Competing with Fad Products: Erroneous Health Beliefs and Market Outcomes

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[Preliminary and incomplete]

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Introduction

In many countries, including most western nations, poor eating habits have led to an increase in the occurrence of nutrition-related diseases such as hypertension, obesity, and diabetes. This has led to substantial efforts by governments and other organizations to promote healthier foods and eating habits. At the same time, individuals have become more health conscious and made efforts to improve their nutrition. This search for healthier foods has given rise to several nutrition fads which have become popular despite scant evidence that these diets are beneficial for the general public. Examples include eating gluten-free, taking supplements such as probiotics and collagen, and choosing low-fat versions of regular food items. Food producers have taken advantage of these fads and offer products tending to devotees of such diets, creating new product categories (and grocery aisles) often with large profit margins (Jargon, 2014). In 2016, sales of gluten-free foods was estimated to have reached $15 billion (Egan, 2016).

Consumption of these products comes with the obvious downside that consumers might incur additional expenses and/or choose from a limited selection of products without achieving the desired health effect. Additionally, the supposedly healthy choice could have undesirable consequences; for instance, many of the gluten-free products available on the market are considered to be of poor quality and nutritional value (Alvarez-Jubete, Arendt, and Gallagher, 2009; Jnawali, Kumar and Tanwar, 2016), there is evidence that gluten-free diets can lead to nutritional deficiencies (e.g., Vici et al., 2016) and that processed foods reformulated to be gluten-free often include more added sugars and fat, which may potentially contribute to diabetes and obesity (Reilly, 2016).

We study how erroneous nutrition assumptions affect profits as well as consumer surplus in the market and how the government could intervene to improve welfare. To be more concrete, we consider a model in which two manufacturers offer products differentiated horizontally based on the product characteristic subject to the fad: 1 a conventional product and a product serving the fad market (e.g., one product containing gluten and one gluten-free product or whole milk versus fat-free milk). Additionally, the products are also differentiated vertically based on taste. Products compete in quality and price where the fad manufacturer faces increasing costs the closer it brings its own quality to the standard set by the conventional product. Consumers are heterogeneous in their willingness to pay for quality and their belief about the health benefit of the fad. They make their purchase decisions based on price, quality, and their preference for complying with the fad.

A substantial body of work has been devoted to analyzing why consumers, particularly in the developed world, often consume nutritionally inferior products and how to combat these behaviors and associated adverse

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1 While fads are not necessarily associated with health claims, in the food sector the two frequently go hand-in-hand. Hence, we use the terms “erroneous health belief” and “fad” interchangeably. Our main interest lies in the consequences of false expectations rather than in the development of the fad.

2 The interpretation of the quality variable could vary, but obvious possibilities include taste and nutritional composition.
health effects. Research in this literature has often analyzed whether government policies such as taxes or information provision can prompt shoppers to choose healthier products (see, e.g., Wang, 2015, Rojas and Wang, 2017, Colantuoni and Rojas, 2015, Fletcher et al, 2010, Elbel et al, 2009, Réquillart et al, 2016, among many others). This level of interest has not extended to the economics of fad products. Saegert et al. (1978) and Saegert and Saegert (1976) discuss the role of consumer mistrust in the rise of fad foods. Ernst (2001) argues that “[functional foods seem] to benefit exclusively the manufacturers of such products [and] should be discouraged,” but does not specify the best way of discouragement. Wansink and Chandon (2006) combine an experimental and a survey study on low-fat foods. They show that consumers, particularly those who are overweight, tend to overeat low-fat foods and that low-fat labels led consumers to increase their estimates of the appropriate serving size. Wansink and Chandon (2006) suggest stricter regulation of the nutrition labeling and associated health claims.

We focus on information provision to overcome the negative effects of erroneous beliefs. As such, our analysis is most related to the work by Wansink and Chandon (2006). Our work differs drastically in methodology, though, as we analyze a theoretical model of competing food manufacturers. We model beliefs over health effects and allow the beliefs to deviate from the scientific evidence. The manufacturer of the fad product first determines a quality level relative to the regular good. Then the two manufacturers compete in prices in a market with consumers differentiated by both their perceived health benefit of the fad and by their willingness to pay for quality. More specifically, we assume two types of consumers, those who believe a health “cost” is associated with the consumption of the conventional product and those who perceive the same health outcome from the two products. We compare the laissez-faire outcome (prices, quality, welfare) with the optimal outcome given two types of social planner: a populist social planner who maximizes social welfare by using consumers’ beliefs about the health effects of the fad, and a paternalistic social planner who maximizes social welfare by adopting the scientific belief. We also compare the laissez-faire outcome to various government interventions such as information provision and minimum quality standards designed to allow consumers to make more informed purchase decisions and raise the nutritional profile of the fad product. We determine changes to product quality, prices, and welfare such policies would effect.

We find that the level of quality differentiation provided by the market is larger than would be provided under both the populist and paternalistic social planner. More specifically, the fad producer chooses a low product quality to avoid price competition and to lower costs. The effect of information provision depends on the extent of the false health perceptions, i.e., the larger the misconception, the larger the likelihood of an increase in consumer surplus following information provision. If consumers perceive the fad product to have a large benefit then the fad manufacturer will reduce its cost and minimize price competition by choosing a low product quality (relative to the conventional good). Consumers who get convinced that the fad’s promises are incorrect will therefore reap great benefit from switching to the conventional product. Put differently, the main driver of product differentiation is the belief in different health effects from the two products. Once this perception gets weakened by additional

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information, the two products will appear more similar to consumers. This increases the competition between the producers, which is beneficial to consumers. Since the differentiation between the products ultimately reduces consumer surplus, it is unsurprising that in situations where the health perception is less strong, information provision is less likely to improve market outcomes.

**Model**

The *true* indirect utility of consumers takes the general form \( U_i = v + \theta x_i - p_i \), with \( i = c, f \) where \( c \) represents the conventional product and \( f \), the product for the fad market. The reserved utility is represented by \( v \), \( \theta \) is the willingness to pay for the quality of product \( i \) (\( x_i \)) with \( x_c > x_f \), and \( p_i \) is the price of product \( i \).

Consumers are heterogeneous in their willingness to pay for quality (\( \theta \)), and we assume \( \theta \) is distributed uniformly over the interval [0; 1], i.e., \( \theta \sim U[0; 1] \). Additionally, consumers are heterogeneous in their perception of the health cost of the conventional product (or health benefit from the fad product). We assume the population is divided into two types of consumers. Consumers in group 1 either perceive no health cost from the conventional product or are unaware or don’t care about the health issues concerning the product. Consumers in group 2 falsely perceive a health cost associated with the consumption of the conventional product (i.e., a health benefit in the consumption of the fad product). We model the perceived health cost by introducing an additive factor \( \lambda \geq 0 \) into the utility function. Since consumers of Group 1 do not perceive a health benefit associated with the fad product, for them \( \lambda = 0 \); consumers of Group 2 do believe in such a health effect, hence \( \lambda > 0 \). Groups 1 and 2 consist of a mass of consumers of sizes \( m \) and \( 1 - m \), respectively. The *perceived* indirect utilities of consumers in groups 1 and 2 are represented as follows:

**Group 1:**
\[
\begin{align*}
V_c &= v + \theta x_c - p_c & \text{if buys the conventional product} \\
V_f &= v + \theta x_f - p_f & \text{if buys the fad product}
\end{align*}
\]  

**Group 2:**
\[
\begin{align*}
V_c &= v + \theta x_c - \lambda - p_c & \text{if buys the conventional product} \\
V_f &= v + \theta x_f - p_f & \text{if buys the fad product}
\end{align*}
\]  

We assume that \( v \) is large enough such that the market is covered, i.e., all consumers buy one unit of either the conventional or fad product, and no consumer choose to buy nothing. For each consumer group, we find the

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3 Anecdotal evidence suggests that most consumers consider products like fat-free yogurts, alcohol-free beer, and gluten-free pastries inferior sensorially than the corresponding conventional products. This perception is backed by several studies, e.g. Tuorila and Cardello (2002). On the nutritional dimension of quality, there is evidence that fad products often substitute the omitted components with other unhealthy ingredients. For example, reduced-fat products often contain more sugar than the corresponding conventional variants (Malnick et al., 2014).

4 Depending on the type of product, there is potentially a third group of consumers for whom consuming the conventional product is medically proscribed, e.g., consumers with celiac disease should not consume products containing gluten. To keep the model general and focused on fads and *perceived* health costs and benefits, we consider only the first two types of consumers.

5 Our main interest lies in the effect of inaccurate perceptions rather than in the development of the fad itself. Hence, we take as exogenous the fraction of the population that buys into the fad.
consumer indifferent between consuming the conventional and fad product. The indifferent consumer in group 1 is
given by $\theta_1 = \frac{p_c - p_f}{\Delta x}$, and the indifferent consumer in group 2 corresponds to $\theta_2 = \frac{p_c - p_f + \lambda}{\Delta x}$, where $\Delta x = x_c - x_f$.

Depending on the location of the indifferent consumer for each consumer group $j (j = 1, 2)$ there can be three
possibilities for the demand of each product ($q_i$):

1) $0 < \theta_j < 1$, thus $q_f = \theta_j$ and $q_c = 1 - \theta_j$,
2) $\theta_j \leq 0$, thus $q_f = 0$ and $q_c = 1$, and
3) $1 \leq \theta_j$, thus $q_f = 1$ and $D_c = 0$.

This gives rise to different possible market configurations in equilibrium and thus, different formulations of total
demand for each product. Note that $\theta_1 < \theta_2$, which serves to eliminate some of the market configurations. We are
left with the following possible cases:

Case A: $0 < \theta_1 < \theta_2 < 1$,
Case B: $\theta_1 \leq 0 < \theta_2 < 1$,
Case C: $0 < \theta_1 < 1 \leq \theta_2$, and
Case D: $\theta_1 \leq 0 < 1 \leq \theta_2$.

In case A, the market is split between product $c$ and $f$ for both consumer groups. Cases B and C have one market
that is preempted; in case D both markets are preempted. In case B, all consumers in group 1 consume product $c$
(preempted market), and consumers in group 2 are split between the two products. In case C, consumers in group 1
are split between the two products, and all consumers in group 2 consume product $f$. In case D, all consumers in
group 1 buy product $c$ and all consumers in group 2 buy product $f$.

On the supply side, there are two manufacturers: firm $c$ producing the conventional product, and firm $f$
producing fad product. They compete in quality and price, in two stages. In the first stage, the manufacturer of the
fad product determines a quality level relative to the conventional good. We assume that the manufacturer of the
conventional product has been in the market for a long time with an established quality, which is assumed to be
exogenous. Both firms are assumed to have the same marginal cost of production, set to zero without loss of
generality. The manufacturer of product $f$ has a sunk cost of developing its product. This cost can be thought of the
cost of developing a recipe of a given quality. Because of the difficulty in bringing fad products’ quality close to
their conventional counterpart (e.g., Gallagher, Gormley and Arendt, 2004; Jnawali, Kumar and Tanwar, 2016;
Monga and Maloney, 2019), we model the sunk cost as $1/\Delta x$, i.e., firm $f$ faces increasing costs the closer it brings
its own quality to the standard set by the conventional product. Once, firm $f$ has chosen its quality, they
simultaneously compete in prices. We solve by backward induction.

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6 The sunk cost’s functional form might seem problematic if one envisions the possibility that manufacturer F produces the
conventional product at the same quality as manufacturer C. In this case the sunk cost would approach infinity. We do not
consider this possibility because such behavior would entail a profit minimum for manufacturer F: In price competition without
The maximization problem of each manufacturer in stage 2 is:

\[ \max_{p_c} \pi_c = p_c [mq_{1c} + (1 - m)q_{2c}], \quad (3) \]

\[ \max_{p_f} \pi_f = p_f [mq_{1f} + (1 - m)q_{2f}] - \frac{1}{\Delta x}, \quad (4) \]

where \( q_{ji} \) is the demand by consumers in group \( j \) for product \( i \). The different cases and associated demand equations are as follows:

**Case A \((0 < \hat{\theta}_1 < \hat{\theta}_2 < 1)\):**

\[
\begin{align*}
q_{1c} &= 1 - \hat{\theta}_1 = 1 - \frac{p_c-p_f}{\Delta x} \\
q_{1f} &= \hat{\theta}_1 = \frac{p_c-p_f}{\Delta x} \\
q_{2c} &= 1 - \hat{\theta}_2 = 1 - \frac{p_c-p_f+\lambda}{\Delta x} \\
q_{2f} &= \hat{\theta}_2 = \frac{p_c-p_f+\lambda}{\Delta x}
\end{align*}
\]

**Case B \((\hat{\theta}_1 \leq 0 < \hat{\theta}_2 < 1)\):**

\[
\begin{align*}
q_{1c} &= 1 \\
q_{1f} &= 0 \\
q_{2c} &= 1 - \hat{\theta}_2 = 1 - \frac{p_c-p_f+\lambda}{\Delta x} \\
q_{2f} &= \hat{\theta}_2 = \frac{p_c-p_f+\lambda}{\Delta x}
\end{align*}
\]

**Case C \((0 < \hat{\theta}_1 < 1 \leq \hat{\theta}_2)\):**

\[
\begin{align*}
q_{1c} &= 1 - \hat{\theta}_1 = 1 - \frac{p_c-p_f+\lambda}{\Delta x} \\
q_{1f} &= \hat{\theta}_1 = \frac{p_c-p_f+\lambda}{\Delta x} \\
q_{2c} &= 0 \\
q_{2f} &= 1
\end{align*}
\]

**Case D \((\hat{\theta}_1 \leq 0 < 1 \leq \hat{\theta}_2)\):**

\[
\begin{align*}
q_{1c} &= 1 \\
q_{1f} &= 0 \\
q_{2c} &= 0 \\
q_{2f} &= 1
\end{align*}
\]

The second stage equilibrium expressions for prices, quantities, indifferent consumers, and profit of each firm for each case are presented in Table 1. Table 1 also shows the conditions for each case to prevail in equilibrium. The conditions are used in Figure 1 to illustrate the separate cases relative to the values of \( m \) and \( \lambda \). We discuss these conditions in what follows.

In case A, both groups of consumers buy the two products. We find that case A generally holds for small enough values of the perceived health cost \( \lambda \) and more precisely, for health cost values that are at least lower than the value of the quality difference. It is worth noting that some consumers in group 1 will buy the fad product, despite their ignorance about its perceived health benefit, because the price of the fad product is lower than the price any differentiation the two manufacturers would get trapped in Bertrand competition. Hence, it is a clearly better strategy to develop the fad product instead.
of the conventional product. Thus, if the perceived health cost is low enough it is profitable for the fad product manufacturer to set its price low enough to capture sales in the two groups of consumers.

In case C some consumers of group 1 and all consumers in group 2 buy the fad product. This case prevails when more than half of the consumers are in group 1, and the rest of the population perceive a relatively large health benefit (cost) from consuming the fad (conventional) product. More precisely, the perceived health benefit from the fad product must be at least larger than 2/3 of the value of the quality difference between the conventional and fad product. As with case A, in case C the fad product is sold at a lower equilibrium price that of the conventional product. Because there is a large concentration of consumers in group 1, it is profitable for those consumers by setting a price lower than the price of the conventional product.

The conditions for case D to prevail are determined by examining the conditions for the other cases. To go from case B to case D, the condition on \( \theta_2 \) should be altered. In case B, for \( \theta_2 < 1 \), \( \Delta x > \frac{B}{2-3m} \) must hold with \( m < 0.5 \). If instead \( \Delta x \leq \frac{B}{2-3m} \), i.e., \( \frac{\Delta x(2-3m)}{1-m} \leq \lambda \), then \( \theta_1 \leq 0 < 1 \leq \theta_2 \), i.e., case D. In Figure 1, this area is located to the right of case B, with \( m < 0.5 \). To go from case C to case D, the condition on \( \theta_1 \) should be altered. In Case C, for \( 0 < \theta_1, m > 0.5 \) must hold. If \( m \leq 0.5 \), then \( \theta_1 \leq 0 < 1 \leq \theta_2 \), i.e., case D. In Figure 1, this area is located below case C \( (m < 0.5) \), but above case B. To go from case A to case D, the conditions on both \( \theta_1 \) and \( \theta_2 \) should be altered. At \( m = 0.5 \), case A holds when \( \Delta x > \lambda \). If \( \Delta x \leq \lambda \), then \( \theta_1 \leq 0 < 1 \leq \theta_2 \), i.e., case D. In Figure 1, this is the horizontal line at \( m = 0.5 \) that begins at \( \lambda = \Delta x \). In summary, case D prevails when \( m \leq 0.5 \) and \( \lambda \geq \frac{\Delta x(2-3m)}{1-m} \). In other words, case D holds if, at the most, half of the population is in group 1 and the perceived health cost is large enough, or at least larger than the value of quality difference between the two goods. In this case, it makes sense for the two firms to set prices such that the two markets are preempted, i.e., firm c sells only to consumers in group 1, and firm f sells only to consumers in group 2.

Next we show that in case D prices are equal. Given that \( q_{1c} = 1 \) and \( q_{1f} = 0 \), then we know that \( V_c \geq V_f \) for all consumers in group 1. Similarly, given that \( q_{2c} = 0 \) and \( q_{2f} = 1 \), then \( V_f \geq V_c \) for all consumers in group 2. If the inequality holds strictly for all consumers in group 1 (i.e., \( \hat{\theta}_1 < 0 \)), then the conventional producer can increase its price without reducing its quantity demanded, a situation that would clearly leave it strictly better off. Similarly, if the inequality holds strictly for all consumers in group 2 (i.e., \( \hat{\theta}_2 > 1 \)), then the fad producer would be increasing its profit by charging a slightly higher price. Hence, in equilibrium \( \hat{\theta}_1 = 0 \) and \( \hat{\theta}_2 = 1 \). For each consumer group, we set the indirect utilities equal for the two goods at \( \hat{\theta}_1 = 0 \) and \( \hat{\theta}_2 = 1 \) to obtain:

Group 1: \( V_c(\hat{\theta}_1 = 0) = V_f(\hat{\theta}_1 = 0) \rightarrow p_c = p_f \),

Group 2: \( V_c(\hat{\theta}_2 = 1) = V_f(\hat{\theta}_2 = 1) \rightarrow \Delta x = p_c - p_f + \lambda \).
Putting the two equations together we have $p_c = p_f$ and $\lambda = \Delta x$, which implies that there can only be a case D equilibrium (in pure strategies) for $x_f^* = x_c - \lambda$.

In the second-stage equilibrium, $p_c > p_f$ for cases A and C, $p_c \leq p_f$ in case B and $p_c = p_f$ in case D. In the market place, we generally observe that fad products are sold at a higher price than their conventional counterpart despite their poorer sensorial and/or nutritional qualities. Thus, case A and C do not reflect well what we observe in reality. In case B, all consumers in group 1 and some consumers in group 2 buy the conventional product, and the price of the fad product is greater or equal to the price of the conventional product in equilibrium. This case prevails when less than half of the consumers are in group 1, and the perceived health cost is neither too small nor too large relative to the quality difference. When this occurs, it is profitable for the manufacturer of the fad product to forgo consumers in group 1 by charging a high price to consumers who perceive a health benefit from the fad product.

We now turn to the first stage of the game in which the fad product manufacturer chooses the quality of its product. For each case, we take the expressions for the equilibrium prices and equilibrium quantities in Table 1, i.e., $p_c = m q_{1i} + (1 - m) q_{2i}$, and insert them in (4), to find the optimal value of $x_f$.

**Proposition 1.** In cases A to C, maximum quality differentiation maximizes the profit firm f and thus it is the minimum value of $x_f$ (or largest value of $\Delta x$) possible in each case that is optimal. In case D, the degree of quality differentiation is equal to the perceived health benefit of the fad product. The equilibrium value of $x_f$ for each case is:

**Case A:** $x_f^* = 0$ and $\Delta x^* = x_c$.

Proof: The first and second derivative of the profit of firm $f$ with respect to $\Delta x$ are $\frac{\partial \pi_f}{\partial \Delta x} = \frac{\Delta x^2 - 9}{9 \Delta x^2}$ and $\frac{\partial^2 \pi_f}{\partial (\Delta x)^2} = \frac{2}{\Delta x^3}$. Setting the first derivative equal to zero and solving for $\Delta x$ gives $\Delta x' = \sqrt{B^2 - 9}$ and $x_f = x_c - \Delta x'$. However, this solution represents a minimum when $B^2 - 9 > 0$ and does not respect the condition for $\theta_2 > 0$, i.e., $\Delta x' < 2B$. Thus, the profit-maximizing solution corresponds to the upper bound value of $\Delta x$, i.e., $\Delta x^* = x_c$ and $x_f^* = 0$, as long as $x_c$ is large enough relative to $\lambda$ (or $\lambda$ is small enough relative to $x_c$, see figure 1). More precisely,

- If $m < 0.5$, $\Delta x > 2B$ must hold (where $B = 2(1 - m)\lambda$). Thus, $x_f^* = 0$ and $\Delta x^* = x_c$, where $x_c > 2B$ or $\lambda < \frac{\Delta x}{2(1-m)}$.
- If $m = 0.5$, $2B = 0.5(3\lambda - 2B)$, and $\Delta x > \lambda$ must hold. Thus, $x_f^* = 0$ and $\Delta x^* = x_c$, where $x_c > \lambda$.
- If $m > 0.5$, $\Delta x > 0.5(3\lambda - 2B)$ must hold. Thus, $x_f^* = 0$ and $\Delta x^* = x_c$, where $x_c > 0.5(3\lambda - 2B)$ or $\lambda < \frac{2\Delta x}{1+2m}$.

**Case B:** $x_f^* = x_c - \Delta x^*$ and $\Delta x^* = 2B$.

Proof: The first and second derivative of the profit of firm $f$ with respect to $\Delta x$ are $\frac{\partial \pi_f}{\partial \Delta x} = \frac{\Delta x^2 - 9}{9(1-m)\Delta x^2}$ and $\frac{\partial^2 \pi_f}{\partial (\Delta x)^2} = \frac{2[9(1-m)\Delta x^2 - 9]}{9(1-m)\Delta x^3}$. Setting the first derivative equal to zero and solving for $\Delta x$ gives $\Delta x' = \sqrt{B^2 - 9(1-m)}$ and $x_f = x_c - \Delta x'$. However, this

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7 For example, the average price of new products in 2018 was $3.66 per 100g among gluten-free cookies but only $1.75 per 100g among cookies containing gluten. (Own calculation based on data from Mintel Academic database)

8 Given that $x_c$ is exogenous, maximizing profit with respect to $x_f$ or $\Delta x$ is equivalent, i.e., $x_f^* = x_c - \Delta x^*$. 
solution represents a minimum when $B^2 - 9(1 - m) > 0$. If $B^2 - 9(1 - m) < 0$, $\pi_f$ increases with $\Delta x$. Thus, the profit-maximizing solution is given by either the lower or upper bound value of $\Delta x$. We can show that $\pi_f(\Delta x = \frac{\mu}{2-3m}) < \pi_f(\Delta x = 2B)$ when $m < 0.5$ (which must hold in this case). In summary:

This case holds if and only if $m < 0.5$, and $\frac{\mu}{2-3m} < \Delta x \leq 2B$. Thus, $\Delta x^* = 2B = 2(1 - m)\lambda$ and $x_f^* = x_c - \Delta x^*$, and $\Delta x^* \in [\lambda, 2\lambda]$.

**Case C:** $x_c^* = x_c - \Delta x^*$ and $\Delta x^* = \frac{3m\lambda}{(1+m)}$.

Proof: The first derivative of the profit of firm $f$ with respect to $\Delta x$ show that profit increases with $\Delta x$ ($\frac{\partial \pi_f}{\partial \Delta x} = \frac{\Delta x^2(2-m)^2 + 9m}{9m\Delta x^2} > 0$). Thus, the profit-maximizing solution corresponds to the upper bound value of $\Delta x$, i.e., $\Delta x^* = \frac{3m\lambda}{(1+m)}$, and $x_f^* = x_c - \Delta x^*$, as long as $m > 0.5$. In summary:

This case holds if and only if $m > 0.5$, and $\Delta x \leq \frac{3m\lambda}{(1+m)}$. Thus, $\Delta x^* = \frac{3m\lambda}{(1+m)}$ and $x_f^* = x_c - \Delta x^*$, and $\Delta x^* \in [\lambda, 1.5\lambda]$.

**Case D:** $x_c^* = x_c - \lambda$ and $\Delta x^* = \lambda$.

Proof: We showed above that $q_{1c} = 1$, $q_{1f} = 0$, and $q_{2c} = 0$, $q_{2f} = 1$ implies that $\theta_1 = 0$ and $\theta_2 = 1$ in equilibrium. Setting the indirect utilities equal at these indifferent consumers implies that $p_c = p_f$ and $\lambda = \Delta x^*$, and thus $x_f^* = x_c - \lambda$. We also showed that this case holds if and only if $m \leq 0.5$ and $\lambda \geq \frac{\Delta x(2-3m)}{1-m}$. Given that $\lambda = \Delta x^*$, these conditions can be satisfied only for $m=0.5$. In summary:

This case holds if and only if $m \leq 0.5$ and $\lambda \geq \frac{\Delta x(2-3m)}{1-m}$. The equilibrium occurs at $m = 0.5$, and $\Delta x^* = \lambda (x_f^* = x_c - \lambda)$.

In what follows, we demonstrate the values of the equilibrium prices in case D.

**Proposition 2.** In case D, the prices of the fad and conventional product is equal to $\lambda$.

Proof:

We have established above that in case D both firms charge identical prices. To determine the value of that price, we look at potential deviations and check whether they are profitable. Each potential deviation comes with a gain and a loss – for example in the case of a price increase, the gain is the increase in revenue due to the higher price on the remaining demand and the loss is the decrease in revenue due a decrease in quantity (because the conditions for consumer choice hold with equality for some consumers, any deviation would get us out of case D into either case B or case C). There are four different cases – each of the manufacturers could deviate by increasing or decreasing the price – and we check for each of them under which conditions such a deviation would be profitable, and determine as a result the condition needed on price to stay within case D.

If the fad producer decreases $p_f$ by $\varepsilon$, the loss in revenue associated with this change is: $Q_f \varepsilon = (1-m)\varepsilon$. To find the gain in revenue associated with this change we need to figure out how many group 1 consumers switch to product $f$: $\nu + \theta x_c - p_c \leq \nu + \theta x_f - (p_f - \varepsilon)$. Plugging in $p_c = p_f$ and $\lambda = \Delta x$ and solving for $\theta: \theta \leq \frac{\varepsilon}{\lambda}$. Hence, this price reduction would lead to an increase in quantity of $m\varepsilon/\lambda$ and an increase in revenue of $m\frac{\varepsilon}{\lambda}(p_f - \varepsilon)$. The deviation is profitable if the gain is greater than the loss: $m\frac{\varepsilon}{\lambda}(p_f - \varepsilon) > (1-m)\varepsilon$. Because we know from above that in equilibrium $m = 1-m = 1/2$ we get the following condition for a profitable deviation: $\varepsilon < p_f - \lambda$. Since firm $f$ can choose any $\varepsilon$ it likes this means that no case D equilibrium exists where $p_f > \lambda$. Thus, the only way to ensure a case D equilibrium and no profitable deviation is $p_f \leq \lambda$.

If the fad producer increases $p_f$ by $\varepsilon$, the loss of quantity: $\nu + \theta x_c - \lambda - p_c \geq \nu + \theta x_f - (p_f + \varepsilon)$. This becomes $\theta \geq \frac{\lambda - \varepsilon}{\lambda} = 1 - \frac{\varepsilon}{\lambda}$. Hence, a fraction $1 - \left(1 - \frac{\varepsilon}{\lambda}\right) = \frac{\varepsilon}{\lambda}$ of group 2 consumers switches to the conventional product. The gain in revenue is $\left(1 - \frac{\varepsilon}{\lambda}\right)\varepsilon$, and the loss in revenue is $\frac{\varepsilon}{\lambda}p_f$. The deviation is beneficial if and only if: $\left(1 - \frac{\varepsilon}{\lambda}\right)\varepsilon > \frac{\varepsilon}{\lambda}p_f$. This simplifies

\[9 \text{Below we omit } m \text{ and } (1-m) \text{ in the revenue comparisons given that they cancel out in equilibrium.}\]
to $\varepsilon < \lambda - p_f$. Again, the only way there is no profitable deviation is that the right-hand side is nonpositive, i.e., $p_f \geq \lambda$. Together with the above this implies $p_f = \lambda$ in equilibrium.

Applying the same procedure to the conventional producer reveals that a necessary and sufficient condition for it not to have any profitable deviations is $p_f = p_c = \lambda$. Thus, in case D equilibrium we must have $p_f = p_c = \lambda$.

In figure 1, the equilibrium outcomes are located on the curves identified by the points $ab$ (orange, case B), point $b$ (purple, case D) and $bc$ (bold green, case C). For case A, any point at a distance of $\varepsilon$ ($\varepsilon \to 0$) to the left of the curves $ab$ and $bc$ (dash blue) can be an equilibrium depending on the value of $x_c$. Table 2 presents a full characterization of the equilibrium in each case as a function of the parameters $m, \lambda,$ and $x_c$.

Consumer surplus in each case is given by the following: $CS = mCS_1 + (1 - m)CS_2$, and $CS_j = CS_{jf} + CS_{jc}$ where $CS_j$ is the consumer surplus of group $j$, which is the sum of the consumer surplus of those buying the fad product, and those buying the conventional product. The total consumer surplus is given by:

$$CS = m \left[ \int_0^{\bar{\theta}_1} (v + \theta x_f - p_f) d\theta + \int_{\bar{\theta}_1}^1 (v + \theta x_c - p_c) d\theta \right]$$

$$+ (1 - m) \left[ \int_0^{\bar{\theta}_2} (v + \theta x_f - p_f) d\theta + \int_{\bar{\theta}_2}^1 (v + \theta x_c - p_c - \lambda) d\theta \right]$$

(5)

**Optimal Quality under Two Types of Social Planners**

Following Réquillart, Soler and Zang (2016), social welfare is evaluated using two types of social planner, i.e., a populist and a paternalistic social planner. The two planners differ in how they evaluate consumer surplus. According to the populist social planner, consumers’ belief regarding the health cost of certain products, whether real or not, should be treated as it is perceived. The populist social planner evaluates consumer welfare of consumers in group 1 and 2 by computing (5) using the utility functions presented in (2). The paternalistic social planner evaluates consumer surplus for those eating conventional products in group 2 using the scientific belief. Given the general lack of scientific evidence behind the benefit of fad products, we assume that scientific belief is $\lambda=0$, i.e., no health cost of the conventional relative to the fad product. The corresponding measure of consumer surplus is $CS_{2c}^{PAT} = \int_{\bar{\theta}_2}^1 (v + \theta x_c - p_c) d\theta$ for consumers in group 2 consuming the conventional product. In other words, the populist planner calculates consumer welfare by adding the $V_i$ (perceived indirect utilities); the paternalistic planner instead adds the $U_i$ (true indirect utilities). Note that we assume the location of the indifferent consumers to be unchanged and thus welfare is examined at the equilibrium quantities and prices presented in table 2. Thus, $CS^{PAT} = mCS_1 + (1 - m)CS_2^{PAT}$, where $CS_1$ is as described above, and $CS_2^{PAT} = CS_{2f} + CS_{2c}^{PAT}$.

We find the optimal quality of the fad product according to the populist and paternalistic social planners by maximizing total welfare with respect to $x_f$. For each case, total welfare under each social planner is computed as:
\[ TW^{POP} = CS + \pi_f + \pi_c, \text{ and } TW^{PAT} = CS^{PAT} + \pi_f + \pi_c. \]

We focus on the results from Case B, which allows the price of conventional product to be smaller or equal to the price of the fad product. \(^{10}\) The total welfare expressions in this case, using the results from Table 1, are:

\[
TW^{POP} = v + \frac{5B^2 - 2B\Delta x(7 - 9m) + \Delta x[x_c(8 - 9m) + x_f] - 18(1 - m)}{18\Delta x(1 - m)}, \text{ and (6)}
\]

\[
TW^{PAT} = v + \frac{\Delta x[x_c(8 - 9m) + x_f] - B^2 - 2B\Delta x - 18(1 - m)}{18\Delta x(1 - m)}. \text{ (7)}
\]

Taking the first-order condition with respect to \(x_f\), we obtain the optimal quality of the fad product when chosen by populist and paternalistic social planners, i.e.,

\[
x_f^{POP} = x_c - \Delta x^{POP}, \text{ with } \Delta x^{POP} = \sqrt{(1 - m)[18 - 5(1 - m)\lambda^2]}, \text{ and (8)}
\]

\[
x_f^{PAT} = x_c - \Delta x^{PAT}, \text{ with } \Delta x^{PAT} = \sqrt{(1 - m)[(1 - m)\lambda^2 - 18]}. \text{ (9)}
\]

We compare these optimal quality levels with the profit-maximizing one in case B, i.e., \(x_f^* = x_c - \Delta x^*\), with \(\Delta x^* = 2(1 - m)\lambda\).

**Proposition 3.** Under case B, the social planners choose a level of vertical differentiation that is less than the market (i.e., optimal \(x_f\) is higher when total welfare is maximized than under profit maximization), and vertical differentiation is higher under the paternalistic than populist social planner.

Proof: First, we can easily show that \(\Delta x^{POP} < \Delta x^{PAT}\) (\(x_f^{POP} > x_f^{PAT}\)). Second, for case B to prevail, the conditions on \(\Delta x\) must be respected (i.e., case B exists if and only if \(m < 0.5\) and \(\frac{B}{2 - 3m} < \Delta x \leq 2B\)). Thus, given that the optimal quality chosen by firm \(f\) corresponds to the maximum level of differentiation possible for this case to hold, then it can only be that \(\Delta x^{POP} < \Delta x^{PAT} \leq \Delta x^*\). The condition for \(\Delta x^{PAT} \leq \Delta x^*\) is \(\lambda \geq \sqrt{\frac{6}{1 - m}}\) with \(m < 0.5\).

The optimal level of differentiation is higher under profit maximization because it implies lower price competition between the two firms. The result \(\Delta x^{POP} < \Delta x^{PAT}\) implies that the populist social planner chooses a quality level for the fad product that is higher than the paternalistic social planner.

To understand this, consider the two planners’ objective functions: For the populist planner, there are two benefits of reducing \(\Delta x\). Consumers who buy the fad product receive a higher quality item and because of reduced differentiation the prices of both products fall. This is counteracted by the lack of differentiation, which means the product offerings are less optimal for consumers with lower valuation. The paternalistic planner recognizes the same effects, but in addition s/he is concerned about consumers making mistakes by purchasing the fad product.

\(^{10}\) It is important to note that the welfare in Case B is affected by the presence of Group 1 consumers, even though they all make the same purchase choice, because the price of the conventional product depends on the actions of the manufacturer of the fad product (and vice versa), and the proportion of consumers in each group. Hence, we cannot simply ignore Group 1.
The greater the quality of the fad product the greater demand for this good and hence the associated welfare loss in the paternalistic planner’s view. Hence, there is less incentive to increase $x_f$ than for the populist planner.

**Effect of Information Provision**

One way for the government to intervene in this market is by providing more information about the true health effect of consuming the conventional product. This could take the form of an information campaign with the collaboration of the medical community. We model the effect of such an information campaign as an increase in the percentage of consumers in group 1, i.e., an increase in $m$. We are interested in analyzing the welfare effects of this intervention. Table 3 shows the equilibrium welfare expressions for case A and case B. We examine the effect of a marginal increase in $m$ when the equilibrium is in case B, i.e., on the curve $ab$ in figure 1. Such increase in $m$ generates a shift to case A.\(^{11}\) In going from case B to case A, we go from a situation where only consumers in group 2 buy the fad product, to a situation where consumers in both groups consume the fad product. This may not seem like a desirable outcome, however we can show that the overall quantity of fad product consumed decreases as a result.\(^{12}\) The welfare outcomes of this intervention are summarized in the following proposition.

**Proposition 4.** When the equilibrium is characterized by case B, a marginal increase in $m$ results in a shift to an equilibrium in case A, and the following consumer welfare effects.

a. $C_{A}^{POP} > C_{B}^{POP}$ when $\frac{47 + 36m - 16m^2 + 3\sqrt{361 - 232m + 176m^2 + 128m^3}}{4(5 + 7m - 20m^2 + 8m^3)} < \frac{\lambda}{x_c} < \frac{1}{2(1 - m)}$.

b. $C_{A}^{PAT} > C_{B}^{PAT}$ when $\frac{53 + 24m - 16m^2 + 3\sqrt{289 - 256m + 64m^2 + 128m^3}}{4(5 + 7m - 20m^2 + 8m^3)} < \frac{\lambda}{x_c} < \frac{1}{2(1 - m)}$.

**Proofs:**

a. There is an increase in consumer surplus (according to populist social planner) if $C_{A}^{POP} > C_{B}^{POP}$ or if

\[
\frac{2\lambda}{x_c} \left(2(2 - m) - (1 - m) \left(\frac{\lambda}{x_c}\right)^2 + 8(4(1 - m)(2m - 1) + 9) + \frac{\lambda}{x_c}(1 - m^2)\right) \left[4(1 - m)(2m - 1) + 9\right] + \frac{\lambda}{x_c} \left[36m - 16m^2 - 47\right] + 13 - 2m < 0.
\]

We set this equation equal to zero and find the following two solutions for $\lambda/x_c$:

\[
\lambda^{-} = \frac{-47 + 36m - 16m^2 - 3\sqrt{361 + 128m^3 - 232m - 176m^2}}{4(5 + 7m - 20m^2 + 8m^3)} \quad \text{and} \quad \lambda^{+} = \frac{-47 + 36m - 16m^2 + 3\sqrt{361 + 128m^3 - 232m - 176m^2}}{4(5 + 7m - 20m^2 + 8m^3)}.
\]

We retain the second solution because $\lambda/x_c$ take positive values for $m \in [0; 0.5]$. Further, we can check that for a given value of $m$, say $m'$, $\lambda/x_c (m')$ makes the inequality (a.1) hold. Further, for case A to be valid with $m \leq 0.5$, the following condition must hold: $\Delta x > 2(1 - m)\lambda$ or $\frac{\lambda}{x_c} < \frac{1}{2(1 - m)}$. Thus, for $C_{A}^{POP} > C_{B}^{POP}$ when $m$ increases, it must be the case that:

\(^{11}\) Another relevant analysis consist in evaluating the effect of a marginal increase in $m$ when the equilibrium is in case D (move from case D to case C).

\(^{12}\) $Q_f$ is greater in case B than case A, i.e., $\frac{1}{2} > \frac{x_c + B}{3x_c}$, because $m < 0.5$ when we move from case B to case A, and $x_c > 2B$. In fact, we can show that case B and D have the highest overall quantities of fad product consumed.
\[-47 + 36m - 16m^2 + 3\sqrt{361 + 128m^3 - 232m - 176m^2} \leq \frac{\lambda}{x_c} \leq \frac{1}{2(1-m)}.\]

b. There is an increase in consumer surplus (according to paternalistic social planner) if $CS_A^{PAT} > CS_B^{PAT}$ or if
\[
\frac{1}{18} \left( 2(2 - m) - (1 - m) \left( \frac{\lambda}{x_c} \right)^2 \left[ 8(1 - m^2) - 9 \right] - 4 \frac{\lambda}{x_c} (1 - m)(1 - 2m) \right) < \frac{\lambda}{x_c} < \frac{1}{2}. \]

Now we have a quadratic expression with $\lambda/x_c$ as variable that can be manipulated to obtain:
\[
\left( \frac{\lambda}{x_c} \right)^2 (1 - m)(1 + 8m^2) = \frac{\lambda}{x_c} \left[ \frac{53 - 24m + 16m^2}{2} \right] + 13 - 2m < 0. \quad (b.1)
\]

We set this equation equal to zero and find the following two solutions for $\lambda/x_c$:
\[
\frac{\lambda^-}{x_c} = \frac{-53 + 24m - 16m^2 + 2\sqrt{289 - 256m + 64m^2 + 128m^3}}{4(m - 8m^2 + 8m^3)} \quad \text{and} \quad \frac{\lambda^+}{x_c} = \frac{-53 + 24m - 16m^2 + 3\sqrt{289 - 256m + 64m^2 + 128m^3}}{4(m - 8m^2 + 8m^3)}.
\]

For both solutions, $\lambda/x_c$ take positive values for $m \in [0; 0.5]$, however, only the second solutions takes values that respect this necessary condition for case A to hold with $\Delta x > 2(1 - m)\lambda$ or $\frac{\lambda}{x_c} < \frac{1}{2(1-m)}$. Thus, we retain the second solution. Further, we can check that for a given value of $m$, say $m'$, $\frac{\lambda}{x_c}(m')$ makes the inequality (b.1) hold. Thus, for $CS_A^{PAT} > CS_B^{PAT}$ when $m$ increases, it must be the case that:
\[
\frac{-53 + 24m - 16m^2 + 2\sqrt{289 - 256m + 64m^2 + 128m^3}}{4(m - 8m^2 + 8m^3)} < \frac{\lambda}{x_c} < \frac{1}{2(1-m)}.
\]

Figure 2 summarizes the effect of an information provision intervention that marginally increases $m$ on consumer surplus. The gray curve shows the maximum value that $\lambda/x_c$ can take under case A with $m < 0.5$. The figure shows that consumer surplus, as measured by the populist social planner, increases with information provision for values of $\lambda/x_c$ between the red and the gray curve, and decreases for values of $\lambda/x_c$ below the red curve. Similarly, the consumer surplus, as measured by the paternalistic social planner, increases with information provision for values of $\lambda/x_c$ between the blue curve and the gray curve, and decreases for values of $\lambda/x_c$ below the blue curve.

We can make the following observations. First, note that for $m < 0.5$, case A holds when $\frac{\lambda}{x_c} < \frac{1}{2(1-m)}$. In this context, $\frac{\lambda}{x_c} \in [0; 1]$. In other words, the perceived health cost from the conventional product is less than the value of the difference in product quality. Thus, the results show that there is an increase in consumer surplus when the ratio between the perceived health cost and the difference in quality is high enough, i.e., greater than half. Second, as the percentage of consumers in group 1 ($m$) increases, the minimum value of $\lambda/x_c$ for an increase in consumer surplus increases. Third, when consumer surplus is measured by a paternalistic social planner a larger range of values of $\lambda/x_c$ generate an increase in consumer surplus. More specifically, the perceived health cost does not have to be as high as when consumer surplus is measured by the populist social planner.

**Conclusion**

The increased focus on better nutrition has brought along not only healthier diets but also numerous fads making questionable and sometimes outright false promises, a trend that has been largely ignored by economists so far. We study the effect these beliefs have on market outcomes such as price, product quality and welfare. In this study, we focus on negative effects associated with paying for supposed benefits that do not exist. We find that the market
underprovides quality compared to the social optimum. This underprovision is more severe when comparing laissez-faire outcomes to the paternalistic social planner than to the populist social planner.

The social planner can improve outcomes by convincing consumers that the health promises associated with the fad are inaccurate. Correcting false perceptions reduces the number of consumers who prefer the fad product and therefore mitigates the effect of overpaying and underprovision of quality. Such information provision would increase consumer surplus when the perceived health benefit of the fad product is sufficiently large but reduce consumer surplus for lower perceived health benefits. The reason for this seems to be that the underprovision of quality by the fad producer is particularly drastic when the perceived positive effect of the fad product is particularly strong. In this case, the manufacturer of the fad product is competitive even when its product’s quality is substantially below that of the conventional product. Note that some fad products are associated with adverse health outcomes, an issue we ignore. More drastic government intervention might be warranted in those circumstances.

This manuscript takes a first step at broadening the literature on nutritional health effects. We add the study of false health beliefs to this body of research that has largely dealt with consumers underestimating negative consequences of eating unhealthy foods. Our work concentrates on the effect of false beliefs rather than its origin. Future work should look into the development of fads and might be able to determine, e.g., optimal intervention times. More generally, additional research is required to further analyze and quantify the negative effects stemming from health fads.
References


Table 1. Second Stage Equilibrium Expressions.

<table>
<thead>
<tr>
<th>Conditions</th>
<th>Case A: $0 &lt; \hat{\theta}_1 &lt; \hat{\theta}_2 &lt; 1$</th>
<th>Case B: $\hat{\theta}_1 \leq 0 &lt; \hat{\theta}_2 &lt; 1$</th>
<th>Case C: $0 &lt; \hat{\theta}_1 &lt; 1 \leq \hat{\theta}_2$</th>
<th>Case D: $\hat{\theta}_1 \leq 0 &lt; 1 \leq \hat{\theta}_2$</th>
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<tbody>
<tr>
<td>$p_c$</td>
<td>$\frac{2\Delta x - B}{3}$</td>
<td>$\frac{2\Delta x - B}{3(1-m)}$</td>
<td>$\Delta x(1 + m)$</td>
<td>$p$</td>
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<tr>
<td>$p_f$</td>
<td>$\frac{\Delta x + B}{3}$</td>
<td>$\frac{\Delta x + B}{3(1-m)}$</td>
<td>$\Delta x(2 - m)$</td>
<td>$p$</td>
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<tr>
<td>$p_c - p_f$</td>
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<td>$\frac{\Delta x - 2B}{3(1-m)}$</td>
<td>$\Delta x(2m - 1)$</td>
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<tr>
<td>$q_{1c}$</td>
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<td>$(1 + m)$</td>
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</tr>
<tr>
<td>$q_{1f}$</td>
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<td>$(2m - 1)$</td>
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<td>$\frac{\Delta x + B}{3\Delta x(1 - m)}$</td>
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<td>1</td>
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<tr>
<td>$Q_c$</td>
<td>$\frac{2\Delta x - B}{3\Delta x}$</td>
<td>$\frac{2\Delta x - B}{3\Delta x}$</td>
<td>$(1 + m)$</td>
<td>$m$</td>
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<td>$Q_f$</td>
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<td>$\frac{\Delta x + B}{3\Delta x}$</td>
<td>$(2 - m)$</td>
<td>$m$</td>
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<td>$\frac{(2\Delta x - B)^2}{9(1-m)\Delta x}$</td>
<td>$\frac{\Delta x(1 + m)^2}{9m}$</td>
<td>$pm$</td>
</tr>
<tr>
<td>$\pi_f$</td>
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<td>$\frac{(\Delta x + B)^2}{9(1-m)\Delta x} \frac{1}{\Delta x}$</td>
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Conditions

- $\hat{\theta}_1 > 0$ \quad $\Delta x > 2B$ \quad $m > 0.5$
- $\hat{\theta}_1 \leq 0$ \quad $\Delta x \leq 2B$ \quad $m \leq 0.5 \rightarrow \text{can't hold}$
- $\hat{\theta}_1 < 1$ \quad Holds!
- $\hat{\theta}_2 > 0$ \quad Holds!
- $\hat{\theta}_2 < 1$ \quad $\Delta x > 0.5(3\lambda - 2B)$ \quad if $m < \frac{2}{3}$, \Delta x > $\frac{B}{2-3m}$
- $\hat{\theta}_2 < 1$ \quad $\Delta x > 0.5(3\lambda - 2B)$ \quad if $m \geq \frac{2}{3}$, \text{can't hold}$
- $\hat{\theta}_2 \geq 1$ \quad $\Delta x \leq \frac{3m\lambda}{(1 + m)}$

Summary

- $\Delta x = \max[0.5(3\lambda - 2B) ; 2B]$ \quad When $m > 0.5$, $\Delta x > 0.5(3\lambda - 2B)$ is the binding condition.
- When $m < 0.5$, $\Delta x > 2B$ is the binding condition.
- When $m = 0.5$, the two conditions are equivalent.
- If $m < \frac{2}{3}$, then $\frac{B}{2-3m} < \Delta x \leq 2B$ must be satisfied.
- If $m \geq \frac{2}{3}$, then $\frac{B}{2-3m} < \Delta x \leq 2B$ must be satisfied.

$B = (1 - m)\lambda$ \quad Weighted average belief about health cost of product c
Table 2. First Stage Equilibrium Expressions.

<table>
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<tr>
<th></th>
<th>Case A: (0 &lt; \hat{\theta}_1 &lt; \hat{\theta}_2 &lt; 1)</th>
<th>Case B: (\hat{\theta}_1 \leq 0 &lt; \hat{\theta}_2 &lt; 1)</th>
<th>Case C: (0 &lt; \hat{\theta}_1 &lt; 1 \leq \hat{\theta}_2)</th>
<th>Case D: (\hat{\theta}_1 \leq 0 &lt; 1 \leq \hat{\theta}_2)</th>
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<td>(x_c - \Delta x^*)</td>
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<td>(\frac{\lambda(2 - m)}{1 + m})</td>
<td>(\lambda)</td>
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<tr>
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<td>(\frac{1}{3}\left(\frac{\lambda(2 - m)^2 - 1 + m}{1 + m - 3m\lambda}\right))</td>
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<tr>
<td>(\hat{\theta}_2)</td>
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<td>(\frac{1}{2(1 - m)})</td>
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\(B = (1 - m)\lambda\)

Weighted average belief about health cost of product \(c\)
Table 3. Consumer welfare in equilibrium

<table>
<thead>
<tr>
<th>Case</th>
<th>Formula</th>
<th>Case A</th>
<th>Case B</th>
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<td>$x_f^*$</td>
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<td>$\Delta x^* = x_c$</td>
<td>$2v(x_c + B)$</td>
<td>$v - \lambda + \frac{x^2}{2}$</td>
<td></td>
</tr>
<tr>
<td>$\frac{v - x_c + B}{2}$</td>
<td>$\left(\frac{x_c - 2B}{3x_c}\right)$</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$\frac{v - x_c + B}{2}$</td>
<td>$\left(\frac{x_c - 2B}{3x_c}\right)$</td>
<td>$\frac{v - 2\lambda + \frac{x_c(3-2m)}{4(1-m)}}{2(1-m)}$</td>
<td></td>
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<td>$\frac{v - 2\lambda + \frac{x_c(3-2m)}{4(1-m)}}{2(1-m)}$</td>
<td></td>
</tr>
</tbody>
</table>
Figure 1. Location of each case with respect to $m$ and $\lambda$. 

\[ m \]

\[ \Delta x \]

\[ 2\Delta x \]

\[ \Delta x \]

\[ 2\Delta x \]

\[ \lambda \]

---

**Case A**

**Case B**

**Case C**

**Case D**
Figure 2. Range of Values of $\lambda/x_c$ for an Increase in Consumer Surplus