

CAN HYSTERESIS AND REAL OPTIONS EXPLAIN THE FARMLAND VALUATION PUZZLE?

by

Calum Turvey

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**Department of Agricultural Economics and Business
University of Guelph
Guelph, Ontario**

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Abstract

This paper proposes that the common finding that land prices are systematically higher than their fundamental value as measured by the present value of future cash might be due to real options arising from uncertainty in cash flows. The paper posits a model in which the seller has a real option to postpone the sale of land. Because the value of land is measured as a present value, the buyer does not hold a similar option to postpone the purchase. It is argued that the seller's option offers a plausible explanation for the wedge between observed farmland prices and the present value model. The paper uses a Dixit and Pindyck (1996) real options framework. Using historical cash flow and land price information for Ontario, it is shown how real options can lead to a land price greater than that predicted by the present value model. The findings also suggest the existence of land price bubbles and shows how a real options framework can be used to detect bubbles.

Keywords: farmland prices, hysteresis, real options, speculative bubbles.

Introduction

Despite the prominence of the present value criteria in determining land values, researchers have been unable to show a time series correspondence between cash flow from agriculture and the market value of farmland, (Featherstone and Baker, 1987; Falk, 1991; Falk and Lee 1998; Clark, Fulton and Scott, 1993; Hanson and Myers, 1995; Weersink et al., 1999). The recent debate between Roche (2001) and Falk, Lee and Susmes (2001) over whether departures of land prices from their fundamental values is caused by bubbles or fads illustrates the variety of explanations applied to, and the economic significance of, the farmland price problem. The notion that land prices are not correlated with fundamental economic information is problematic when the present value model is used in economic or applied contexts. The conventional approach to pricing farmland is to estimate current and future cash flows and discount those cash flows using an appropriate, risk adjusted, discount rate to its present value (e.g. Baker, Ketchabaw and Turvey, 1991). If the NPV rule is not sufficiently robust to explain the most rudimentary of agricultural investments, then there is a need to explore a more general theory of asset valuation.

One of the key elements largely ignored in the land capitalization formula is the impact of uncertainty on land prices. How the sellers and buyers of farmland deal with uncertainty plays an important role in the market pricing of farm assets. For example Turvey, Baker, and Weersink (1992) use a portfolio choice model to show how the rental rate of farmland will increase as the expected marginal value product of farmland increases with uncertainty. Featherstone and Baker (1988) have shown that land values increase with rental rates. If rental rates increase with uncertainty, and farmland values increase with rental rates, then it follows that farmland prices increase with uncertainty. However, neither the portfolio selection model or the econometric land price models adequately explain the farmland price puzzle caused by a persistent wedge between what is believed to be economically rational by the present value model, and actual farmland prices.

New models of capital investment based on the theory of real options have been able to explain many peculiar anomalies in investment theory. A real option has a broad definition as summarized in Trigeorgis (1993) and Amran and Kulatilaka (1999). In general a real option represents the value to a firm of having the flexibility to accept, reject, or postpone new investment opportunities. As new information arrives, uncertainty about future cash flows

gradually resolve and management may have significant flexibility to alter its strategy in order to capitalize on improved upside potential while limiting or mitigating the downside relative to the manager's initial expectations (Trigeorgis, 1993). This flexibility is represented by (put or call) options imbedded in the investment opportunity with an underlying asset being the gross project values of expected operating cash flows (Trigeorgis, 1993). The theory of real options can show how uncertainty explains the acceptance of projects that have a negative net present value or the rejection of projects with a positive net present value (Dixit and Pindyck, 1996).

A special case of the real options framework includes a behavioural characteristic called hysteresis. While hysteresis has several economic interpretations (Katzner, 1999) it mostly relates to economic conditions in which the past is related to the present in a number of ways. Persistent or ergodic price structures for example can relate past prices to the present and onto the future. The belief that a reduction in the price of a traded security will be followed eventually by a rise in its value is an example of a hysteresis phenomenon that would cause resistance to selling immediately. Bubbles and fads that result in extra fundamental valuations result from an extreme form of hysteresis where rational beliefs about future outcomes become a self fulfilling prophecy. The importance of hysteresis in a real options framework is that it creates a zone of inactivity. For example models by Dixit (1989), Krugman (1989) and Martzoukas (2001) explore models in which the presence of sunk costs brought about by irreversible investments, in the presence of large exchange rate fluctuations, can affect firms' decisions about exit and entry into foreign markets that are not reversed when the exchange rate returns to its previous level. Ansic and Pugh (1999) note that a firm's exit decision from a market is determined not only by its current trading position, but also by the expected value of remaining in the market. While in the market, exiting means that expected future profits cannot fall below zero, but by doing so the firm also forgoes any opportunity to increase future firm value. In an industry influenced by exchange rate uncertainty this means that the higher the volatility in exchange rates the higher is the option to remain in the industry

This paper presents an economic framework in which owners of farmland, having already made a sunk cost investment for the perpetual rights to cash flow from the land and other property rights, own an option on future capital gains relative to current fundamental values. Like the Dixit (1989), Krugman (1989), Martzoukas (2001) and Ansic and Pugh (1999) trade models, the failure of farmers to sell land at the current present value of cash flows is determined

by the opportunity costs associated with future capital gains. Hysteresis thus creates a wedge between the present value of current cash flows and the possibility of increased farmland values in the future. This put option provides the owner with the right, but not the obligation to sell the farmland at some future date when operating cash flows reach or exceed a specific value. If these options exist, then the present value model usually applied to valuing farmland will be incorrectly specified since it ignores this option value. The true value of farmland under a real options framework is the fundamental (present) value of the land based on currently observed operating cash flows, including expected growth, plus an option on future capital gains above expectations. The farmland price puzzle can plausibly be explained by an agricultural economy in which the buyers of farmland have to purchase all, or part of, the seller's option in order to induce a sale. This is the proposition discussed in this paper.

The purpose of this paper is to explore the proposition that the market value of land includes real options on future growth and capital gains. If uncertainty is a significant component of land value then the traditional present value capitalization formula is misspecified. Using a Dixit and Pindyck (1996) framework, the real options valuation of farmland is examined using agricultural data for Ontario. The present value bid price of land and its real option value is calculated and then compared to observed land values. The results provide some support for a new theory of real asset pricing, one that can explain observed deviation of market farmland values from the present value model, including the possibility of bubbles.

Background

The application of real options theory to farmland investments has previously been discussed. For example, Cappozza and Helsley (1990) and Cappozza and Sick (1994) cast the urban-rural land price relationship in a real options framework. In their models, farmland prices increase with the real option to convert farmland to urban uses. Risk is determined by urban rents, and the real option value diminishes as the distance to the urban centers increase. Titman (1985) and Quigg (1993) provide examples for pricing options to develop undeveloped land by setting development costs as the strike price and rents, or sale, as the state variable. Bailey (1991) examines how shares of rubber and palm oil companies in Malaysia are valued when production can be temporarily suspended when marginal costs exceed marginal revenues. Under shut-down provisions the present value of the firm without generating any positive cash flow is

zero, yet the option to resume production at some future date when uncertainty drives prices above marginal costs results in a positive share value. In a similar context, Dixit (1992) notes that in the early 1980's there must have been many farm families with asset returns below their marginal cost of labour and other costs, yet these farms did not immediately sell the land and exit farming, but instead kept the farm alive on the chance that future cash flow increased. This is the basis of the hysteresis argument used in this paper. Under the conventional present value rule negative cashflows will result in an asset with no value, yet in agriculture we do not observe zero-valued land assets. Even land taken out of production because of low productivity will be put into production if prices rise to some trigger level. One can view marginal costs as the strike price on an option to produce agricultural commodities. When prices fall below marginal costs, production is abandoned. But there is always the possibility that price will rise at some future date so the option to produce has value. With this option in place land has value in excess of its present value, which is why we do not observe landowners accepting zero-valued bids for farmland, even when that land generates no cash flow. Likewise, when prices are above marginal costs and productive land has a positive present value we still do not observe land being sold at its present value bid price, even with growth expectations included.

The variety of explanations available for the discrepancies take the present value model as given and solve for possible external influences. Fads and bubbles result from econometric models, which include conventional capitalization as part of the econometric and then attribute sustained deviations from the model as fads or bubbles (Featherstone and Baker 1987, Falk, Lee and Sumses 2001, Roche 2001). Models that attribute government programs examine the incremental cashflow from stabilization policies and the possibility of a reduced risk premium in the discount rate as exogenous and endogenous solutions to the capitalization model (Featherstone and Baker, 1988, Weersink et al 1999). Other models that examine time varying discount rates essentially consider the problem as one of the present value model being correct and solving the discount rate as the internal rate of return on the market value given cash flow expectations (Hanson and Meyers 1995, Weersink et al 1999).

None of the many studies on farmland values has explicitly considered uncertainty as a source of the discrepancy between observed land values and the land capitalization model. Yet when uncertainty is included in the present value framework in the form of real options, it is found that the conventional present value model fails to represent the full value of the asset or

investment. Bailey's (1991) result is a case in point. The failure of idle land to sell for zero dollars is another. And in a similar context, the observation that some Ontario dairy farmers have paid excessive premiums for farmland on the expectation that at some future date, environmental restrictions may tie herd size to their land base suggests that the premium paid by these farmers can be interpreted as the value of a real option to produce milk at some future date. And while not considered explicitly in the various studies on farmland values, a finding of fads or bubbles is entirely consistent with Dixit's (1992) contention that bubbles arise from an extreme form of real options arising from hysteresis. In fact, Roche (2001) states that a bubble arises when the anticipation of rising prices includes more market participants in pursuit of short-term capital gains. If a fad or bubble is viewed as a form of hysteresis then the fervor can only arise if the owners of land postpone sales so that demand exceeds supply. Furthermore since the conventional land capitalization model includes growth expectations, then surely fads and bubbles can arise only from speculation that actual growth will exceed expected growth. Such speculation can only arise from uncertainty in future outcomes and for buyers and sellers to recognize these outcomes as optimists. Therefore, any model that concludes in favour of fads or bubbles must also admit that hysteresis, uncertainty, and the possibility of extremely good outcomes, is not a trivial component of farmland pricing.

The Farmland Investment Problem

The bid price, or fundamental value of farmland is often based on the following present value structure.

$$(1) \quad V(\pi) = \frac{\pi^*}{r - \alpha}$$

where π^* is a rational expectation of cash flows generated from the land, α is the anticipated growth rate in cash flow, and r is an appropriate discount rate. This model has been tested quite extensively in the agricultural economics literature under the null hypothesis that there is no significant difference between the land capitalization model and the observed market price for farmland, V^* , i.e. $H_0: V(\pi) = V^*$. Rejecting this null hypothesis implies that some other, unknown, economic is driving market values of farmland. The correct, but unknown, model can be specified as

$$V_t^* = V(\pi_t) + F_t,$$

where F_t represents a persistent economic that drives a wedge between the fundamental value of farmland $V(\pi_t)$ and the market value V_t^* at some particular point in time, t . The exact source of this wedge is unclear from the existing literature. Speculative bubbles and fads (Featherstone and Baker, 1987; Falk and Lee 1998), time varying discount rates (Hanson and Myers 1995; Weersink et al., 1999) or government programs (Featherstone and Baker, 1998; Weersink et al., 1999) have all been proposed, but none have provided clear-cut explanations.¹

Real options, may provide a plausible explanation for the farmland price wedge. In essence, this paper proposes that the difference between market values and fundamental values is a problem of the optimal timing of a market transaction given uncertainty about future values of the asset (or the underlying state variable from which asset values are derived). The key proposition of this paper is that owners of land have a positive valued option to postpone the sale of land in the hopes of higher future capital gains. In order to induce a market transaction the buyer must purchase all, or part, of this option from the seller. The option value will be above and beyond the land's fundamental value and, therefore, the proposition offers a plausible explanation for the land price puzzle. The key question asked in this paper is whether uncertainty over future outcomes (cash flow) can cause the owners of capital to delay the sale of capital in the hope of increasing capital gains and real wealth?

To put the timing problem into perspective, reconsider what is being assumed in the classical farmland bid price model defined by equation (1). In order for (1) to be valid it must be assumed that all variables are known with reasonable certainty. As such, the owner of the land has the right to receive the present value benefit of all future cash flows from the land. With this present value known, there is no benefit to postponing the decision since the present value of

¹ In the finance literature an emerging set of research is focussing on what Campbell (2000) refers to as the equity puzzle. The equity puzzle refers to persistent departures of market returns on stock from fundamental value. Heaton and Lucas (1999) attempt to reconcile the departure by calculating the implied returns or growth rates that would match fundamental value to market value. Campbell (2000) and Sundaresan (2000) review the equity puzzle in terms of uncertainty with Sundaresan (2000) focusing on dynamic risks. Cecchetti et al. (1998) built an economic model based on subjective and fluctuating beliefs about growth and use this to explain economic expansions and contractions. Hong and Stein (1999) note that the failure of asset pricing models to explain short run departures and long run reversions suggest that behavioural dynamics can also influence asset values. Shiller's (2000) book entitled "Irrational Exuberance" provides a similar explanation. These models suggest a type of hysteresis fundamental in which departures from value are perpetuated by reinforced herd mentality. The limit of this behaviour is a speculative bubble.

selling now or one year from now is the same in present value terms.² Likewise, if the buyer can invest the amount $V(\pi)$ at the rate r , then he too will be indifferent towards buying now or later.

When future cash flows are uncertain the present value model no longer holds, at least in terms of the timing of the sale. For example, if cash flows evolve randomly over time according to the Brownian motion,

$$(3) \quad d\pi = \pi[\alpha dt + \sigma dZ]$$

where π is current cash flow, α is the expected natural growth rate in cashflows per unit of time, and σ is the volatility of cash flow as measured by the standard deviation of its percentage change, and Z is a standard wiener process, then by Ito's lemma the value of farmland will evolve stochastically according to

$$(4) \quad dV(\pi) = \frac{\pi}{r - \alpha} [\alpha dt + \sigma dZ] \\ = V(\pi) [\alpha dt + \sigma dZ]$$

For convenience we will denote $V(\pi)$ and $dV(\pi)$ for the present value of cash flow and the change in this present value given a change in cash flow expectations (and other variables)³. Likewise, I and dI represent the value of farmland and the change in the value of farmland given an expectation and change in expectation of π . We can write the land price dynamic as

$$dI = I [\alpha_1 dt + \sigma_1 dZ_1].$$

If $dI = 0$ then the value of the land is fixed even though $dV(\pi) \neq 0$. When I is a present value then $\alpha_1 = \alpha$, $\sigma_1 = \sigma$, $dZ_1 = dZ$ and $dI = dV$ and the change in the value of the assets is perfectly correlated with the change in the present value of cash flow. The identity $dI=dV$ is the standard assumption of the present value model and the condition being evaluated in this paper. We also leave open, for the purpose of discussion, the possibility that $dV(\pi)$ and dI follow correlated Brownian motions, but not exact present values. Then $dI \neq dV(\pi) \neq 0$.

² To see this suppose that the sale is postponed one year. In that year π is received in cash flow and the value of land 1 year hence is $\pi(1+g)/(r-g)$. The present value is $PV = (\pi(r-g) + \pi(1+g)/(r-g) (1+r) = \pi(1+r)/(r-g) (1+r) = \pi/(r-g)$ which equals equation 1. Postponement has no impact on real wealth.

³ $dV(\pi) = dV(\pi, t+1) - dV(\pi, t)$ so using the expected value of (4) $V(\pi, t+1) = V(\pi, t)(1+\alpha) = \pi(1+\alpha)/(r-\alpha)$ which is common form of the bid price growth formula (e.g Baker et al 1991) when π is an observation rather than an expectation as defined in equation (1).

The Emergence of Real Options on Farmland

In this paper, the operating hypothesis is that the owners of capital own an option to postpone the sale of the asset under conditions of risk. This option is similar to an American put option, which provides the owner with the right, without obligation, to sell the land at some (unspecified) future date⁴. The option value arises from a number of economic conditions (Dixit and Pindyck, 1976; Able et al., 1998; McDonald and Seigal, 1988; Trigeorgis, 1993; Amran and Kulatilaka, 1999). First, the landowner has already made a sunk-cost investment in the land and this entitles the owner to all future capital gains. Second, the decision to sell land is irreversible in that it cannot costlessly be reversed. Once the land is sold, all rights to unanticipated future capital gains are forgone. Third, the decision to defer the sale is reversible, which means that any (capital gain) losses that might arise from misjudging future cash flows and probabilities can be mitigated in an economically significant way, and fourth, the future sale of land will occur only if incremental capital gains exceed zero by a certain amount.

⁴ The subject of this paper and the forgoing model is on the option value of delaying a sale in order to achieve a higher possible capital gain when cash flows are uncertain. The option on the capital gain component is actually a lower bound on the option value to postpone the sale since the intervening rents, which accrue to the land owner, are not explicitly considered. To see that the option on the gain is a lower bound assume that a perfect market exists that allows for a replicating portfolio (See Dixit and Pindyck (1996) or Copeland and Antikarov (2001)) of marketable securities that matches the cash flow and capital value of the underlying land market. For simplicity, assume current cash flow is \$100/acre, discounted at 10%. With zero drift the instantaneous present value bid price is \$1,000/acre. Over a one period time step cash flows can increase to \$150 or fall to \$50 and at each state the instantaneous present value of land would be \$1,500 and \$500 respectively. In the good state the owner receives \$150 in cash and a capital gain of \$500 for a total benefit of \$650, and in the bad state faces a capital loss of \$500 and \$50 cash flow for a net benefit of -\$450. To calculate the option to postpone construct a replicating portfolio with a payoff of MAX(500,0) when only capital gains are considered and MAX(650,0) when cash flow and capital gains are considered. With x representing the position in the replicating risky asset and B representing the amount of risk free bonds earning a rate of 5% the portfolios are constructed as follows; for the option on the capital gain

- a) $X \cdot 1500 + B(1.05) = 500$ in the good state and
b) $X \cdot 500 + B(1.05) = 0$ in the bad state.

And for the option on the cash flow plus capital gain

- c) $X \cdot 1500 + B(1.05) = 650$ in the good state and
d) $X \cdot 500 + B(1.05) = 0$ in the bad state.

Solving a) and b) simultaneously gives $X = .5$ and $B = -238.10$ which means that a replicating portfolio with a payoff schedule of Max [500, 0] occurs if the farmer borrows -238.10 at the risk free rate and buys .5 units of the spanning security. The present value of this portfolio will simply be $X \cdot 1000 + B = .5 \times 1000 - 128.10 = \261.90 . Simply put this means that it is worth \$261.90 to wait one period and sell the land at that time. Solving c) and d) in the same way gives $x = .65$ and $B = -309.52$ which says to borrow more and buy more of the spanning asset. The option value is then $x \cdot 1000 + B = .65 \cdot 1000 - 309.52 = 340.48$. Because of the intervening cash flow the option to postpone is higher than the option on the capital gain, with the latter being equivalent to selling idle or vacant land for its best alternative use. The implication of this result suggests that the option to postponing the sale as presented in this paper may in fact understate the true real option value.

In the short run, the option to postpone does not guarantee a capital gain and decreases in land values may be observed. In fact, the risk of capital losses is not totally eliminated until the land is sold. However, under the third condition there exists, with known probability, a level of cash flow, π^* , that optimizes the capital gain. Under the real options hypothesis land owners will delay the sale of land until π^* occurs, even if in the short run, π and hence $V(\pi)$, declines. Dixit's (1992) hysteresis argument explains this behaviour⁵. The value of the option to postpone the sale of land until π^* occurs is defined by $F(\pi^*)$. The problem stipulates only at what level of cash flow (given current levels) it is best to sell at. It does not determine when either π^* or the sale will occur. In this context the real option is similar to a perpetual option as described by Merton (1973). Later we derive and calculate formulas that determine π^* and $F(\pi^*)$.

Let $I=V(\pi)$ define the current land value evaluated at current cash flow π , and $I(\pi^*)$ the present value of land evaluated at π^* . The capital gain when π^* occurs is then $(I(\pi^*) - I) > 0$. The decision faced by the landowner is to sell the land immediately, an irreversible decision, netting zero additional capital gains, or postpone the sale until π^* occurs and sell at that time. The value of the option is then determined by the boundary condition $F(\pi^*) = \text{Max}[0, I(\pi^*) - I]$.

The Option to Buy Farmland

To this point we have ignored the buyer's position. In much of the literature on real options the position is taken that the buyer-investor can postpone a capital purchase when future outcomes are uncertain. This is equivalent to an American call option on the real asset (the right to purchase). This literature requires the same conditions of irreversibility, or at least costly reversibility (Abel et al. 1996). For example, the investment in a plant is irreversible in that it cannot be used for any other purpose. If cash flows on the project decrease then the investment will have lost value. Hence, by delaying the investment, future cash flow uncertainties and ambiguities can be somewhat resolved (McDonald and Siegel, 1988). If the market becomes too competitive then the postponement decision can be reversed and the investment made immediately (Dixit and Pindyck, 1996). The risk to the buyer is that by postponing the investment, interim cash flow will have been lost.

⁵ The hysteresis argument has been used in other agricultural contexts. Richards and Patterson (1998) use the hysteresis argument to explain labour movement between urban and rural economies.

The boundary condition for the real option to buy an asset is isomorphic to an option to sell (McDonald and Seigel, 1988; Trigeorgis, 1993), that is $F(\pi^*) = \text{Max}[0, V(\pi^*) - I]$.

For the call option the investment is made when the net present value equals $V(\pi^*) - I$, or it is not made at all. For the put option, the sale is made when land prices exceed current price by $V(\pi^*) - I$, or it is not sold.

The problem with the call option is that it assumes first that the buyer acts as a monopolist and, second, that the investment can be postponed indefinitely while I is held constant (Dixit and Pindyck, 1996). That is, in reference to equation (4), $dI=0$ while $dV>0$. If I is a present value, as is the case of capital assets like farmland, then I will not remain constant at all⁶. In fact, when π^* occurs $I(\pi^*) = V(\pi^*)$ and $dI(\pi^*) = dV(\pi^*)$. The call option value is based on

$$F(\pi^*) = \text{Max}[0, V(\pi^*) - I(\pi^*)] = \text{Max}[0, V(\pi^*) - V(\pi^*)] = 0.$$

Hence, when the fundamental value of an asset is equal to the present value of future cash flows, given the prevailing economic conditions, there is no (or little) option value to waiting.⁷ In contrast, the value of the seller's put option is measured relative to $F(\pi^*) = \text{Max}[0, V(\pi^*) - I]$ where I represents the current value of farmland (or a fixed point in time). The economic result suggests that in farmland markets, the owners of land hold a valuable option to postpone the sale while buyers hold no such option to postpone the purchase.⁸

Real Options and the Market Value of farmland

The existence of the put option suggests that the value of the land to the owner is not simply $V(\pi)$, but $V(\pi)$ plus the option to future (unanticipated) capital gains, i.e. $V^* = V(\pi) + F(\pi^*)$, while the value to the buyer is $V^* = V(\pi)$. If both the buyer and the seller have symmetric

⁶ Again, it is important to distinguish between the real option discussed here and a financial option such as Black and Scholes (1973). If the buyer and the seller were to agree to a contract whereby the buyer could postpone the purchase of the farmland until some future date then the value of that option would be priced using formula similar (but not exactly the same as) a conventional European call option on non-dividend paying stock. If at some future date, T , the market value exceeds the current value, I , then the buyer can buy the land for I , otherwise the buyer can (without obligation) buy the land at a lower price. Using a Black-Scholes type model to price real options over a defined time horizon is discussed by Amran and Kulatilaka (1999) and further notes to this paper.

⁷ In McDonald and Seigel (1988) and Dixit and Pindyck (1996) an intermediate option will exist if the correlated Brownian motion defined by dI and $dV(\pi)$ are not perfectly correlated. McDonald and Seigel (1988) define the boundary condition $\text{Max}[0, V/I - C^*]$. When V and I are both Brownian motion the decision is to invest when the ratio V/I exceeds a critical value C^* . If $I=V$ then clearly C^* can only equal 1, and the decision is to invest immediately.

⁸ Other conditions could result in a similar conclusion. For example, property rights provide a natural monopoly to landowners.

information and can agree upon $V(\pi)$ then negotiating a land transaction involves only $F(\pi^*)$. What is not known is how negotiations will evolve and what proportion of the option must the buyer purchase in order to induce a sale.

The farmland transaction can (as in Lambrecht (2001)) be viewed in terms of a Stackelberg leader-follower game with symmetric information. In this game the seller, being the leader, will only agree to a sale if he receives a bid premium. The purchaser is the follower, and the first pass bid will equal V_t , the current fundamental (present) value of the land. The leader (as a monopolist) sets $V_t^* = V_t + F(\pi)$, where $F(\pi)$ is the current value of $F(\pi^*) = \text{Max}[0, E[V_{t+n} - V_t]] > 0$, as the initial offer⁹. The bid-ask spread after the initial pass through is equal to the option value. Since neither V_t^* or V_t are acceptable equilibrium solutions, a second best solution must be sought. The second-best solution emerges from a sequential solution of each counterparty's reaction function. The seller will internalize the buyer's requirement for an immediate sale at some price $V_t < v_t < V_t^*$ while the buyer internalizes the seller's requirements for some price $V_t > v_t \geq V_t^*$; that is the second best equilibrium will result for some λ ($0 < \lambda < 1$), such that the seller receives a bid premium of $\lambda F(V)$. A value of $\lambda = .5$ would represent a Nash equilibrium while a value $\lambda > .5$ would represent a greater degree of market power by the seller (the land demand curve may be highly inelastic) and $\lambda < .5$ would represent a greater degree of market power by the buyer (the land demand curve may be highly elastic). With the bid premium $\lambda F(V)$, the transacted value of land will be

$$v_t = V_t + \lambda F(V_t).$$

The reduced bid premium $\lambda F(V_t)$ represents the certainty equivalent value of receiving v_t immediately and with certainty, rather than V_t^* with uncertainty. From the buyer's perspective, the seller has not fully extracted all of the option value. The buyer pays v_t for an asset with an expected worth of V_t^* . The difference $(1 - \lambda) F(V)$ represents the expected value of the capital gain above v_t that accrues to the buyer.

Why the option that leads to such a negotiation or equilibrium exists for farmland is of course debatable. If commodity prices are low relative to history then farmers might see an increase as being imminent, and when commodity prices are rising farmers may examine the

⁹ If $F(\pi^*)$ is the option value to waiting for π^* to occur, then $F(\pi) = F(\pi|\pi^*) < F(\pi^*)$ represents the opportunity costs of selling immediately at π . If $\pi \geq \pi^*$, then $F(\pi) \geq F(\pi^*)$ and the land will be sold immediately at its present value bid price.

economic conditions that led to the increase and conclude that the demand forces are more than transient and thus optimistically conclude that prices will continue to rise. In terms of stock market behaviour, Shiller (2000) argues that factors such as the internet, baby-boom demand, and herd mentality all contributed to the recent rise of the DOW index. Likewise in agriculture, new cost reducing technologies, free trade agreements, biotechnologies and pharmaceuticals, unfound nutritional values and future conversion of lands outside of agriculture are all possibilities that can give rise to option values. In the 1970's and 1980's the common belief that land provided a hedge against inflation gave rise to an option on the real purchasing power of the dollar, and this option value got built into the price of land. One cannot discount the role that government programs might play either. For example when prices fall, stabilization or even disaster relief are available to limit the downside risk so that there will always be a floor to the price of land, leaving the upside to roam with the markets¹⁰.

Market Prices, Bid Prices and Real Options Values

This section develops the real options pricing model along the lines of Dixit and Pindyck (1996). (It is assumed throughout that $\lambda=1$.) In their Chapter 6 they provide a solution based on market arbitrage and contingent claims analysis as well as a solution using dynamic programming. The dynamic programming approach is developed in this paper. There are two steps involved in the solution to the real options valuation. The first step is to determine a formula that describes the value of land as a function of the stochastic variables. The value of land is denoted by $V(\pi)$ where π , defined as cash flow, is the stochastic variable that determines

¹⁰ It has also been argued in the finance literature that market signaling can give rise to real options in a number of different ways. For example, Leland and Pyle (1977) develop a model whereby movement by one party signals a forthcoming benefit or cost to the second party. Applied to the farmland problem a potential buyer placing a bid on a parcel of land may signal to the seller an imminent increase in cash flow and land values. Such a signal may cause the land owner to postpone the sale giving rise to the real option value. Likewise, a tender offer for the shares of a corporation will signal to the owners a higher future, perhaps synergistic value, i.e. with a tender offer in place, the shareholder has the option to postpone acceptance until the true value to the acquirer is revealed. Fishman (1989) argues that an all cash offer signals a high value for the target and this may deter other bidders. Myers and Mijluf (1984), Hansen (1987) and Martin (1996) argue that if a firm is perceived to be undervalued then the tender offer is made on a cash basis, while a perceived overvaluation results in an equity based tender offer. When information is asymmetric such signaling can result in the emergence of real options (Lambrecht, 2001). As a hypothesis, cash offers would signal larger capital gains in the future so that the shareholders of the target firm have a higher valued option to postpone the sale. An equity deal on the other hand would signal a lower chance of future capital gains so shareholders would likely accept the tender offer sooner than later. The models by Lambrecht (2001) are based on symmetric information. In this paper it is argued that the timing and terms of mergers, stock offers and cash offers evolve from a real options framework.

value. Once the basic land price formula is determined, the second step determines the real option value. It is assumed that cash flows evolve over time according to equation (3) and by Ito's lemma the value of farmland evolves according to equation (4).

The Dixit-Pindyck solution is based on the notion that the real option value fluctuates with time and risk. Given that $V(\pi)$ describes the value of land given π , α and r , the option value $F(\pi,t)$ is given by the following Bellman equation.

$$(5) \quad F(\pi,t) = E[F(\pi,t) + d F(\pi,t)]e^{-rdt}$$

Note that Equation (5) includes the time designate variable t in the option value $F(\pi,t)$, whereas the original Dixit-Pindyck model does not. I will derive the general partial differential equation for option prices with t included. (This will be used later on to price real options on finite lived investments using a Black- Scholes option pricing model). In (5) the current options price is given by the expected value of the option price plus the change in its value over time. This is then discounted to the present. Applying Ito's lemma to (5), using $d\pi$, and using the fact that $(1-rdt)$ is equivalent to e^{-rdt} ,

$$F(\pi) = F(\pi) + [1/2 \sigma^2 \pi^2 F'' + \alpha \pi F' + F'_t] (1-r) F dt,$$

or

$$(6) \quad 1/2 \sigma^2 \pi^2 F'' + \alpha \pi F' + F'_t - r F = 0.$$

Setting $F'_t = 0$ in Equation (6) provides the stochastic differential equation used to solve for the real option price $F(\pi)$. (Note that $F(\pi) \neq F(\pi,t)$.)¹¹ To obtain this solution we add three boundary conditions. These are

¹¹ Note that Equation (6) with $F'_t \neq 0$ is very similar to the partial differential equation in Black and Scholes (1973). The solution to the call option price is

$$C(\pi,t) = \pi e^{(a-r)T} N(d_1) - X e^{-rT} N(d_2) \text{ Where } d_1 = \frac{1}{\sigma \sqrt{T}} \left[\ln \left[\frac{\pi}{X} \right] + (\alpha + 1/2 \sigma^2) T \right] \text{ and } d_2 = d_1 - \sigma \sqrt{T}$$

This option is priced relative to the diffusion of cash flow not land. Since cash flow (or land for that matter) is not a traded asset, a strong assumption must be made about the (potential) existence of a spanning portfolio of traded assets that is perfectly correlated with the volatility of cash flow, and that can be used to create a riskless hedge in the cash position. If this assumption is acceptable then $\alpha=0$, r equals the risk free rate of return, and $C(\pi,t)$ is the Black-Scholes model. If a spanning asset does not exist then a risk free hedge is impossible (Dixit and Pindyck, 1996; Stokes et al., 1997) and the option pricing model will have to include the market price of risk as in Gorman (1976) and Cox, Ingersoll and Ross (1985). In this case one can still set r equal to the risk-free rate, but the term α

$$(7) \quad F(0) = 0$$

$$(8) \quad F(\pi^*) = V(\pi^*) - V(\pi)$$

and

$$(9) \quad F'(\pi^*) = V'(\pi^*).$$

Condition (7) says that if cash flows are zero the option will be zero. Condition (8) is a value matching condition. It says that at some level of cash flow π^* , the value of the option must equal

the present value of the investment. Also in (8), $V(\pi) = I = \frac{\pi}{r - \alpha}$ which is the current present

value of expected cash flows. Since V is the current reference point, the right hand side of equation 8 gives the capital gain when the π^* trigger is reached. Condition (9) is the smooth pasting condition. It says that the optimal time to sell occurs for some π^* such that the incremental gain in options value exactly equals the incremental gain in net present value. The smooth pasting condition ensures that at some point the option value will become tangent to the options payoff curve (as in Figure 1).

Appendix I shows that the solution to (6) is

$$(10) \quad F(\pi^*) = A\pi^{*\beta}$$

with β given by A3 and A given by A5. The number given by π^* , and therefore the values of $F(\pi)$ and $F(\pi^*)$ depend upon initial conditions. These initial conditions are represented or captured by A . This means that the value of $\pi^*_{t-1} \neq \pi^*_t \neq \pi^*_{t+1}$ if $\pi_{t-1} \neq \pi_t \neq \pi_{t+1}$. This implies that the real option value is continually changing value over time since the initial bid price changes with π . If π_t is high but then decreases then subsequent calculations of π^* and $F(\pi^*)$ will be lower, and if π increases in the next period then the values for π^* and $F(\pi^*)$ will increase. It follows that the current value of the option to wait, $F(\pi)$, will also be changing over time. Keep in mind that $F(\pi) \neq F(\pi^*)$. $F(\pi)$ is the value of the option to wait when current cash flow

would become $\alpha - \theta\sigma$, where $\theta\sigma$ is the market price of risk. Finally, while it is possible to write a real option on cash flow the most likely real option would be on land itself. For example, suppose an investor wanted to buy farmland 3 years hence but wanted protection against land price increases. The current price of land is V_0 so $X = V_0$ is the strike price. If in 3 years $V_T > V_0$ then the investor will buy the land at the prevailing price V_T but will receive from the call option a payoff of $V_T - V_0$. The total payoff is $-V_0 = (-V_T + (V_T - V_0))$. If $V_T \leq V_0$ then the option expires without value and the investor pays the lower price for the land. In essence the option on the real asset with a payoff $E[\text{Max}[0, V_T - V_0]]$ results in an investment payoff of $E[\text{Min}[V_T, V_0]]$.

expectations are π . That is, it is the current value of an option to postpone the sale until π rises to π^* . $F(\pi^*)$, in contrast, is the intrinsic value of the option at π^* .

The calculus of the option values (Dixit and Pindyck 1996) reveals that the option value will increase as risk increases, growth rates increase, or costs of capital decrease. As risk increases (due to commodity price volatility for example), the optimal trigger, π^* , will be higher which in turn implies that the sale will be postponed even longer. Likewise an increase in the expected growth rate in cash flow from farming or a decrease in the discount rate will increase the value of the option.

An Illustrative Example of Real Option Values

How uncertainty creates the option value, $F(\pi)$, will be discussed in relation to Figure 1. In Figure 1 there are 2 curved lines $0W_1$ and $0W_2$ and a hatched line. The hatched line represents the net present value (NPV) of the farmland investment for a given rate of discount and cash flows as indicated on the x axis. Suppose that current cash flows from 1 acre of land is \$100/acre and the market is fair in the sense that the maximum bid price will simply be the perpetual present value of the cash flows with zero growth. For cash flows of \$100/acre the NPV will equal zero ($NPV = \pi/r - V = 0$). From this position, cash flows in the future can increase or decrease. In Figure 1 the range of possible outcomes is 0 to \$220/acre. Using a discount rate of 6% and zero growth ($\alpha = 0$) the bid price for land is $\$1,666 = \$100/.06$. If cash flow falls to zero the land is worthless and the value of the investment will be lost. This is why the present value line intersects the Y-axis at $-\$1,666$. On the up-side the value of land could increase to $\$3,666$ if cash flow increased to $\$220/acre$. This implies a $\$2,000/acre$ gain over the initial condition.

Suppose that when cash flow is \$100/acre a land owner is offered the maximum bid price of \$1,666. The farmer must decide on whether the offer price is fair enough to induce a sale. Assume that variance in the annual percentage change in cash flow, $\sigma = .1637$. The farmer realizes that by selling immediately he eliminates the downside risk associated with a decrease in cash flow but also recognizes that uncertainty can also cause an increase in the price of land, and that by postponing the sale there is a real possibility of capital gains.

The farmer's decision is comprised of a real option to postpone the sale of the land until some future date or trigger. This real option gives the farmer the right but not the obligation to

sell the land at some price in the future. The curves $0W_1$ and $0W_2$ represent the value of this option under two separate (and mutually exclusive) possibilities. The first option exogenously sets a goal or target. The land will be sold if cash flows equal (or exceed) \$120/acre. At \$120/acre the value of land will increase to \$2,000/acre and the capital gain (the intrinsic value of the option) is equal to \$333/acre.

But what is this option worth when the current conditions are \$100/acre? In Figure 1, at point C, which is read off the option curve $0W_1$ and calculated by setting π^* in equations (A5) and (A6 or 10) equal to \$120. The option value is \$212/acre. The difference between the \$333 and \$212 is due to uncertainty. However, a conventional interpretation applies; if current expected cash flow is \$100 per acre then the expected value of postponing the sale until expected cash flow increases to \$120 is \$212/acre. What this suggests is that the farmland is not simply worth its \$1,666 fundamental value, but the fundamental value plus the value of the option to wait. In other words, the value of the farmland is \$1,878 (\$1,666 + \$212).

Setting a target at \$120 is arbitrary. The curve $0W_1$ intersects the present value line, but the optimality condition from smooth pasting forces the slope of the option value curve to equal the slope of the present value curve. (In Figure 1 twist the option value curve upwards until it is tangent to the present value line.) Using equation (A4) the calculated value of π^* is \$167/acre. At this point of tangency, the value of farmland is \$2,782 with an option value of \$1,116/acre. The value of this put option can be verified by substituting $\pi^* = \$167$ into equation (A5) and then solving (A6) or (10). In this instance the farmer has the right, but not the obligation, to postpone selling the land until expected cash flows increase from \$100/acre to \$167/acre.

If the optimal strategy to maximizing capital gains is to postpone the sale decision until cash flow equals \$167/acre, what is the value of the option when cash flow is \$100? To calculate the current value of the option solve (A5) using π^* . This gives the value for A. Next using this value for A, substitute \$100 for π^* in (A6) or (10). This real option value is \$311 and it corresponds to point d in figure 1 when cash flow is \$100/acre.

The interpretation of this option value is as follows. The landowner has the right to postpone the sale of the land into the future. Given current conditions it is optimal to postpone the sale until cash flow equals \$167/acre. But \$167 occurs with uncertainty, and may not occur at all. Since cash flow of \$167 will occur further off into the future than any value below it, the seller would want to know what the equivalent price of the option is in current dollars. The

option price of \$311 is the current value of receiving a gain of \$1,116 in the future. That is, given current cash flows of \$100/acre, risk of 16.37%, zero growth, and a discount rate of 6% the owner would be willing to give up a future gain of \$1,116 for \$311 current dollars. The value $F(100)=\$311/\text{acre}$ is therefore the opportunity cost of forgoing future capital gains by selling immediately.

Because the target at point b is optimal, the real value of farmland when expected cash flows are \$100/acre is equal to its fundamental value plus its real option value. This equals \$1,977 (1,666 + 311). Essentially this means that the seller would be indifferent to receiving \$1,977 today for the land or postponing the sale under uncertain conditions until expected cash flows approach \$167/acre¹².

Now consider the buyer's position. At \$100 the farmer's maximum bid price is \$1,666 which is also equal to the present value cash flows. It will not be lower because the seller has the same information as the buyer. The buyer's NPV is zero. Because of the option value, the seller does not have to accept the bid but can wait to see if cash flows increase. If cash flow increases to \$120 the present value and bid price will increase to \$2,000. The seller has achieved a \$334 capital gain but the NPV to the buyer is still zero. If the seller holds off until cash flows increase to the optimum cash flow of \$167, the bid price facing the buyer will be \$2,783, the seller will have gained \$1,117 and the NPV facing the buyer is still zero. The key point here is that the buyer is not advantaged by postponing the purchase, since the buyer will always face at most a zero NPV investment. The seller on the other hand has every incentive to postpone the sale. If cash flow falls below \$100/acre in the short run, with hysteresis the seller will delay the sale while waiting for cash flow and land values to increase.

The graphical approach indicates several key observations regarding the value of farmland. The most obvious is that the true value of farmland is not just the present value of cash flows. It must include the option value of future capital gains. In Figure 1 the expected value of these gains is \$311. Under this theory, a bid of \$1,666 using conventional bid pricing rules will understate the lands' true value. That is, while the buyer may bid \$1,666/acre the seller can justify asking \$1,977. The real option value has created a wedge between the bid and ask price. In order for a transaction to take place the buyer will have to purchase (at least part of) the

¹² The optimal target is at point b since any points to the right of point b have no meaningful interpretation (Dixit 1992) since the option to postpone the sale increases at a rate greater than value of the land. Transactions valued to the right of point b are considered to be speculative in nature and suggest the formation of a speculative bubble.

option to future capital gains from the seller. Consequently, the actual value at which farmland will transact will be in excess of the conventional bid price.

Real Options and the Farmland Investment

In the previous section, it was argued that the seller of land owns an option to postpone its sale until cash flows increase. The option to wait has value since by waiting future capital gains may be higher. When the strike value (π^*) is reached the seller will sell the land for $V(\pi^*)$, its exact present value, and the buyer will purchase the land for $V(\pi^*)$. Consequently the buyer has no option to postpone since no matter what the level of cash flow is he will always pay its fundamental bid price value for a zero-NPV investment. It was then argued that since the buyer had no option to wait while the seller did, then in order to induce a sale the buyer would have to purchase part of the option from the seller. Buying the option, or part thereof, implies that observed land prices will always be higher, or at least no lower, than the fundamental value of future cash flows. In this section, this proposition is evaluated using historical land, cash flow, and interest rates for the province of Ontario.

The data are presented in Table 1 for the years 1971 to 1998. The 2nd, 3rd and 4th columns are the nominal per acre value of land and cash income from farming. The prime rate is reported in nominal terms. Since the prime rate represents the best rate to borrowers, 3% was added to account for risk and to ensure a positive expected return to equity¹³.

The 5th column in Table 1 is the deterministic present value $V(\pi)$. The first bid price is calculated for 1975 and the expected cash income, $E[\pi]$, equals a (.4, .3, .2, .1) weighted average of the previous four years (from column 1) with the most recent year having the highest weight. The discount rate is the rate in column 4 plus 3%. The annual growth rate α , equaled the average value of $\ln(\pi_t/\pi_{t-1})$ over the 1971 to 1998 period (4.6%). Likewise, volatility $\sigma = .164$, was computed as the standard deviation of $\ln(\pi_t/\pi_{t-1})$ over the 1971 to 1998 period.

Column 6 calculates the difference between the market value of farmland (column 2) and the fundamental value (column 5). In only 2 years did the market value fall below the fundamental value. On average the market price exceeded the fundamental price by \$618 with a

¹³ The risk premium reflects the risk premium on a low beta stock in the sense of the Capital Asset Pricing Model. A higher risk premium will reduce the fundamental value and increase the option value while a lower risk premium will increase the fundamental value and lower the real option value.

maximum of \$1,329/acre. Column 7 presents the value of the real option at current cash flows, i.e. $F(\pi) = A \pi^B$ from Equation (10) and A is calculated from equation (A5) using π^* . Column 8 is the economic value of land. It is equal to the fundamental value plus the option value $V(\pi) + F(\pi)$ from columns 5 and 7.

The 9th column presents the optimal cash income level, π^* , that would trigger a sale. This is the smooth pasting condition. It is calculated from equation (A4) where the variable $V(\pi)$ was set equal to the prevailing fundamental land value as computed in column 5. In column 10 the ratio of $\pi^*/E[\pi]$ provides a multiple of the current expected cash income to induce the selling of land (optimally).

Column 11 calculates $V(\pi^*)$ the value of land evaluated at π^* , and column 12 gives the real option values $F(\pi^*)$. The option values are calculated from equation (10) but it is easy to see that column 12 is simply the difference between column 11 and column 5. The results, as summarized in Table 1, indicate that the trigger cash level averaged \$192/acre while actual cash averaged \$83/acre. On average, expected cash flow would have to increase by 2.38 times with a minimum of 1.54 and a maximum of 5.38 in order to induce an optimal sale. This result indicates that (relative to fundamental value) there has been a persistent real option on farmland in Ontario. The mean real option value evaluated at π^* was \$1,933/acre with a range from \$202/acre to \$11,223/acre.

Column 13 calculates the ratio $(V(\pi) + F(\pi))/ V(\pi)$ which measures the percentage increase in the fundamental land value due to real options. The results indicate that on average the option value will increase fundamental land prices by about 29%. But there is variability in the ratio which ranged from a low of about 16% to a high of 55%. According to the Stackelberg theory, these values should represent a maximum since the transaction price is hypothesized to be lower than $V(\pi) + F(\pi)$ for $\lambda \neq 0$.

To interpret these results further, the data for 1998 in Table 1 tell the following story. In 1998 the market price of land was \$2,538. Based on expected cash flows of \$89.12/acre ($=.1*76.34+.2*84.41+.3*92.38+.4*92.17$) and a discount rate of 9.6% ($= 6.6\% + 3\%$), and anticipated growth of 4.6%, the fundamental value of land was calculated to be \$1,783. There was a \$754/acre wedge between the land's market and fundamental value. Based on current cash flow expectations of \$89.12 the optimal strategy for the land owner would be to wait until cash

flow increased to \$279.73 before selling the land. The value of this option is \$3,814.29. However, instead of waiting for cash flow to rise, the present value of the option given current cash flow expectations is \$711.83. That is, given that the optimum sell trigger at some future, but unknown date, is \$279.73, the value of the option at current cash flow expectations is \$576.36. In terms of conventional pricing of a perpetual American option, the option value at smooth pasting (\$3,814) can be viewed as the intrinsic value of the option at expiration, while \$711.83 is the current value of the option. This implies that the maximum value of farmland is the current fundamental value of \$1,783.29 plus the current value (or a negotiated portion thereof) of the real option on capital gains, i.e. \$2,495.12 (\$1,783.29 + \$711.83). In 1998 the market value of land exceeded the economic value by \$43 (\$2,538 - \$2,495)¹⁴.

These values are of interest because they indicate the range of possible error in valuation when uncertainty is excluded from the capital budgeting problem. Figure 2 illustrates the relationship between $V(\pi)$, $V(\pi) + F(\pi)$ and actual land values between 1975 and 1998. The graph shows that actual land values are, more often than not, higher than the economic value ($V(\pi) + F(\pi)$). Since $V(\pi) + F(\pi)$ represents a theoretical maximum, any excess in market value above this sum may represent a speculative bubble. The graph indicates the possibility of bubbles between 1978 and 1986 and 1988 and 1991. Over these periods the market price of farmland is well in excess of the real optioned value suggesting the existence of a speculative bubble.

Real Options and Land Price Bubbles

The question now is to make some determination as to the source of the bubbles. To see how the bubble fits in a real options framework return to Figure 1. In Figure 1 a speculative bubble will arise from option prices on the OW_2 curve to the right of point b. Mathematically the interpretation is that a higher value is placed on the option to postpone than is warranted by the value of the investment at any π right of π^* . The higher the value of π the higher the option value, with each incremental increase in expectation of π leading to "successively larger bubbles." The economic interpretation is that the seller has expectations of extra-fundamental capital gains. The seller sees economic value where none exists and the evidence suggests that buyers agree. The belief that in times of inflation land will retain value regardless of cash flow

¹⁴ Not shown in Table 1.

generation, or that no matter what happens government support will always be forthcoming, or demand from non-farm sources driven by non-farm valuations (e.g. commercial or residential construction) are all examples of things that could cause a bubble. The results indicate two extended bubbles. Even if it is assumed that market to value ratios of 1.5 or less are due to short-term shocks (as posited by Falk and Lee, 1998) two distinct bubbles are still identifiable 1979-1984 and 1989-1991.

Of course there are other explanations for the results. The values for a and σ , as long-run variables, were assumed constant in the Table 1 calculations. The assumed volatility for σ was 16.4%. An increase in volatility would increase π^* , $V(\pi^*)$, $F(\pi^*)$ and $V(\pi) + F(\pi)$, but an examination of the data using rolling 5 year volatility measures for the period between 1978 and 1985 was actually lower than 16.4% with an average of about 12%. So volatility cannot explain the 1979-1984 bubble. The average (5 year rolling) volatility for 1989-1991 was about 22% , which does indicate a higher volatility than was used in Table 1. Therefore, it is possible that the 1989-1991 bubble does not really exist because the volatility was underestimated. However, the years 1987 and 1988 both had short run volatility measures of 23% but no bubble.

The second possibility is the assumption of a 4.6% expected long run growth rate. Using short run growth estimates (5 year rolling averages) we find that the 1979-1984 rates are 11.8%, 7.9%, 2.4%, 4.3%, 2.8% and 7.0% respectively. Empirically π^* decreases with increased growth so it is possible that underestimates of growth led to the bubble conclusion at least for 1979, 1980 and 1984. This cannot explain the bubble for the remaining years (1981-1983) so underestimating growth is not a good explanation for the 1979-1984 bubble finding. For the 1989-1991 bubble, growth rates were 2.0%, -1.1%, and 1.7%. Since these are lower than the long run average, their effect would have been to increase π^* shifting the smooth pasting condition to the right, but the curvature of the option valuation function changes through growth's effects on A and β so overall the option value decreases. Therefore, underestimates of growth cannot explain the 1989-1991 bubble. The indications are that in all likelihood there have been two speculative bubbles in Ontario farmland prices since 1975.

Discussion and Conclusions

This paper set out asking a very fundamental question: Can real options explain the discrepancies between observed land values and the land capitalization model? The emerging theories and literature on hysteresis and real options provides a rich and reasonable approach to explaining many of the discrepancies that agricultural economists have discovered when positive theories such as the pricing of farm land do not hold under rigorous empirical testing. Real options emerge when investors have the option to act or postpone making an investment decision when the investment has uncertain outcomes unto its self, or is derived from an underlying stochastic process. This real option emerges because the seller always has an option to sell immediately or postpone the sale in the hopes that cash flow and hence land values and capital gains will ultimately increase. The emergence of options in the agricultural land markets emerges when long run positive growth trends and uncertainty signal an expectation of capital gains at some point in the future. Because of the growth trend farmers may not be willing to sell even if cash flows decrease below the trend in the short run. For market determined assets such as land this means that the observed land price does not always match the price predicted from the simple discounting of future cash flows.

The results tend to support the hypothesis. Using Ontario data from 1975 to 1998 it was shown that for about 9 years the market value of land was very close to the fundamental value plus the real option value. In 15 of 24 years the market price was no more than 1.5 times the economic value of farmland. This finding does not refute the existence of real options since evidence by Falk and Lee (1998) shows that from time to time short run shocks can push land prices away from equilibrium, but eventually they will return. In 9 of 24 years the results support the existence of a speculative bubble in farmland prices. A speculative bubble does not refute the existence of real options since a bubble by definition is simply a real option gone awry. But the finding of bubbles does refute the implicit notion of rationality in real option pricing and in this respect we must reject the idea that real options can explain away bubbles. In answer to the query that econometricians actually discovered real options rather than bubbles the evidence in this research suggests that they did indeed discover bubbles.

There are a number of issues that need further investigation. One direction of research would be to survey landowners and determine if in fact they purposefully delay selling land in the hopes of land price increases or if they attempt to extract a premium from the buyer as

compensation for future capital gains. In other words, how persistent is the hysteresis argument used in the proposition? In addition, specific examinations need to clarify why from time to time bubbles persist and what causes real option values to rise beyond reason. Behavioural models such as those described in Shiller (2000) provide some foundation principles, but the behavioural theories of asset overvaluation have yet to consider option values explicitly. It is easy to examine equation (6) and its roots to show that option values increase with growth, but by any definition of a random walk, growth is a long-term parameter. Still, one can imagine a short-run increase in growth that in turn increases land and option values. In a model such as Hong and Stein (1999) a cascading of growth expectations, initially real and later imagined, can explain the result.

We must also be cautious about some of the underlying assumptions of the paper. The assumption of perfect correlation between cash flows and land values at each moment in time can easily be questioned. This assumption plays a critical role in theory because without it, it would be difficult to argue that the buyer has no option. Indeed, further research may want to examine option values under the weaker assumption of less than perfect correlation, and defining uncorrelated Brownian motions between two or more random influences could do this. Cash flow and inflation as independent or weakly correlated Brownian motions for example could be used to explain the inflation driven land prices of the 1970's, and the collapse of the inflation option may explain the dramatic decline of land prices in the late 1980's. The existence of agricultural stabilization programs and disaster relief may also create defensible option values. Finally, the whole notion of hysteresis needs to be explored. From a classical standpoint the notion that a farmer might postpone a sale, that is giving up certainty today for uncertainty in the future, is as much a function of psychology as economics since it requires certain beliefs that may not be justified by a purely economic model. Without this psychology the real options discussed in this paper may not be transacted at all. However, with the assumption of hysteresis in place, this paper has provided the first step in a new avenue for investigating the dynamics of agricultural land values.

Appendix I

Following Dixit and Pindyck (1996), the solution to (6) is obtained by assuming that the option price is described by

$$(A1) \quad F(\pi) = A\pi^\beta$$

where A and β are parameters to be determined. Applying the appropriate calculus to (A1) and substituting these into (6) yields the quadratic equation

$$(A2) \quad \frac{1}{2} \sigma^2 \beta(\beta-1) + \beta\alpha - r = 0.$$

Solving for the positive root of β gives

$$(A3) \quad \beta = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \left(\left(\frac{\alpha}{\sigma} - 1/2 \right)^2 + \frac{2r}{\sigma^2} \right)^{1/2}$$

Solving the smooth pasting condition (9) in terms of A, and substituting A into the value matching condition (8) yields

$$(A4) \quad \pi^* = \left[\frac{\beta}{\beta-1} \right] V(\pi)(r-\alpha).$$

Equation (A4) gives the optimal level of cash flow (π^*) at which the option to sell should be exercised (e.g. \$167 in Figure 1). It is a function of interest rates, volatility, growth and fundamental value as defined by $V(\pi) = \pi / (r - \alpha)$. To maximize expected NPV the optimal decision rule is to postpone the sale until π^* occurs. Finally

$$(A5) \quad A = \frac{\pi^*}{(r-\alpha)\beta\pi^{*\beta}}, \text{ and}$$

the value of the option can be solved by

$$(A6) \quad F(\pi^*) = A\pi^{*\beta}$$

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Table 1: Real Options Valuation of Ontario Farmland (Means and Standard Deviations from 1975-1998)

1	2	3	4	5	6	7	8	9	10	11	12	13
Year	Land Price/Acre	Cash flow/Acre (π)	Prime Rate (I)	PV Bid Price $V(\pi)=I$	Market - $V(\pi)$	Option Value $F(\pi)$	$V(\pi)+F(\pi)$	Optimum Put Option Strike Value π^*	$\pi^*/E[\pi]$	$V(\pi^*)$	Option Value $F(\pi^*)$	$(V(\pi)+F(\pi))/V(\pi)$
1971	325	25.36	6.48									
1972	352	31.83	6.00									
1973	440	45.62	7.65									
1974	583	56.82	10.75									
1975	749	59.63	9.42	580.26	168.74	164.10	744.36	99.71	2.20	1,276.76	696.49	1.28
1976	893	60.08	10.04	630.77	262.23	168.38	799.15	111.76	2.10	1,325.01	694.24	1.27
1977	995	56.31	8.50	839.21	155.79	260.87	1,100.09	138.31	2.39	2,006.45	1,167.24	1.31
1978	1138	64.77	9.69	719.69	418.31	198.38	918.08	125.31	2.15	1,550.71	831.02	1.28
1979	1301	73.09	12.90	538.40	762.60	115.99	654.38	110.11	1.81	975.35	436.95	1.22
1980	1528	65.72	14.25	521.51	1,006.49	103.54	625.05	113.87	1.73	900.64	379.13	1.20
1981	1695	77.27	19.29	377.71	1,317.29	59.19	436.90	102.61	1.54	580.20	202.50	1.16
1982	1659	71.05	15.81	504.86	1,154.14	92.25	597.11	118.47	1.65	833.94	329.08	1.18
1983	1542	85.65	11.17	753.73	788.27	183.36	937.09	141.34	1.96	1,478.50	724.77	1.24
1984	1509	97.19	12.06	742.21	766.79	169.07	911.28	145.62	1.88	1,392.75	650.54	1.23
1985	1402	73.62	10.58	963.73	438.27	245.62	1,209.34	175.46	2.03	1,954.71	990.98	1.25
1986	1288	107.56	10.52	929.34	358.66	238.08	1,167.42	168.67	2.04	1,892.20	962.86	1.26
1987	1288	103.61	9.52	1,176.55	111.45	329.43	1,505.98	203.20	2.18	2,567.55	1,391.01	1.28
1988	1489	105.61	10.83	1,063.81	425.19	265.66	1,329.48	196.15	2.00	2,125.95	1,062.13	1.25
1989	1908	96.85	13.33	871.54	1,036.46	182.66	1,054.20	182.10	1.78	1,552.90	681.36	1.21
1990	2147	90.70	14.06	818.11	1,328.89	164.18	982.29	177.05	1.74	1,421.43	603.32	1.20
1991	2303	81.63	9.94	1,162.18	1,140.82	313.13	1,475.31	204.86	2.12	2,459.09	1,296.91	1.27
1992	2184	104.91	7.48	1,529.05	654.95	537.73	2,066.78	242.92	2.71	4,136.65	2,607.60	1.35
1993	2144	83.18	5.94	2,176.91	-32.91	974.74	3,151.65	346.81	3.68	8,008.20	5,831.29	1.45
1994	2134	76.54	6.88	1,710.98	423.02	655.05	2,366.03	268.67	2.98	5,099.92	3,388.94	1.38
1995	2188	84.41	8.65	1,203.47	984.53	368.19	1,571.65	199.59	2.36	2,835.45	1,631.98	1.31
1996	2384	92.38	6.06	1,881.88	502.12	823.44	2,705.33	298.17	3.56	6,691.94	4,810.05	1.44
1997	2471	92.17	4.96	2,563.07	-92.07	1,420.90	3,983.97	462.05	5.38	13,786.45	11,223.38	1.55
1998	2538	87.97	6.60	1,783.29	754.71	711.83	2,495.12	279.73	3.14	5,597.58	3,814.29	1.40
Mean	1,703.21	82.99	10.35	1,085.09	618.11	364.41	1,449.50	192.19	2.38	3,018.76	1,933.67	1.29
Std. Dev.	524.29	15.46	3.41	575.16	420.26	332.07	901.64	88.23	0.86	3,010.79	2,474.20	0.10
Minimum	325.00	25.36	4.96	377.71	-92.07	59.19	436.90	99.71	1.54	580.20	202.50	1.16
Maximum	2,538.00	107.56	19.29	2,563.07	1,328.89	1,420.90	3,983.97	462.05	5.38	13,786.45	11,223.38	1.55

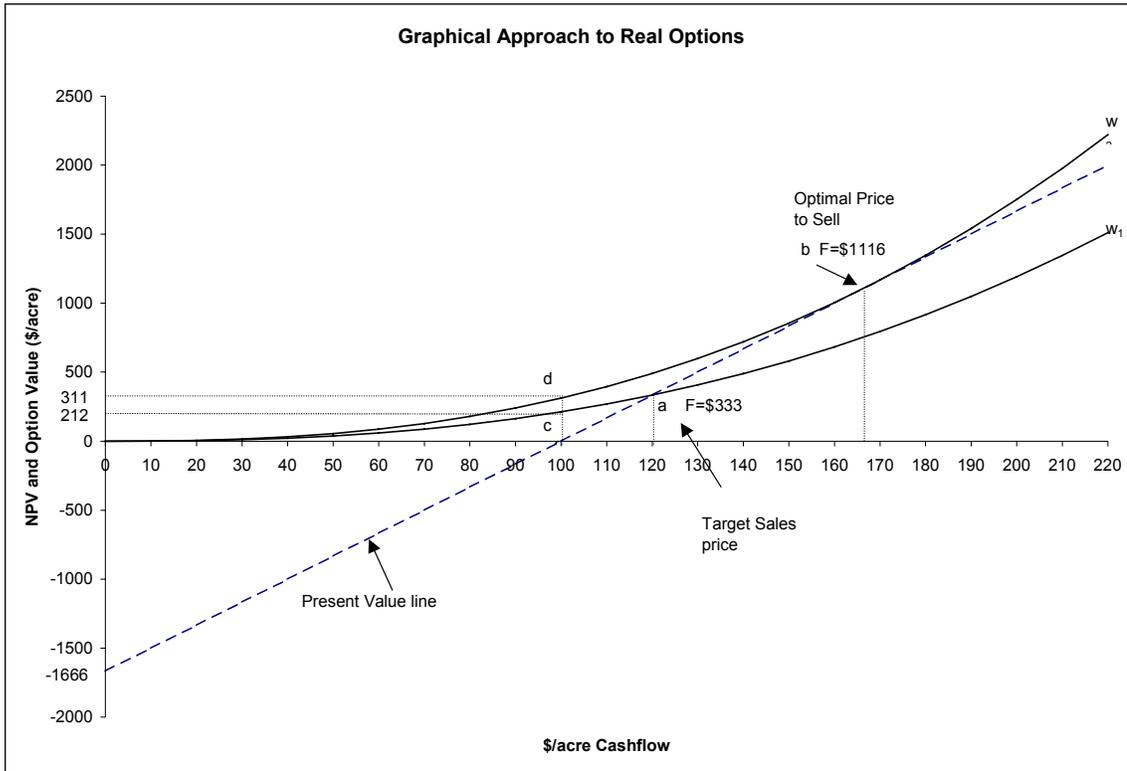


Figure 1: A Graphical Approach to Real Options

Real Options and the Value of Farmland

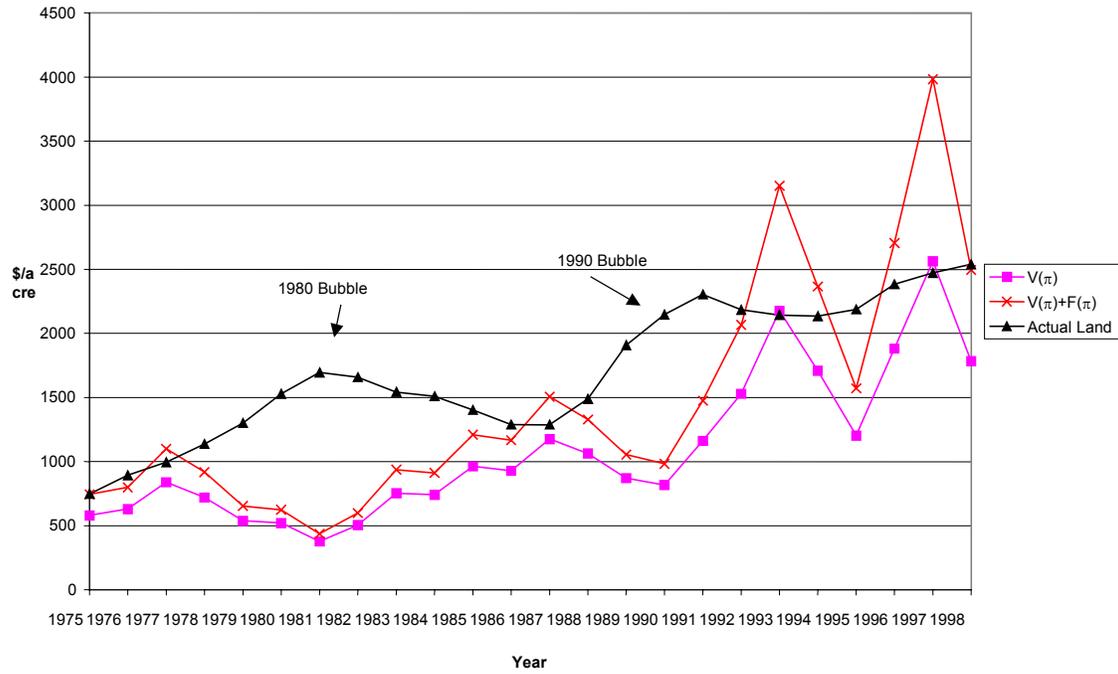


Figure 2: Real Options and the Value of Farmland in Ontario