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"Internal Instability" of Peasant Households : A Further Analysis of the de Janvry, Fafchamps, and Sadoulet Model

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Based on the model by de Janvry, Fafchamps, and Sadoulet (1991), this study gives a two-step interpretation to response of the shadow or internal prices in a peasant household model with missing markets for both food and labor. In the first step, changes in exogenous variables have initial effects on the internal prices of food and labor in their respective "internal markets". In the second step, these effects are combined through interactions between the internal markets that are caused by cross-price effects between food and labor. Response of the internal prices under the two missing markets can be thought of as resulting from these interactions. This interpretation, as well as the sensitivity analysis based on it, reveals not only how the marked "internal instability" (or sharp fluctuation of the perceived scarcity) of food and labor arises but also which assumptions are crucial to produce it in the simulation analysis of de Janvry et al.

Key words : internal instability, missing markets, internal prices, perceived scarcity, initial effects, interactions between the internal markets.

1. Introduction

de Janvry, Fafchamps, and Sadoulet [2] focused on two anomalous characteristics commonly observed in peasant households, "internal instability" and "external stability", and explained these characteristics using a peasant household model with missing markets for food and labor.

When the markets for food and labor are missing, changes in exogenous variables, such as market prices of other commodities, compel the household to adjust the demand and supply for food and labor. This adjustment causes fluctuations in the "internal prices" (or perceived scarcity) for food and labor, which equilibrate the demand and supply of these commodities within the household. Because fluctuations of internal prices are unobservable from outside the household, it may be referred to as "internal instability".

On the other hand, changes in exogenous variables influence the household's demand and supply of factors and commodities in two ways. One is a direct effect of changes in exogenous variables where the internal prices are fixed, which represents the household's response under competitive markets for food and labor. The other is an indirect effect of changes in exogenous variables via changes in the internal prices, which represents the household's unique response to the case of missing markets. Because the latter tends to have an opposite effect to the former and to make the observable response of the household less elastic, the inelastic response may be referred to as "external stability".

The simulation analysis by de Janvry et al. succeeds in demonstrating marked internal instability and external stability especially for the case where the markets for both food and labor are missing. In this case, however, the analysis of de Janvry et al. fails to give a detailed account of (a) how these anomalous characteristics arise and (b) which assumptions regarding the household's production and consumption patterns are crucial in producing these characteristics. These two points deserve close examination because

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answering them not only provides a more detailed rationale for the paradoxes posed by de Janvry et al. but also helps to identify the key assumptions made in the simulation.

A close examination of internal instability requires an intuitive interpretation of the response of the internal prices under two missing markets. However, the complicated expression of this response has prevented existing studies of peasant household models from offering such an interpretation.[1] External stability, on the other hand, can be analyzed in a similar way to the existing analysis under one missing market once the response of the internal prices is interpreted. On this basis, the present study focuses on internal instability and attempts to answer points (a) and (b) by examining interactions between the "internal markets" for food and labor within the household.

The following section introduces a peasant household model with missing markets for both food and labor and gives an intuitive interpretation to the response of their internal prices. The third section uses this interpretation to explain how the marked internal instability arose in the simulation analysis of de Janvry et al. The fourth section presents a sensitivity analysis to identify the key assumptions in their simulation analysis. The final section offers some concluding remarks.

2. An Intuitive Interpretation to the Response of Internal Prices Under Two Missing Markets

de Janvry et al. assume that a peasant household maximizes its utility function $U(c_f, c_l, c_m, \boldsymbol{z})$ subject to the following constraints:

$$p_m c_m \leq p_c q_c + p_x q_x + S \quad \text{(cash income constraint)},$$

$$G(q_c, q_f, q_l, q_x, \boldsymbol{z}) = 0 \quad \text{(production technology)},$$

$$q_i + T_i \geq c_i (i = f, l) \quad \text{(equilibrium for food and labor)}.$$

Quantities q_c and q_f denote the amounts produced of cash and food crops, respectively, and quantities $-q_l$ and $-q_x$ denote the amounts demanded of labor and other inputs, respectively. Quantities c_f, c_l, and c_m denote the amounts of food, leisure, and manufactured goods consumed, respectively. Expressions p_i and T_i denote the market price and the endowment of commodity i, respectively, and S denotes a cash endowment. Vector $\boldsymbol{z}$ denotes shift factors of the functions $U(\cdot)$ and $G(\cdot)$.[2]

Because the markets for food and labor (or leisure) are missing, we define their internal prices, p_f and p_l, which balance the demand and supply in the respective "internal markets" for food and labor within the household. Then, as de Janvry et al. show for the above problem, the optimality conditions associated with production organization yield factor demand and output supply functions $q_i = q_i(\boldsymbol{p}, \boldsymbol{z})$ $(i = c, f, l, x)$, with the two endogenous internal prices of food and labor being included in the price vector $\boldsymbol{p}$.[3] Similarly, the optimality conditions associated with consumption choice yield uncompensated commodity demand functions $c_i = c_i(\boldsymbol{p}, Y, \boldsymbol{z})$ $(i = f, l, m)$ with $Y = p_c q_c + p_f q_f + p_l(q_l + T_l) + p_x q_x + S$ representing full income.

We first define the elasticity $\varepsilon_1(p_i, b)$ of the internal price $p_i (i = f, l)$ with respect to an exogenous variable b. The variable b chosen by de Janvry et al. includes the market prices p_c and p_m of cash crops and manufactured goods, monetary head tax τ, and an index ϕ of food productivity. The meaning of the numerical subscript "1" will be clarified below. To define the elasticity $\varepsilon_1(p_i, b)$, we substitute the demand and supply functions of commodity $i (= f, l)$ into the corresponding equilibrium condition to obtain $q_i(\boldsymbol{p}, \boldsymbol{z}) + T_i = c_i(\boldsymbol{p}, Y, \boldsymbol{z})$. After differentiating both sides of this condition with respect to $b (\neq T_i)$ with the internal price of the other commodity j $(= f, l; j \neq i)$ being fixed, we solve the resulting equation for elasticity of the internal price to define

$$\begin{aligned} \varepsilon_1(p_i, b) &\equiv (\partial \ln p_i / \partial \ln b)_{dp_j = 0} \\ &= -D_i(b) / D_i(p_i), \end{aligned} \quad (1)$$

where $D_i(b) \equiv \partial \ln q_i / \partial \ln b - (\partial \ln c_i / \partial \ln b)(c_i / q_i)$ and $D_i(p_i) \equiv \partial \ln q_i / \partial \ln p_i - (\partial \ln c_i / \partial \ln p_i)(c_i / q_i)$.

It should be noted that unlike de Janvry et al., this study, for convenience of the subsequent analysis, defines the price elasticity $\partial \ln c_i / \partial \ln p_j$ of the uncompensated demand for commodity i as follows :

$$\begin{aligned} \partial \ln c_i / \partial \ln p_j = h_{ij} + (s_{j\pi} s_{\pi Y} + s_{jY} - s_j) \eta_i \\ (i, j = f, l, m), \end{aligned} \quad (2)$$

where $\pi \equiv \sum_{i \in \{c, f, l, x\}} p_i q_i$, $s_{j\pi} \equiv p_j q_j / \pi$, $s_{\pi Y} \equiv \pi / Y$,

$s_{jY} \equiv p_j T_j / Y$, $s_j = p_j c_j / Y$, $\eta_j = \partial \ln c_j / \partial \ln Y$, and $h_{ij} = \partial \ln h_i / \partial \ln p_j$, with h_i denoting the compensated demand for commodity i. Specifically, price elasticity $\partial \ln c_i / \partial \ln p_j$ in equation (2) includes profit and endowment effects, $(s_{j\pi} s_{\pi Y} + s_{jY}) \eta_i$, in this study.

It should also be noted that the system of equations (1) and (2) is the equivalent of equation (4) of de Janvry et al. :

$$E(p_i / p_j) = -\{E_{ij} - \theta_{ij} r_i - \eta_i r_i (s_{\pi Y} s_{j\pi} + s_{jY})\} / \{E_i - \theta_i r_i - \eta_i r_i (s_{\pi Y} s_{i\pi} + s_{iY})\},$$

where $E(p_i/p_j) = \varepsilon_1(p_i, p_j)$, $E_i \equiv \partial \ln q_i / \partial \ln p_i$, $E_{ij} \equiv \partial \ln q_i / \partial \ln p_j (i \neq j)$, $r_j \equiv c_j / q_j$, $\theta_{ij} \equiv h_{ij} - s_j \eta_i$, $\theta_i \equiv \theta_{ii}$, and the lowercase letter π is used for profit. This equation and the system of equations (1) and (2) above are equivalent because equation (2) and the definition of $D_i(p_j)$ imply that

$$\begin{aligned} D_i(p_j) &= E_{ij} - \{h_{ij} + (s_{\pi Y} s_{j\pi} + s_{jY} - s_j) \eta_i\} r_i \\ &= E_{ij} - \{\theta_{ij} + (s_{\pi Y} s_{j\pi} + s_{jY}) \eta_i\} r_i \\ &= E_{ij} - \theta_{ij} r_i - \eta_i r_i (s_{\pi Y} s_{j\pi} + s_{jY}) \\ &\quad \text{for } i \neq j, \\ D_i(p_i) &= E_i - \theta_i r_i - \eta_i r_i (s_{\pi Y} s_{i\pi} + s_{iY}). \end{aligned}$$

The elasticity of the internal price p_i in equation (1) is identified as the elasticity obtained under a single missing market, so that the numerical subscript of the elasticity $\varepsilon_1(p_i, b)$ indicates the number of missing markets. The denominator $D_i(p_i)$ on the right-hand side of equation (1) represents the difference between the slopes of the demand and supply functions in the internal market for food or labor, while the numerator $D_i(b)$ represents the difference between shifts of these functions caused by a change in exogenous variable b. Equation (1) shows that a change in exogenous variable b causes shifts of the demand and supply functions in the internal market for food or labor to make its internal price fluctuate.

Next, we define and interpret elasticity $\varepsilon_2(p_i, b)$ of the internal price p_i with respect to an exogenous variable b under missing markets for both food and labor, where the numerical subscript "2" indicates the number of missing markets. To this end, we substitute the demand and supply functions of both food and labor into the respective equilibrium conditions to obtain $q_i(\boldsymbol{p}, \boldsymbol{z}) + T_i = c_i(\boldsymbol{p}, Y, \boldsymbol{z})$ $(i = f, l)$. After differentiating both sides of these two conditions with respect to $b (\neq T_i)$ with the two internal prices now being regarded as endogenous, we solve the resulting equations for elasticities of the internal prices to define

$$\varepsilon_2(p_i, b) \equiv \{D_i(p_j) D_j(b) - D_j(p_j) D_i(b)\} / |\boldsymbol{D}|, \tag{3}$$

where $\boldsymbol{D}$ is a 2×2 matrix of $D_i(p_j)$ $(i, j = f, l)$.

To give an intuitive interpretation of the elasticity $\varepsilon_2(p_i, b)$, consider the following interactions between the internal markets for food and labor. A change in exogenous variable b has initial effects $\varepsilon_1(p_f, b)$ and $\varepsilon_1(p_l, b)$ on the internal prices of food and labor, respectively, just as in the preceding analysis of a single missing market. Due to cross-price effects between food and labor, the change $\varepsilon_1(p_j, b)$ in the internal price of commodity j $(= f, l)$ shifts the demand and supply functions of the other commodity $i (= f, l; i \neq j)$ to change its internal price. Recalling equation (1), this change in the internal price p_i may be expressed as $\alpha_{ij} \varepsilon_1(p_j, b)$, where

$$\alpha_{ij} \equiv -D_i(p_j) / D_i(p_i) = d \ln p_i / d \ln p_j, \quad i, j = f, l.^{4)} \tag{4}$$

The initial change $\varepsilon_1(p_f, b)$ in the internal price p_f of food changes the internal price p_l of labor by $\alpha_{lf} \varepsilon_1(p_f, b)$ in the internal market for labor, which in turn changes the internal price p_f of food by $\alpha_{fl}(\alpha_{lf} \varepsilon_1(p_f, b))$ in the internal market for food, and so on. On the other hand, the initial change $\varepsilon_1(p_l, b)$ in the internal price p_l of labor changes the internal price p_f of food by $\alpha_{fl} \varepsilon_1(p_l, b)$ in the internal market for food, which in turn changes the internal price p_l of labor by $\alpha_{lf}(\alpha_{fl} \varepsilon_1(p_l, b))$ in the internal market for labor, and so on. Thus, when we define $\varepsilon_2^*(p_i, b)$ as the sum of these changes in the internal price of commodity $i (= f, l)$, it can be expressed as

$$\begin{aligned} \varepsilon_2^*(p_f, b) &= \varepsilon_1(p_f, b) + \alpha_{fl} \varepsilon_1(p_l, b) \\ &\quad + \alpha_{fl}(\alpha_{lf} \varepsilon_1(p_f, b)) \\ &\quad + \alpha_{fl}(\alpha_{lf}(\alpha_{fl} \varepsilon_1(p_l, b))) + \cdots, \\ \varepsilon_2^*(p_l, b) &= \varepsilon_1(p_l, b) + \alpha_{lf} \varepsilon_1(p_f, b) \\ &\quad + \alpha_{lf}(\alpha_{fl} \varepsilon_1(p_l, b)) \\ &\quad + \alpha_{lf}(\alpha_{fl}(\alpha_{lf} \varepsilon_1(p_f, b))) + \cdots, \end{aligned} \tag{5}$$

which can be rearranged as

$$\varepsilon_2^*(p_i, b) = \sigma \varepsilon_1(p_i, b) + \alpha_{ij} \sigma \varepsilon_1(p_j, b) \quad (i, j = f, l; i \neq j), \tag{5'}$$

$$\sigma = \sum_{t=0}^{\infty}(\alpha_{fl}\alpha_{lf})^t$$

To take this analysis further, the infinite series σ is required to converge, which can be proved under plausible conditions. To prove this, let $\pi(\boldsymbol{p}, \boldsymbol{z}) = \sum_{i \in \{c, f, l, x\}} p_i q_i(\boldsymbol{p}, \boldsymbol{z})$, and $e(\boldsymbol{p}, u, \boldsymbol{z}) = \sum_{i \in \{f, l, m\}} p_i h_i(\boldsymbol{p}, u, \boldsymbol{z})$ represent profit and expenditure functions, respectively, where $h_i(\cdot)$ denotes the compensated demand function for commodity i and u denotes a utility level. Then, we can prove the following proposition.

Proposition 1 : Suppose that the profit function $\pi(\boldsymbol{p}, \boldsymbol{z})$ is strictly convex and the expenditure function $e(\boldsymbol{p}, u, \boldsymbol{z})$ is strictly concave in the internal prices of food and labor. Then, $0 \leq \alpha_{fl}\alpha_{lf} < 1$.

Proof : Let $\pi_{ij} = \partial^2\pi/\partial p_i \partial p_j$ and $e_{ij} = \partial^2 e/\partial p_i \partial p_j$ denote the second derivatives of the functions $\pi(\cdot)$ and $e(\cdot)$ with the internal prices being regarded as exogenous. Hotelling's lemma implies $\partial q_i/\partial p_j = \pi_{ij}$, and Shephard's lemma implies $\partial c_i/\partial p_j = \partial h_i/\partial p_j = e_{ij}$ for $i, j = f, l$, where we use the fact that the income effect in $\partial c_i/\partial p_j$ should vanish for changes in the internal prices. Then,

$$\begin{aligned} D_i(p_j) &\equiv \partial \ln q_i/\partial \ln p_j - (\partial \ln c_i/\partial \ln p_j)(c_i/q_i) \\ &= (p_j/q_i)(\partial q_i/\partial p_j - \partial c_i/\partial p_j) \\ &= (p_j/q_i)(\pi_{ij} - e_{ij}). \end{aligned}$$

Using this relation, we obtain

$$\begin{aligned} |\boldsymbol{D}| &= \begin{vmatrix} (p_f/q_f)(\pi_{ff} - e_{ff}) & (p_l/q_f)(\pi_{fl} - e_{fl}) \\ (p_f/q_l)(\pi_{lf} - e_{lf}) & (p_l/q_l)(\pi_{ll} - e_{ll}) \end{vmatrix} \\ &= (p_f p_l / q_f q_l)|\boldsymbol{\pi}_{pp} - \boldsymbol{e}_{pp}|, \end{aligned} \qquad (6)$$

where

$$\boldsymbol{\pi}_{pp} = \begin{bmatrix} \pi_{ff} & \pi_{fl} \\ \pi_{lf} & \pi_{ll} \end{bmatrix} \text{ and } \boldsymbol{e}_{pp} = \begin{bmatrix} \boldsymbol{e}_{ff} & \boldsymbol{e}_{fl} \\ \boldsymbol{e}_{lf} & \boldsymbol{e}_{ll} \end{bmatrix}.$$

Since the profit and expenditure functions are strictly convex, and strictly concave in the internal prices of food and labor, we obtain

$$\begin{aligned} \pi_{ii} > 0, \quad e_{ii} < 0 \quad \text{for } i = f, l, \\ |\boldsymbol{\pi}_{pp}| > 0, \quad |\boldsymbol{e}_{pp}| > 0. \end{aligned} \qquad (7)$$

These relations imply that the matrices $\boldsymbol{\pi}_{pp}$ and $\boldsymbol{e}_{pp}$ are positive definite and negative definite, respectively. Hence, for any two-dimensional row vector $\boldsymbol{x} \neq 0$,

$$\boldsymbol{x}'(\boldsymbol{\pi}_{pp} - \boldsymbol{e}_{pp})\boldsymbol{x} = \boldsymbol{x}'\boldsymbol{\pi}_{pp}\boldsymbol{x} - \boldsymbol{x}'\boldsymbol{e}_{pp}\boldsymbol{x} > 0.$$

This in turn implies that the matrix $\boldsymbol{\pi}_{pp} - \boldsymbol{e}_{pp}$ is positive definite, so that $|\boldsymbol{\pi}_{pp} - \boldsymbol{e}_{pp}| > 0$. Hence, we use equation (6) and the definition of the matrix $\boldsymbol{D}$ to obtain

$$|\boldsymbol{D}| = D_f(p_f)D_l(p_l) - D_f(p_l)D_l(p_f) < 0, \qquad (8)$$

where $p_i (i = f, l)$ and q_f are positive and q_l is negative by definition. Dividing both sides of equation (8) by $D_f(p_f)D_l(p_l)$ and noting that $D_f(p_f) = (p_f/q_f)(\pi_{ff} - e_{ff}) > 0$, $D_l(p_l) = (p_l/q_l)(\pi_{ll} - e_{ll}) < 0$, and $\alpha_{ij} \equiv -D_i(p_j)/D_i(p_i)$, we obtain

$1 - \alpha_{fl}\alpha_{lf} > 0$.

Furthermore, $\pi_{ij} = \pi_{ji}$ and $e_{ij} = e_{ji}$ according to the symmetry conditions and $\pi_{ii} - e_{ii} > 0 (i = f, l)$ according to condition (7), so that

$$\begin{aligned} \alpha_{fl}\alpha_{lf} &= D_f(p_l)D_l(p_f) / D_f(p_f)D_l(p_l) \\ &= (\pi_{fl} - e_{fl})(\pi_{lf} - e_{lf}) / (\pi_{ff} - e_{ff})(\pi_{ll} - e_{ll}) \\ &= (\pi_{fl} - e_{fl})^2 / (\pi_{ff} - e_{ff})(\pi_{ll} - e_{ll}) \geq 0. \end{aligned}$$

Q.E.D.

Because the profit and expenditure functions used in the empirical analysis usually satisfy the conditions in Proposition 1, we will assume these conditions in the analysis below. Proposition 1 therefore implies that the infinite series σ converges to $1/(1 - \alpha_{fl}\alpha_{lf})$, so that equation (5′) can be written as

$$\begin{aligned} \varepsilon_2^*(p_i, b) &= \varepsilon_1(p_i, b) / (1 - \alpha_{fl}\alpha_{lf}) \\ &\quad + \alpha_{ij}\varepsilon_1(p_j, b) / (1 - \alpha_{fl}\alpha_{lf}) \\ &\qquad (i, j = f, l; i \neq j). \end{aligned} \qquad (9)$$

It can be shown that the elasticity $\varepsilon_2(p_i, b)$ in equation (3) and the sum $\varepsilon_2^*(p_i, b)$ in equation (9) are identical. This means that the elasticity $\varepsilon_2(p_i, b)$ of the internal price is interpreted as the consequence of interactions between the internal markets for food and labor that start from the initial effects, and that the interactions are characterized by the sign and magnitude of the "coefficient" α_{ij} in the interactive process. A detailed interpretation of the coefficient α_{ij} is given in the next section in a more specific context.

3. The Marked Internal Instability in the Simulation Analysis of de Janvry et al.

This section uses the interpretation above to answer the point (a) posed in the first section, i. e., how the marked internal instability arises in the simulation analysis of de Janvry et al. To begin with, we need to evaluate some elasticities using Table 1 of de Janvry et al. From this table, we can direct-

Table 1. The assumed values of elasticities in the simulation analysis of de Janvry et al.

	Elasticity with respect to						
	p_f	p_l	p_c	p_m	τ	ϕ	Y
Food demand c_f	−0.50	0.20	0.21	0.17	−0.70	0.30	0.70
Food supply q_f	0.80	−0.22	−0.54	0.00	0.00	1.80	—
Labor demand q_l	0.37	−0.60	0.17	0.00	0.00	0.37	—
Labor supply $T_l - c_l$	−0.33	0.69	−0.40	−0.09	1.35	−0.59	−1.35
Internal price of food $p_f(n=1)$	—	—	0.58	0.13	−0.54	−1.14	—
Internal price of food $p_f(n=2)$	—	—	0.88	0.19	−1.07	−1.10	—
Internal price of labor $p_l(n=1)$	—	—	0.45	0.07	−1.05	0.74	—
Internal price of labor $p_l(n=2)$	—	—	0.93	0.17	−1.64	0.13	—

Note : The elasticity of the internal price p_i for the case "$n=1$" represents the initial effect $\varepsilon_1(p_i, b)$, and the one for the case "$n=2$" represents the elasticity $\varepsilon_2(p_i, b)$ under the two missing markets.

ly obtain the price elasticities of production factors and outputs, $\partial \ln q_i/\partial \ln p_j (i, j=f, c, x, l)$, and the price elasticities of uncompensated demand excluding profit and endowment effects, $\theta_{ij} \equiv h_{ij} - s_j \eta_i$. The price elasticities of uncompensated demand including profit and endowment effects, $\partial \ln c_i/\partial \ln p_j (i, j=f, l, m)$, are evaluated using the values in the table and the Slutsky-type equation (2).[5] Values of the price elasticities thus obtained, together with the values of income elasticities in the table, are presented in Table 1 of this study, where the elasticities of labor supply are evaluated by multiplying those of leisure demand by ratio $c_l/q_l = -1.5$.[6] Table 1 of this study also presents values for the elasticities with respect to monetary head tax τ and an index ϕ of food productivity, which are directly obtained from the columns "None" in Table 2 of de Janvry et al.

Substitution of the elasticities thus evaluated into equation (1) gives values for the initial effect $\varepsilon_1(p_i, b)$, which are presented in Table 1 as the elasticity of the internal price p_i for the case "$n=1$". Moreover, substitution of those elasticities into equation (3) or (9) gives values for the elasticity $\varepsilon_2(p_i, b)$, which are presented in Table 1 as the elasticity of the internal price p_i for the case "$n=2$".

Large absolute values of the elasticity $\varepsilon_2(p_i, b)$ can be thought of as indicating marked internal instability under two missing markets. Hence, changes in the price p_c of cash crops and monetary head tax τ cause this type of instability for both food and labor, while changes in the price p_m of manufactured goods have a negligible effect. Furthermore, changes in food productivity ϕ cause marked internal instability only for food, and cause $|\varepsilon_1(p_i, \phi)| > |\varepsilon_2(p_i, \phi)|$ unlike changes in the other exogenous variables.

According to the interpretation in the previous section, factors determining the sign and magnitude of the elasticity $\varepsilon_2(p_i, b)$ $(i=f, l)$ may be classified into two groups. One is related to the initial effects, $\varepsilon_1(p_f, b)$ and $\varepsilon_1(p_l, b)$, in the respective internal markets for food and labor, and the other is related to the coefficients, α_{fl} and α_{lf}, representing how the two internal markets interact.

We first examine the initial effects, $\varepsilon_1(p_f, b)$ and $\varepsilon_1(p_l, b)$, defined by equation (1). Table 1 shows that their absolute values are moderate or large for $b=p_c, \tau, \phi$, but are very small for $b=p_m$, which are explained by the slopes and shifts of the demand and supply functions in the internal markets. Table 1 shows that the slope of the demand function in the internal market is evaluated at −0.50 $(=\partial \ln c_f/\partial \ln p_f)$ for food and −0.60 $(=\partial \ln q_l/\partial \ln p_l)$ for labor, respectively, whereas the slope of the supply function is evaluated at

0.80 $(=\partial \ln q_f/\partial \ln p_f)$ for food and 0.69 $(=-(\partial \ln c_l/\partial \ln p_l)(c_l/q_l))$ for labor, respectively.[7] The table also shows various shifts of the demand and supply functions in the internal markets: a change in the price p_c of cash crops causes relatively large shifts in the supply functions for food and labor; a change in the price p_m of manufactured goods causes very small or no shifts in all the functions; a change in monetary head tax τ causes large shifts in the food demand and labor supply functions; and a change in productivity ϕ causes large shifts in the supply functions for food and labor. These patterns of shifts, as well as the relatively steep slopes of the demand and supply functions, determine the sign and magnitude of the initial effects.

For convenience in the subsequent analysis, we also note in Table 1 that the two initial effects, $\varepsilon_1(p_f, b)$ and $\varepsilon_1(p_l, b)$, have the same sign for $b=p_c$, p_m, and τ under the assumptions made by de Janvry et al. A close examination of the elasticities in Table 1 shows that these relations reflect assumptions commonly made regarding the behavior of peasant households both as producers and consumers. For example, it is found from equation (1) and Table 1 that $\varepsilon_1(p_f, p_c)$ and $\varepsilon_1(p_l, p_c)$ are both positive because food and leisure are normal goods, the demand for labor increases with rising prices for cash crops, and the two crops are competitive.

Now, we examine the coefficients α_{fl} and α_{lf} which characterize interactions between the two internal markets. For the elasticity values in Table 1, α_{fl} and α_{lf} are evaluated to be 0.32 and 0.54, respectively. Equation (4) and condition (7) imply that the two coefficients are positive only when $D_f(p_l)<0$ and $D_l(p_f)>0$, which in turn are satisfied under the three assumptions made in Table 1. That is, the demand for labor increases with the price of food, the supply of food decreases with wage, and food and leisure are substitutes. Among these three assumptions, the first two are generally held to be true because the producer's factor demand is likely to increase with output prices and its output supply is likely to decrease with factor prices, all other things being equal.

The appropriateness of the last assumption may be explained as follows.[8] Homogeneity and symmetry conditions for the compensated demand functions $h_i(\boldsymbol{p}, u, \boldsymbol{z})$ imply that

$$s_f h_{fi} + s_l h_{li} + s_m h_{mi} = 0 \quad \text{for } i = f, l, m,$$

where $h_{ij}=\partial \ln h_i/\partial \ln p_j$. Since the expenditure share s_i is positive and the own price elasticity $h_{ii}=e_{ii}$ is negative according to condition (7), the relation above, together with the symmetry condition in elasticity form, $s_i h_{ij} = s_j h_{ji}$, implies the following relations among the three cross price elasticities, h_{fl}, h_{fm}, and h_{lm}:

$$\begin{aligned} &h_{fl} + h_{fm} = -h_{ff} > 0, \\ &s_f h_{fl} + s_l h_{lm} = -s_l h_{ll} > 0, \\ &s_f h_{fm} + s_l h_{lm} = -s_m h_{mm} > 0. \end{aligned}$$

The first relation, for example, implies that either h_{fl} or h_{fm} should be positive. By the same reasoning, it is eventually found that two of the three cross-price elasticities should be positive. In addition, even in the case where h_{fm} and h_{lm} are definitely positive and where the sign of h_{fl} is indefinite, we find from the inequalities above that $h_{fl}>\max\{-h_{fm}, -(s_l/s_f)h_{lm}\}$. Hence, when either of the price elasticities, h_{fm} or h_{lm}, is sufficiently small in absolute value, h_{fl} is also likely to be positive. Thus, the conditions, $D_f(p_l)<0$ and $D_l(p_f)>0$, are likely to be satisfied, and hence the coefficients α_{fl} and α_{lf} are also likely to be positive.

As equation (4) shows, the positive coefficients α_{fl} and α_{lf} mean that the internal price of food (labor) responds positively to a rise in the internal price of labor (food), all other things being equal. In other words, under the present assumptions, severer perceived scarcity of labor causes higher production costs to reduce the supply of food on the one hand, and causes substitution of food for leisure to increase the demand for food on the other, which combine to yield severer perceived scarcity of food. At the same time, severer perceived scarcity of food causes substitution of leisure for food to decrease the supply of labor on the one hand, and causes higher production revenue to increase the demand for labor on the other, which combine to yield severer perceived scarcity of labor.

Having explained the important assumptions in the simulation analysis of de Janvry et al., we can explain why internal instability is more marked under the missing markets for both food and labor.

Proposition 2 : Suppose that the profit function $\pi(\boldsymbol{p}, \boldsymbol{z})$ is strictly convex, and the expenditure function $e(\boldsymbol{p}, u, \boldsymbol{z})$ is strictly concave in the internal prices of food and labor. Suppose also that $\alpha_{\mathrm{fl}}>0$, $\alpha_{\mathrm{lf}}>0$, and the initial effects, $\varepsilon_1(p_{\mathrm{f}}, b)$ and $\varepsilon_1(p_{\mathrm{l}}, b)$, of a change in an exogenous variable b have the same sign. Then, $|\varepsilon_2(p_{\mathrm{i}}, b)|\geq|\varepsilon_1(p_{\mathrm{i}}, b)|$ for $i=f, l$.

Proof : If the profit and expenditure functions are strictly convex, and strictly concave in the internal prices of food and labor, then the condition $0\leq\alpha_{\mathrm{fl}}\alpha_{\mathrm{lf}}<1$ follows from Proposition 1. Since the two initial effects $\varepsilon_1(p_{\mathrm{i}}, b)$ and $\varepsilon_1(p_{\mathrm{j}}, b)$ have the same sign and $\alpha_{\mathrm{ij}}>0$ by assumption, equation (9) implies that elasticity $\varepsilon_2(p_{\mathrm{i}}, b)(=\varepsilon_2^*(p_{\mathrm{i}}, b))$ has the same sign as the first and second terms on the right hand side of equation (9). Hence, we obtain[9)]

$$|\varepsilon_2(p_{\mathrm{i}}, b)| = |\varepsilon_1(p_{\mathrm{i}}, b) / (1-\alpha_{\mathrm{fl}}\alpha_{\mathrm{lf}})| + |\alpha_{\mathrm{ij}}\varepsilon_1(p_{\mathrm{j}}, b) / (1-\alpha_{\mathrm{fl}}\alpha_{\mathrm{lf}})|.$$

Using the conditions $\alpha_{\mathrm{ij}}>0$ and $0\leq\alpha_{\mathrm{fl}}\alpha_{\mathrm{lf}}<1$ again, we obtain

$$\begin{aligned}|\varepsilon_2(p_{\mathrm{i}}, b)| &= |\varepsilon_1(p_{\mathrm{i}}, b)| / (1-\alpha_{\mathrm{fl}}\alpha_{\mathrm{lf}}) \\ &\quad + \alpha_{\mathrm{ij}}|\varepsilon_1(p_{\mathrm{j}}, b)| / (1-\alpha_{\mathrm{fl}}\alpha_{\mathrm{lf}}) \qquad (10)\\ &\geq |\varepsilon_1(p_{\mathrm{i}}, b)| / (1-\alpha_{\mathrm{fl}}\alpha_{\mathrm{lf}}) \\ &\geq |\varepsilon_1(p_{\mathrm{i}}, b)|.\end{aligned}$$

Q.E.D.

For changes in $b=p_{\mathrm{c}}, p_{\mathrm{m}}, \tau$, the two initial effects have the same sign as noted above. Hence, Proposition 2 implies that internal instability is more marked in the case of two missing markets than in the case of one missing market. For $b=\phi$, on the other hand, the initial effects of changes in food productivity have the opposite signs, and the proposition does not apply.

In addition, the following proposition provides some insight into the effect of the coefficients α_{fl} and α_{lf} on the magnitude of $|\varepsilon_2(p_{\mathrm{i}}, b)|$.

Proposition 3 : Under the same conditions as in Proposition 2, $\partial|\varepsilon_2(p_{\mathrm{i}}, b)|/\partial\alpha_{\mathrm{ij}}\geq 0\ (i, j=f, l;\ i\neq j)$.

Proof : Under the same conditions as in Proposition 2, equation (10) holds and is rearranged as

$$|\varepsilon_2(p_{\mathrm{i}}, b)| = \{|\varepsilon_1(p_{\mathrm{i}}, b)| + \alpha_{\mathrm{ij}}|\varepsilon_1(p_{\mathrm{j}}, b)|\} / (1-\alpha_{\mathrm{fl}}\alpha_{\mathrm{lf}}).$$

Then, we obtain

$$\begin{aligned}\partial|\varepsilon_2(p_{\mathrm{i}}, b)| / \partial\alpha_{\mathrm{ij}} &= [|\varepsilon_1(p_{\mathrm{i}}, b)|(1-\alpha_{\mathrm{fl}}\alpha_{\mathrm{lf}}) + \alpha_{\mathrm{ji}}\{|\varepsilon_1(p_{\mathrm{i}}, b)| \\ &\quad + \alpha_{\mathrm{ij}}|\varepsilon_1(p_{\mathrm{j}}, b)|\}] / (1-\alpha_{\mathrm{fl}}\alpha_{\mathrm{lf}})^2.\end{aligned}$$

This derivative is nonnegative because $\alpha_{\mathrm{ij}}>0$ and $\alpha_{\mathrm{ji}}>0$ by assumption, and because $0\leq\alpha_{\mathrm{fl}}\alpha_{\mathrm{lf}}<1$ according to Proposition 1. *Q.E.D.*

This proposition means that given the two initial effects of the same sign, greater coefficients α_{fl} and α_{lf} result in a larger absolute value for the elasticity $|\varepsilon_2(p_{\mathrm{i}}, b)|$ and hence cause more marked internal instability under the present assumptions.

As we have seen, the initial effects in Table 1 take on moderate or large absolute values for $b=p_{\mathrm{c}}, \tau, \phi$, while they take on small absolute values for $b=p_{\mathrm{m}}$. Given these initial effects and the moderate values of the coefficients α_{fl} and α_{lf}, Propositions 2 and 3 explain why marked internal instability arises for changes in the price of cash crops and for changes in monetary head tax, but does not for changes in the price of manufactured goods. On the other hand, marked internal instability is expected to arise for changes in food productivity if at least one of the initial effects is sufficiently large.

4. Sensitivity Analysis

This section presents a sensitivity analysis to address point (b) raised in the first section, i. e., which assumptions on the household's production and consumption patterns are crucial in producing marked internal instability. de Janvry et al. determined their assumed values of elasticities by taking the most likely values for the demand and supply elasticities from literature on peasant households and by imposing on the elasticities constraints derived from utility maximization and profit maximization. The various assumptions in Table 1 of de Janvry et al. may be classified into two groups. One includes assumptions affecting only the initial effects (or shifts of the demand and supply functions caused by changes in exogenous variables) and the other includes assumptions also affecting interactions between the internal markets. In this section, we present a sensitivity analysis conducted on the latter to identify key assumptions in the simulation analysis of de Janvry et al.

Among the assumed values of elasticities in

Table 2. Results of the sensitivity analysis for case (i)

Changes in assumed values of elasticities				
$\partial \ln q_l / \partial \ln p_l$	−0.60	−1.00	−1.50	−2.00
$\partial \ln q_l / \partial \ln p_c$	0.17	0.57	1.07	1.57
$\partial \ln q_c / \partial \ln p_l$	−0.15	−0.49	−0.92	−1.35
$\partial \ln q_c / \partial \ln p_c$	1.00	1.34	1.77	2.20
Changes in coefficients α_{fl} and α_{lf}				
α_{fl}	0.32	0.32	0.32	0.32
α_{lf}	0.55	0.42	0.32	0.26
Changes in initial effects $\varepsilon_1(p_f, b)$ and $\varepsilon_1(p_l, b)$				
$\varepsilon_1(p_f, b)$ for $b = p_c$	0.57	0.57	0.57	0.57
$b = p_m$	0.13	0.13	0.13	0.13
$b = \tau$	−0.54	−0.54	−0.54	−0.54
$b = \phi$	−1.15	−1.15	−1.15	−1.15
$\varepsilon_1(p_l, b)$ for $b = p_c$	0.45	0.58	0.67	0.73
$b = p_m$	0.07	0.05	0.04	0.03
$b = \tau$	−1.05	−0.80	−0.62	−0.50
$b = \phi$	0.74	0.57	0.44	0.36
Changes in elasticities $\varepsilon_2(p_f, b)$ and $\varepsilon_2(p_l, b)$ of the internal prices				
$\varepsilon_2(p_f, b)$ for $b = p_c$	0.87	0.88	0.88	0.89
$b = p_m$	0.19	0.17	0.16	0.15
$b = \tau$	−1.06	−0.92	−0.82	−0.76
$b = \phi$	−1.11	−1.12	−1.13	−1.13
$\varepsilon_2(p_l, b)$ for $b = p_c$	0.92	0.94	0.96	0.97
$b = p_m$	0.17	0.13	0.09	0.07
$b = \tau$	−1.64	−1.19	−0.88	−0.70
$b = \phi$	0.14	0.10	0.07	0.06

Table 1, the following two sets may be worth examining and could be suitable for conducting the analysis. One relates to the small price elasticities of labor demand and output supply. Some other studies including Lau and Yotopoulos [4] and Kuroda and Yotopoulos [3], by contrast, estimate similar elasticities at much larger values (about 1.0-2.0 in absolute value), though the larger values may come from Cobb-Douglas technology specified in these studies.[10] The other relates to the net substitution relation between food and leisure. Although this relation seems plausible, as explained in the previous section, it may not be satisfied and seems to be one of the important assumptions in determining the magnitude and sign of the coefficients α_{fl} and α_{lf}.

Thus, we examine the cases where (i) the own-price elasticity, $\partial \ln q_l / \partial \ln p_l$, of labor demand varies from −0.60 (assumed by de Janvry et al.) to −2.00, and (ii) the elasticity h_{fl} of compensated food demand with respect to wage varies from 0.20 (assumed by de Janvry et al.) to −0.20.[11] We will observe the impact of these variations on the response of the internal prices. In the following sensitivity analysis, we take a simple way of varying the values of the elasticities directly, instead of obtaining them after varying parameter values of the profit and indirect utility functions originally set by de Janvry et al.[12]

For case (i), as elasticity $\partial \ln q_l / \partial \ln p_l$ varies from −0.60 to −2.00, homogeneity and symmetry conditions of factor demand and output supply functions are maintained in the following way. We first increase the absolute value of the own-price elasticity $\partial \ln q_l / \partial \ln p_l$ (<0) of labor demand and also increase its elasticity $\partial \ln q_l / \partial \ln p_c (>0)$ with respect to the cash crop price by the same amount to maintain homogeneity of the labor demand func-

Table 3. Results of the sensitivity analysis for case (ii)

Changes in assumed values of elasticities			
h_{ff}	−0.50	−0.30	−0.10
h_{fl}	0.20	0.00	−0.20
h_{lf}	0.22	0.00	−0.22
h_{ll}	−0.45	−0.23	−0.01
Changes in coefficients α_{fl} and α_{lf}			
α_{fl}	0.32	0.20	0.02
α_{lf}	0.55	0.39	0.05
Changes in initial effects $\varepsilon_1(p_f, b)$ and $\varepsilon_1(p_l, b)$			
$\varepsilon_1(p_f, b)$ for $b = p_c$	0.57	0.68	0.83
$b = p_m$	0.13	0.15	0.19
$b = \tau$	−0.54	−0.64	−0.78
$b = \phi$	−1.15	−1.36	−1.66
$\varepsilon_1(p_l, b)$ for $b = p_c$	0.45	0.60	0.94
$b = p_m$	0.07	0.10	0.15
$b = \tau$	−1.06	−1.43	−2.21
$b = \phi$	0.75	1.01	1.56
Changes in elasticities $\varepsilon_2(p_f, b)$ and $\varepsilon_2(p_l, b)$ of the internal prices			
$\varepsilon_2(p_f, b)$ for $b = p_c$	0.87	0.86	0.85
$b = p_m$	0.19	0.19	0.19
$b = \tau$	−1.06	−0.99	−0.82
$b = \phi$	−1.11	−1.26	−1.63
$\varepsilon_2(p_l, b)$ for $b = p_c$	0.92	0.94	0.98
$b = p_m$	0.17	0.17	0.16
$b = \tau$	−1.64	−1.82	−2.26
$b = \phi$	0.14	0.52	1.48

tion : $\sum_j(\partial \ln q_l/\partial \ln p_j)=0\,(j=f, c, x, l)$. We then increase the absolute value of the elasticity $\partial \ln q_c/\partial \ln p_l (<0)$ of the cash crop supply with respect to wage to maintain the symmetry condition : $\partial \ln q_c/\partial \ln p_l = (\partial \ln q_l/\partial \ln p_c)(p_l q_l/Y)/(p_c q_c/Y)$, and also increase the own-price elasticity $\partial \ln q_c/\partial \ln p_c\ (>0)$ by the same amount to maintain homogeneity of the cash crop supply function : $\sum_j(\partial \ln q_c/\partial \ln p_j)=0\,(j=f, c, x, l)$. For simplicity of the analysis, we use cash crop alone as an output whose supply elasticities vary with the elasticities of labor demand. Different sets of the elasticities $\partial \ln q_i/\partial \ln p_j$ thus obtained, which are presented in Table 2 as "changes in assumed values of elasticities", are used to evaluate the corresponding elasticities of the internal prices, with the other assumed values of elasticities kept invariant.

For case (ii), as elasticity h_{fl} varies from 0.20 to −0.20, we maintain homogeneity and symmetry conditions on the price elasticity h_{ij} in the following way, with expenditure share s_i and income elasticity η_i being fixed. We first decrease the value of the cross-price elasticity h_{fl} of food demand and also increase the value of the own-price elasticity h_{ff} (<0) by the same amount to maintain homogeneity of the food demand function : $\sum_j h_{fj}=0\,(j=f, l, m)$.[13] We then decrease the value of the elasticity h_{lf} of leisure demand with respect to food price to maintain the symmetry condition : $h_{lf}=\theta_{lf}+s_f\eta_l=(s_f/s_l)(\theta_{fl}+s_l\eta_f)=(s_f/s_l)h_{fl}$, and also increase the value of the own price elasticity $h_{ll}(<0)$ to maintain homogeneity of the leisure demand function : $\sum_j h_{lj}=0\,(j=f, l, m)$. Different sets of the elasticities h_{ij} thus obtained, which are presented in Table 3 as "changes in assumed values of elasticities", are used to evaluate the corresponding elasticities of the internal prices, with the other assumed values of elasticities kept invariant.

As shown in the previous sections, response $\varepsilon_2(p_i, b)(i=f, l)$ of the internal prices is characterized by the initial effects, $\varepsilon_1(p_f, b)$ and $\varepsilon_1(p_l, b)$, and by the coefficients α_{fl} and α_{lf}. As also shown in the previous sections, the initial effects and the two coefficients are expressed by the slopes and shifts of the demand and supply functions in the internal markets. Hence, the discussion below will examine results of the sensitivity analysis in terms of these slopes and shifts.

Table 2 presents results of the sensitivity analysis for case (i). In this case, only the slope of the labor demand function and its shift to changes in the cash crop price vary in the internal markets for food and labor.

The initial effect $\varepsilon_1(p_f, b)$ on the internal price of food exhibits no variation for the four exogenous variables because no change arises in the internal market for food. The initial effect $\varepsilon_1(p_l, b)$ on the internal wage for $b=p_m$, τ, and ϕ is influenced only by the changing slope of the demand function in the internal market for labor, so that it is substantially decreased as the slope changes from -0.60 to -2.00. The initial effect $\varepsilon_1(p_l, b)$ for $b=p_c$ is influenced by the changing shift $\partial \ln q_l/\partial \ln p_c$ as well as the changing slope of the demand function in the internal market for labor. Since the former dominates the latter, this initial effect is increased to some extent as the shift expands from 0.17 to 1.57.

Furthermore, response α_{fl} of the internal price of food to changes in wage does not vary at all for the same reason as given above. Response α_{lf} of the internal wage to changes in food price is influenced only by the changing slope of the demand function in the internal market for labor, and is substantially decreased as the slope changes from -0.60 to -2.00. Then, Proposition 3 suggests that given the initial effects $\varepsilon_1(p_f, b)$ and $\varepsilon_1(p_l, b)$ of the same sign, interactions between the internal markets have weaker effects on expanding response $\varepsilon_2(p_i, b)$ of the internal prices as the slope of the demand function becomes more gradual.

Combining the discussion about the initial effects and the two coefficients, elasticity $\varepsilon_2(p_i, b)(i=f, l)$ for $b=p_c$ hardly varies because the larger initial effects are almost offset by the weaker interactions. Elasticity $\varepsilon_2(p_i, b)$ for $b=p_m$ and $b=\tau$ becomes moderately or substantially smaller in absolute value because the invariant or smaller initial effects are combined with the weaker interactions. We cannot apply a similar examination of Propositions 2 and 3 to elasticity $\varepsilon_2(p_i, \phi)$, but it was found that the difference between $\varepsilon_2(p_i, \phi)$ and $\varepsilon_1(p_i, \phi)$ becomes smaller as α_{lf} becomes smaller. To sum up, the changing slope of the labor demand function moderately affects the internal instability of food arising from changes in the price of manufactured goods and monetary head tax, and substantially affects the internal instability of labor arising from changes in the price of manufactured goods, monetary head tax, and food productivity.

Table 3 presents the results of the sensitivity analysis for case (ii). In this case, not only the slope of the food demand function and its shift to changes in wage, but also the slope of the labor supply function and its shift to changes in food price vary in the internal markets for food and labor.

The initial effect $\varepsilon_1(p_f, b)$ on the internal price of food is influenced only by the changing slope of the demand function in the internal market for food, so that it is increased to some extent as the slope changes from -0.50 to -0.10. The initial effect $\varepsilon_1(p_l, b)$ on the internal wage is influenced only by the changing slope of the supply function in the internal market for labor, so that it is substantially increased as the slope changes from 0.68 to 0.02 (or as h_{ll} changes from -0.45 to -0.01).

Furthermore, response α_{fl} of the internal price of food to changes in wage is influenced by the changing slope and shift of the demand function in the internal market for food. More specifically, the demand function slopes more steeply and shifts leftward (instead of rightward) reflecting complementarity (instead of substitutability) between food and leisure, as the assumed elasticities change. It can be seen from Table 3 that despite the steeper slope, the leftward shift makes response α_{fl} of the internal price of food negligible. Similarly, it is found that response α_{lf} of the internal wage to changes in food price becomes negligible when substitutability between food and leisure is replaced by complementarity between them. Then, Proposition 2 suggests that interactions between the two internal markets have much weaker effects on expanding response $\varepsilon_2(p_i, b)$ of the internal prices as the designated elasticities change. In other words, the two internal markets work almost independently.

Combining the discussion about the initial effects and the two coefficients, elasticity $\varepsilon_2(p_i, b)(i=f, l)$ for $b=p_c$ and $b=p_m$ hardly varies because the larger initial effects are almost offset by much weaker interactions. Elasticity $\varepsilon_2(p_i, b)$ for $b=\tau$ is moderately decreased or increased in absolute value because the moderately or substantially larger initial effects are combined with much weaker interactions. As in case (i), it is found for $b=\phi$ that the difference between $\varepsilon_2(p_i, b)$ and $\varepsilon_1(p_i, b)$ becomes smaller. To sum up, replacing substitutability with complementarity between food and leisure makes interactions of the internal markets much weaker. At the same time, the accompanying changes in the own-price elasticities cause steeper slopes of the food demand and labor supply functions and magnify the two initial effects in the respective internal markets. These changes in the internal markets moderately affect the internal instability of food arising from changes in monetary head tax and food productivity, moderately affect the internal instability of labor arising from changes in monetary head tax, and substantially affect the internal instability of labor arising from changes in food productivity.

5. Concluding Remarks

This study provides a two-step interpretation of the response of internal prices in a peasant household model with missing markets for both food and labor. In the first step, changes in exogenous variables have initial effects on the internal prices of food and labor, just as in the existing analysis under a single missing market for food or labor. In the second step, these initial effects are combined through interactions between the internal markets for food and labor that are caused by the cross-price effects of these commodities. The response of the internal prices under the two missing markets can be thought of as resulting from these interactions, which are characterized by the coefficients $\boldsymbol{\alpha}_{fl}$ and $\boldsymbol{\alpha}_{lf}$.

Close examination of these two coefficients reveals how the marked internal instability arises under plausible conditions. A change in an exogenous variable causes tightness of the respective internal markets for food and labor, and makes the household perceive a severe scarcity of these commodities. The severe perceived scarcity of food (labor) causes further tightness of the internal market for labor (food), and makes the household perceive severer scarcity of labor (food), and so on. Such an interactive process concerning the household's perceived scarcity shows a typical way of causing the marked internal instability under the two missing markets.

Close examination of the two coefficients also reveals which assumptions about the household's production and consumption patterns are crucial in producing the marked internal instability. The sensitivity analysis shows, at least, that some of the price elasticities forming the coefficients $\boldsymbol{\alpha}_{fl}$ and $\boldsymbol{\alpha}_{lf}$ (in particular, slopes of the demand and supply functions in the internal markets) seem to be important in producing the marked instability in the simulation analysis of de Janvry et al.

Finally, it should be noted that the values of elasticities assumed by de Janvry et al. do not seem to be estimated by use of appropriate statistical methods which explicitly allow for missing markets for food and labor. Hence, for available data on peasant households, we should try to verify the plausibility of the assumed values by use of the more appropriate methods proposed by Lopez [5] and Sonoda and Maruyama [6], [7].

1) Besley [1] makes a comparative statics analysis of peasant household models with market failures for two or three commodities, and directly examines the response of the internal prices in the same form as (3) to be shown later in this study.

2) We follow de Janvry et al. in defining the vector $\boldsymbol{z}$. It includes, for example, an index ϕ of food productivity in the production function $G(\cdot)$.

3) We follow de Janvry et al. in defining the vector $\boldsymbol{p}$ for notational convenience. Specifically, it includes the prices of cash crops, food

crops, labor, and other inputs in the factor demand and output supply functions, $q_i=q_i(\boldsymbol{p}, \boldsymbol{z})$, while it includes the prices of food, leisure, and manufactured goods in the commodity demand functions, $c_i=c_i(\boldsymbol{p}, Y, \boldsymbol{z})$. For simplicity, we denote the exogenous market prices by p_i in place of $\overline{p_i}$ unlike de Janvry et al.

4) Note that changes in the internal prices have no income effects in evaluating α_{ij} because $q_j+T_j-c_j=0$ or $s_{j\pi}s_{\pi Y}+s_{jY}-s_j=0(j=f, l)$ holds in the Slutsky-type equation (2).

5) For example, to evaluate the own-price elasticity $\partial \ln c_f/\partial \ln p_f$ of the demand for food, we can use the values $\theta_{ff}=-0.80$, $s_{f\pi}s_{\pi Y}=p_f q_f/Y=0.426$, and $\eta_f=0.70$ in Table 1 of de Janvry et al. as well as the definition $T_f=0$. Then we obtain $\partial \ln c_f/\partial \ln p_f=\theta_{ff}+s_{f\pi}s_{\pi Y}\eta_f=-0.80+0.426\times 0.70=-0.50$.

6) Note that $c_l/q_l=(p_l c_l/Y)/(p_l q_l/Y)$ and that $p_l c_l/Y$ and $p_l q_l/Y$ are evaluated at 0.383 and -0.255, respectively, in Table 1 of de Janvry et al.

7) The own-price elasticity of labor supply is obtained by using the relation :

$$\{\partial(T_l - c_l) / \partial p_l\} \{p_l / (T_l - c_l)\} \\ = - (\partial c_l / \partial p_l)(p_l / q_l) \\ = - (\partial \ln c_l / \partial \ln p_l)(c_l / q_l)$$

8) In the fourth section, we will also examine the case where food and leisure are complements.

9) $|x+y|=|x|+|y|$ if x and y have the same sign.

10) For Cobb-Douglas technology $q_f=A(-q_l)^{\alpha}(-q_x)^{\beta}(-q_k)^{\gamma}$ (q_l and q_x : variable inputs ; q_k : fixed input ; A : constant), it is shown that $\partial \ln q_l/\partial \ln p_l=-(1-\beta)/(1-\alpha-\beta)$, which should be less than -1 under conditions $\alpha>0$, $\beta>0$ and $\alpha+\beta<1$. In addition, since $\partial \ln q_l/\partial \ln p_l=(\beta-1)(\partial \ln q_l/\partial \ln p_f)$, the elasticities $\partial \ln q_l/\partial \ln p_l$ and $\partial \ln q_l/\partial \ln p_f$ take a close absolute value for small values of β.

11) de Janvry et al. do not make explicit assumptions about the price elasticity h_{ij} of compensated demand but do for the price elasticity θ_{ij} of uncompensated demand excluding profit and endowment effects. However, these two elasticities vary by the same amount if expenditure share s_j and income elasticity η_i are fixed in the relation : $h_{ij}=\theta_{ij}+s_j\eta_i$. Hence, we can vary h_{fl} from 0.20 to -0.20 by varying θ_{fl} from -0.07 (assumed by de Janvry et al.) to -0.47. We do not decrease the elasticity h_{fl} below -0.20 to maintain conditions $h_{ii}<0$ for $i=f, l$.

12) This is partly because de Janvry et al. do not report parameter values of the profit and indirect utility functions in their paper.

13) If we vary cross-price elasticity h_{fm} in place of own-price elasticity h_{ff}, we need to control homogeneity and symmetry conditions across the three commodities, which is much more complicated.

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