The role of expected protein levels in determining the impact of protein premiums and discounts

by

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ABSTRACT

This paper investigates the role of expected protein levels in determining the impact of protein premiums and discounts on firstly, a grower's income stream, and secondly, a grower's willingness-to-pay for a forward contract. The impact is examined for a range of expected protein levels (9%-13%). When considering a grower's income stream, for expected protein levels less than approximately 10.2% expected income, E(I), and the variance of income, Var(I), decrease, and E(I) is the dominating effect causing an overall decrease in expected utility (EU). There exists a small protein window (approximately 10.2%-10.3%) where E(I) and Var(I) decrease and the Var(I) effect is dominant causing an overall increase in EU. For expected protein levels greater than 10.3%, E(I) increases and Var(I) decreases, both working to positively affect EU. Hence, growers with low expected protein levels are disadvantaged by the scheme. A sensitivity analysis is conducted on key parameter values to understand their impact on this relationship and it is found that although the window changed slightly in size and level, it did not significantly alter this relationship.

Expected protein levels also have a significant role in determining the impact of protein premiums and discounts on a grower's willingness-to-pay for a forward contract. This paper shows that in the presence of protein premiums and discounts growers with expected protein less than approximately 10% are willing to pay more for a forward contract, and growers with expected protein greater than approximately 10% are willing to pay less for a forward contract. A sensitivity analysis conducted on key parameter values did not significantly modify this relationship.

INTRODUCTION

In 1989 the Australian Wheat Board (AWB) introduced a scheme by which wheat growers are paid for the protein content of their wheat. Generally, growers are paid a premium for high protein wheat (above 10%) and are discounted for low protein wheat (below 10%) (AWB 1998). This scheme significantly affects a grower's income stream, particularly due to the impact of seasonal variability which creates an inverse relationship between wheat yield and protein (for a given nitrogen level) (Robinson 1995). More specifically, seasonal conditions in which yields are relatively high, the protein content is relatively low and a lower price is received. Conversely, seasonal conditions in which yields are relatively low, the protein content is relatively high and a higher price is received.

Fraser (1997) showed that, for a protein payment system which is centred on a grower's existing expected protein level, this negative correlation between price and yield decreases both the expected level and variance of income. This paper extends these results to consider whether a protein premiums and discounts system not centred on a grower's existing expected protein level (i.e. expected protein not equal to 10%) plays a role in determining the impact of protein premiums and discounts. This impact is examined over a realistic range of expected protein levels for West. Australian wheat growers (9%-13%) and it is shown that, depending on this level, both the expected level and variance of income effects can be negative, or the former positive and the latter negative. In addition, this analysis determines not only the expected protein level at which these effects are in conflict, but also which impact dominates in determining the overall effect on grower utility.

The impact of protein premiums and discounts on a grower's willingness-to-pay for a forward contract is also analysed. Fraser (1997) showed that, for a protein payment system centred on a grower's existing expected protein level, protein premiums and discounts increased the grower's willingness-to-pay for a forward contract, and that this relationship is positively related to both the size of the payment and the grower's level of seasonal variability. This paper extends these results to investigate the effect of a grower's expected protein level in determining this impact.

A sensitivity analysis is conducted on a number of uncertain factors which may effect the role of expected protein levels in determining the impact of protein premiums and discounts both on a grower’s income stream and on the willingness-to-pay for a forward contract. These factors include the level of seasonal variability, the nature of the relationship between yield and protein, and the size of the protein payment.

The structure of the paper is as follows. Section 1 develops the model of the impact of protein premiums and discounts on the grower's income stream which allows for a range of levels of expected protein. Section 2 reports the results of a numerical analysis of this model, including details of the sensitivity analysis of key parameter values. The paper concludes with a brief summary.

SECTION 1: The Model

The model is based on that developed in Fraser (1997), with two main modifications. The first is a simplification of the specified relationship between yield and protein. In Fraser (1997) this relationship was represented by a hyperbolic form:

$$y = \gamma / r \tag{1}$$

where: r = uncertain protein level
γ = parameter relating the joint variability of yield and protein
y = uncertain yield per hectare

In what follows this relationship is simplified to a linear form:

$$y = a - br \tag{2}$$

where: a = notional maximum yield
b = parameter relating the joint variability of yield and protein

Note that the accuracy with which this linear form can substitute for the more realistic hyperbolic form depends on the extent of seasonal variation. For example, a coefficient of yield variation of 20% means that 70% of the probability distribution lies within one standard deviation of the mean. As noted in Fraser (1997), in this situation the two forms will be 'similar' (p 143). Consequently, given that the empirical focus of our analysis is on wheat-growing regions of Western Australia where coefficients of yield variation have been estimated to be typically of this magnitude, our view is that this simplification does not significantly weaken the applicability of our analysis.

The second modification is a generalisation of the relationship between price and protein. In Fraser (1997) this relationship was restricted to only three discrete grades of wheat: 'high, medium and low protein' (p 142). It has subsequently become apparent that a weakness of this specification is that it substantially inhibits analysis of the role of differing protein levels in determining the impact of protein premiums and discounts on the wheat grower's income stream. Consequently, in what follows the relationship between price and protein is specified to represent more accurately the AWB's existing protein payment scales. Since these scales are based on protein payment increments for each 0.1 per cent of protein, the (uncertain) price the grower receives (p) can be represented by:

$$p = p_B + (r - 0.1)x \qquad (3)$$

where: p_B = uncertain base price per tonne for wheat

x = premium or discount per unit of protein above or below 10%

On this basis, the grower's uncertain income per hectare in the absence of protein premiums and discounts (I_0) is given by:

$$\begin{aligned} I_0 &= py \\ &= p_B(a - br) \qquad (4) \end{aligned}$$

so that expected income ($E(I_0)$) and the variance of income ($Var(I_0)$) are given by[1]:

$$E(I_0) = \bar{p}_B(a - b\bar{r}) \qquad (5)$$

$$Var(I_0) = (a - b\bar{r})^2 Var(p_B) + \bar{p}_B{}^2 b^2 Var(r) + b^2 Var(p_B) Var(r) \qquad (6)$$

where: $\bar{p}_B$ = expected base price

$\bar{r}$ = expected protein level

$Var(p_B)$ = variance of base price

$Var(r)$ = variance of protein level

Note that, as in Fraser (1997), the grower's uncertain base price and protein level (as determined by seasonal uncertainty) have been assumed to be independent.

In addition, the grower's uncertain income per hectare in the presence of protein premiums and discounts (I_1) is given by:

$$
\begin{aligned}
I_1 &= py \\
&= (p_B + (r - 0.1)x)(a - br) \qquad (7)
\end{aligned}
$$

so that expected income ($E(I_1)$) and the variance of income ($Var(I_1)$) can be approximated by[2]:

$$
\begin{aligned}
E(I_1) &= \bar{p}_B(a - b\bar{r}) + E((a - br)(r - 0.1)x) \\
&= \bar{p}_B(a - b\bar{r}) + a(\bar{r} - 0.1)x + b0.1x\bar{r} - bx(Var(r) + \bar{r}^2) \qquad (8)
\end{aligned}
$$

$$
\begin{aligned}
Var(I_1) \approx\ & (a - b\bar{r})^2 Var(p_B + (r - 0.1)x) + (\bar{p}_B + (\bar{r} - 0.1)x)^2 b^2 Var(r) \\
& + 2(\bar{p}_B + (\bar{r} - 0.1)x)(a - b\bar{r})Cov((p_B + (r - 0.1)x), (a - br)) \qquad (9)
\end{aligned}
$$

Since[3]:

$$
Var(p_B + (r - 0.1)x) = Var(p_B) + x^2 Var(r) \qquad (10)
$$

and:

$$
\begin{aligned}
& Cov((p_B + (r - 0.1)x), (a - br)) \\
&\qquad = E((p_B + (r - 0.1)x) - (\bar{p}_B + (\bar{r} - 0.1)x)((a - br) - (a - b\bar{r})) \\
&\qquad = -bxVar(r) \qquad (11)
\end{aligned}
$$

it follows that (9) can be rewritten as:

$$
\begin{aligned}
Var(I_1) \approx\ & (a - b\bar{r})^2 (Var(p_B) + x^2 Var(r)) + (\bar{p}_B + (\bar{r} - 0.1)x)^2 b^2 Var(r) \\
& - 2(\bar{p}_B + (\bar{r} - 0.1)x)(a - b\bar{r})bxVar(r) \qquad (12)
\end{aligned}
$$

In order to analyze the impact of the protein premiums and discounts system on a grower's income stream, consider first the difference between equations (5) and (8):

$$E(I_0) - E(I_1) = -a\bar{r}x + 0.1x(a - b\bar{r}) + bx(Var(r) + \bar{r}^2) \tag{13}$$

The first term on the right-hand-side of (13) is negative, while the second and third terms are positive. Consequently, the impact of protein premiums and discounts on expected income would appear to be analytically ambiguous. However, note that if:

$$\bar{r} = 0.1$$

then (13) simplifies to:

$$E(I_0) - E(I_1) = bxVar(r) > 0 \tag{14}$$

confirming the result of Fraser (1997) that a protein payment system which is centred on the grower's existing expected protein level will reduce expected income. Nevertheless, the negative first term on the right-hand-side of (13) raises the possibility that a grower's expected income may be increased by a protein premiums and discounts system which is not centred on the grower's existing expected protein level. In particular, note that the magnitude of the first term is positively related to the level of $\bar{r}$, while that of the second term is negatively related. Consequently, the numerical analysis of the next section will explore whether for large enough $\bar{r}$ the impact of protein premiums and discounts on expected income is positive. Next consider the impact of protein premiums and discounts on the variance of income. Subtracting equation (12) from (6) gives:

$$\begin{aligned} Var(I_0) - Var(I_1) = {} & b^2Var(r)(\bar{p}_B{}^2 - (\bar{p}_B + (\bar{r} - 0.1)x)^2) \\ & + 2(\bar{p}_B + (\bar{r} - 0.1)x)(a - b\bar{r})bxVar(r) \\ & + Var(r)(b^2Var(p_B) - x^2(a - b\bar{r})^2) \end{aligned} \tag{15}$$

The first term on the right-hand-side of (15) is positive or negative as:

$$\bar{r} \lesseqgtr 0.1$$

In addition, the second term is positive for all $\bar{r}$, although its size is of indeterminate relationship to the level of $\bar{r}$, while the third term may be positive or negative, but is increasing in $\bar{r}$. Consequently, for the special case of :

$$\bar{r} = 0.1$$

in which case the first term on the right-hand-side of (15) is zero, the result of Fraser (1997) that:

$$Var(I_1) < Var(I_0)$$

can once again be verified analytically for small values of x. However, for other values of $\bar{r}$ the analytical ambiguity of (15) cannot be resolved. Therefore, the numerical analysis of the next section will also explore the role of the level of $\bar{r}$ in determining the impact of protein premiums and discounts on the variance of income.

Finally, in this section recall the observation in Fraser (1997) that the grower's willingness-to-pay for a forward contract will be increased by the introduction of protein premiums and discounts centred on the grower's existing level of expected protein, where the willingness-to-pay was estimated using the mean-variance form of the grower's utility of income (U(I)):

$$E(U(I)) = U(E(I)) + \frac{1}{2}U''(E(I))Var(I) \qquad (16)$$

Note that for the specification developed in this section, the grower's willingness-to-pay for an ungraded forward contract (in the absence of protein premiums and discounts) can be found by using (16) with and without the $Var(p_B)$ terms in equation (6). In addition, the grower's willingness-to-pay for a multigrade forward contract (in the presence of protein premiums and discounts) can be found by using (16) with and without the $Var(p_B)$ term in equation (12). In the numerical analysis to follow the risk management results of Fraser (1997) will also be generalised by an assessment of the role of the level of expected protein in determining the impact of protein premiums and discounts on a grower's willingness-to-pay for a forward contract.

SECTION 2: Numerical Analysis

This section is presented in two parts. Part A analyses the role of the expected protein level in detemining the effect of protein premiums and discounts on a grower's income stream, and Part B considers this role in determining the effect of protein premiums and discounts on a grower's willingness-to-pay for a forward contract.

In order to undertake the numerical analysis it is necessary to specify a functional form for the grower's utility of income. As in Fraser (1997), it is assumed in what follows that this utility function is given by the constant relative risk aversion form:

$$U(I) = \frac{I^{1-R}}{1-R} \tag{17}$$

where: R = coefficient of relative risk aversion.

In addition, the following parameters are chosen for a base case:
$\overline{p}_B$ = 150
CVp_B = coefficient of variation of base price = 0.2
a = 3.5
b = 15
CVr = coefficient of variation of protein = 0.2
x = 500 ($5/% protein)
R = 0.5

Note that $\overline{p}_B$, a and b have been chosen to approximate actual values. Anderson et al. (1988) provide supporting estimates of seasonal variability for WA wheat growers, and Bardsley and Harris (1987) provide supporting estimates of attitudes to risk in the wheatbelt of Australia. Furthermore, x is based on the current payment level.

Also note that a sensitivity analysis is subsequently conducted to investigate the effect of key parameter values in modifying the role of the expected protein level. This involves the following parameter changes (these changes are not made simultaneously, rather each change is a separate analysis):

CVr= 0.25

b = 10, 20

x = 1000 ($10/% protein)

Part A

Consider first the base case results as presented in Table 1. For $\bar{r}$ less than 10.19%, the changes in E(I) and Var(I) are both negative. However, the overall negative change in EU shows that the impact on E(I) is dominating. Whereas for $\bar{r}$ between 10.19% and 10.33%, the changes in E(I) and Var(I) are both negative, but the positive change in EU shows that the impact on Var(I) is dominating. For $\bar{r}$ greater than 10.33%, E(I) is increased and Var(I) is decreased, and so both effects cause a positive impact on the grower's EU.

Table 1 The effect of protein premiums and discounts on income and expected utility for different levels of expected protein (b = 15, CVr = 0.2, x = 500).

$\bar{r}$ (%)	$E(I_0)$	$Var(I_0)$	EU_0	$E(I_1)$	$Var(I_1)$	EU_1
9.00	322.5	5866	35.66	309.3	4552	34.97
10.00	300.0	5706	34.37	297.0	4225	34.26
10.19	295.7	5685	34.12	294.5	4177	34.12
10.33	292.6	5671	33.93	292.6	4145	34.01
10.50	288.8	5657	33.70	290.3	4109	33.87
11.00	277.5	5629	33.01	283.1	4029	33.44
11.50	266.3	5621	32.31	275.6	3986	32.98
13.00	232.5	5721	30.09	250.7	4116	31.41

These results both confirm those of Fraser (1997) that a protein payment system centred on the grower's average protein level (i.e. 10%) reduces the expected level and variance of income, and generalises them for growers with average protein levels less than 10.33%. However, for growers with average protein levels greater than this, expected income increases. Moreover, allowing for the fact that the effect on expected income is not always the dominating effect, it can be seen that grower's with expected protein levels less than 10.19% are disadvantaged by the scheme while grower's with expected protein levels greater than this level are advantaged.

Now consider the case where CVr is increased to 0.25 (Table 2). The relationship is largely unchanged except that the 'window' where the variance effect is dominant has slightly increased in size (0.14% cf 0.23%) and in level (ie average protein in the window has increased from 10.26% to 10.43%). This finding reflects both that the value of the risk benefits associated with protein premiums and discounts is greater under increased seasonal uncertainty, and that a grower's average protein must be higher before a negative expected income effect is dominated by the risk benefits.

Table 2 The effect of protein premiums and discounts on income and expected utility for different levels of expected protein under increased CVr (b = 15, CVr = 0.25, x = 500).

$\bar{r}$ (%)	$E(I_0)$	$Var(I_0)$	EU_0	$E(I_1)$	$Var(I_1)$	EU_1
9.00	322.5	6826	35.62	308.0	4773	34.88
10.00	300.0	6891	34.31	295.3	4577	34.14
10.31	293.0	6932	33.90	291.1	4551	33.90
10.50	288.8	6963	33.63	288.4	4545	33.73
10.54	287.8	6970	33.57	287.8	4544	33.70
11.00	277.5	7062	32.93	281.1	4563	33.29
11.50	266.3	7187	32.22	273.4	4633	32.81
13.00	232.5	7723	29.95	247.8	5215	31.15

Tables 3 and 4 present the results under increased and decreased b (b = 20 and 10 respectively). Under these scenarios, the window where the variance effect is dominant increases and decreases in both size and level respectively. Hence, an increase or decrease in b has a similar impact to an increase or decrease in CVr. This similarity can be explained by noting that in both cases the change modifies the strength of the negative relationship between price and yield, and therefore modifies the strength of the effect of protein premiums and discounts on the expected level and variance of income.

Table 3 The effect of protein premiums and discounts on income and expected utility for different levels of expected protein under increased b (b = 20, CVr = 0.20, x = 500).

$\bar{r}$ (%)	$E(I_0)$	$Var(I_0)$	EU_0	$E(I_1)$	$Var(I_1)$	EU_1
9.00	255.0	5634	31.59	243.3	3963	30.93
10.00	225.0	5769	29.57	221.0	4050	29.42
10.33	215.1	5845	28.87	213.2	4140	28.87
10.50	210.0	5892	28.50	209.1	4199	28.57
10.67	204.9	5941	28.13	204.9	4266	28.27
11.00	195.0	6057	27.37	196.7	4426	27.65
11.50	180.0	6247	26.19	183.7	4736	26.63
13.00	135.0	7056	22.11	141.7	6220	22.89

Table 4 The effect of protein premiums and discounts on income and expected utility for different levels of expected protein under decreased b (b = 10, CVr = 0.20, x = 500).

$\bar{r}$ (%)	$E(I_0)$	$Var(I_0)$	EU_0	$E(I_1)$	$Var(I_1)$	EU_1
9.00	390.0	6842	39.27	375.4	6091	38.54
10.00	375.0	6561	38.50	373.0	5650	38.43
10.12	373.2	6531	38.41	372.6	5601	38.41
10.17	372.5	6518	38.37	372.5	5580	38.40
10.50	367.5	6434	38.11	371.4	5442	38.35
11.00	360.0	6317	37.72	369.6	5243	38.26
11.50	352.5	6208	37.32	367.5	5055	38.16
13.00	330.0	5938	36.08	359.6	4560	37.76

The effect of a doubling in the size of the payment is illustrated in Table 5. The size and level of the window in which the variance effect is dominant does not change significantly. Nevertheless, the associated strengthening of the negative relationship between price and yield means not only that growers with $\bar{r}$ less than 10.23% are disadvantaged by the scheme, while growers with $\bar{r}$ greater than 10.23% are advantaged by the scheme (compared with 10.19% for the base case), but also the strength of the impacts is increased.

Table 5 The effect of protein premiums and discounts on income and expected utility for different levels of expected protein under increased x (b = 15, CVr = 0.20, x = 1000).

$\bar{r}$ (%)	$E(I_0)$	$Var(I_0)$	EU_0	$E(I_1)$	$Var(I_1)$	EU_1
9.00	322.5	5866	35.66	296.1	4161	34.21
10.00	300.0	5706	34.37	294.0	3625	34.11
10.23	294.8	5681	34.06	293.1	3520	34.06
10.33	292.6	5671	33.93	292.6	3477	34.04
10.50	288.8	5657	33.70	291.8	3406	33.99
11.00	277.5	5629	33.01	288.7	3227	33.82
11.50	266.3	5621	32.31	284.9	3095	33.60
13.00	232.5	5721	30.09	268.9	3056	32.62

Part B

Table 6 presents a grower's willingness-to-pay for a forward contract in the presence and absence of protein premiums and discounts in terms of the certainty equivalent of income. The effect of protein premiums and discounts is to increase the grower's willingness-to-pay for a forward contract where $\bar{r}$ is less than 9.94%. Where $\bar{r}$ is greater than 9.94%, the effect of protein premiums and discounts is to decrease the grower's willingness-to-pay for a forward contract .

Table 6 Percent change in Certainty Equivalent (CE) derived from a forward contract in the absence and presence of protein premiums and discounts.

$\bar{r}$ (%)	$\% \Delta$ in CE_0	$\% \Delta$ in CE_1
9.00	1.026	1.097
9.94	1.033	1.033
10.00	1.033	1.029
11.00	1.044	0.969
13.00	1.081	0.869

This result is inconsistent with the findings of Fraser (1997) that this willingness-to-pay increases for an expected protein level of 10%. However, as the sensitivity analysis in the top part of Table 7 shows, this inconsistency may be attributed to our re-specification of the protein-yields relationship (i.e. see b=10 compared with b=15). More specifically, the results in Table 6 generalise the findings of Fraser to show that willingness-to-pay increases only in situations where the negative impact of protein premiums and discounts on expected income is clearly dominant. In the situations where the risk benefits of protein premiums and discounts are relatively important, or where expected income increases, it is clear that willingness-to-pay for a forward contract decreases. This latter finding reflects the associated perception of a reduced relative value from the risk benefits of a forward contract.

Table 7 Values of $\bar{r}$ for which a grower is willing to pay the same amount for a forward contract in the presence of protein premiums and discounts as in their absence ($\bar{r}*$).

	$\bar{r}*$ (%)
b=15, CVr = 0.2, x = 500	9.94
b=10, CVr = 0.2, x = 500	10.05
b=15, CVr = 0.25, x = 500	9.92
b=15, CVr = 0.2, x = 1000	10.12

In general, Table 7 contains a sensitivity analysis for $\bar{r}*$, where $\bar{r}*$ is the level of $\bar{r}$ at which the grower's willingness to pay for a forward contract in the absence of protein premiums and discounts is equal to that in their presence. For $\bar{r}$ less than and greater than these values, the change in a grower's willingness-to-pay is positive and negative respectively. For each case $\bar{r}*$ varies little. Consequently, in general terms it can be concluded that for $\bar{r} \prec 10\%$ a grower is willing to pay more for a forward contract in the presence of protein premiums and discounts then in their absence and vice versa.

Finally, note that Fraser (1997) suggested that the potential exists for the AWB to discriminate between regions with different levels of seasonal uncertainty in terms of the price of a forward contract, with the implementation of such discrimination seeing growers with less reliable seasonal conditions paying more for this type of contract. The analysis here suggests that the potential also exists for the AWB to discriminate between regions with different average protein levels, in that growers with lower average protein levels are willing to pay more for a forward contract. However, because, in the wheat growing areas of Western Australia higher seasonal variability also corresponds with higher average protein levels, perhaps such discrimination is not appropriate[4].

CONCLUSIONS

This paper has considered the role of expected protein levels in determining the impact of protein premiums and discounts on a grower's income stream and on their willingness-to-pay for a forward contract. When considering the effect on income, it was found using the base case set of parameter values that for an expected protein level less than 10.33% protein premiums and discounts caused E(I) and Var(I) to decrease. Moreover, for protein levels less than 10.19% the effect on E(I) is the dominant effect and EU decreased, while for protein levels between 10.19% and 10.33% the variance effect is dominant and EU increased. For protein levels greater than 10.33%, E(I) increased and Var(I) decreased, both making a positive impact on EU.

A sensitivity analysis was conducted on key parameter values to understand their effect in modifying these outcomes. Increased seasonal variability and the strength of the tradeoff between protein and yield increased the size and the level of the expected protein window for which the variance effect is dominant. Hence, in these situations the risk benefit of the protein premiums and discounts increases in importance in determining the overall impact. The effect of a doubling in size of the protein premiums and discounts had little effect on the window where the variance effect is dominant. However, if a grower was disadvantaged by the scheme, they were even more so under an increase in the size of the payment.

Finally, when considering the role of expected protein levels in determining the effect of protein premiums and discounts on a grower's willingness-to-pay for a forward contract, under all scenarios considered a grower with $\bar{r}$ less than approximately 10% is willing to pay more for a forward contract and less in the case of $\bar{r}$ above 10%. This was found to be a robust result as a sensitivity analysis conducted on key parameter values did not significantly modify this outcome.

It may be concluded that the grower's existing expected protein level plays a crucial role in determining whether the instrument of protein premiums and discounts is viewed favourably or unfavourably.

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FOOTNOTES

1 See Mood, Graybill and Boes (1981), p180

2 Note that: $E(X^2) = Var(X) + (E(X))^2$,

$$Var(XY) \approx \bar{Y}^2 Var(X) + \bar{X}^2 Var(Y) + 2\bar{X}\bar{Y}Cov(X,Y)$$

See Mood, Graybill and Boes (1981), p181

3 See Mood, Graybill and Boes (1981), p178

4 This paper does not consider the effect of changes in a grower's attitudes to risk. However, unreported results show that different levels of risk aversion do not significantly affect the relationships found in this analysis.