equivalent, whether producers are risk-neutral or risk-averse, that is, $\pi^{nt} = \pi^{at} = \pi^Q$, and $E\pi^{nt} = E\pi^X$, and $EU[\pi^{at}] = EU[\pi^X]$.

An Example: Wheat in Israel

To estimate the production function F in (2) and the nitrogen uptake rate, δ, we used experimental data for an irrigated wheat (cultivar "Miriam 1") from Kafkafi and Bar Yosef. The wheat was seeded in the middle of November 1969 and harvested in mid-May 1970. The rainy season relevant for our analysis is from the end of November to the end of March. The end of March was chosen as the end of period one and the beginning of period two. Using data on plant-available nitrogen in the root zone (0–40 cm), $A + X$, at the beginning of December ("beginning" of period one) and at the beginning of April (period two), we first estimated the production function by using ordinary least squares (OLS) for three polynomial functions: the quadratic, the square root, and the three halves. The polynomial forms were previously recommended and chosen by Hexem and Heady to represent response to nitrogen and water for a few selected crops. The quadratic specification performed best among the polynomials examined for yield with 15 observations and 10 degrees of freedom. The estimated yield-response function (with t-values in parentheses) is

$$F = -4342.61 + 53.659(A + X) - 0.1210(A + X)^2 + 71.157N - 0.3255N^2,$$
$$ (3.666) (-3.056) (2.471) (-2.188)$$
$$R^2 = 0.7912,$$

where the units of F, A, X, and N are in kilograms per hectare (kg/ha). Unfortunately, biological theory does not provide much guidance in determining appropriate functional form and the quadratic function is selected on statistical grounds (highest R^2 and t-statistic values). Obviously, it may result in a positive yield even when no nitrogen is available at the early growth stage. However, in practice, A and X are both positive and hence this case is unlikely and may be ignored. It should be emphasized that the cross product term, $(A + X)N$, was initially included in the regression equation, but the estimated value of its parameter was close to zero and was statistically insignificant ($t < 0.1$). In other words, in the specific example considered, $(A + X)$ and N are roughly independent inputs ($F_{NX} \approx 0$).

The value of nitrogen uptake, δ, chosen for our illustrative example was 0.6, which is approximately a simple average of nitrogen uptake of the experimental plots. Unfortunately, the experimental results of Kafkafi and Bar-Yosef do not include information on nitrogen leaching below the root zone. Following a personal communication with Kafkafi, the value of the leaching parameter was assumed to be 0.0012, which implies $\beta = \theta(1 - \delta) = 0.0005$. Additional parameter values chosen for the example were $p = \$0.18$ per kg of grain yield (which represents the market price net of nonnitrogen and nonwater variable costs per kg of wheat); $s = \$1.07$ per kg of pure inorganic nitrogen fertilizer; $w = \$0.10$ per cubic meter of irrigation water; $W^* = 650$ mm, and $E(Z) = 528.75$ mm, which is a simple average of 41 years of rainfall data (1950/51–1990/91) in the experimental area. The assumed value of the pre-planting plant-available nitrogen in the soil, A, was 100 kg/ha.

The ex ante and ex post optimization problems for various values of t were solved for a risk-neutral farmer and a risk-averse farmer.[6] A constant relative risk-aversion (CRRA) utility function, $U = -\pi^{1-R}$, was used for the example where R is the relative risk-aversion measure. To illustrate the impact of risk aversion, the chosen value of R was 123 so that a

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[6] Observe that under a first-best quota X^* there is no ex ante optimization problem. Moreover, the ex post optimization problem under an ex ante quota X^* is exactly the same as that under an action equivalent tax, as shown in the section on the first-best quotas. Thus, the empirical results for the quota problem are ommited here.

Table 1. Empirical Findings for a Risk-Neutral Farmer

Tax t ($/kg)	Early Nitrogen Application X^{nt} (kg/ha)	Plant-Available Nitrogen in Period Two N (kg/ha)	Wheat Grain Yield q (kg/ha)	Expected Nitrogen Leaching $\bar{L}$ (kg/ha)	Expected Profits/ha $E\pi^{nt}$ Before Rebate	After Rebate
0.00	100.46	100.16	5412.41	53.10	648.05	648.05
0.05	99.32	"	5406.35	52.79	643.05	648.02
0.10	98.17	"	5399.98	52.49	638.11	647.93
0.15	97.02	"	5393.28	52.19	633.24	647.79
0.20	95.87	"	5386.26	51.88	628.42	647.59
0.23	95.18	"	5381.90	51.70	625.55	647.44
0.25	94.73	"	5378.93	51.58	623.65	647.33
0.30	93.57	"	5371.27	51.27	618.94	647.01
0.35	92.43	"	5363.30	50.97	614.29	646.64
0.40	91.28	"	5355.00	50.67	609.70	646.21
0.50	88.99	"	5337.47	50.06	600.69	645.18
0.60	86.69	"	5318.65	49.45	591.91	643.92
0.65	85.54	"	5308.77	49.15	587.60	643.20
0.70	84.40	"	5298.56	48.84	583.34	642.42
0.80	82.10	"	5277.20	48.23	575.02	640.70
1.00	77.51	"	5230.64	47.02	559.06	636.57

risk-averse farmer applies about 30% more nitrogen ex ante than a risk-neutral farmer when no tax or quota is imposed. The results for the first-best tax for a risk-neutral and a risk-averse farmer are summarized in tables 1 and 2, respectively.

Several points in the example are worth noting:

1. The value of plant-available nitrogen in period two, $N = 100.16$ kg/ha is the same for the risk-neutral and risk-averse farmers. From (5g), dN/dX is positive, zero, or negative according to whether F_{NX} is positive, zero, or negative. For example, if F_{NX} were positive, $X^{at} > X^{nt}$ implies $N^{at} > N^{nt}$. In our example, $F_{NX} = 0$, and hence $dN/dX = 0$, and thus the plant-available nitrogen in period two is independent of risk attitude. Moreover, $F_{NX} = 0$ implies that the optimal level of N is independent of the tax rate.

2. Observe that since $\pi_Z = 0.10 - 1.07 \times 0.0005X$ is positive for all levels of X in the relevant range, X^{at} is greater than X^{nt} for all levels of t, as predicted by Proposition 1. This is particularly true for low values of t. For example, X^{at} is greater than X^{nt} by 29% when $t = 0$, and only by about 3.5% when $t = 0.5$. Obviously, both X^{at} and X^{nt} decrease when t increases. Specifically, X^{nt} decreases linearly with t at a low rate for all levels of t; whereas X^{at} decreases at a high rate for low values of t and at a low rate for high values of t (e.g., X^{at} decreases by 31.45% when t increases from 0 to $0.25 and only by 8.5% when t increases from $0.70 to $1).

Table 2. Empirical Findings for a Risk-Averse Farmer

Tax t ($/kg)	Early Nitrogen Application X^{at} (kg/ha)	Plant-Available Nitrogen in Period Two N (kg/ha)	Wheat Grain Yield q (kg/ha)	Expected Nitrogen Leaching $\bar{L}$ (kg/ha)	Expected Profits/ha $E\pi^{nt}$	
					Before Rebate	After Rebate
0.00	129.68	100.16	5459.32	60.84	629.45	629.45
0.05	123.56	"	5466.60	59.22	630.25	636.43
0.10	115.93	"	5462.98	57.19	631.25	642.84
0.15	112.92	"	5457.68	56.40	627.73	644.67
0.20	103.48	"	5426.82	53.90	627.15	647.85
0.23	100.94	"	5414.84	53.22	624.76	647.98
0.25	98.65	"	5402.69	52.62	622.42	647.08
0.30	97.98	"	5398.90	52.44	618.51	647.91
0.35	96.11	"	5387.24	51.95	613.99	647.63
0.40	94.44	"	5377.05	51.50	609.48	647.26
0.50	92.09	"	5360.88	50.88	600.48	646.52
0.60	89.69	"	5342.98	50.24	591.71	645.52
0.65	88.53	"	5333.82	49.94	587.41	644.95
0.70	87.38	"	5324.43	49.63	583.15	644.32
0.80	85.08	"	5305.45	49.02	574.83	642.89
1.00	80.49	"	5261.45	47.81	558.86	639.35

3. Expected nitrogen leaching, $\bar{L} = \beta E(Z)X$, for the risk-neutral farmer decreases linearly with t at a very low rate; whereas, for the risk-averse farmer it declines fast at low rates of t and becomes relatively insensitive to tax for values of t above $0.25. Note further that when t increases from zero to $1.00, $\bar{L}$ decreases by 27% (from 60.84 to 47.81 kg of nitrogen) for the risk-averse farmer and only by 13% (from 53.10 to 47.02 kg) for the risk-neutral farmer.

4. For the sake of illustration, assume first that the leaching target set by the policy maker is equal to $\bar{L}$ for the risk-neutral farmer when no tax is imposed, that is, $\bar{L}$ = 53.10 kg/ha of available nitrogen (table 1). The tax level that yields the same target for the risk-averse farmer t^a is about $0.23/kg (table 2). These results demonstrate Proposition 2 that to attain a target level of expected leaching (or a target level of X^*), a higher tax must be imposed on a risk-averse farmer than on a risk-neutral farmer ($t^a > t^n$). Moreover, note from table 1 that $E\pi^{nt}(t^n = 0) = E\pi^{at}(t^a \approx \$0.23) \approx \$648/ha$ after rebate as stated in Proposition 3. If the target leaching level reduces to 50 kg/ha, or the target ex ante application is $X^* = 89.13$ kg/ha (or $A + X^* = 189.13$), then the taxes that should be imposed to attain this target level are about $t^n = \$0.50/kg$ for the risk-neutral farmer and $t^a = \$0.65/kg$ for the risk-averse farmer, respectively. Again, $t^a > t^n$ and $E\pi^{nt}(t^n \approx \$0.50) = E\pi^{at}(t^a \approx \$0.65) \approx \$645/ha$ after the rebate.

5. Obviously, the expected profit before rebate for the risk-neutral farmer is higher than for the risk-averse farmer, especially under the lower tax levels. For $t > \$0.15/kg$, the

differences between the expected profits before rebate for a risk-neutral and a risk-averse farmers are negligible. But when $t \le \$0.15$, the expected profit after rebate is higher for the risk-neutral farmer under the low tax level. It is interesting to note that while the after-rebate expected profit for the risk-neutral farmer decreases monotonically as t increases, this is not true for the risk-averse farmer. When t increases from zero to \$0.23, $E\pi^{at}$ after rebate increases and then remains stable for a wide range of t values.

Concluding Remarks

This article investigates the effects of first-best policies to regulate nitrogen use at the farm level. Yield is assumed to depend on the plant-available nitrogen at the beginning and after the rainy season. Due to random rainfall, some portion of nitrogen applied at the beginning of the season leaches below the root zone and contaminates the groundwater aquifer. Leaching of sidedressed nitrogen applied after the rainy season is assumed to be negligible.

"First-best" policy is a tax or a quota imposed on ex ante application because ex post application is not subject to leaching. It is shown that a risk-averse farmer uses more nitrogen ex ante than a risk-neutral one to attain a target level of expected leaching, and therefore, a higher tax must be imposed on the former than on the latter ($t^a > t^r$). A first-best tax and a quota that are action equivalent will yield the same level of welfare, whether the farmer is risk-neutral or averse.

An empirical model for wheat in Israel was used to illustrate the analytical findings. It is shown that the gap between X^{at} and X^{nt} is higher for low tax values and that the impact of a tax increase from the no-tax situation on the reduction in expected leaching is greater for the risk-averse farmer than for the risk-neutral one. The expected profit *after rebate* is higher for the risk-neutral farmer under low tax levels ($t \le \$0.15$/kg) and slightly lower when t exceeds \$0.15. The numerical example also shows that under certain conditions, expected profit (not utility) of a risk-averse farmer may be higher with taxes than without them, provided that taxes are rebated.

Although first-best taxes and quotas are efficient measures to achieve a target level of leaching, it may be difficult to enforce them in practice. First-best quotas or taxes are applied on ex ante nitrogen application and not on ex post application, because leaching of nitrogen applied ex post is negligible. If the storage cost of nitrogen is not prohibitively high, farmers may purchase and store nitrogen at the end of a season and use it at the beginning of the next season, thereby effectively evading nitrogen tax or quota. This possibility of tax or quota evasion points to the need for studying second-best policies to regulate nitrogen use.

If the split between ex ante and ex post applications is not easily observed by the policy maker, total nitrogen use may be monitored by controlling the supply of nitrogen. When the storage cost is not prohibitively high, it may be practical to control total nitrogen use, which is a second-best policy. Analysis of second-best policies can be found in Feinerman and Choi. Many public utilities (e.g., electricity) can monitor the timing of consumption and charge different rates accordingly. Thus, if the government can monitor early nitrogen use or if the storage cost is high, then first-best policy is not only feasible but may also be practical.

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