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DIVISION OF AGRICULTURE AND NATURAL RESOURCES  
UNIVERSITY OF CALIFORNIA *Berkeley*

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ON THE EXISTENCE AND OPTIMALITY OF COMPETITIVE  
EQUILIBRIA IN NONRENEWABLE RESOURCE INDUSTRIES

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Anthony C. Fisher and Larry S. Karp

California Agricultural Experiment Station  
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Anthony C. Fisher and Larry S. Karp

Department of Agricultural and Resource Economics  
University of California  
Berkeley, California 94720

If average costs in a nonrenewable resource industry are U-shaped, a competitive equilibrium may not be optimal and, indeed, may not exist. Although the differential equation that describes the change in the rate of extraction is the same for planner and firm, the boundary conditions obtained from the transversality conditions for the respective problems (for planner and firm) will not, in general, be the same. If costs are convex, or if there exists a backstop technology which can produce the resource services at sufficiently low cost, the boundary conditions are, however, the same.

**Keywords:** nonrenewable resources, competitive equilibrium, welfare optimum, transversality conditions.

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## 1. Introduction

A widely accepted folk-theorem of resource economics maintains that, in the absence of externalities or other market failures, the "socially optimal" (Pareto efficient) extraction profile is reproduced by a decentralized economy. That is, competitive resource owners extract at the same rate as the social planner. The conditions under which this conclusion is correct are more limited than is commonly recognized. The Second Theorem of welfare economics states that, given convex production and indifference sets (and the absence of externalities or other market failures), any Pareto optimal allocation can be supported as a competitive equilibrium. The folk-theorem alluded to above appears to be a special case of the Second Theorem of welfare economics.

If average costs of an industry are U-shaped, which occurs where there are fixed costs or where marginal costs first fall and then rise, there is a nonconvexity in that industry. Provided that the point of minimum average costs occurs at a sufficiently small level of production, it is still the case that the competitive industry produces the socially optimal quantity. This provides an example where local rather than global convexity is enough to insure that the social planner's allocation can be decentralized. This conclusion is reassuring, since the existence of U-shaped average costs is important both empirically and theoretically. However, this conclusion does not carry over to extractive industries. In certain cases, U-shaped average costs in these industries imply that the socially optimal extraction path cannot be reproduced as a competitive equilibrium. There is no reason to suppose that the existence of U-shaped average costs are less important in extractive industries than in other parts of the economy. Therefore, the recognition that in this circumstance a competitive

equilibrium (often) either fails to exist or is not socially optimal may have important implications for the way we think about nonrenewable resource industries.

The reason for the qualitative difference in the welfare properties between static industries and extractive industries, where both have U-shaped average costs, has a very simple explanation. We mentioned above the well-known fact that the competitive equilibrium in the static industry reproduces the socially optimal allocation provided that that level of production is greater than the level at which average costs are minimized. This is another way of saying that the industry is not a "natural monopoly." The analogous requirement is unlikely to be met in a nonrenewable resource model. Except in a special case, discussed below, where a backstop technology can produce the resource services at a sufficiently low cost, the planner wants to produce at a rate where average costs exceed marginal costs, near the final part of the extraction trajectory. Competitive firms want to cease production where average costs equal marginal costs. Consequently, the social planner's preferred trajectory and the competitive trajectory do not, in general, coincide in the case where average costs are U-shaped.

There is an additional problem which makes the existence of a competitive trajectory problematic. If all producers intend to stop producing at the point where average and marginal costs are equal, aggregate production drops from a positive level to 0. If the demand curve is negatively sloped, this causes price to jump at the time production drops to 0. This jump in price violates the price arbitrage equation which must hold in a competitive equilibrium. Therefore, if producers are identical, in the sense that they all use the same technology and have the same expectations, there cannot be a competitive equilibrium in which they all adopt the same plans.

We should note that his result depends on the micro-foundations of the industry cost curve. For example, suppose capital in the resource industry is perfectly mobile in the sense that firms costlessly enter and exit the industry. Then, as shown

by Schulze (1974), each firm produces at the point of minimum average cost as long as it is in the industry. As price rises, the decline in industry output is accomplished by the continuing exit of firms. This is a situation where firms are identical but adopt different plans in equilibrium. Here another difficulty arises, however. If firms cease to behave as price takers when there are just a few left, the competitive equilibrium discussed by Schulze breaks down. Our conjecture is that, with perfectly mobile capital, any equilibrium would not be competitive.<sup>1</sup>

Alternatively, one might assume, as we do, that capital in the resource industry is fixed, due to the presence of costs that must be incurred before extraction begins, when it ceases, and if and when it resumes. This seems more realistic. Note, by the way, that it does not require a constant output over time from each firm. Rather, each moves along its U-shaped curve. We then adopt the standard convention of an industry cost curve as the aggregation of cost curves of identical firms. This is equivalent to modeling the competitive industry as if it consisted of a single, representative firm.

Although nonrenewable resource models have been widely studied since Hotelling's (1931) classic paper, the importance of assuming that extraction costs are not U-shaped does not appear to have been explicitly recognized in the literature, with the exception of a recent unpublished manuscript by Rees (1989). Models of nonrenewable resources are usually studied using the Maximum Principle. The necessary condition for the maximization of the Hamiltonian, together with the equation of motion of the costate variable, suggest that the social planner's and the competitive equilibrium are identical under quite general circumstances. However, those two conditions only imply that the differential equation that describes the change in the rate of extraction is the same for both problems. In order for the solutions to the respective differential equations to also be the same, that is, in order for the extraction paths to be the same, it is also necessary that the respective

boundary conditions be the same. These boundary conditions are obtained from the transversality conditions to the respective maximization problems (that of the social planner and of the representative competitive firm). Our previous remarks follow from analysis of the transversality conditions.

The next section provides some detail on our assumptions about the nature of the resource industry. Section 3 analyzes the social planner's optimization problem for a general case which includes constant, increasing, and U-shaped average costs and the possibility of a backstop technology which provides a replacement to the nonrenewable resource. Section 4 considers the maximization problem of the representative firm and shows the conditions under which the solution to that problem reproduces the socially optimal extraction path. A concluding section provides some further insight into the results.

## 2. Structure of the Resource Industry: Cost and Demand

We posit a general cost function of the form

$$\left. \begin{array}{l} g + h(q) \\ 0 \end{array} \right\} \text{ for } \begin{array}{l} q > 0 \\ q = 0 \end{array}$$

where  $g$  is some constant and  $q$  is quantity (flow) of output. A  $g > 0$  implies the presence of fixed costs so that average costs are U-shaped. For  $g = 0$ , average costs are also U-shaped if  $h(q)$  is not convex near  $q = 0$ . Note that  $g = 0$  and  $h(q)$  convex imply constant or increasing average costs. Our focus is on the cases where average costs are U-shaped as these lead to the somewhat surprising results described in the introduction. Finally, we define the quantity  $q^*$  by

$$(1) \quad \frac{g + h(q^*)}{q^*} = h'(q^*);$$

that is, average and marginal costs are equal at  $q^*$ . These relationships are shown in Figure 1.

On the demand side, we assume a (planner's) utility function  $U(q)$ , with  $U'(q) \equiv p$ , where  $p$  is price. This is consistent with the usual representation of utility, or welfare, as  $U(q) = \int p(q) dq$ —the area under an inverse demand curve. Consider a price  $\bar{p}$ , which may be interpreted as the price of a substitute or backstop for the resource. Then, the quantity  $\bar{q}$  is defined by

$$\begin{aligned}
 & U'(\bar{q}) = \bar{p} \\
 (2) \quad & \text{or} \\
 & \bar{p} \bar{q} + k = U(\bar{q})
 \end{aligned}$$

where  $k$  is the area between the demand curve  $U'(q)$  and price  $\bar{p}$ , from  $q = 0$  to  $q = \bar{q}$ . These relationships are shown in Figure 2.

### 3. Optimal Depletion: The Planner's Problem

The planner's problem is assumed to be one of allocating the resource over time in such a fashion as to maximize the present value of utility, net of costs. In symbols, this is

$$\max_{\{q\}, T} \int_0^T e^{-rt} [U^0(q) - h(q)] dt - \int_0^T e^{-rt} g dt + e^{-rT} \frac{k}{r}$$

where  $r$  is the discount rate and  $U^0$  is defined by

$$U^0 = \begin{cases} U(q) & \text{for } q \geq \bar{q} \\ U(\bar{q}) - \bar{p}(\bar{q} - q) & \text{for } q < \bar{q} \end{cases}$$

subject to

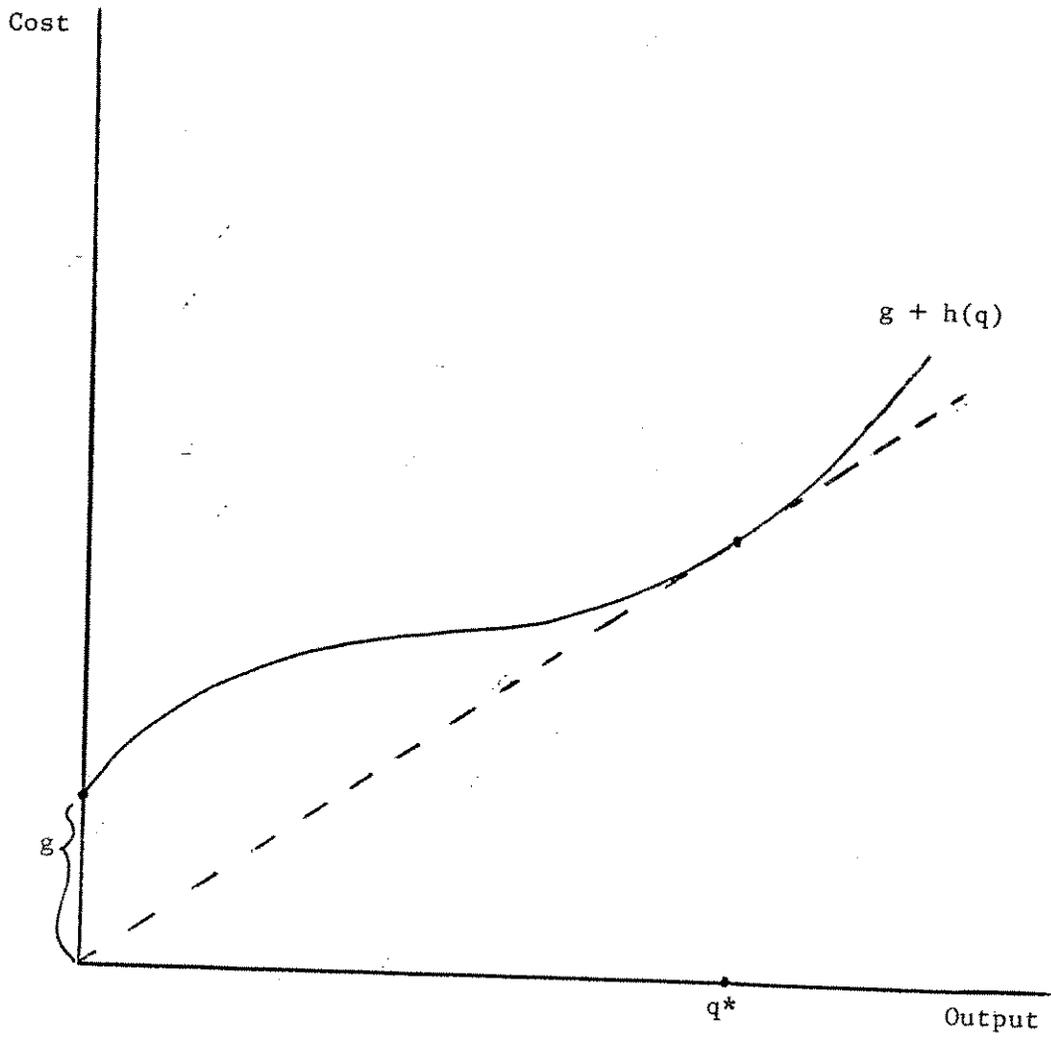


Figure 1

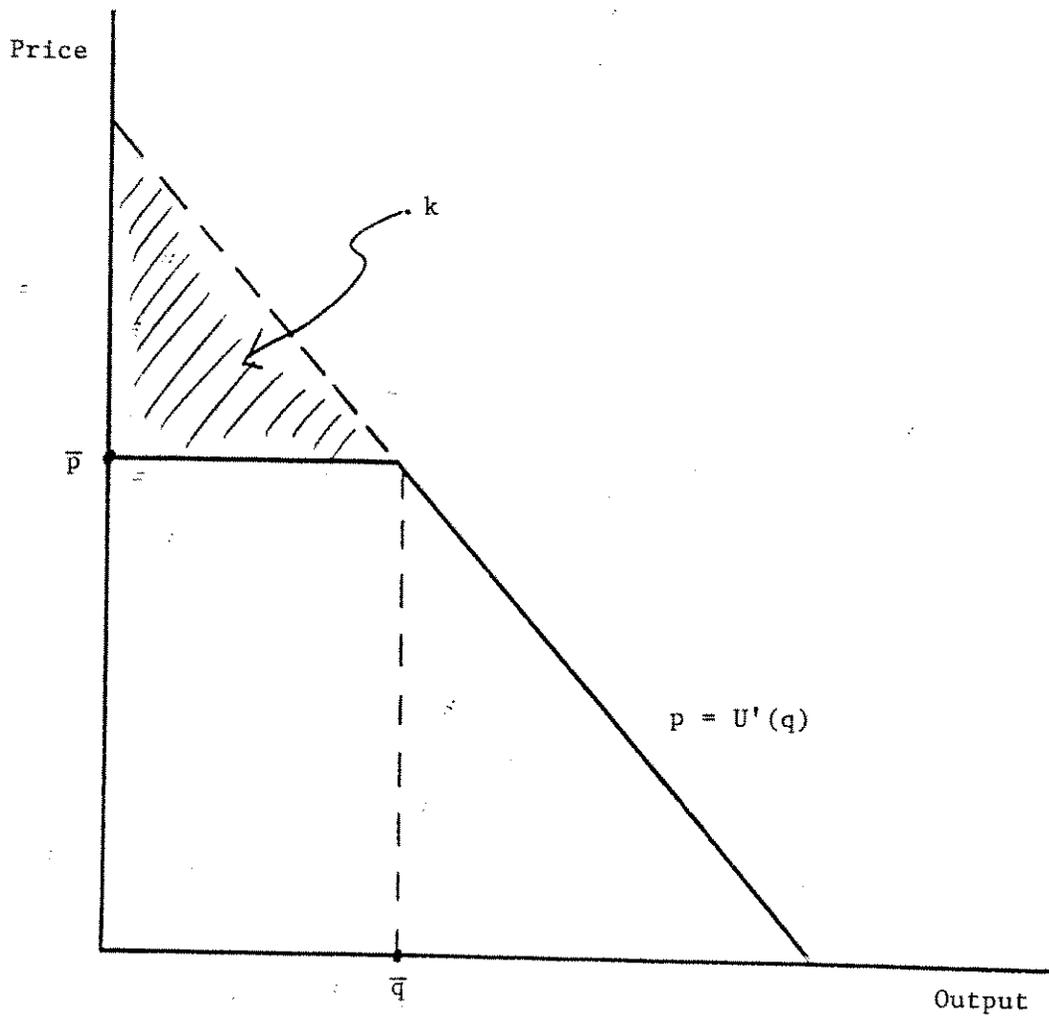


Figure 2

$$-\dot{S} = q$$

where  $S$  is the resource stock. Noting that

$$\int_0^T g e^{-rt} dt = \frac{g(1 - e^{-rT})}{r},$$

the constrained optimization problem can be rewritten as

$$\max_{\{q\}_T} \int_0^T e^{-rt} [U^0(q) - h(q)] dt + e^{-rT} \left( \frac{g+k}{r} \right) - \frac{g}{r}.$$

The (current-value) Hamiltonian for this problem is

$$H = U^0(q) - h(q) - \lambda q$$

where  $\lambda$  is the costate variable. Necessary conditions are

$$(3) \quad U^{0'}(q) - h'(q) - \lambda = 0$$

and

$$(4) \quad \dot{\lambda} = r \lambda.$$

The transversality condition, in terms of the present-value Hamiltonian, is

$$e^{-rT} [U^0(q_T) - h(q_T)] - e^{-rT} \lambda_T q_T - e^{-rT} (g+k) = 0.$$

Canceling the exponential factor in each term, we obtain

$$(5) \quad U^0(q_T) - h(q_T) - \lambda_T q_T - (g+k) = 0.$$

The second-order condition for maximization of the Hamiltonian is

$$(6) \quad U^{0''}(q) - h''(q) < 0.$$

Now, let us define

$$(7) \quad f(q) \equiv U^0(q) - h(q) - [U^{0'}(q) - h'(q)] q - (g + k),$$

with

$$(8) \quad f'(q) = -[U^{0''}(q) - h''(q)] q > 0$$

for  $q > 0$ , by equation (6). Substituting equation (3) into equation (5), and using the definition in equation (7), we obtain

$$(9) \quad f(q_T) = 0.$$

Turning now to the relationship between the quantities  $q^*$  and  $\bar{q}$  defined in equations (1) and (2), we can distinguish three possible cases:

$$(i) \quad q^* = \bar{q}$$

$$(ii) \quad q^* > \bar{q}$$

$$(iii) \quad q^* < \bar{q}.$$

We consider each in turn. For case (i), define  $\hat{q}$  by

$$q^* = \bar{q} \equiv \hat{q}.$$

Then,

$$f(\hat{q}) = [U^0(\bar{q}) - U^{0'}(\bar{q}) \bar{q} - k] - [h(q^*) - h'(q^*) q^* + g].$$

Notice that the expression in the first square brackets on the right-hand side is equal to zero, from the definition in equation (2). Similarly, the expression in the second set of square brackets is equal to zero, from the definition in equation (1). We conclude  $f(\hat{q}) = 0$  and, from equation (9),

$$q_T = q^* = \bar{q}.$$

For case (ii), we have

$$f(q^*) = U^0(q^*) - U^{0'}(q^*) q^* - k > 0$$

as shown in Figure 3. Also,

$$\begin{aligned} f(\bar{q}) &= -[h(\bar{q}) - h'(\bar{q}) \bar{q} + g] \\ &= \bar{q} \left[ h'(\bar{q}) - \frac{h(\bar{q}) + g}{\bar{q}} \right] < 0 \end{aligned}$$

as shown in Figure 4. We conclude, for case (ii), that

$$\bar{q} < q_T < q^*.$$

The planner stops producing where average costs exceed marginal costs; price jumps at T from  $p(q_T)$  to  $\bar{p}$ , as in Figure 5, suggesting that a competitive equilibrium will not reproduce the social optimum. We shall have more to say on this point in the next section, where properties of a competitive equilibrium are explicitly considered.

For case (iii), we have

$$\begin{aligned} f(q^*) &= U^0(q^*) - U^{0'}(q^*) q^* - k \\ &= U(\bar{q}) - \bar{p}(\bar{q} - q^*) - \bar{p} q^* - k = 0 \end{aligned}$$

from the definition of  $U^0$  when  $q < \bar{q}$ . Furthermore,

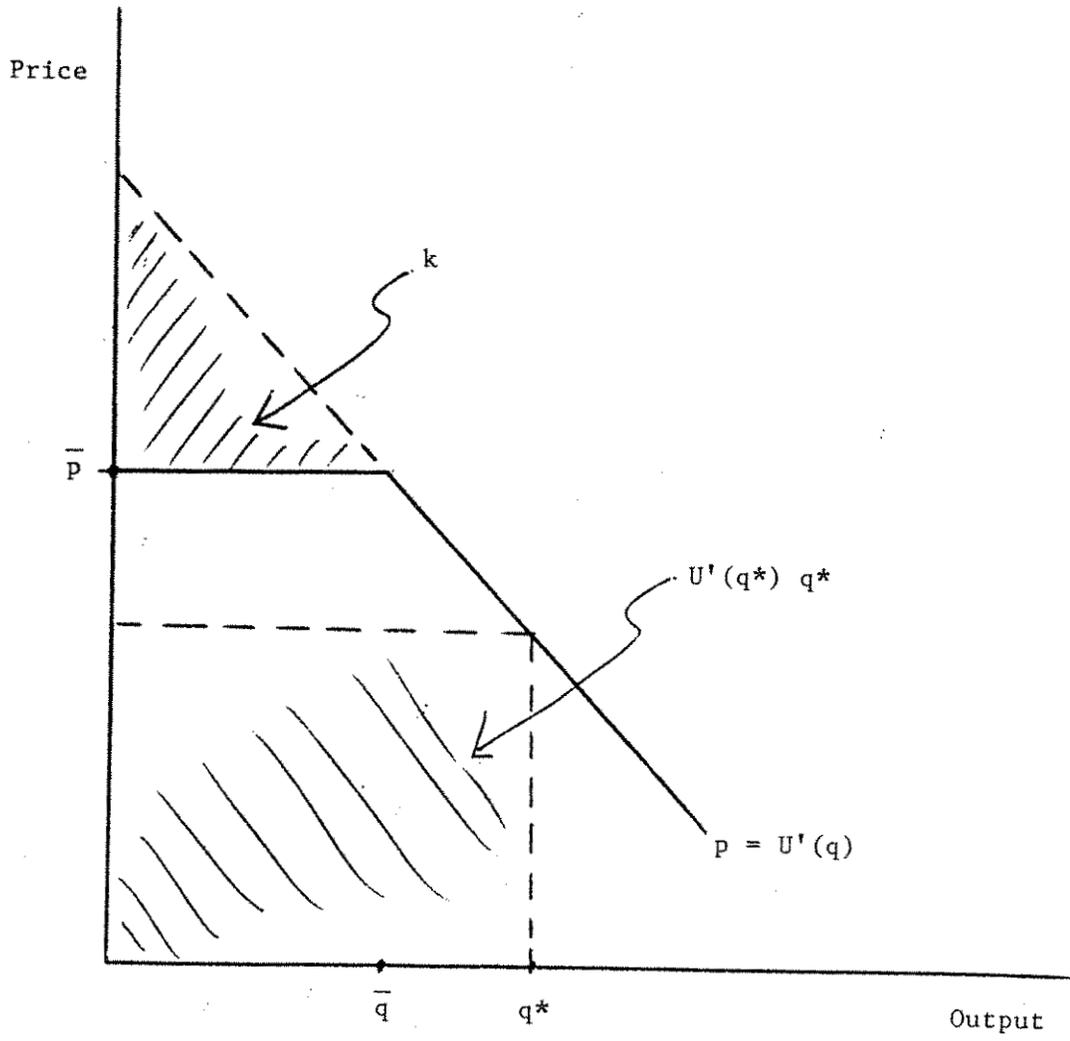


Figure 3

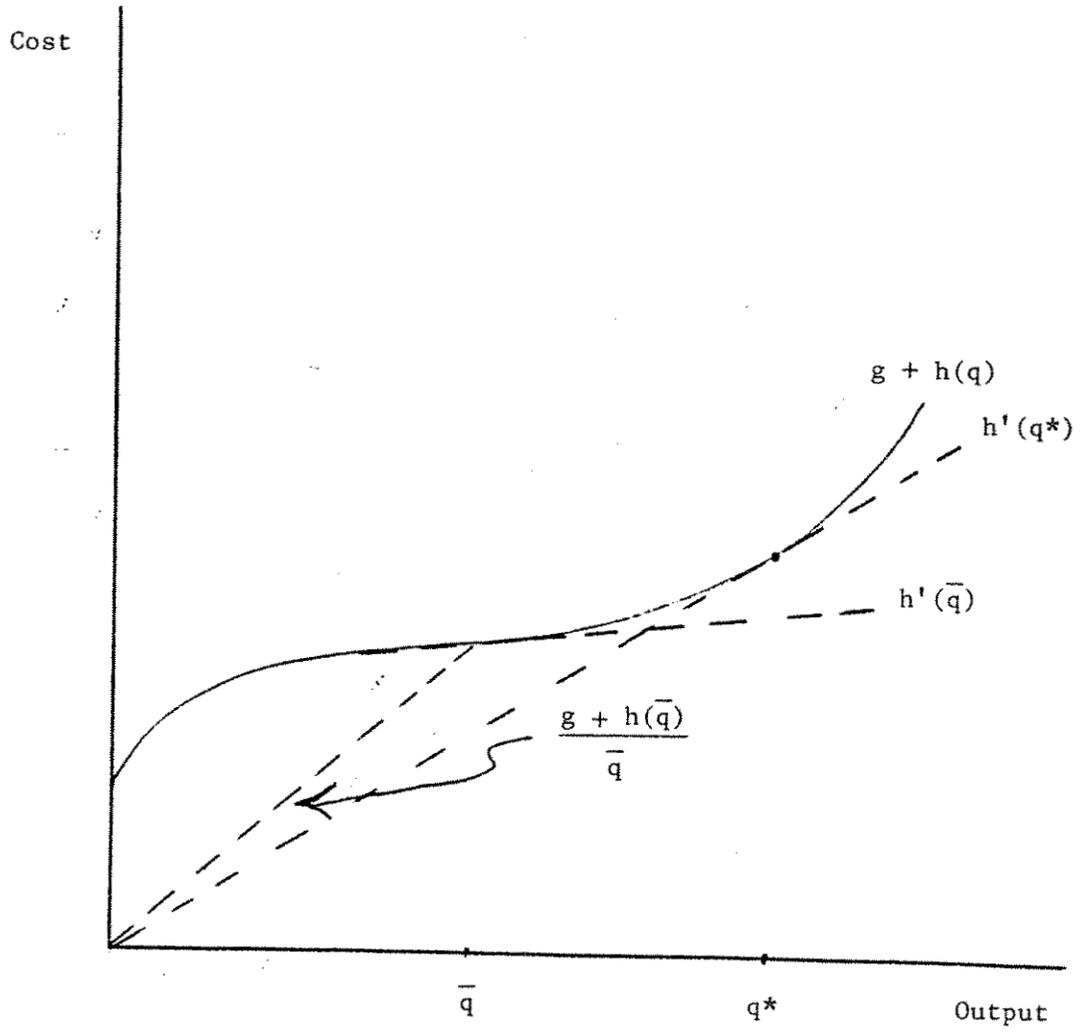


Figure 4

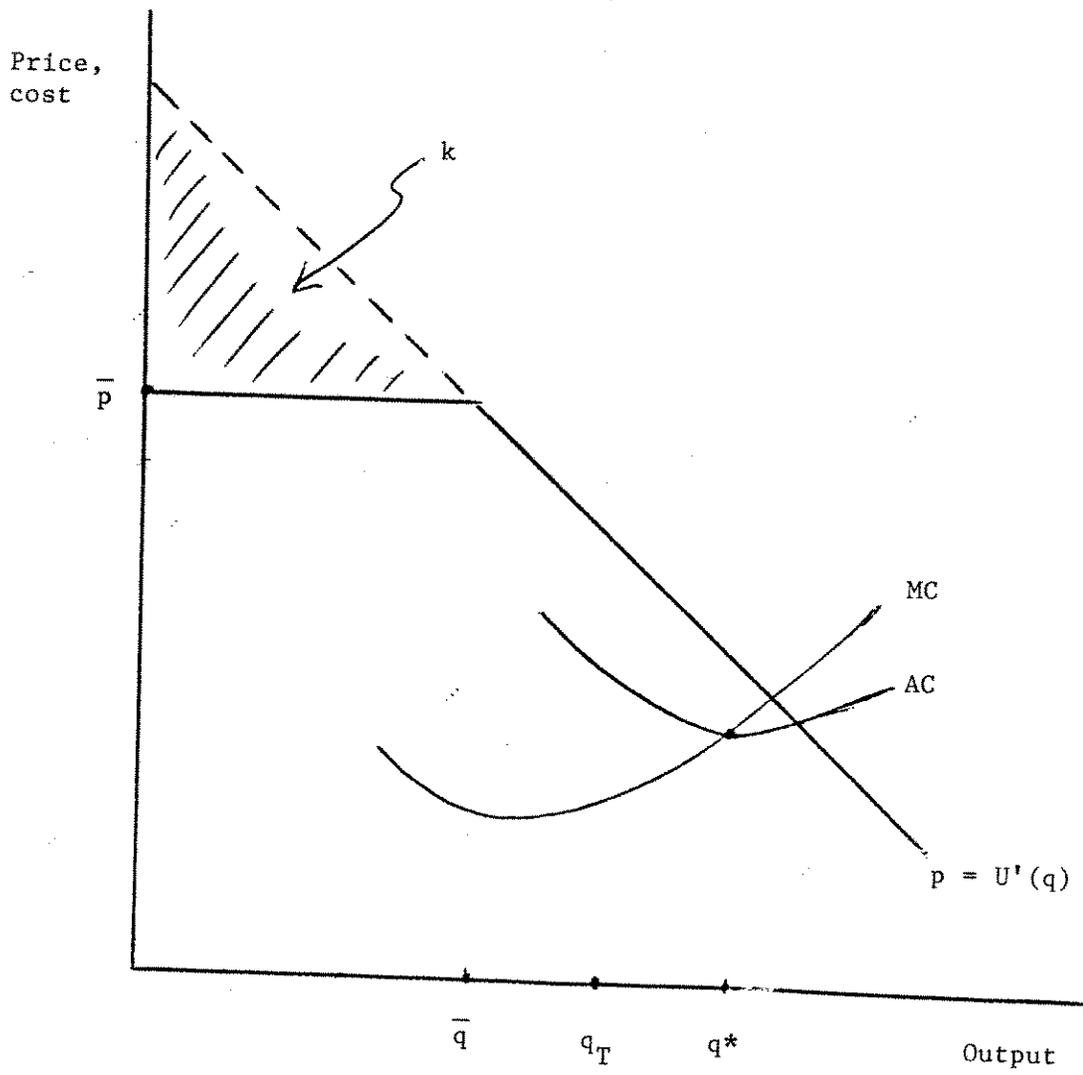


Figure 5

$$f(\bar{q}) = \bar{q} \left[ h'(\bar{q}) - \frac{h(\bar{q}) + g}{\bar{q}} \right] > 0.$$

In this case then

$$q_T = q^* < \bar{q}.$$

As in case (i), production ceases when average costs equal marginal costs (at  $q^*$ ) and price remains at  $\bar{p}$ . Note that  $q^* \leq \bar{q}$  [which includes cases (i) and (iii) but not (ii)] implies the existence of a backstop, since otherwise  $\bar{q} = 0$ . Case (ii) is also consistent with a backstop but a backstop with a relatively high cost—high enough that it does not affect the solution in that case:  $\bar{q} < q_T < q^*$  and the breakdown of a competitive equilibrium.

#### 4. Optimal Depletion: The Firm's Problem

The competitive firm's problem is assumed to be one of allocating the resource over time to maximize the present value of profits. In symbols, this is

$$\max_{(q), T} \int_0^T e^{-rt} [p_t q_t - h(q_t)] dt - \int_0^T e^{-rt} g dt$$

subject to

$$-\dot{S} = q.$$

Simplifying, the maximand becomes

$$\int_0^T e^{-rt} [p_t q_t - h(q_t)] dt + e^{-rT} \frac{g}{r} - \frac{g}{r}.$$

The Hamiltonian is

$$H = p_t q_t - h(q_t) - \lambda q_t.$$

First-order conditions are

$$(10) \quad p_t - h'(q_t) - \lambda = 0$$

and

$$(11) \quad \dot{\lambda} = r \lambda.$$

The transversality condition is

$$(12) \quad p_T q_T - h(q_T) - \lambda_T q_T - g = 0.$$

Now, let us define

$$(13) \quad y(q) \equiv p_t q_t - h(q_t) - (p_t - h'(q_t)) q_t - g.$$

Using equations (10) and (12), we obtain

$$(14) \quad y(q_T) = 0.$$

Rewriting equation (13) as

$$h'(q_t) q_t - [h(q_t) + g]$$

yields  $y(q^*) = 0$ , and we conclude

$$q_T = q^*.$$

Recall that this result, with production ceasing where average costs equal marginal costs, holds in the planner's problem if and only if  $q^* \leq \bar{q}$ . Conversely, if  $\bar{q} < q^*$ , the (identical) competitive firms do not reproduce the social optimum.

## 5. Concluding Remarks

We can get some further insight into these results by studying specializations of the model. For example, consider

case A:  $g = 0, k = \gamma$

case B:  $g = \gamma, k = 0$

where  $\gamma$  is a positive number. In A,  $\bar{q} > 0$  so that we may (though we need not) be in case (i), in which  $q^* = \bar{q}$ , or case (iii), in which  $q^* < \bar{q}$ . Recall that in both cases a competitive equilibrium exists and is optimal (in the absence of market failures not considered here). A backstop to the resource can provide the same services at sufficiently low cost that the socially optimal price trajectory does not exhibit a jump that would violate the arbitrage condition for a competitive equilibrium.

In B, on the other hand,  $\bar{q} = 0$  so that we must be in case (ii) in which  $\bar{q} < q^*$ . In this case we have shown that a competitive equilibrium does not reproduce the social optimum. No backstop can provide the resource services at a cost low enough to avoid the price jump at time T. Although the planner's problem with a fixed cost ( $g > 0$ ) or a low-cost backstop ( $k > 0$ ) is analytically the "same," the existence and optimality of a competitive equilibrium depend critically on which of these relations holds.

Note, finally, that, if  $g = 0$  and  $h(q)$  is everywhere convex, then  $q^* = 0 \leq \bar{q}$  and a competitive equilibrium has the "normal" welfare properties.

### Footnote

<sup>1</sup>Noncompetitive behavior near the final part of the trajectory, when there are only a few active firms, alters the equilibrium during the phase when there are many active firms. Therefore, even when many firms are in the industry, the extraction path may not lie close to the social planner's path.

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