Water-Storage Capacities versus Water-Use Efficiency: Substitutes or Complements?

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We investigate the economic relation between two common approaches to tackling water scarcity and adapting to climate change, namely expanding water-storage capacities and improving water-use efficiency. We build, analyze, and extend a simple model for capacity choices of dams, incorporating stochastically dynamic control of water inventories and efficiency in water use. We show that expanding water-storage capacities could encourage water users to improve water-use efficiency and improving water-use efficiency could increase optimal dam sizes even if water-use efficiency improvement decreases the water demand. The possibility of complementarity is numerically illustrated by an empirical example of the California State Water Project. Our analysis implies that, if complementarity holds, resources should be distributed in a balanced way between water-storage expansions and water-use efficiency improvement instead of being concentrated on one side with the other side being ignored.

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1 Introduction

Water scarcity is among the most important constraints limiting social and economic development throughout the world. Climate change will make the constraint even tighter: As reported by the Intergovernmental Panel on Climate Change (Jiménez Cisneros et al., 2014, p. 251), “water resources are projected to decrease in many mid-latitude and dry subtropical regions, and … even where increases are projected, there can be short-term shortages due to more variable streamflow (because of greater variability of precipitation) and seasonal reductions of water supply due to reduced snow and ice storage.”

Two technological approaches are among the most frequently considered to tackle water scarcity and adapt to climate change: One is to build dams, reservoirs, and other water projects to move water intra-annually from wet to dry seasons against seasonality of water and store water inter-annually against uncertainty of water in future years (e.g., surveys by Yeh, 1985; Simonovic, 1992). The other is to improve water-use efficiency, for example, adopting conservation technologies like drip irrigation instead of flood irrigation in agriculture (e.g. surveys by Caswell, 1991; Sunding and Zilberman, 2001; Schoengold and Zilberman, 2007) and reducing the leaking and evaporation loss in water conveyance (e.g., Chakravorty et al., 1995). An important policy question then emerges: How should limited resources be allocated between the policies that provide incentive to the two approaches? The key to the question is the economic relation between water-storage capacities and water-use efficiency: Are they substitutes or complements?

This paper investigates this relation theoretically with empirical illustrations. More specifically, we focus on two questions, which, as we shall show, are the two sides of the same coin and the answer to one of them leads to the answer to the other:

1. Will improvement in water-use efficiency increase or decrease optimal dam capacities?
2. Will larger dam capacities encourage or discourage water users to improve efficiency?

People might intuitively believe that larger dams should discourage water-use efficiency improvement and that the improvement should make larger dams less desirable (e.g., the World Wide Fund for Nature, 2014; Beard, 2015). In other words, water-use efficiency and water-storage capacities should be substitutes. We shall prove analytically that water-use efficiency and water-storage capacities could be complements—larger dams could encourage water users to adopt more efficient technologies and the adoption could lead to more investment in water-storage capacities—and the relation depends on purposes of dams: First, if the main purpose of the dam is to cope with intra-annual seasonality of water, then complementarity will appear when water-use efficiency improvement increases the water demand. Second, if the dam is also designed to manage with inter-annual
uncertainty of water, then complementarity could still appear even when water-use efficiency improvement does decrease the water demand.

Behind the second, probably more counterintuitive result lies a subtle but intuitive mechanism: If the marginal productivity of effective water declines fast and the decline does not get much slower as effective water increases, then water-use efficiency improvement will decrease the water demand, but it will also increase the future probability of the dam reaching the full capacity, because it will increase the optimal amount of water stored for the future. The increase in the future probability of the dam reaching the full capacity could eventually raise the marginal benefit and optimal choices of dam capacities. This mechanism would never be uncovered if stochastic, dynamic control of water inventories were not considered.

The first result about dams against intra-annual seasonality is derived from a minimalistic single-period, two-season, deterministic model for capacity choices of dams, incorporating efficiency in water use, while holding site selection (e.g., Bıçak et al., 2002; the International Commission on Large Dams, 2007) and other important issues in dam design (e.g., the International Union for Conservation of Nature and World Bank, 1997; Hurwitz, 2014) constant. We then extend the model to a two-period, stochastic model with dynamic management of water inventories to derive the second result about dams against also inter-annual uncertainty. We then confirm the insight and analysis in the two-period, stochastic model by further considering an infinite planning horizon and proving parallel results.

To illustrate the analytical results, we specify the infinite-horizon, stochastic model to the irrigation water-inventory management problem of the California State Water Project. According to the California Department of Water Resources (1963–2013), in 2010, the Project is “the largest state-built, multipurpose, user-financed water project” in the United States and its water benefits “approximately 25 million of California’s estimated 37 million residents” and “irrigates about 750000 acres of farmland.” The significance of the Project in American and global agriculture establishes the practical significance of our analysis. Illustrations confirm the empirical relevance of our theoretical results and the existence of complementarity between dam capacities and water-use efficiency improvement in the particular case. In the illustrations, the complementarity is more prominent in the positive impact of water-use efficiency improvement on water-storage expansions, but not the other way around.

**Contribution to policy debates and literature.** Our analysis directly contributes to a lasting and important debate about water-infrastructure investment in water-resource management. Dams, reservoirs, and other water storage facilities have been an important contributor to human civilizations. They have been providing huge benefits in the agricultural, energy, and urban sectors, but
frequently accompanied with huge environmental, ecological, social, and economic cost. Without fully recognizing these costs, dams have been overbuilt, causing major struggles across the world, e.g., in the western United States, as depicted in Reisner’s (1993)’s *Cadillac Desert*. Improving water-use efficiency is then increasingly perceived as an important alternative to dam building. The cost-benefit analysis method and the symbol of its success in practice, the United States Water Resources Council’s (1983)’s *Principles and Guidelines* for the United States Army Corps of Engineers, have also been criticized as often overemphasizing structural measures but overlooking the alternative approach (e.g., Zilberman et al., 1994; the World Commission on Dams, 2000). Some scholars further advise policymakers not to build large dams (e.g., Ansar et al., 2014). Our analysis suggests that the evolution of ideas should be met with a balanced management of dam building and water-use efficiency improvement instead of overlooking either approach, especially when complementarity holds.

More specifically, our paper would provide implications for many major water-policy debates, because these debates are usually about fierce competition between the policies favoring water-project expansions and the policies encouraging water-use efficiency improvement. For example, in response to the devastating drought since 2012 in the western United States and, especially, California, lawmakers have been working at both federal and state levels to authorize and fund expansions of water-project capacities, as many studies have documented huge benefit from water-project capacities in reducing the drought impact in the area (e.g., Hansen et al., 2011, 2014, Howitt et al., 2011; Zilberman et al., 2011). People who are not fans of new dams and reservoirs, however, think that money should be spent only to subsidize recycling projects and conservation-technology adoption, as they believe that the efficiency improvement will have smaller and fewer dams demanded and that dam expansions would severely discourage conservation effort. Our analysis

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1. Dams have turned deserts in California’s Central Valley into one of the most productive agricultural regions in the world, have survived large cities in Northern China like Beijing through the periodic droughts in the area, and, in Reisner (1993, p. 162–164)’s words, have produced “American hydroelectric capacity that could turn out sixty thousand aircraft in four years,” which “simply outproduced” the Axis and helped the Allies win the Second World War. The benefits are not costless. For example, when dams are built, the natural environment is seriously altered, in many cases irreversibly, and the salmon and other aquatic species are endangered. Sometimes numerous families are displaced and historic and cultural sites are covered. Huge potential loss associated with the dam failure risk is also created. For the recent debate on the cost-benefit accounting about large dams, see Ansar et al. (2014), Nombre (2014).

2. For the politics about the United States Central Valley Project Improvement Act of 1992, see Fischhendler and Zilberman (2005). For the controversy about China’s Three Gorges Dam, see Jackson and Sleigh (2000). For the disappointment of dams in India, see McCully (2001) and Dufo and Pandel (2007).


4. Goodhue and Martin (2014) and Howitt et al. (2014, 2015) present estimates of the loss caused by the drought. As a result of the drought, in January 2014, the California Department of Water Resources announced the first zero water allocation from the California State Water Project in the Project’s 54-year history. In April 2015, the Governor of California, Jerry Brown, directed the first ever statewide mandatory water reductions. For an example of media coverage on the severeness of the drought, see Serna (2014) and Walton (2015).

5. For examples of the debate, see Calefati (2014), Dunning and Machtinger (2014), and Hanson (2015).
implies, however, that resources should be distributed in a balanced way between dam expansions and water-use efficiency improvement, instead of being concentrated on either side.

This paper connects the literature about water-infrastructure investment, water-inventory management, and conservation-technology adoption. To our knowledge, our two-period, stochastic model is the first capacity-choice model for water projects in literature (e.g., Miltz and White, 1987; Tsur, 1990; Fisher and Rubio, 1997; Schoengold and Zilberman, 2007; Haddad, 2011; Houba et al., 2014; Xie and Zilberman, 2014) to incorporate water-use efficiency and stochastically, dynamic control of water inventories, which allows us to answer the two questions. We are therefore the first to identify water-use efficiency as a potential factor affecting water-storage investment. The results from our infinite-horizon, stochastic model are the first analytical comparative statics about the marginal benefit of storage capacities for the whole equilibrium in the economic literature of dynamic, sometimes stochastically, control of inventories of water (e.g., Burt, 1964; Riley and Scherer, 1979; Gisser and Sánchez, 1980; Dudley and Musgrave, 1988; Tsur and Graham-Tomasi, 1991; Fisher and Rubio, 1997; Chatterjee et al., 1998; Truong, 2012) and other storable commodities (e.g., Williams and Wright, 1991). Our analysis also adds water-storage capacities to the long list of potential factors affecting conservation-technology adoption in literature (e.g., David, 1975; Caswell and Zilberman, 1986; Caswell et al., 1990; Dinar and Yaron, 1992; Dinar et al., 1992; Shah et al., 1995; Green et al., 1996; Khanna and Zilberman, 1997; Carey and Zilberman, 2002; Koundouri et al., 2006; Baerenklau and Knapp, 2007; Schoengold and Suding, 2014). Apparently, few studies have discussed implications of water-storage capacities though large dams and reservoirs usually affect a large number of water users.

Both sides of our complementarity result are further related to resource economics in a broader perspective. On the one hand, the positive impact of water-storage capacities on water-use efficiency improvement is linked to the literature on underinvestment in efficiency improvement of energy and other resource use (e.g., surveys by Jaffe and Stavins, 1994; Jaffe et al., 2004; Gillingham et al., 2009; Linares and Labandeira, 2010; Allcott and Greenstone, 2012; Gerarden et al., 2015a,b). Our result adds underinvestment in storage of resources to the list of potential factors inducing underinvestment in resource-use efficiency. On the other hand, the positive impact of water-use efficiency on optimal water-storage capacities is also related to the rebound effect, which has other names, such as the Jevons (1865) paradox and the Khazzoom (1980)–Brookes (1992) postulate. In the literature, a positive rebound effect on resource use could offset the resourcesaving effect of efficiency improvement in energy use (e.g., Chan and Gillingham, 2015; surveys by Greening et al., 2000; Alcott, 2005; Hertwich, 2005; Sorrell, 2009) and water use (e.g., Ward and Pulido-Velazquez, 2008; Pfeiffer and Lin, 2014; Cobourn, 2015). We extend the literature by showing that efficiency improvement could still increase the demand for storage investment even if it decreases the temporary demand for resource use.
Last, but not least, our results could generate some counterintuitive implications about environmental concern, conservation effort, infrastructure investment, and conservation outcomes, contributing to a rich body of literature on the relation between infrastructure investment and resource conservation (e.g., on roads and deforestation, Chomitz and Gray, 1996; Nelson and Hellerstein, 1997; Pfaf, 1999; Cropper et al., 2001; Deng et al., 2011; on roads and groundwater depletion, Chakravorty et al., 2015). On the one hand, increasing concern about environmental externality would lead to smaller dams, which, when water-storage capacities and water-use efficiency are complements, would discourage water users from adopting conservation technologies. Given that the function of the benefit from consumptive use of water is weakly increasing, the smaller dams and less conservation effort, however, will eventually decrease consumptive use of water, which is a positive outcome of conservation. On the other hand, huge progress of adoption of more-efficient irrigation technologies has been made and still has potential to continue across the world (e.g., Postel, 2013), but the conservation effort could increase the demand for water-storage investment and will eventually increase consumptive use of water and environmental damage, even though both of the outcomes are optimal from the efficiency perspective that takes market and environment considerations into account. This implication is consistent with and more than the general agreement among water economists that adoption of efficient irrigation technologies usually leads higher consumptive use of water (e.g., the International Water Resource Economics Consortium, 2014).

The paper is unfolded as follows. Section 2 builds and analyzes the single-period, two-season, deterministic model for dams against intra-annual seasonality. Section 3 establishes the two-period, stochastic model for dams against also inter-annual uncertainty and derives the main results of this paper. Section 4 extends the two-period, stochastic model with an infinite horizon. Section 5 shows the numerical illustrations and Section 6 concludes the paper.

2 Water-Storage Capacities against Intra-Annual Seasonality

Recall that we consider two main purposes of dams—against intra-annual seasonality and inter-annual uncertainty. We start in this section with a simple model for capacity choices of dams only with the first purpose—to transfer water from a wet season to a dry season. The model has two stages. As illustrated by Figure 1, in the second stage, there is only one period with two seasons. In the first, wet season, given the amount of water availability, \( a_0 \leq 0 \), and the dam capacity, \( \bar{a} \geq 0 \), the dam captures water as much as its capacity allows. In the second, dry season, there is no water added to the dam, and the dam releases all the captured water, \( w_0 = \min \{ a_0, \bar{a} \} \geq 0 \), into a distribution and allocation system. The water release, \( w_0 \), generates the benefit of \( B(w_0, \alpha) \), which
is a function in the water release and a parameter of water-use efficiency, \( \alpha \in (0,1) \). Note that we have introduced the two key parameters for our purpose—the dam capacity, \( \bar{a} \), and water-use efficiency, \( \alpha \).

We further assume the function of water benefit, \( B(w, \alpha) \equiv B(\alpha w) \), where \( B(\cdot) \) is the benefit generated by effective water. In other words, \( \alpha \) measures input efficiency—the proportion of applied water that is effectively used. Adopting more-efficient irrigation technologies or better conveyance technologies would then increase \( \alpha \). This assumption follows the idea of Caswell and Zilberman (1986) that modern irrigation technologies increase water-use efficiency. To make the model solvable, we also assume \( B''(\cdot) < 0 \) and \( 0 < B'(\alpha \min \{a_0, \bar{a}\}) < \infty \). It is also important to recall that the marginal benefit of or inverse demand for water is \( B_1(w, \alpha) \equiv \alpha B'(\alpha w) \), and the impact of higher water-use efficiency on it is \( B_{12}(w, \alpha) \equiv B'(\alpha w) + \alpha w B''(\alpha w) \). Therefore, higher water-use efficiency will decrease the (inverse) demand for water if and only if the marginal productivity of effective water, \( B'(\alpha w) \), declines sufficiently fast. Also, if the decline does not get much slower as effective water increases, the negative impact of higher water-use efficiency on the (inverse) demand for water will be larger for larger water use.

The dam generated value given the dam capacity, initial water availability, and water-use efficiency is then

\[
V^*(\bar{a}, a_0, \alpha) \equiv B(w_0, \alpha) \quad \text{s.t.} \quad w_0 = \min \{a_0, \bar{a}\} ,
\]

or simply

\[
V^*(\bar{a}, a_0, \alpha) = B(\min \{a_0, \bar{a}\} , \alpha).
\]

At the first stage of the model, the dam designer is maximizing the dam generated value, \( V^*(\bar{a}, a_0, \alpha) \), net of the construction and maintenance cost, \( C(\bar{a}) \), and the environmental-damage cost, \( D(\bar{a}) \), by choosing the dam capacity. Readers could also consider the decision to be how the policymaker should adjust the total water-storage capacity of a huge water system by introducing a new dam or removing an old dam. Especially for large dams, the construction cost should also include social cost, for example, displacement of residents and demolishing of historical and cultural sites. The environmental-damage cost should also include the opportunity cost of the water that is captured by the dam and would be used instead for other environmental and ecological purposes, for example, surviving aquatic species, in the form of overflows. The marginal-cost functions

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\(^6\)Chakravorty et al. (1995, 2009) have discussed the optimal design of the distribution and allocation system. As the economics of the distribution and allocation system is not our paper’s main focus, we leave the functioning of the system out of the model. The function \( B(\cdot, \cdot) \) can include agricultural, industrial, and environmental benefit and any other outcomes of the dams that depend on water storage or release, e.g., drought relief and flood control. For a general description of the various benefit generated by dams, see the World Commission on Dams (2000). The function of the benefit of water release has already accounted for any downstream economic distortions.
are assumed positive and increasing, which means that $C'(\cdot) > 0$, $C''(\cdot) > 0$, $D'(\cdot) > 0$, and $D''(\cdot) > 0$.\footnote{The assumption is not too unrealistic, since the resource for dam building and maintenance is always limited. As larger dams make the ecological system more vulnerable to further human actions, it is fair to assume an increasing marginal environmental-damage cost. Furthermore, the assumption makes the dam-capacity problem have solutions.}

We then model the capacity choice of the dam as follows:

$$
\max_{\bar{a} \geq 0} V^*(\bar{a}, a_0, \alpha) - C(\bar{a}) - D(\bar{a}).
$$

(3)

The first-order condition is then

$$
V_1^*(\bar{a}^*, a_0, \alpha) = C'(\bar{a}^*) + D'(\bar{a}^*).
$$

(4)

The left-hand side of the condition is the marginal benefit of dam capacities, while the right-hand side is the marginal cost of dam capacities. To avoid triviality, we rule out the cases in which the first-order condition does not have any solution or has only negative solutions. The solution of the first-order condition, $\bar{a}^*$, is then the optimal dam capacity.

The condition suggests that $\bar{a}^*$ should make the marginal benefit and the marginal cost of dam capacities intersect with each other. It also hints that the key to the economic analysis of optimal dam capacities is to investigate the property of the marginal benefit of dam capacities, $V_1^*(\bar{a}, a_0, \alpha)$, since any shifts, rotations, or other changes in the marginal benefit will move the intersection between the marginal benefit and the marginal cost, which means that the optimal dam capacity changes.\footnote{Readers might want to think $a_0 = 0$ as there is no water in the dam when the dam is built. Readers can also think $a_0 = e_0$, the inflow into the dam in the first season. The difference between the interpretations is minor in our analysis. For simplicity, we leave $a_0$ in the dam generated value function without specifying it.}

The marginal benefit of dam capacities is

$$
V_1^*(\bar{a}, a_0, \alpha) = \frac{dB(\min \{a_0, \bar{a}\}, \alpha)}{d\bar{a}} = I_{a_0 > \bar{a}} \cdot B_1(\bar{a}, \alpha),
$$

(5)

where $I$ represents an indicator function. This expression carries important intuition. Considering only the purpose of dams in transferring water between seasons, the marginal benefit of dam capacities will always be zero, if the dam capacity constraint is not binding and there is no spill ($a_0 \leq \bar{a}$). When the dam capacity is binding and there is spill ($a_0 > \bar{a}$), one additional unit of dam capacities allows the dam to capture one additional unit of water in the wet season and generate benefit in the dry season. The marginal benefit of dam capacities is then the marginal benefit of water in the dry season, when $w_0 = \bar{a}$. 
Recall the two questions that we have asked: 1) Will water-use efficiency improvement increase or decrease the optimal dam capacity? 2) Will larger dam capacities increase or decrease the incentive for water users to improve the efficiency? The questions are different, but the key to them is eventually the same: It is the sign of the cross partial derivative (CPD) of the dam generated value with respect to dam capacities and water-use efficiency, \( V_{13}^*(\bar{a}, a_0, \alpha) \). To see this point, on the one hand, about Question 1, the sign of the CPD tells whether an increase in water-use efficiency will shift the marginal benefit of dam capacities up or down and whether the dam designer should choose larger or smaller dam capacities. On the other hand, about Question 2, given a dam capacity, if the representative water user can choose whether to improve water-use efficiency, the program will be

\[
\max_{\alpha \in (0,1)} V^*(\bar{a}, a_0, \alpha) - G(\alpha),
\]

where \( G(\alpha) \) is an increasing, convex function, representing the cost of the technology or investment by which the water user can make water-use efficiency reach \( \alpha \). The first-order condition of the program is then

\[
V^*_3(\bar{a}, a_0, \alpha^*) = G'(\alpha^*).
\]

The left-hand side is the marginal contribution of water-use efficiency to the dam generated value, and the right-hand side is the marginal cost of water-use efficiency improvement. Assuming interior solutions, the water user’s optimal choice of water-use efficiency, \( \alpha^* \), should make the marginal contribution and the marginal cost intersect with each other. Therefore, the sign of the CPD, \( V_{13}^*(\bar{a}, a_0, \alpha) \), tells whether a larger dam capacity will shift the marginal contribution of water-use efficiency up or down and whether the water user should choose higher or lower water-use efficiency.

As the two questions have the same key, we should focus on the sign of the CPD to answer both questions. When it is positive, water-use efficiency improvement will increase the optimal dam capacity and larger dam capacities will increase the incentive for the improvement. In this case, we can state that there is complementarity between dam capacities and water-use efficiency. When the CPD is negative, both questions will have the opposite answers. In this case, we can state that dam capacities and water-use efficiency are substitutes.

The CPD turns out to be

\[
V_{13}^*(\bar{a}, a_0, \alpha) = \frac{d(I_{a_0 > \bar{a}} \cdot B_1(\bar{a}, \alpha))}{d\alpha} = I_{a_0 > \bar{a}} \cdot B_{12}(\bar{a}, \alpha).
\]

Its sign is determined by the sign of \( B_{12}(\bar{a}, \alpha) \). We then have the following proposition.

**Proposition 1.** Considering only the purpose of dams against intra-annual seasonality, dam capacities and water-use efficiency will be complements if and only if water-use efficiency improvement
increases the marginal benefit of water (or inverse demand for water) in water use. More precisely, 
\( V'_{13}(\bar{a}, a_0, \alpha) \geq 0 \) if and only if \( B_{12}(\bar{a}, \alpha) \geq 0 \).

Proposition 1 is straightforward. Intuitively, if water-use efficiency improvement increases the 
marginal benefit of water in water use, then the additional water that is captured by the additional 
dam capacity will generate larger benefit, so the marginal benefit of dam capacities will increase, 
which will suggest a larger optimal choice of dam capacities, given any marginal cost function of 
dam capacities. As we have a well-behaved problem, larger dam capacities will also increase the 
incentive of water-use efficiency improvement.

Proposition 1 emphasizes the impact of water-use efficiency improvement on the marginal ben-
efit of water (or inverse demand for water) in water use. Which characteristic of the water benefit 
is determining the impact? As mentioned earlier, if the marginal productivity of effective water 
declines slow as effective water increases, then water-use efficiency improvement will increase the 
inverse demand for water. In other words, the slope of the downward-sloping marginal productivity 
is flat. Mathematically, we have

\[
B_{12}(w, \alpha) = \frac{d^2B(\alpha w)}{d\alpha dw} = B'(\alpha w) + \alpha w B''(\alpha w). 
\]

Therefore, \( B_{12}(w, \alpha) \geq 0 \) is equivalent to

\[
\text{EMP} \equiv -\frac{\alpha w B''(\alpha w)}{B'(\alpha w)} \leq 1, 
\]

where EMP represents the elasticity of the marginal productivity of effective water. We document 
this result as a corollary.

**Corollary 1.** *Considering only the purpose of dams against intra-annual seasonality, dam ca-
pacities and water-use efficiency will be complements if and only if the elasticity of the marginal 
productivity (EMP) of effective water is smaller than one.*

Proposition 1 and Corollary 1 contribute to discussion on the economic relation between water-
project capacities and water-use efficiency. Intuitively, it is tempting to conclude that water-project 
capacities and water-use efficiency should always be substitutes: If water-use efficiency is higher, 
water users seem to be less urgent for water-supply and water-project capacities. If water-project 
capacities are larger and water supply is always abundant, water users seem to have less incentive to 
 improve water-use efficiency. Many studies, however, recognize that the relation between water use 
or availability and conservation should depend on the EMP and will be complementary if EMP < 1. 
This recognition is first modeled in *Caswell and Zilberman* (1986) and well noted in other studies 
(e.g., surveys by *Feder and Umali*, 1993; *Lichtenberg*, 2002). The potential complementarity has
also been shown empirically relevant in studies by Peterson and Ding (2005), Ward and Pulido-Velazquez (2008), Pfeiffer and Lin (2014), and Cobourn (2015) among others. Together with Xie and Zilberman (2014), Proposition 1 and Corollary 1 extend the relation to water projects only against intra-annual seasonality.

As Vaux et al. (1981) recognize, the isoelastic and the linear water demands are convenient in econometric studies and influential in policy related researches. As Caswell and Zilberman (1986) recognize, linear water demands are also consistent with the classical three-stage model of the marginal productivity of water. We then apply Proposition 1 and Corollary 1 to the two important specifications of the water demand.

**Corollary 2.** Consider only the purpose of dams against intra-annual seasonality:

- When the water demand is isoelastic, dam capacities and water-use efficiency will be complements if and only if the water demand is elastic.

- When the water demand is linear, dam capacities and water-use efficiency will be complements if and only if the existing dam capacity is sufficiently small. More precisely, there exists a critical level of dam capacities, \( \hat{a} \) such that \( V_{13}(\hat{a}, a_0, \alpha) \geq 0 \) if and only if \( \hat{a} < \hat{a} \), where \( \hat{a} \) solves \( \frac{\alpha \hat{a} B''(\hat{a})}{B'(\hat{a})} = 1 \).

The intuition of Corollary 2 is simple. The result about isoelastic water demands follows the classic result in water resource economics that water-use efficiency improvement will increase the inverse demand for water if and only if the isoelastic water demand is elastic. For linear water demands, another classic result indicates that water-use efficiency improvement will increase the inverse demand for water if and only if the initial water use is small. As the initial water use in our problem is equal to the dam capacity, the result about linear water demands follows.

All of the results in this section suggest that, if we only consider the purpose of dams against intra-annual seasonality, whether dam capacities and water-use efficiency are complements or substitutes is exactly determined by the first-order impact of water-use efficiency on the water demand. A natural question follows: What factors are affecting whether water-use efficiency improvement will increase or decrease the marginal benefit of water or inverse demand for water, or, equivalently, whether the marginal productivity of effective water decreases fast or slow in water use?

Two factors deserve special attentions. The first is land constraints—it is easy to expect that the marginal productivity of effective water should decline much slower, when irrigable land is not constrained and irrigators can expand planted areas, than it does when irrigators have to exploit the constrained irrigable land. This factor could be important in the developing world. The second

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9For studies with similar results but in a more hydrological perspective, see Scheierling et al. (2006b) and Huffaker and Whittlesey (2003).
factor is the stage of the development of water resources. In areas like Western Europe and India where water resources have already been exploited by infrastructure investments (e.g., Shah and Kumar, 2008; Hasanain et al., 2013), it is likely that water-use efficiency improvement will decrease the inverse demand for water use and, therefore, lead to smaller water projects against intra-annual seasonality. For areas like sub-Saharan Africa where agriculture is still mainly fed by rain (e.g., Kadigi et al., 2013), the opposite is more likely to hold. Some scholars have already been seeing that, given unconstrained irrigable areas and small initial water-catchment and storage capacities, adoption of efficient-irrigation technologies is increasing the demand for water and the demand for water-storage projects (e.g., about Xinjiang, a major area of irrigated agriculture in China, Xu, 2015).

3 Water-Storage Capacities against Intra-Annual Seasonality and Inter-Annual Uncertainty

Not only overcoming intra-annual seasonality, dams usually also transfer water from wet years to dry years to overcome uncertainty in inter-annual variation. In this section we extend the model to two periods with stochastic inflows in the second period and optimal management of water inventories between the two years. We shall show how new intuition is added and how the earlier results are altered.

As illustrated by Figure 2, we assume that there are two periods, 0 and 1. In period 0, given the amount of water availability, \( a_0 \leq 0 \), and the dam capacity, \( \bar{a} \geq 0 \), the dam still captures water as much as its capacity allows, but the dam operator can store and carry some water, \( s_0 \geq 0 \), to carry to period 1, instead of release all the captured water. The water release is then \( w_0 = \min\{a_0, \bar{a}\} - s_0 \geq 0 \). For clarification, we call \( s_0 \) the water storage and \( \bar{a} \) the dam or water-storage capacities. In period 1, the water availability, \( a_1 \), is equal to the stochastic inflow to the dam in period 1, \( e_1 \in [\underline{e}, \bar{e}] \), where \( \underline{e} > 0 \), plus the water storage carried from period 0 net of evaporation, \( (1 - d)s_0 \in [0, s_0] \). Given the dam capacity, \( \bar{a} \), the dam can capture \( \min\{a_1, \bar{a}\} \geq 0 \), and the operator will release all the captured water, \( w_1 \equiv \min\{a_1, \bar{a}\} \). The water release, \( w_t \) with \( t \in \{0, 1\} \), generates the benefit of \( B(w_t, \alpha) \equiv B(\alpha w_t) \), where \( B''(x) < 0 \) and \( 0 < B'(\cdot) < \infty \).

We then have a stochastic, dynamic control problem of water-inventory management,

\[
V^*(\bar{a}, a_0, \alpha) = \max_{w_0, s_0} \left\{ B(w_0, \alpha) + \rho E_0 [B(w_1, \alpha)] \right\} \quad \text{s.t.} \quad (11)
\]

\[
s_0 \geq 0, \quad w_0 = \min\{a_0, \bar{a}\} - s_0 \geq 0, \quad a_1 = (1 - d)s_0 + e_1, \quad w_1 = \min\{a_1, \bar{a}\}, \quad (12)
\]
where $\rho$ is the discount factor. The problem is equivalent to

$$V^*(\bar{a}, a_0, \alpha) \equiv \max_{s_0} \left\{ B(\min\{a_0, \bar{a}\} - s_0, \alpha) + \rho \mathbf{E}_0 \left[ B(\min\{(1 - d)s_0 + e_1, \bar{a}\}, \alpha) \right] \right\} \text{ s.t. } (13)$$

$$s_0 \geq 0, \min\{a_0, \bar{a}\} - s_0 \geq 0.\quad (14)$$

Recall that our goal is still to sign the CPD, $V_{13}^*(\bar{a}, a_0, \alpha)$. To do that we still need to first examine the marginal benefit of dam capacities, $V_1^*(\bar{a}, a_0, \alpha)$.

First observe that it is always suboptimal to store all of the captured water in period 0. To see this point, suppose that it is optimal to store all of the captured water in period 0 for period 1, which means $s_0^* = \min\{a_0, \bar{a}\}$. Then an Euler inequation,

$$B_1(0, \alpha) \leq \rho(1 - d)\mathbf{E}_0 \left[ I_{(1-d)\min\{a_0, \bar{a}\} + e_1 \leq \bar{a}} \cdot B_1((1 - d)\min\{a_0, \bar{a}\} + e_1, \alpha) \right],\quad (15)$$

must hold, which is impossible, because

$$\rho(1 - d)\mathbf{E}_0 \left[ I_{(1-d)\min\{a_0, \bar{a}\} + e_1 \leq \bar{a}} \cdot B_1((1 - d)\min\{a_0, \bar{a}\} + e_1, \alpha) \right] \leq B_1(\min\{e, \bar{a}\}, \alpha) < B_1(0, \alpha).\quad (16)$$

In other words, if all of the capture water in period 0 is stored, the marginal benefit of water release in period 0 will be so high that releasing water of the amount of a little bit more than zero will be beneficial. Therefore, store all of the captured water in period 0 cannot be optimal.

As the extension is about water-inventory management, it would be trivial for us to consider the case in which the optimal water storage is zero. This case indeed generates the same results as in Section 2. We then only focus on the case in which the optimal water storage and the optimal water release in period 0 are both strictly positive. In this case, the dam generated value is

$$V^*(\bar{a}, a_0, \alpha) = B(\min\{a_0, \bar{a}\} - s_0^*, \alpha) + \rho \mathbf{E}_0 \left[ B(\min\{(1 - d)s_0^* + e_1, \bar{a}\}, \alpha) \right]\quad (17)$$

and an Euler equation,

$$B_1(\min\{a_0, \bar{a}\} - s_0^*, \alpha) = \rho(1 - d)\mathbf{E}_0 \left[ I_{(1-d)s_0^* + e_1 \leq \bar{a}} \cdot B_1((1 - d)s_0^* + e_1, \alpha) \right],\quad (18)$$

must hold. The left-hand side of the equation is the cost that increasing an additional unit of water storage will incur, which is the current marginal benefit of water. The right-hand side is the benefit that increasing water storage will generate, which is the discounted, expected additional benefit of water of the $1 - d$ additional unit of water in the future. The optimal water storage, $s_0^*$, should make the marginal cost and benefit equal, because, otherwise, the dam operator would be able to improve
the dam generated value by adjusting water storage.

With some algebra and by the Envelope Theorem, the marginal benefit of dam capacities turns out to be

\[ V_1^*(\bar{a}, a_0, \alpha) = I_{a_0 > \bar{a}} \cdot B_1(\bar{a} - s_0^*, \alpha) + \rho B_1(\bar{a}, \alpha) P_0 [(1 - d)s_0^* + e_1 > \bar{a}] . \]  

(19)

This expression carries important intuition. Considering the purposes of dams against both intra-annual seasonality and inter-annual uncertainty, first, when the dam currently reaches the full capacity, an additional unit of dam capacities allows us to capture one additional unit of water, generating additional benefit of \( B_1(\bar{a} - s_0^*, \alpha) \). Second, the additional unit of dam capacities will also allow us to capture one additional unit of water, generating additional expected, discounted benefit of \( \rho B_1(\bar{a}, \alpha) \), if the dam reaches the full capacity in the future, which will happen with a probability of \( P_0 [(1 - d)s_0^* + e_1 > \bar{a}] \).

With some algebra, we can derive the CPD,

\[ V_{13}^*(\bar{a}, a_0, \alpha) = I_{a_0 > \bar{a}} \cdot \left( B_{12}(\bar{a} - s_0^*, \alpha) - B_{11}(\bar{a} - s_0^*, \alpha) \frac{\partial s_0^*(\bar{a}, a_0, \alpha)}{\partial \alpha} \right) \\
+ \rho B_{12}(\bar{a}, \alpha) [1 - F_{e_1}(\bar{a} - (1 - d)s_0^*)] \\
+ \rho(1 - d) B_1(\bar{a}, \alpha) f_{e_1}(\bar{a} - (1 - d)s_0^*) \frac{\partial s_0^*(\bar{a}, a_0, \alpha)}{\partial \alpha} , \]  

(20)

where \( s_0^* \equiv s_0^*(\bar{a}, a_0, \alpha) \) is the optimal water storage given the dam capacity, the initial water availability, and water-use efficiency, \( F_{e_1}(\cdot) \) is the cumulative distribution function of the future inflow, and \( f_{e_1}(\cdot) \) is the probability density function of the future inflow. This expression shows that there are three channels through which water-use efficiency improvement affects the marginal benefit of dam capacities:

1. It changes the additional benefit that is generated by the additional unit of water captured by the additional unit of dam capacities when the dam currently reaches the full capacity, \( B_1(\bar{a} - s_0^*, \alpha) \). The channel is represented by the first term of the CPD, \( I_{a_0 > \bar{a}} \cdot \left( B_{12}(\bar{a} - s_0^*, \alpha) - B_{11}(\bar{a} - s_0^*, \alpha) \frac{\partial s_0^*(\bar{a}, a_0, \alpha)}{\partial \alpha} \right) \).

2. It changes the additional benefit that will be generated if the dam reaches the full capacity in the future, \( \rho B_1(\bar{a}, \alpha) \). This channel is represented by the second term of the CPD, \( \rho B_{12}(\bar{a}, \alpha) [1 - F_{e_1}(\bar{a} - (1 - d)s_0^*)] \).

The first two channels are both about the impact of water-use efficiency on the marginal benefit of water release when the dam reaches the full capacity.

3. It changes the likelihood that the dam will reach the full capacity in the future, \( P_0[(1 - d)s_0^* + e_1 > \bar{a}] \).
This channel is represented by the third term of the CPD, \( \rho(1 - d)B_1(\bar{a}, \alpha)f_{e_1}(\bar{a} - (1 - d)s_v) \frac{\partial\phi_v(\bar{a},a_0,\alpha)}{\partial \alpha} \).

Examining the directions of the three channels, we can have the main result of this paper.

**Proposition 2.** Consider the purposes of dams against both intra-annual seasonality and inter-annual uncertainty. Assume the optimal water storage is not zero. If the marginal productivity of effective water declines fast and the decline does not get much slower as effective water increases, then water-use efficiency improvement will 1) decrease the marginal benefit of water release when the dam reaches the full capacity and 2) make it more likely for the dam to reach the full capacity in the future by increasing the optimal water storage. Whether water-use efficiency and dam capacities are substitutes or complements is then indeterminate.

More precisely, assume \( s_v^* \neq 0 \). If \( B_{12}(w, \alpha) \leq 0 \) for any \( w \in [\bar{e}, \max\{\bar{a}, (1 - d)\bar{s} + \bar{e}\}] \) and \( B_{121}(w, \alpha) \leq 0, B_{111}(w, \alpha) \geq 0 \) for any \( w \in [\bar{e}, (1 - d)\bar{s} + \bar{e}] \), then the first two terms of \( V_{13}^*(\bar{a}, a_0, \alpha) \) are negative and the third term is positive. The sign of \( V_{13}^*(\bar{a}, a_0, \alpha) \) is then indeterminate.

Appendix A.1 proves Proposition 2. For the first point, first, note that whether water-use efficiency improvement will increase the inverse demand for water determines the direction of the second channel through which the improvement affects the marginal benefit of dam capacities. Second, the direction of the first channel is determined by 1) whether the improvement will increase the inverse demand for water and 2) whether the improvement will increase the optimal water storage and, therefore, the optimal water release when the dam currently reaches the full capacity. It turns out that the indirect effect through the optimal water storage will not reverse the direction that is settled by the direct effect through the inverse demand for water. Therefore, a similar result to Proposition 1 is derived.

[Figure 3 about here.]

For the second point, first, note that water-use efficiency improvement will increase the likelihood of the dam reaching the full capacity in the future if and only if the improvement increases the optimal water storage carried from the current period to the future period. Second, if the impact of the improvement on the marginal benefit of water is negative and if the impact is stronger with larger water use, then the optimal water storage will increase.

Figure 3 illustrates a case in which improvement in water-use efficiency increases the optimal water storage. First, note that the storage decision, \( s_v^*(\bar{a}, a_0, \alpha) \), makes the marginal benefit of water storage, \( \rho(1 - d)E_0 \left[I_{(1-d)s_0 + e_1} \cdot B_1((1 - d)s_0 + e_1, \alpha)\right] \), and the marginal cost of water storage, \( B_1(\min\{a_0, \bar{a}\} - s_0, \alpha) \), intersect. Recall that, if the marginal productivity of effective
water declines fast, then an increase in water-use efficiency, $\alpha$, will decrease the inverse demand for water, $B_1(w, \alpha)$. Therefore, it will shift both of the marginal benefit and the marginal cost of water storage, and both shifts will be downward. The question is then which shift is larger.

For an intuitive interpretation, we can roughly consider

$$\mathbb{E}_0 [B_1((1 - d)s_0 + e_1, \alpha)] \approx B_1((1 - d)s_0 + \mathbb{E}_0 [e_1], \alpha)$$

(21)

and approximate the Euler equation as

$$B_1(\min\{a_0, \bar{\alpha}\} - s_0^*, \alpha) \approx \rho(1 - d) I_{(1 - d)s_0 + e_1 \leq \bar{\alpha}} \cdot B_1((1 - d)s_0 + \mathbb{E}_0 [e_1], \alpha).$$

(22)

It suggests that the current water release, $\min\{a_0, \bar{\alpha}\} - s_0^*$, should roughly be larger than the mean of the future water release, $(1 - d)s_0 + \mathbb{E}[e_1]$, because, otherwise, the marginal benefit of water in the future would be lower than that now and it would not be beneficial to store any water. Recall that, if the decline of the marginal productivity of effective water does not get much slower as effective water increases, then the negative impact of higher water-use efficiency on the marginal benefit of water should be larger with larger water release. Therefore, the downward shift in the marginal cost of water storage will be larger than the downward shift in the marginal benefit of water storage. We then know that the intersection between the marginal cost and the marginal benefit of water storage will move rightward, suggesting a larger choice of water storage, $s_0^*$, which will increase the likelihood of the dam reaching the full capacity in the future.\(^{10}\)

We have two important remarks about the mechanism. First, it will hold even if $\rho(1 - d)$ is close to one, which means that the different magnitudes of the shifts in the marginal benefit and the marginal cost of water storage comes from not only the regular “discount-factor effect” but also the properties of the marginal productivity of effective water. Second, the mechanism would not exist if there were no stochastically dynamic control of water inventories: The mechanism depends on the probability of the dam reaching the full capacity in the future, which would become meaningless if inflows were not stochastic. The mechanism also relies on the water storage decision, which will be assumed away if the dam does not control water inventories dynamically.

Proposition\(^2\) emphasizes the monotonicity of the impact of water-use efficiency improvement on the marginal benefit of water with respect to water use—the sign of $B_{121}(w, \alpha)$. Which characteristic of the water benefit is determining the monotonicity? As mentioned above, if the decline of the marginal productivity of effective water does not get much slower as effective water increases, then $B_{121}(w, \alpha) \leq 0$. In other words, the marginal productivity of effective water is not extremely

\(^{10}\)Conditions such as $B_{111}(w, \alpha) \leq 0$ and $B_{1211}(w, \alpha) \geq 0$ polish the argument with technical details.
convex. Mathematically, we have

$$B_{121}(w, \alpha) = 2\alpha B''(\alpha w) + \alpha^2 w B''(\alpha w).$$  \hfill (23)

Therefore, $B_{121}(w, \alpha) \leq 0$ is equivalent to

$$\text{SEMP} \equiv -\frac{\alpha w B''(\alpha w)}{B''(\alpha w)} \leq 2, \hfill (24)$$

where SEMP represents the second-order elasticity of the marginal productivity of effective water. We document this result as a corollary.

**Corollary 3.** Consider the purposes of dams against both intra-annual seasonality and inter-annual uncertainty. Assume the optimal water storage is not zero. If the elasticity of the marginal productivity (EMP) of effective water is larger than one and the second-order elasticity of the marginal productivity (SEMP) is smaller than two, then water-use efficiency improvement will 1) decrease the marginal benefit of water release when the dam reaches the full capacity and 2) make it more likely for the dam to reach the full capacity in the future by increasing the optimal water storage. Whether water-use efficiency and dam capacities are substitutes or complements is then indeterminate.

Corollary 3 extends the literature on the impact of water-use efficiency on economic behaviors of water-resource management. As mentioned earlier, whether water-use efficiency improvement will increase the marginal benefit of water or inverse demand for water and, therefore, water use, is determined by the EMP. Corollary 3 shows that the impact of the improvement on water storage behaviors is determined by not only the EMP but also the SEMP.

Comparing the two sets of results—Proposition 1 and Corollary 1 and Proposition 2 and Corollary 3 shows that different purposes of dams imply different economic relations between dam capacities and water-use efficiency. If we incorporate the purpose of dams against inter-annual uncertainty, then we must examine whether water-use efficiency improvement will increase the optimal water storage and, therefore, the likelihood of the dam reaching the full capacity in the future. Therefore, we do not only care about whether water-use efficiency improvement will increase or decrease the marginal benefit of water but also care about whether the impact of the improvement on the marginal benefit of water is increasing or decreasing in water use.

We can also apply Proposition 2 and Corollary 3 to the isoelastic and the linear specifications of the water demand.

**Corollary 4.** Consider the purposes of dams against both intra-annual seasonality and inter-annual uncertainty. Assume the optimal water storage is not zero.
• When the water demand is isoelastic, dam capacities and water-use efficiency will be complements if and only if the water demand is elastic.

• When the water demand is linear, water-use efficiency improvement will always decrease the marginal benefit of water release when the dam reaches the full capacity and increase the likelihood of the dam reaching the full capacity in the future, if the minimum inflow is sufficiently large. Whether dam capacities and water-use efficiency are complements or substitutes is then indeterminate. More precisely, there exists a critical level of the minimal inflow, \( \hat{e} \), such that \( B_{12}(w, \alpha) \leq 0 \) for any \( w \geq \hat{e} \), \( \frac{\partial B_0(\bar{a}, a_0, \alpha)}{\partial \alpha} \geq 0 \), and the sign of \( V_{13}^*(\bar{a}, a_0, \alpha) \) is indeterminate if \( e \geq \hat{e} \), where \( \hat{e} \) solves \( \frac{\partial B_0(\bar{a}, a_0, \alpha)}{\partial \alpha} = 1 \).

The intuition of Corollary 4 is simple. For isoelastic water demands, water-use efficiency improvement will shift up the marginal benefit of water if and only if the demand is elastic. More importantly, this shift will be proportional, so it will not change the optimal water storage and, therefore, the likelihood of the dam reaching the full capacity in the future. We can then identify a positive impact of the improvement on optimal choices of dams and the complementarity between dam capacities and water-use efficiency. For linear water demands, first, water-use efficiency improvement will decrease the inverse demand for water if and only if the initial water use is large, which is guaranteed by the sufficiently large inflow minimum. Second, the negative impact does become stronger as water use increases. Therefore, the conditions in Proposition 2 are satisfied and the result is derived.

As Corollary 4’s result for isoelastic water demands highlights the elasticity of the water demand, it would be worthy to discuss a little bit more about the factors affecting the elasticity. It is natural to speculate that the elasticity of the water demand is highly correlated with the economic properties of the water produced commodity. We can think that the benefit of effective water, \( B(x) \), is equal to the production function of the water produced commodity, e.g., irrigated agricultural products or hydropower, in effective water, multiplied by the inverse demand for the commodity—the revenue of the commodity production. It is easy to see that the elasticity of the demand for the commodity and the elasticity of the water demand are positively correlated as long as the production function is increasing in water.

This observation carries important policy implications. On the one hand, many small, developing countries are exporting agricultural commodities, and the sector is important for the economy. When their production is small in the world market, they face an almost perfectly elastic demand for the commodity, so the irrigation demand for water could be elastic. In this case, improvements in water-use efficiency, which could result from international aid, could optimally lead to larger

\footnote{For example, the meta-analysis by \cite{Scheierling2006} finds that the irrigation water demand is more inelastic if the irrigated crop is high valued.}
dams for the irrigation needed for the commodity production. This point suggests that the aid tackling water challenges in developing countries should have a joint perspective about international trade, conservation, and water infrastructures. On the other hand, in cases of dams used to produce nonexported commodities or commodities with low demand elasticities, e.g., electricity and staple food for domestic consumption, the derived demand for water could be inelastic, so dam capacities and water-use efficiency could be substitutes. This point suggests that the joint policy about conservation and water infrastructures should critically depend on the property of the water produced commodity.

Corollary 4 also suggests that the functional form of the water demand is critical in determining the economic relation between dam capacities and water-use efficiency. Studies find that the irrigation water demand is usually inelastic (e.g., Moore et al., 1994; Schoengold et al., 2006; Hendricks and Peterson, 2012). An isoelastic, elastic specification of the water demand suggests that irrigation-dam expansions and conservation-technology adoption should be substitutes. Water-demand elasticities do vary with respect to water use, however, and it is possible for the linear specification that has the same elasticity as the isoelastic, inelastic specification when the water price is at its mean level to suggest complementarity when the inflow minimum is sufficiently large. The difference in functional forms could therefore leads to opposite answers to the two questions.

All of the results in this section suggest that whether dam capacities and water-use efficiency are complements or substitutes is determined by 1) the purposes of water-storage capacities and 2) the first- and the second-order shapes of the water demand. Once policymakers have good knowledge about these factors, careful simulations could help to answer the two questions we asked. We conclude this section by discussing the general policy implications of knowing complementarity or substitution.

**General policy implications.** When dam capacities and water-use efficiency are complements, first, public water-storage capacities could be expanded without discouraging improvement in water-use efficiency, e.g., adopting more-efficient irrigation technologies and better conveyance technologies. Second, policymakers might believe that subsidizing water users to improve water-use efficiency could make expanding water storage unnecessary, but the subsidies could backfire by increasing the demand for investment in water storage. When dam capacities and water-use efficiency are substitutes, some opposite policy implications would follow.

Last but not least, in the case of complementarity, assuming a policymaker maximizing the social welfare that is related to water-storage capacities, limited resources should be distributed in a balanced way between dam building and water-use efficiency improvement, instead of being concentrated on either side with the other side being ignored. To see this point, consider the problem
of resource allocation between increasing dam capacities by $\Delta \bar{a}$ and improving water-use efficiency by $\Delta \alpha$:

$$\max_{\Delta \bar{a} \geq 0, \Delta \alpha \geq 0} V^*(\bar{a} + \Delta \bar{a}, a_0, \alpha + \Delta \alpha) \quad \text{s.t.} \quad p_{\bar{a}} \cdot \Delta \bar{a} + p_\alpha \cdot \Delta \alpha \leq b,$$  \hspace{1cm} (25)

where $p_{\bar{a}} \equiv C'(\bar{a}) + D'(\bar{a})$ is the price for dam expansion, $p_\alpha \equiv G'(\alpha)$ is the price for water-use efficiency improvement, and $b$ is the policy budget. An interior solution with $\Delta \bar{a} > 0$ and $\Delta \alpha > 0$ corresponds to a balanced distribution of the budget, while a corner solution with $\Delta \bar{a} = 0$ or $\Delta \alpha = 0$ corresponds to concentrating the budget on either dam expansion or water-use efficiency improvement with the other being ignored. An interior solution will be reached as long as the isovalue curve, $V^*(\bar{a} + \Delta \bar{a}, a_0, \alpha + \Delta \alpha) = v$, is tangent with the budget-constraint line, $p_{\bar{a}} \cdot \Delta \bar{a} + p_\alpha \cdot \Delta \alpha = b$, at a point with $\Delta \bar{a} > 0$ and $\Delta \alpha > 0$, in a $\Delta \bar{a}$-$\Delta \alpha$ span. It is equivalent to say that the slope of the isovalue curve in $\Delta \bar{a}$, $-\frac{V^*_1(\bar{a} + \Delta \bar{a}, a_0, \alpha + \Delta \alpha)}{V^*_3(\bar{a} + \Delta \bar{a}, a_0, \alpha + \Delta \alpha)}$, increases and becomes less negative as $\Delta \bar{a}$ increases. Mathematically, it is equivalent to

$$d \left( -\frac{V^*_1(\bar{a} + \Delta \bar{a}, a_0, \alpha + \Delta \alpha)}{V^*_3(\bar{a} + \Delta \bar{a}, a_0, \alpha + \Delta \alpha)} \right) \cdot V^*_1(\bar{a} + \Delta \bar{a}, a_0, \alpha + \Delta \alpha) > 0.$$  \hspace{1cm} (26)

Note that complementarity between dam expansion and water-use efficiency improvement is equivalent to $V^*_1(\bar{a} + \Delta \bar{a}, a_0, \alpha + \Delta \alpha) > 0$, so the complementarity will guarantee an interior solution to the resource allocation problem, which means that balanced distribution between the policies will be optimal. Only extremely strong substitution with $V^*_1(\bar{a} + \Delta \bar{a}, a_0, \alpha + \Delta \alpha) \ll 0$ could make ignoring either dam expansions or water-use efficiency improvement optimal.

4 Extension with an Infinite Horizon

In this section we extend the simple two-period, stochastic model by incorporating an infinite horizon of the dam operator. The extension is for two purposes. First, as the simple model assumes only two periods in the dam operation, we shall use the extended model to show that the insight and results from the simple model are robust if a longer horizon is introduced to the dam operation. Second, as the horizon of dam operators is usually long in reality (e.g., Reilly, 1995), the extended model can help us in empirical illustrations.
The extension then turns the water-inventory management problem into

\[ V^*(\bar{a}, a_0, \alpha) = \max_{\{w_t\}_{t=0}^\infty, \{s_t\}_{t=0}^\infty} \mathbb{E}_0 \left[ \sum_{t=0}^\infty \rho^t B(w_t, \alpha) \right] \quad \text{s.t. (27)}\]

\[ s_t \geq 0, w_t \geq 0, w_t + s_t = \min\{a_t, \bar{a}\} \text{ for any } t \geq 0; \]

\[ a_0 \text{ is given; } a_t = (1 - d)s_{t-1} + e_t \text{ for any } t \geq 1, \quad (28)\]

where \( e_t \in [\underline{e}, \bar{e}] \sim e, \text{ i.i.d., and all the variables have the same meaning as in the simple model.} \]

Assume \( \underline{e} > 0 \). Assume \( B'(\underline{e}) < \infty, B'(\cdot) > 0, \text{ and } B''(\cdot) < 0 \). Equivalently, \( B_1(w, \alpha) > 0 \) and \( B_{11}(w, \alpha) < 0 \).

The extended model is close to the economic literature about dynamic, sometimes stochastic control of water inventories (e.g., Burt, 1964; Riley and Scherer, 1979; Gisser and Sánchez, 1980; Tsur and Graham-Tomasi, 1991; Fisher and Rubio, 1997; Chatterjee et al., 1998; Truong, 2012). Our model can contribute with some analytical comparative statics about the marginal benefit of water-storage capacities, as it is difficult to be done with stochasticity and long planning horizons in the literature. Another thread of studies has an institutional focus on the optimal design of water storage rights (e.g., Dudley and Musgrave, 1988; Freebairn and Quiggin, 2006; Brennan, 2008; Hughes and Goesch, 2009; Truong and Drynan, 2013). Different from their approach, our paper adopts a macroperspective by assuming a centralized water-inventory management. After all, with the exception of Fisher and Rubio (1997), the literature on optimal inventory management of water and other storable commodities generally focuses on the optimal management of existing storage systems while our model focuses on the optimal adjustment of existing storage systems or optimal design of new storage systems.

In particular, the three most closely related models in the literature to the extended model are Fisher and Rubio (1997), Hughes and Goesch (2009), and Truong (2012). Fisher and Rubio (1997)’s analysis of the variability impact of inflows on the real-time renovation of dams is done in a continuous-time setting. The dam in their model is not against seasonal inequality, and spills are assumed to generate benefit as regulated water release does. As inflows to dams are highly seasonal with water years being the unit of time, we use a discrete-time approach, and assume that spills are not used and do not generate benefit. As we focus less on the real-time dam renovation, we simplify the capacity choice as a one-shot decision. Also, our analytical results are about the whole equilibrium, while Fisher and Rubio (1997) only look at the mean level.

Hughes and Goesch (2009) focus numerically on the design of the capacity sharing rule given a

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12 The Office of Management and Budget (2011) recommends a constant but not declining discount factor for project evaluation. We assume an infinite planning horizon for analytical simplicity, instead of a horizon of 50–100 years, which is recognized by Reilly (1995) as more realistic for dams.
storage capacity, different from our theoretical focus on the relation between capacity choices and water-use efficiency. Similar to Fisher and Rubio (1997), they assume that the storage capacity constrains only the dam’s capacity to store the water inventory and that spills generate irrigation benefit, but use a discrete-time setting.

Truong (2012) focuses theoretically on the impact of dam capacities on water-inventory management, which is, again, different from our focus. Our model is also different from Truong (2012) as we assume away the stochasticity of water demand, which is caused by stochastic rainfalls, for simplicity, but introduce water-use efficiency—a key parameter for our purpose.\footnote{Truong (2012)’s model assumes that the storage capacity constrains the total amount of water that is available for inventory management. It also assumes that spills do not generate irrigation benefit. This approach is the same as us and implicitly considering the purposes of dams against both seasonal inequality and annual uncertainty. This approach is also seen in dam models in applied probability theory (e.g., Moran (1959)).}

The extended model can also be regarded as an extension of the competitive storage model for commodity markets (e.g., Working, 1933; Gustafson, 1958; Samuelson, 1971; Gardner, 1979; Newbery and Stiglitz, 1981; Scheinkman and Schechtm, 1983; Williams and Wright, 1991; Deaton and Laroque, 1992; Bobenrieth et al., 2002). Truong (2012) is the first work with a theoretical focus on the impact of storage capacities on the model equilibrium. Asche et al. (2014) incorporate an upper bound only capping the storage decision. Our model extends the literature by analyzing the impact of other parameters on choices of storage capacities.

The extended model carries the same logic as in the simple two-period, stochastic model. The Euler (in)equations of the water-inventory management problem are

\[ B_1(w_t^*, \alpha) \geq \rho(1 - d)\mathbb{E}_t[V_2^*(\bar{\alpha}, (1 - d)s_t^* + e_{t+1}, \alpha)] \text{ if } s_t^* = 0; \]
\[ B_1(w_t^*, \alpha) = \rho(1 - d)\mathbb{E}_t[V_2^*(\bar{\alpha}, (1 - d)s_t^* + e_{t+1}, \alpha)] \text{ if } s_t^* > 0 \text{ and } w_t^* > 0; \]
\[ B_1(w_t^*, \alpha) \leq \rho(1 - d)\mathbb{E}_t[V_2^*(\bar{\alpha}, (1 - d)s_t^* + e_{t+1}, \alpha)] \text{ if } w_t^* = 0, \] (29)

where \( w_t^* \) and \( s_t^* \) are the optimal water release and water storage at \( t \) given the water availability, \( a_t \), respectively, and \( w_t^* + s_t^* \equiv \min\{a_0, \bar{\alpha}\} \), the amount of water that is captured at \( t \). The left-hand sides of the (in)equations are the marginal benefit of water release or the marginal cost of water storage and the right-hand sides are the marginal benefit of water storage. The equation only holds when the optimal water release and the optimal water storage are both positive.

[Figure 4 about here.]

Figure 4 shows an example of the solution to the water-inventory management problem. The dashed line is the marginal benefit of water release if the dam releases all of the captured water, which could be suboptimal. The solid line shows the marginal benefit of water under optimal water-inventory management as a function in the current water availability, \( a_t \). When \( a_t < a < \bar{\alpha} \),
all the water is captured by the dam and the marginal benefit of water release will be higher than
the marginal benefit of water storage even if the dam does not store any water. Therefore, it is
optimal to release all of the captured water. The marginal benefit of water, $V^*_2(\bar{a}, a_t, \alpha)$, then goes
exactly the same as the marginal benefit of water release valued at the level of the current water
availability, $B_1(a_t, \alpha)$, so the dashed line and the solid line overlap. When $\bar{a} < a_t < \bar{a}$, all the water
is still captured by the dam, but a positive amount of water storage, $s^*_t > 0$, will make the marginal
benefit of water release and the marginal benefit of water storage break even. The marginal benefit
of water, $V^*_2(\bar{a}, a_t, \alpha) = B_1(w^*_t, \alpha)$, is then higher than the marginal benefit of water release valued
at the level of the current water availability, $B_1(a_t, \alpha)$, because $w^*_t \equiv a_t - s^*_t < a_t$. When $a_t > \bar{a}$, the
dam reaches the full capacity and can capture no more than $\bar{a}$. Therefore, any additional water
will spill and the marginal benefit of water is zero. We denote the optimal water storage in this
case as $\bar{s}$. The marginal benefit of water release when the dam reaches the full capacity is then
$p \equiv B_1(\bar{a} - \bar{s}, \alpha)$.

The extended model derives parallel results to Proposition 2 and Corollaries 3 and 4. Appendix A.2 lists the results and shows the proof. Our effort contributes to the literature on the competitive storage model, as the comparative statics about long-run behaviors of the model is analytically so difficult that only few exceptions have attempted (e.g., Deaton and Laroque, 1992; Truong, 2012).

5 An Empirical Example with Numerical Illustrations

In this section, we present numerical illustrations of our results by simulating the extended model. The simulation is about the empirical example of the irrigation water-inventory management problem of the California State Water Project. We use three specifications of the water demand in the illustrations: 1) isoelastic, elastic with the elasticity being $-1.21$ as estimated by Frank and Beattie (1979); 2) isoelastic, inelastic with the elasticity being $-0.79$ as estimated by Schoengold et al. (2006); and 3) linear with the same elasticity as the second isoelastic, inelastic demand when the demand is equal to the 1975–2010 mean of the annual water deliveries from the Project to agricultural use. The three specifications help to confirm our theoretical results and show their empirical relevance. Table 1 summarizes the three demand functions, while Table 2 summarizes the specification of the whole simulation. For more details about the specification, see Appendix A.3.

[Table 1 about here.]

[Table 2 about here.]

For each of the three water demands, we focus on two questions—whether more-efficient technology adoption in irrigation, which induces higher water-use efficiency (larger $\alpha$), will increase
or decrease the marginal benefit of dam capacities, \( V_1^* (\bar{a}, a_0, \alpha) \), and whether water-storage expansions, which induce larger dam capacities (larger \( \bar{a} \)), will increase or decrease the marginal contribution of water-use efficiency to the dam generated value, \( V_3^* (\bar{a}, a_0, \alpha) \). The two questions are equivalent to the two questions we have asked, respectively, and are eventually about the sign of the CPD, \( V_{13}^* (\bar{a}, a_0, \alpha) \).

Table 3 shows results with the benchmark level of storage capacities, 2025335 acre-feet, zero initial water availability, and the benchmark level of water-use efficiency, 0.7135. Panel A is for the isoelastic, elastic demand. A 1% improvement in water-use efficiency from 0.7135 to 0.7206 will increase the marginal benefit of dam capacities by 0.17%. This positive impact confirms the prediction of complementarity in Corollary 4 (and Corollary 6 in Appendix A.2) for isoelastic, elastic water demands. Moreover, the 0.17% increase in the marginal benefit of dam capacities is solely caused by a 0.17% increase in the marginal benefit of water release when the dam reaches the full capacity, while the net present frequency of the dam reaching the full capacity in the future does not change. This observation confirms the logic of Corollary 4 (and Corollary 6).

How will the 0.17% increase in the marginal benefit of dam capacities be reflected on the optimal choice of storage capacities? Without information about the marginal cost of dam capacities, the most we can do is to estimate the range of the impact: It is obvious that if the marginal cost of dam capacities are perfectly vertical, then the optimal choice of storage capacities will not change. If the marginal cost of dam capacities are assumed perfectly horizontal, then we can derive the upper bound of the increase in optimal storage capacities caused by the 1% improvement in water-use efficiency. Mathematically, totally differentiating both side of the first-order condition of the dam-capacity choice gives

\[
V_{11}^*(\bar{a}^*, a_0, \alpha) d\bar{a}^* + V_{13}^*(\bar{a}^*, a_0, \alpha) d\alpha = (C''(\bar{a}) + D''(\bar{a}^*)) d\bar{a}^*,
\]

which derives

\[
0 < \frac{d\bar{a}^*}{d\alpha} = \frac{V_{13}^*(\bar{a}^*, a_0, \alpha)}{-V_{11}^*(\bar{a}^*, a_0, \alpha) + C''(\bar{a}) + D''(\bar{a}^*))} < \frac{-V_{13}^*(\bar{a}^*, a_0, \alpha)}{V_{11}^*(\bar{a}^*, a_0, \alpha)}. \tag{31}
\]

A little bit algebra can express the upper bound of the elasticity of the optimal choice of storage capacities with respect to water-use efficiency, \( \frac{d\bar{a}^*}{d\alpha} \cdot \frac{\alpha}{\bar{a}^*} \), as the elasticity of the marginal benefit of dam capacities with respect to water-use efficiency, \( \epsilon_{\alpha} V_1^*(\bar{a}, a_0, \alpha) \), divided by the elasticity of the

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In this case, the upper bound for the elasticity of the optimal choice of storage capacities with respect to water-use efficiency will be \( \frac{\epsilon V_1^{*}(\tilde{a},a_0,\alpha)}{\epsilon V_1^{*}(\tilde{a},a_0,\alpha)} \):

\[
0 < \frac{\partial \tilde{a}^*}{\partial \alpha} \cdot \frac{\alpha}{\tilde{a}^{*}} < -\frac{V_{13}^{*}(\tilde{a}^*, a_0, \alpha)}{V_{11}^{*}(\tilde{a}^*, a_0, \alpha)} \cdot \frac{\alpha}{\tilde{a}^{*}} = -\frac{\alpha V_{13}^{*}(\tilde{a}^*, a_0, \alpha)}{V_{11}^{*}(\tilde{a}^*, a_0, \alpha)} \cdot \left( \frac{V_{11}^{*}(\tilde{a}^*, a_0, \alpha)}{V_{11}^{*}(\tilde{a}^*, a_0, \alpha)} \right)^{-1}
\]

\[
\equiv -\frac{\epsilon V_1^{*}(\tilde{a},a_0,\alpha)}{\epsilon V_1^{*}(\tilde{a},a_0,\alpha)} \cdot \left( \frac{V_{11}^{*}(\tilde{a}^*, a_0, \alpha)}{V_{11}^{*}(\tilde{a}^*, a_0, \alpha)} \right)
\]

(32)

In other words, the 1% improvement in water-use efficiency will generate at most a negligible but still positive increase in the optimal storage capacity from the benchmark level if we assume the water demand is isoelastic and elastic.

How will a 1% increase in dam capacities change the optimal water-use efficiency? A similar exercise can express upper bound of the elasticity of the optimal choice of use efficiency with respect to dam capacities as the elasticity of the marginal contribution of water-use efficiency with respect to dam capacities divided by the elasticity of the marginal contribution with respect to water-use efficiency. In this case, the upper bound will be a small but still positive number, \(6.83 \times 10^{-3}\). In other words, a 1% increase in dam capacities will generate at most a 0.007% improvement in water-use efficiency if we assume the water demand is isoelastic and elastic.

Panel B reports results for the isoelastic, inelastic water demand. They confirm the prediction and the logic of Corollary 4 (and Corollary 6), again: For isoelastic, inelastic demands, dam capacities and water-use efficiency are substitutes, and water-use efficiency improvement decreases the marginal benefit of dam capacities without changing the frequency of the dam reaching the full capacity in the future.

Panel C reports results for the linear water demand. Because around 72% of the possible values of the inflow distribution in the empirical example is larger than the critical level of water release beyond which water-use efficiency improvement will decrease and steepen the linear inverse demand for water, the conditions for linear demands in Corollary 4 (and Corollary 6) are then almost satisfied. Consistent with theoretical predictions, water-use efficiency improvement will decrease the marginal benefit of water release when the dam reaches the full capacity but will also increase the frequency of the dam reaching the full capacity in the future. Moreover, the second impact does dominate and a 1% improvement in water-use efficiency will increase the marginal benefit of dam capacities by 0.99%. The positive impact suggests complementarity between dam capacities and water-use efficiency if we assume the water demand is linear.

Comparing Panels B and C now confirms the importance of the specification of water demands and the purpose of dams against inter-annual uncertainty in the economic relation between dam capacities and water-use efficiency. The underlying water demands of the two panels both have a
price elasticity of $-0.79$ if the demand is equal to the mean of the 1975–2010 annual water deliveries from the California State Water Project to agricultural use, but differ in their functional forms: The water demand of Panel B is isoelastic while the demand of Panel C is linear. The difference in functional forms leads to different predictions about the economic relation between dam capacities and water-use efficiency. The reason for the difference in predictions happens is just that water-use efficiency improvement could increase the frequency of the dam reaching the full capacity in the future by optimally increasing water storage, and we can only recognize this impact by recognizing the purpose of dams against inter-annual uncertainty.

As we have discussed earlier, irrigation demand for water is usually inelastic. As Caswell and Zilberman (1986) recognize, the linear water demand is more consistent than the isoelastic demand with the classic three-stage model of the marginal productivity of water. The two points suggests that, for the irrigation water-inventory management problem of the California State Water Project, the linear water demand and Panel C should be empirically more relevant than the other two isoelastic specifications and Panels A and B. Panel C does suggest complementarity between dam capacities and water-use efficiency, which implies balanced distribution of limited resources on water-storage expansions and water-use efficiency improvement.

[Table 4 about here.]

Table 4 tests the robustness of Table 3 by calculating responses of the variables of interest to a 5% increase in water-use efficiency or water-storage capacities. All the results in Table 3 qualitatively hold and their magnitude becomes larger in Table 4. Panel C in Table 4 shows that a 5% increase, a reasonable improvement, in water-use efficiency will at most increase the optimal dam capacity by around 1%, while a 5% increase in water-storage capacities will at most increase the optimal water-use efficiency by around 0.2%.

Both Panel Cs in Tables 3 and 4 show asymmetry in the complementarity between water-storage capacities and water-use efficiency: The impact of dam capacities on optimal water-use efficiency is always quite small. The smallness is because that the existing dam capacity is large: First, the contribution of water-use efficiency to the dam generated value depends on the amount of water release in the long run, and so does the marginal contribution—the incentive of water-use efficiency improvement. Second, when the existing dam capacity is large, the amount of water release is large, so the relative increase in the amount of water release by additional dam capacities will be small. Therefore, the impact of the small increase in dam capacities on the incentive of water-use efficiency improvement and the optimal water-use efficiency will be weak. The complementarity between dam capacities and water-use efficiency is then more prominent in the impact of water-use efficiency improvement on water-storage expansions, but not the other way around.
Using the linear demand for water, we finally illustrate the comparison between the cost-minimization logic in the engineering literature (e.g., surveys by Yeh, 1985; Simonovic, 1992) and the value-maximization logic of economists. To reach the value generated by the benchmark dam capacity with the benchmark water-use efficiency, if there is a 5% increase in water-use efficiency, the minimal dam capacity, which will incur the minimal cost, will be 29.59% smaller than the benchmark dam capacity. This result confirms the intuition that, since the function of the benefit from effective water is increasing, higher water-use efficiency means a higher dam generated value given any dam capacity, so it will lead to a smaller capacity choice along the cost-minimization logic. In contrast, weighing the marginal benefit and the marginal cost of dam capacities, the optimal dam capacity with the same water-use efficiency improvement, as shown in Table 4, will be larger than the benchmark dam capacity by at most 0.97%.

6 Conclusion

In this paper, we analyze the relation between two technological approaches in water-resource management, namely, expanding water-storage capacities and enhancing water-use efficiency, under optimal control of water inventories. This relation is the key to the policy design to tackle water scarcity and adapt to climate change. Recognizing the purposes of dams against not only intra-annual seasonality but also inter-annual uncertainty, we show that water-storage capacities and water-use efficiency could be complements if the marginal productivity of water declines fast and if the decline does not get much slower as water use increases.

There are also dams in areas where the peaks of water endowment and demand generally overlap. The purpose of these dams are primarily against inter-annual uncertainty but not intra-annual seasonality. For these water-storage facilities, our insight that water-use efficiency improvement could increase the likelihood of dams reaching their full capacities by changing control rules of water inventories is still applicable.

Our results imply that the policies encouraging public or private water storage could encourage water users to improve water-use efficiency, e.g., adopt more efficient technologies in irrigation or invest in conveyance systems, and the policies subsidizing the improvement could also increase the demand for water-storage capacities. After all, policymakers should not separately design the two categories of policies—expanding water storage and improving water-use efficiency. In the case of complementarity, resources should not be concentrated only on one category with the other being ignored. This implications is especially important for the countries with small initial water-storage capacities, by which water-use efficiency improvement will increase the demand for water, and the countries with generally abundant inflows and large initial water-storage capacities, by which water-use efficiency improvement will increase the likelihood that dams reach their full capacities.
As the relation between the policies is important in policy debates and could be counterintuitive, it deserves more serious theoretical modeling and empirical investigations. Further effort could be made to specify the improvement in water-use efficiency, e.g., model conservation-technology adoption with heterogeneous water users and specific land constraints. The cost of dams that will be correlated with water-use efficiency improvement, for example, displacement or introduction of specific water users should also be considered. Our model can also serve as a starting point for a research agenda on the relation between water-storage expansions and other approaches in water-resource management, e.g., introducing water markets to existing systems of water rights and adopting drought-tolerant varieties in agriculture. In a more general perspective, our analysis on the marginal benefit of storage capacities can be applied and extended to investigate investment decisions in other contexts, such as the joint management of water and food inventories. Ultimately, introducing political economy into the discussion between water infrastructure and conservation effort would be necessary.

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References


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A Appendices

A.1 Proof of Proposition 2

Proof. To sign the CPD we need to look at $\frac{\partial s^*_0(a_0, a, \alpha)}{\partial \alpha}$. By the Euler equation,

$$B_1(\min\{a_0, \bar{a}\} - s^*_0, \alpha) = \rho(1-d)E_0 \left[I_{(1-d)s^*_0 + e_1 \leq \bar{a}} \cdot B_1((1-d)s^*_0 + e_1, \alpha)\right]$$

$$= \rho(1-d) \int_{-\infty}^{\bar{a} - (1-d)s^*_0} f_{e_1}(x)B_1((1-d)s^*_0 + x, \alpha)dx,$$

we have

$$- B_{11}(\min\{a_0, \bar{a}\} - s^*_0, \alpha)ds^*_0 + B_{12}(\min\{a_0, \bar{a}\} - s^*_0, \alpha)d\alpha$$

$$= \rho(1-d)^2E_0 \left[I_{(1-d)s^*_0 + e_1 \leq \bar{a}} \cdot B_{11}((1-d)s^*_0 + e_1, \alpha)\right]ds^*_0$$

$$- \rho(1-d)^2 f_{e_1}(\bar{a} - (1-d)s^*_0)B_1(\bar{a}, \alpha)ds^*_0$$

$$+ \rho(1-d)E_0 \left[I_{(1-d)s^*_0 + e_1 \leq \bar{a}} \cdot B_{12}((1-d)s^*_0 + e_1, \alpha)\right]d\alpha,$$

so

$$\frac{\partial s^*_0(\bar{a}, a_0, \alpha)}{\partial \alpha} = \left(B_{12}(\min\{a_0, \bar{a}\} - s^*_0, \alpha) - \rho(1-d)E_0 \left[I_{(1-d)s^*_0 + e_1 \leq \bar{a}} \cdot B_{12}((1-d)s^*_0 + e_1, \alpha)\right]\right)$$

$$\cdot \left[B_{11}(\min\{a_0, \bar{a}\} - s^*_0, \alpha) + \rho(1-d)^2E_0 \left[I_{(1-d)s^*_0 + e_1 \leq \bar{a}} \cdot B_{11}((1-d)s^*_0 + e_1, \alpha)\right]\right]^{-1}.$$

(35)
We then know

\[ I_{a_0 > a} \cdot \left( B_{12}(\bar{a} - s_0^*, \alpha) - B_{11}(\bar{a} - s_0^*, \alpha) \frac{\partial s_0^* (\bar{a}, a_0, \alpha)}{\partial \alpha} \right) \]

\[ = I_{a_0 > a} \cdot \left\{ B_{12}(\bar{a} - s_0^*, \alpha) - B_{11}(\bar{a} - s_0^*, \alpha) \\
\quad \cdot \left( B_{12}(\bar{a} - s_0^*, \alpha) - \rho(1 - d)E_0 \left[ I_{(1 - d)s_0^+ + e_1 \leq \bar{a}} \cdot B_{12}((1 - d)s_0^* + e_1, \alpha) \right] \right) \right\} \\
\quad \cdot \left[ B_{11}(\bar{a} - s_0^*, \alpha) + \rho(1 - d)^2E_0 \left[ I_{(1 - d)s_0^+ + e_1 \leq \bar{a}} \cdot B_{11}((1 - d)s_0^* + e_1, \alpha) \right] \right] \\
\quad - \rho(1 - d)^2f_{e_1}(\bar{a} - (1 - d)s_0^*)B_1(\bar{a}, \alpha) \right\}^{-1} \}

\[ = I_{a_0 > a} \cdot \left\{ \rho(1 - d)^2B_{12}(\bar{a} - s_0^*, \alpha)E_0 \left[ I_{(1 - d)s_0^+ + e_1 \leq \bar{a}} \cdot B_{11}((1 - d)s_0^* + e_1, \alpha) \right] \\
\quad - \rho(1 - d)^2B_{12}(\bar{a} - s_0^*, \alpha)f_{e_1}(\bar{a} - (1 - d)s_0^*)B_1(\bar{a}, \alpha) \\
\quad + \rho(1 - d)B_{11}(\bar{a} - s_0^*, \alpha)E_0 \left[ I_{(1 - d)s_0^+ + e_1 \leq \bar{a}} \cdot B_{12}((1 - d)s_0^* + e_1, \alpha) \right] \\
\quad \cdot \left[ B_{11}(\bar{a} - s_0^*, \alpha) + \rho(1 - d)^2E_0 \left[ I_{(1 - d)s_0^+ + e_1 \leq \bar{a}} \cdot B_{11}((1 - d)s_0^* + e_1, \alpha) \right] \right] \\
\quad - \rho(1 - d)^2f_{e_1}(\bar{a} - (1 - d)s_0^*)B_1(\bar{a}, \alpha) \right\}^{-1} \}

(36)

Denote \( s_0^*(\bar{a}, a_0, \alpha) \equiv \bar{s}(\bar{a}, \alpha) \) or just \( \bar{s} \) when \( a_0 \geq \bar{a} \). Then we have

\[ I_{a_0 > a} \cdot \left( B_{12}(\bar{a} - \bar{s}, \alpha) - B_{11}(\bar{a} - \bar{s}, \alpha) \frac{\partial \bar{s}^* (\bar{a}, a_0, \alpha)}{\partial \alpha} \right) \]

\[ = I_{a_0 > a} \cdot \left\{ \rho(1 - d)^2B_{12}(\bar{a} - \bar{s}, \alpha)E_0 \left[ I_{(1 - d)\bar{s} + e_1 \leq \bar{a}} \cdot B_{11}((1 - d)\bar{s} + e_1, \alpha) \right] \\
\quad - \rho(1 - d)^2B_{12}(\bar{a} - \bar{s}, \alpha)f_{e_1}(\bar{a} - (1 - d)\bar{s})B_1(\bar{a}, \alpha) \\
\quad + \rho(1 - d)B_{11}(\bar{a} - \bar{s}, \alpha)E_0 \left[ I_{(1 - d)\bar{s} + e_1 \leq \bar{a}} \cdot B_{12}((1 - d)\bar{s} + e_1, \alpha) \right] \\
\quad \cdot \left[ B_{11}(\bar{a} - \bar{s}, \alpha) + \rho(1 - d)^2E_0 \left[ I_{(1 - d)\bar{s} + e_1 \leq \bar{a}} \cdot B_{11}((1 - d)\bar{s} + e_1, \alpha) \right] \right] \\
\quad - \rho(1 - d)^2f_{e_1}(\bar{a} - (1 - d)\bar{s})B_1(\bar{a}, \alpha) \right\}^{-1} \}

(37)

By the Euler equation we know

\[ B_1(\bar{a} - \bar{s}, \alpha) = \rho(1 - d)E_0 \left[ I_{(1 - d)\bar{s} + e_1 \leq \bar{a}} \cdot B_1((1 - d)\bar{s} + e_1, \alpha) \right] \leq B_1((1 - d)\bar{s} + e_1, \alpha), \]
so \( \bar{a} - \bar{s} \geq (1 - d)\bar{s} + \epsilon \). Therefore, we can sign the first term of the CPD:

\[
I_{a_0 > a} \cdot \left( B_{12}(\bar{a} - s_0^*, \alpha) - B_{11}(\bar{a} - s_0^*, \alpha) \frac{\partial s_0^*(\bar{a}, a_0, \alpha)}{\partial \alpha} \right) \geq 0 \tag{39}
\]

if \( B_{12}(w, \alpha) \geq 0 \) for any \( w \in [(1 - d)\bar{s} + \epsilon, \bar{a}] \);

\[
I_{a_0 > a} \cdot \left( B_{12}(\bar{a} - s_0^*, \alpha) - B_{11}(\bar{a} - s_0^*, \alpha) \frac{\partial s_0^*(\bar{a}, a_0, \alpha)}{\partial \alpha} \right) \leq 0 \tag{40}
\]

if \( B_{12}(w, \alpha) \leq 0 \) for any \( w \in [(1 - d)\bar{s} + \epsilon, \bar{a}] \). We also know that the second term of the CPD will be positive if and only if \( B_{12}(\bar{a}, \alpha) \geq 0 \). Therefore, we can sign the sum of the first two terms of the CPD: It will be positive if \( B_{12}(w, \alpha) \geq 0 \) for any \( w \in [(1 - d)\bar{s} + \epsilon, \bar{a}] \). It will be negative if \( B_{12}(w, \alpha) \leq 0 \) for any \( w \in [(1 - d)\bar{s} + \epsilon, \bar{a}] \).

Now continue considering \( \frac{\partial s_0^*(\bar{a}, a_0, \alpha)}{\partial \alpha} \) to sign the third term of the CPD. When \( B_{12}(w, \alpha) \leq 0 \) for any \( w \in [\epsilon, (1 - d)\bar{s} + \bar{e}] \),

\[
B_{12}(\min\{a_0, \bar{a}\} - s_0^*, \alpha) - \rho(1 - d)E_0 \left[ I_{(1-d)s_0^* + e_1 \leq \bar{a}} \cdot B_{12}((1 - d)s_0^* + e_1, \alpha) \right]
\leq B_{12}(\min\{a_0, \bar{a}\} - s_0^*, \alpha) - E_0 \left[ I_{(1-d)s_0^* + e_1 \leq \bar{a}} \cdot B_{12}((1 - d)s_0^* + e_1, \alpha) \right]
\leq B_{12}(\min\{a_0, \bar{a}\} - s_0^*, \alpha) - E_0 \left[ B_{12}((1 - d)s_0^* + e_1, \alpha) \right]. \tag{41}
\]

When \( B_{12}(w, \alpha) \leq 0 \) and \( B_{111}(w, \alpha) \geq 0 \) for any \( w \in [\epsilon, (1 - d)\bar{s} + \bar{e}] \), by Jensen (1903)'s inequality,

\[
B_{12}(\min\{a_0, \bar{a}\} - s_0^*, \alpha) - \rho(1 - d)E_0 \left[ I_{(1-d)s_0^* + e_1 \leq \bar{a}} \cdot B_{12}((1 - d)s_0^* + e_1, \alpha) \right]
\leq B_{12}(\min\{a_0, \bar{a}\} - s_0^*, \alpha) - E_0 \left[ B_{12}((1 - d)s_0^* + e_1, \alpha) \right]
\leq B_{12}(\min\{a_0, \bar{a}\} - s_0^*, \alpha) - B_{12}((1 - d)s_0^* + E_0 [e_1], \alpha). \tag{42}
\]

When \( B_{111}(w, \alpha) \leq 0 \) for any \( w \in [\epsilon, (1 - d)\bar{s} + \bar{e}] \), by the Euler equation and Jensen (1903)'s inequality,

\[
B_1(\min\{a_0, \bar{a}\} - s_0^*, \alpha) = \rho(1 - d)E_0 \left[ I_{(1-d)s_0^* + e_1 \leq \bar{a}} \cdot B_1((1 - d)s_0^* + e_1, \alpha) \right]
\leq E_0 \left[ I_{(1-d)s_0^* + e_1 \leq \bar{a}} \cdot B_1((1 - d)s_0^* + e_1, \alpha) \right]
\leq E_0 \left[ B_1((1 - d)s_0^* + e_1, \alpha) \right]
\leq B_1((1 - d)s_0^* + E_0 [e_1], \alpha), \tag{43}
\]

so \( \min\{a_0, \bar{a}\} - s_0^* \geq (1 - d)s_0^* + E_0 [e_1] \).

When \( B_{12}(w, \alpha) \leq 0, B_{121}(w, \alpha) \leq 0, B_{111}(w, \alpha) \leq 0, \) and \( B_{1211}(w, \alpha) \geq 0 \) for any \( w \in \)
\[ e, (1 - d)\bar{s} + \bar{e} \],

\[
B_{12}(\min \{a_0, \bar{a} \} - s^*_0, \alpha) - \rho(1-d)E_0 \left[ I_{1-d} s^*_0 + e_1 \cdot B_{12}((1-d)s^*_0 + e_1, \alpha) \right] 
\leq B_{12}(\min \{a_0, \bar{a} \} - s^*_0, \alpha) - B_{12}((1-d)s^*_0 + E_0 [e_1], \alpha) 
\leq 0.
\]  

Therefore, \( \frac{\partial s^*_0(\bar{a}, a_0, \alpha)}{\partial \alpha} \geq 0 \), so the third term of the CPD is positive. \( \square \)

### A.2 Results for the Extended Model

**Proposition 3.** Assume the optimal water storage is not zero when the dam reaches the full capacity. Water-use efficiency improvement will make the dam capacity tighter given the frequency of future spill, if \( B_{12}(w, \alpha) \leq 0 \) for any \( w \in [e, \bar{a} - \bar{s}] \), where \( \bar{s} \) is the optimal water storage when the dam reaches the full capacity. Water-use efficiency improvement will make future spill more often if \( B_{12}(w, \alpha) \leq 0 \), \( B_{121}(w, \alpha) \leq 0 \), \( B_{111}(w, \alpha) \geq 0 \) for any \( w \in [e, (1-d)\bar{s} + \bar{e}] \). Therefore, the impact of water-use efficiency improvement on the marginal benefit of dam capacities, or, equivalently, whether water-use efficiency and dam capacities are complements or substitutes, is indeterminate if \( B_{12}(w, \alpha) \leq 0 \) for any \( w \in [e, \max \{\bar{a} - \bar{s}, (1-d)\bar{s} + \bar{e}\}] \) and \( B_{121}(w, \alpha) \leq 0 \), \( B_{111}(w, \alpha) \leq 0 \), and \( B_{1211}(w, \alpha) \geq 0 \) for any \( w \in [e, (1-d)\bar{s} + \bar{e}] \).

**Corollary 5.** Water-use efficiency improvement will make the dam capacity tighter given the frequency of future spill but will also make future spill more often, if the elasticity of the marginal productivity (EMP) is sufficiently large and the second-order elasticity (SEMP) is sufficiently small. The critical levels are 1 and 2, respectively. Whether water-use efficiency and dam capacities are complements or substitutes is indeterminate.

**Corollary 6.** Given an isoelastic water demand, dam capacities and water-use efficiency will be complements if and only if the water demand is elastic. More precisely, assume that \( \mu \equiv \left( \frac{wB_{11}(w, \alpha)}{B_{11}(w, \alpha)} \right)^{-1} \) is a constant. Then, \( V_{13}^* (\bar{a}, a_0, \alpha) > 0 \) if \( \mu < -1 \); \( V_{13}^* (\bar{a}, a_0, \alpha) = 0 \) if \( \mu = -1 \); \( V_{13}^* (\bar{a}, a_0, \alpha) < 0 \) if \( -1 < \mu < 0 \). Given a linear water demand, water-use efficiency improvement will make the dam capacity tighter given the frequency of future spill but will also make future spill more often, if the existing capacity is sufficiently large and the lowest possible inflow is sufficiently large. Whether water-use efficiency and dam capacities are complements or substitutes is indeterminate.
A.2.1 Proof of Proposition

The Bellman (1957) equation is

\[
V^*(\bar{a}, a_0, \alpha) \equiv \max_{s_0} \left\{ B(\min\{a_0, \bar{a}\} - s_0, \alpha) + \rho E_0 \left[ V^*(\bar{a}, (1 - d)s_0 + e_1, \alpha) \right] \right\} \quad \text{s.t.} \quad (45)
\]

\[s_0 \geq 0, \min\{a_0, \bar{a}\} - s_0 \geq 0, a_0 \text{ is given.} \quad (46)
\]

The marginal benefit of dam capacities is

\[
V_1^*(\bar{a}, a_0, \alpha) = I_{a_0 > \bar{a}} \cdot B_1(\bar{a} - s_0^*, \alpha) - B_1(\min\{a_0, \bar{a}\} - s_0^*, \alpha) \frac{\partial s_0^*(\bar{a}, a_0, \alpha)}{\partial \bar{a}} + \rho E_0 \left[ V_1^*(\bar{a}, (1 - d)s_0^* + e_1, \alpha) \right]
+ \rho (1 - d) E_0 \left[ V_2^*(\bar{a}, (1 - d)s_0^* + e_1, \alpha) \right] \frac{\partial s_0^*(\bar{a}, a_0, \alpha)}{\partial \bar{a}} \quad (47)
\]

Suppose \(s_0^* = \min\{a_0, \bar{a}\}\). Then an Euler inequation,

\[
B_1(0, \alpha) \leq \rho (1 - d) E_0 \left[ V_2^*(\bar{a}, (1 - d) \min\{a_0, \bar{a}\} + e_1, \alpha) \right]
= \rho (1 - d) E_0 \left[ I_{(1-d)\min\{a_0, \bar{a}\} + e_1 \leq \bar{a}} \cdot B_1(w_1^*, \alpha) \right],
\]

must hold, but it is impossible, because

\[
B_1(0, \alpha) \leq \rho (1 - d) E_0 \left[ I_{(1-d)\min\{a_0, \bar{a}\} + e_1 \leq \bar{a}} \cdot B_1(w_1^*, \alpha) \right] < B_1(0, \alpha) \quad (49)
\]

makes a contradiction. Therefore, \(s_0^* \in [0, \min\{a_0, \bar{a}\}]\).

Suppose \(s_0^* = 0\). Then the marginal benefit of dam capacities is

\[
V_1^*(\bar{a}, a_0, \alpha) = I_{a_0 > \bar{a}} \cdot B_1(\bar{a}, \alpha) + \rho E_0 \left[ V_1^*(\bar{a}, e_1, \alpha) \right]. \quad (50)
\]

Suppose \(s_0^* \in (0, \min\{a_0, \bar{a}\})\), an Euler equation,

\[
B_1(\min\{a_0, \bar{a}\} - s_0^*, \alpha) = \rho (1 - d) E_0 \left[ V_2^*(\bar{a}, (1 - d)s_0^* + e_1, \alpha) \right], \quad (51)
\]

must hold. Then the marginal benefit of dam capacities is

\[
V_1^*(\bar{a}, a_0, \alpha) = I_{a_0 > \bar{a}} \cdot B_1(\bar{a} - s_0^*, \alpha) + \rho E_0 \left[ V_1^*(\bar{a}, (1 - d)s_0^* + e_1, \alpha) \right]. \quad (52)
\]
Collecting the two cases of $s^*_0 \in [0, \min\{a_0, \bar{a}\})$, the marginal benefit of dam capacities is

$$V^*_1(\bar{a}, a_0, \alpha) = I_{a_0 > \bar{a}} \cdot B_1(\bar{a} - s^*_0, \alpha) + \rho E_0 [V^*_1(\bar{a}, (1 - d)s^*_0 + e_1, \alpha)]$$

$$\equiv I_{a_0 > \bar{a}} \cdot B_1(\bar{a} - s, \alpha) + \rho E_0 [V^*_1(\bar{a}, (1 - d)s^*_0 + e_1, \alpha)], \quad (53)$$

where we denote the optimal storage when the dam reaches the full capacity as $s(\bar{a}, \alpha)$ or simply $s$. By iteration,

$$V^*_1(\bar{a}, a_0, \alpha) = I_{a_0 > \bar{a}} \cdot B_1(\bar{a} - s, \alpha) + \sum_{t=1}^{\infty} \rho^t E_0 [I_{a^*_t > \bar{a}} \cdot B_1(\bar{a} - s, \alpha)]$$

$$= I_{a_0 > \bar{a}} \cdot B_1(\bar{a} - s, \alpha) + B_1(\bar{a} - s, \alpha) \sum_{t=1}^{\infty} \rho^t E_0 [I_{a^*_t > \bar{a}}]$$

$$= I_{a_0 > \bar{a}} \cdot B_1(\bar{a} - s, \alpha) + B_1(\bar{a} - s, \alpha) \sum_{t=1}^{\infty} \rho^t \left(1 - F_{a^*_t | \bar{a}, a_0, \alpha}(\bar{a}; \bar{a}, a_0, \alpha)\right)$$

$$= B_1(\bar{a} - s, \alpha) \left[I_{a_0 > \bar{a}} + \sum_{t=1}^{\infty} \rho^t \left(1 - F_{a^*_t | \bar{a}, a_0, \alpha}(\bar{a}; \bar{a}, a_0, \alpha)\right)\right] \quad (54)$$

The CPD is then

$$V^*_{13}(\bar{a}, a_0, \alpha) = \left(B_{12}(\bar{a} - s, \alpha) - B_{11}(\bar{a} - s, \alpha) \frac{\partial s(\bar{a}, \alpha)}{\partial \alpha}\right)$$

$$\cdot \left[I_{a_0 > \bar{a}} + \sum_{t=1}^{\infty} \rho^t \left(1 - F_{a^*_t | \bar{a}, a_0, \alpha}(\bar{a}; \bar{a}, a_0, \alpha)\right)\right]$$

$$- B_1(\bar{a} - s, \alpha) \sum_{t=1}^{\infty} \rho^t \frac{\partial F_{a^*_t | \bar{a}, a_0, \alpha}(\bar{a}; \bar{a}, a_0, \alpha)}{\partial \alpha}. \quad (55)$$

The first term is the impact of water-use efficiency on the tightness of the dam capacity when it is binding now and will be binding in the future. The second term is the impact on the frequency of the constraint being binding in the future.

We have known that $s(\bar{a}, \alpha) \in [0, \bar{a})$. Suppose $s(\bar{a}, \alpha) = 0$. Then the optimal storage will always be zero. Therefore, the CPD becomes

$$V^*_{13}(\bar{a}, a_0, \alpha) = B_{12}(\bar{a}, \alpha) \cdot \left[I_{a_0 > \bar{a}} + \sum_{t=1}^{\infty} \rho^t (1 - F_{e_t}(\bar{a}))\right], \quad (56)$$

whose sign is determined by the sign of $B_{12}(\bar{a}, \alpha)$. 

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Now consider the case in which \( s(\bar{a}, \alpha) \in (0, \bar{a}) \). First focus on \( \partial s(\bar{a}, \alpha) / \partial \alpha \). By the Euler equation,

\[
B_1(\bar{a} - \bar{s}, \alpha) = \rho (1 - d) E_0 \left[ V_2^*(\bar{a}, (1 - d)\bar{s} + e_1, \alpha) \right]
\]

\[
= \rho (1 - d) \int_{-\infty}^{\bar{a} - (1 - d)\bar{s}} f_{e_1}(x) V_2^*(\bar{a}, (1 - d)\bar{s} + x, \alpha) dx,
\] (57)

we have

\[
B_{12}(\bar{a} - \bar{s}, \alpha) d\alpha - B_{11}(\bar{a} - \bar{s}, \alpha) d\bar{s}
\]

\[
= - \rho (1 - d)^2 f_{e_1}(\bar{a} - (1 - d)\bar{s}) V_2^*(\bar{a}, \bar{a}, \alpha) d\bar{s}
+ \rho (1 - d)^2 \left[ \int_{-\infty}^{\bar{a} - (1 - d)\bar{s}} f_{e_1}(x) V_{22}^*(\bar{a}, (1 - d)\bar{s} + x, \alpha) dx \right] d\bar{s}
+ \rho (1 - d) \left[ \int_{-\infty}^{\bar{a} - (1 - d)\bar{s}} f_{e_1}(x) V_{23}^*(\bar{a}, (1 - d)\bar{s} + x, \alpha) dx \right] d\alpha
\]

\[
= - \rho (1 - d)^2 f_{e_1}(\bar{a} - (1 - d)\bar{s}) V_2^*(\bar{a}, \bar{a}, \alpha) d\bar{s}
+ \rho (1 - d)^2 E_0 \left[ I_{(1 - d)\bar{s} + e_1 \leq \bar{a}} \cdot V_{22}^*(\bar{a}, (1 - d)\bar{s} + e_1, \alpha) \right] d\bar{s}
+ \rho (1 - d) E_0 \left[ I_{(1 - d)\bar{s} + e_1 \leq \bar{a}} \cdot V_{23}^*(\bar{a}, (1 - d)\bar{s} + e_1, \alpha) \right] d\alpha
\] (58)

Therefore, we know

\[
\frac{\partial s(\bar{a}, \alpha)}{\partial \alpha} = \left[ B_{12}(\bar{a} - \bar{s}, \alpha) - \rho (1 - d) E_0 \left[ I_{(1 - d)\bar{s} + e_1 \leq \bar{a}} \cdot V_{22}^*(\bar{a}, (1 - d)\bar{s} + e_1, \alpha) \right] \right]
\]

\[
\cdot \left[ B_{11}(\bar{a} - \bar{s}, \alpha) + \rho (1 - d)^2 E_0 \left[ I_{(1 - d)\bar{s} + e_1 \leq \bar{a}} \cdot V_{22}^*(\bar{a}, (1 - d)\bar{s} + e_1, \alpha) \right] \right]
\]

\[
- \rho (1 - d)^2 f_{e_1}(\bar{a} - (1 - d)\bar{s}) V_2^*(\bar{a}, \bar{a}, \alpha) \right]^{-1},
\] (59)
\[
B_{12}(\bar{a} - \bar{s}, \alpha) - B_{11}(\bar{a} - \bar{s}, \alpha) \frac{\partial \bar{s}(\bar{a}, \alpha)}{\partial \alpha} \\
= B_{12}(\bar{a} - \bar{s}, \alpha) - B_{11}(\bar{a} - \bar{s}, \alpha) \\
\cdot \left[ B_{12}(\bar{a} - \bar{s}, \alpha) - \rho(1 - d)E_0 \left( I_{(1-d)\bar{s}+e_1 \leq \bar{a}} \cdot V_{23}^*(\bar{a}, (1 - d)\bar{s} + e_1, \alpha) \right) \right] \\
\cdot \left[ B_{11}(\bar{a} - \bar{s}, \alpha) + \rho(1 - d)^2E_0 \left( I_{(1-d)\bar{s}+e_1 \leq \bar{a}} \cdot V_{22}^*(\bar{a}, (1 - d)\bar{s} + e_1, \alpha) \right) \right] \\
- \rho(1 - d)^2 f_{e_1}(\bar{a} - (1 - d)\bar{s})V_{22}^*(\bar{a}, \bar{a}, \alpha) \\
= \left[ \rho(1 - d)^2 B_{12}(\bar{a} - \bar{s}, \alpha)E_0 \left( I_{(1-d)\bar{s}+e_1 \leq \bar{a}} \cdot V_{22}^*(\bar{a}, (1 - d)\bar{s} + e_1, \alpha) \right) \\
- \rho(1 - d)^2 B_{12}(\bar{a} - \bar{s}, \alpha)f_{e_1}(\bar{a} - (1 - d)\bar{s})V_{22}^*(\bar{a}, \bar{a}, \alpha) \\
+ \rho(1 - d) B_{11}(\bar{a} - \bar{s}, \alpha)E_0 \left( I_{(1-d)\bar{s}+e_1 \leq \bar{a}} \cdot V_{23}^*(\bar{a}, (1 - d)\bar{s} + e_1, \alpha) \right) \right] \\
\cdot \left[ B_{11}(\bar{a} - \bar{s}, \alpha) + \rho(1 - d)^2E_0 \left( I_{(1-d)\bar{s}+e_1 \leq \bar{a}} \cdot V_{22}^*(\bar{a}, (1 - d)\bar{s} + e_1, \alpha) \right) \right] \\
- \rho(1 - d)^2 f_{e_1}(\bar{a} - (1 - d)\bar{s})V_{22}^*(\bar{a}, \bar{a}, \alpha) \right]^{-1} \\
\text{(60)}
\]

Note \(V_{2}^*(\bar{a}, a, t, \alpha) = I_{a_1 \leq a} \cdot B_1(w^*(\bar{a}, a, t, \alpha), \alpha)\), where \(w^*(\bar{a}, a, t, \alpha)\) is the optimal current water release when the current water availability is \(a_t\). Therefore,
\[
E_0 \left( I_{(1-d)\bar{s}+e_1 \leq \bar{a}} \cdot V_{23}^*(\bar{a}, (1 - d)\bar{s} + e_1, \alpha) \right) \\
= E_0 \left( I_{(1-d)\bar{s}+e_1 \leq \bar{a}} \cdot B_{12}(w^*(\bar{a}, (1 - d)\bar{s} + e_1, \alpha), \alpha) \right) \text{(61)}
\]

and
\[
E_0 \left( I_{(1-d)\bar{s}+e_1 \leq \bar{a}} \cdot V_{22}^*(\bar{a}, (1 - d)\bar{s} + e_1, \alpha) \right) \\
= E_0 \left( I_{(1-d)\bar{s}+e_1 \leq \bar{a}} \cdot B_{11}(w^*(\bar{a}, (1 - d)\bar{s} + e_1, \alpha), \alpha) \right) \\
\leq 0. \text{(62)}
\]

We then can sign \(B_{12}(\bar{a} - \bar{s}, \alpha) - B_{11}(\bar{a} - \bar{s}, \alpha)\frac{\partial \bar{s}(\bar{a}, \alpha)}{\partial \alpha}\) and the first term of the CPD: They are positive if \(B_{12}(w, \alpha) \geq 0\) for any \(w \in [\bar{e}, \bar{a} - \bar{s}]\). They are negative if \(B_{12}(w, \alpha) \leq 0\) for any \(w \in [\bar{e}, \bar{a} - \bar{s}]\).

Now focus on the second term of the CPD and, equivalently, \(\frac{\partial F_{a_1}^*(\bar{a}, a_0, \alpha)}{\partial \alpha} \frac{(\bar{a}; \bar{a}, a_0, \alpha)}{\partial \alpha} \). First consider
\[
\frac{\partial F_{a_1}^*(\bar{a}, a_0, \alpha)}{\partial \alpha} = \frac{dF_{e_1}(\bar{a} - (1-d)\bar{s}_0^*(\bar{a}, a_0, \alpha))}{\partial \alpha} = -(1 - d)f_{e_1}(\bar{a} - (1 - d)\bar{s}_0^*)\frac{\partial \bar{s}_0^*(\bar{a}, a_0, \alpha)}{\partial \alpha}. \text{ We have already known that } \bar{s}_0^*(\bar{a}, a_0, \alpha) \in [0, \min\{\bar{a}, a_0\})
\]

Suppose \(\bar{s}_0^* = 0\). Then \(\frac{\partial \bar{s}_0^*(\bar{a}, a_0, \alpha)}{\partial \alpha} = 0\).
Suppose \( s_0^* \in (0, \min\{\bar{a}, a_0\}) \). By the Euler equation,

\[
B_1(\min\{a_0, \bar{a}\} - s_0^*, \alpha) = \rho(1 - d)E_0[V_2^*(\bar{a}, (1 - d)s_0^* + e_1, \alpha)]
\]

\[
= \rho(1 - d) \int_{-\infty}^{\bar{a} - (1 - d)s_0^*} f_{e_1}(x)V_2^*(\bar{a}, (1 - d)s_0^* + x, \alpha)dx,
\]

we know

\[
B_{12}(\min\{a_0, \bar{a}\} - s_0^*, \alpha) d\alpha - B_{11}(\min\{a_0, \bar{a}\} - s_0^*, \alpha) ds_0^*
\]

\[
= -\rho(1 - d)^2 f_{e_1}(\bar{a} - (1 - d)s_0^*)V_2^*(\bar{a}, \alpha) ds_0^*
\]

\[
+ \rho(1 - d)^2 \left[ \int_{-\infty}^{\bar{a} - (1 - d)s_0^*} f_{e_1}(x)V_{22}^*(\bar{a}, (1 - d)s_0^* + x, \alpha)dx \right] ds_0^*
\]

\[
+ \rho(1 - d) \left[ \int_{-\infty}^{\bar{a} - (1 - d)s_0^*} f_{e_1}(x)V_{23}^*(\bar{a}, (1 - d)s_0^* + x, \alpha)dx \right] d\alpha
\]

so

\[
\frac{\partial s_0^*}{\partial \alpha}(\bar{a}, a_0, \alpha) = \left[ B_{12}(\min\{a_0, \bar{a}\} - s_0^*, \alpha) - \rho(1 - d)E_0[I_{(1-d)s_0^*+e_1\leq \bar{a}} \cdot V_{23}^*(\bar{a}, (1 - d)s_0^* + e_1, \alpha)] \right]
\]

\[
\cdot \left[ B_{11}(\min\{a_0, \bar{a}\} - s_0^*, \alpha) + \rho(1 - d)^2E_0[I_{(1-d)s_0^*+e_1\leq \bar{a}} \cdot V_{22}^*(\bar{a}, (1 - d)s_0^* + e_1, \alpha)] \right]
\]

\[
- \rho(1 - d)^2 f_{e_1}(\bar{a} - (1 - d)s_0^*)V_2^*(\bar{a}, \bar{a}, \alpha)^{-1},
\]

\[
(65)
\]

Note again \( V_2^*(\bar{a}, a_t, \alpha) = I_{a_t\leq \bar{a}} \cdot B_1(w^*(\bar{a}, a_t, \alpha), \alpha) \). Therefore,

\[
E_0 \left[ I_{(1-d)s_0^*+e_1\leq \bar{a}} \cdot V_{23}^*(\bar{a}, (1 - d)s_0^* + e_1, \alpha) \right]
\]

\[
= E_0 \left[ I_{(1-d)s_0^*+e_1\leq \bar{a}} \cdot B_{12}(w^*(\bar{a}, (1 - d)s_0^* + e_1, \alpha), \alpha) \right]
\]

\[
(66)
\]

and

\[
E_0 \left[ I_{(1-d)s_0^*+e_1\leq \bar{a}} \cdot V_{22}^*(\bar{a}, (1 - d)s_0^* + e_1, \alpha) \right]
\]

\[
= E_0 \left[ I_{(1-d)s_0^*+e_1\leq \bar{a}} \cdot B_{11}(w^*(\bar{a}, (1 - d)s_0^* + e_1, \alpha), \alpha) \right]
\]

\[
\leq 0.
\]

(67)

With the similar analysis as in the two-period stochastic model, we know that, when \( B_{12}(w, \alpha) \leq 0, \)
\[B_{121}(w, \alpha) \leq 0, B_{111}(w, \alpha) \leq 0, \text{ and } B_{1211}(w, \alpha) \geq 0 \text{ for any } w \in [\bar{\epsilon}, \bar{s} + \bar{\epsilon}],\]

\[B_{12}(\min\{a_0, \bar{a}\} - s_0^*, \alpha) - \rho(1-d)E_0 \left[ I_{(1-d)s_0^* + \epsilon_1 \leq \bar{a}} \cdot V_{23}^*(\bar{a}, (1-d)s_0^* + \epsilon_1, \alpha) \right] \leq 0. \quad (68)\]

Therefore, \( \frac{\partial s_0^*(\bar{a}, a_0, \alpha)}{\partial \alpha} \geq 0. \)

Collecting the two cases of \( s_0^*(\bar{a}, a_0, \alpha) \in [0, \min\{\bar{a}, a_0\}] \), we see \( \frac{\partial s_0^*(\bar{a}, a_0, \alpha)}{\partial \alpha} \geq 0 \) and \( \frac{\partial F_{a_1}^{*}(\bar{a}, a_0, \alpha)}{\partial \alpha} \) = \( -(1-d)f_{e_1}(\bar{a} - (1-d)s_0^*) \frac{\partial s_0^*(\bar{a}, a_0, \alpha)}{\partial \alpha} \leq 0. \)

Now consider \( \frac{\partial F_{a_1}^{*}(\bar{a}, a_0, \alpha)}{\partial \alpha} \). For any realization of \((e_1, e_2)\), see

\[a_2^* = (1-d)s_1^*(\bar{a}, a_1^*, \alpha) + e_2 = (1-d)s_1^*(\bar{a}, (1-d)s_0^*(\bar{a}, a_0, \alpha) + e_1, \alpha) + e_2. \quad (69)\]

Therefore,

\[
\frac{da_2^*}{d\alpha} = (1-d) \left[ \frac{\partial s_1^*(\bar{a}, a_1^*, \alpha)}{\partial \alpha} + (1-d) \frac{\partial s_0^*(\bar{a}, a_1^*, \alpha)}{\partial \alpha} \frac{\partial s_0^*(\bar{a}, a_0, \alpha)}{\partial \alpha} \right] \quad (70)
\]

With the similar analysis as for \( \frac{\partial s_0^*(\bar{a}, a_0, \alpha)}{\partial \alpha} \), we know that \( \frac{\partial s_1^*(\bar{a}, a_0, \alpha)}{\partial \alpha} \geq 0. \) We also know that in equilibrium \( \frac{\partial s_1^*(\bar{a}, a_0, \alpha)}{\partial \alpha} \geq 0. \) Therefore, \( \frac{da_2^*}{d\alpha} \geq 0. \) Therefore, there is a first-order stochastic shift in the distribution of \( a_2^* \) conditional on \( a_0 \), so \( \frac{\partial F_{a_2}^{*}(\bar{a}, a_0, \alpha)}{\partial \alpha} \leq 0. \)

Similarly, we know \( \frac{\partial F_{a_2}^{*}(\bar{a}, a_0, \alpha)}{\partial \alpha} \leq 0 \) for any \( t \geq 1. \) Therefore, the second term of the CPD is positive.

### A.3 Specification of the Numerical Illustrations

The California State Water Projects captures water from the Sierra Nevada through the Feather River into Lake Oroville, the main storage facility of the Project.\footnote{The Project starts from three reservoirs in the Upper Feather area—Antelope Lake, Frenchman Lake, and Lake Davis. Spills and releases from the three reservoirs flow into the Feather River.} In each year, inflows and spills are predominately during winter and spring (January–May). Water stored in Lake Oroville is released into the Oroville-Thermalito Complex (Thermalito Forebay), then transported from the Complex southward through the Feather River, the Sacramento River, and the California Aqueduct, and stored in reservoirs locating along the Project from the north to the south. Around May–June, the Project decides water allocation for contractors in the current year, which generates irrigation benefit in the second half of the year. Around November–December, observing storage in principal reservoirs, the Project announces a preliminary plan for water allocation in the next year. This operation pattern fits our model and we can use the calendar year as the time unit in the specification of the model.
The 1974–2010 data of the end-of-calendar-year storage in principal reservoirs of the California State Water Project are available from the California Department of Water Resources (1963–2013, 1976–2014). The Department (1963–2013) reports the 1975–2010 data of the project wide deliveries. According to the Department (1976–2014), the average annual evaporation-loss rate of the water storage in the five primary storage facilities—Antelope Lake, Frenchman Lake, Lake Davis, Lake Oroville, and the San Luis Reservoir—in 1976, 1981, 1986, 1991, 1996, and 2001 is 0.038, which is approximately 0.04. The Department (1976–2014; 1990–2014) also reports the 1975–2010 data of the amount of spills from Lake Oroville. Given the evaporation-loss rate, the 1974–2010 end-of-calendar-year storage data, the 1975–2010 delivery data, and the 1975–2010 spill data, we can find the corresponding 1975–2010 inflows by calculation, which have a mean of 3891587 acre-feet and a corrected sample standard deviation of 1444480 acre-feet. The total amount of water that is captured by the Project, which is the end-of-calendar-year storage plus the project wide deliveries, has a mean of 7285378 acre-feet for the 20 years that saw positive spills among the 36 years. We set the storage capacity that is equivalent to our model as 7285378 acre-feet.15

The Department (1963–2013) records the 1975–2010 data of the annual deliveries to agricultural use, which have a mean of 936098 acre-feet or, equivalently, 27.80% of the total delivery. We use this percentage to adjust the inflow distribution and the storage capacity, which means that, for agricultural use, the baseline storage capacity is 0.2780 × 7285378 = 2025335 acre-feet and the inflow distribution has a mean of 0.2780 × 3891587 = 1081861 acre-feet and a corrected sample standard deviation of 0.2780 × 1444480 = 401565 acre-feet. The distribution of the adjusted, estimated historical inflows, which we use in the illustrations, is uniform with 36 possible values.16

The Department (1998–2005) publishes its annual estimates of irrigated crop areas, consumed fractions, and applied water per unit of area. The latest data available online are for 2005. We calculate the benchmark water-use efficiency in the following procedure: First, we focus on the county-level data for the 18 counties that were served by the 29 long-term contracting agencies of the California State Water Project at the end of 2010.17 Second, for each county and each crop among the 20 categories of crops, we calculate the total amount of applied water in 2005 by multiplying the irrigated crop area with the applied water per unit area.18 Third, for each county and each

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15The Department (1963–2013) reports that the project wide storage capacity is 5.4038 million acre-feet at the end of 2010. This is not the capacity equivalent to our model.

16The 36 values are 239001, 345959, 538214, 584182, 611960, 632764, 683223, 794128, 824611, 824867, 846706, 888651, 984498, 928424, 968210, 999585, 1052469, 1059629, 1106896, 1108130, 1111920, 1151602, 1186559, 1210988, 1309659, 1336180, 1398546, 1403399, 1409113, 1432491, 1473347, 1486822, 1609242, 1761617, 1813942, and 191946.

17The 18 counties include Alameda, Butte, Kern, Kings, Los Angeles, Napa, Orange, Plumas, Riverside, San Bernardino, San Diego, San Luis Obispo, Santa Barbara, Santa Clara, Solano, Stanislaus, Ventura, and Yuba Counties. The 29 agencies are listed in the California Department of Water Resources (1963–2013, Bulletin 132-11, p. 11).

18The 20 categories include grain, rice, cotton, sugar beets, corn, beans, safflower, other field crops, alfalfa, pasture,
crop, we calculate the total amount of effective water by multiplying the total amount of applied water with the consumed fraction. Finally, we aggregate the total amounts of applied and effective water by counties and crops, and calculate the overall water-use efficiency by dividing the total amount of effective water over the total amount of applied water, which is 0.7135.

A recent estimate of the price elasticity of the water demand for irrigation in California by Schoengold et al. (2006) is $-0.79$ with panel data in which the mean price is $46.49$ per thousand cubic meters, which is approximately $57$ per acre-foot. We then assume that, in our specification, the water demand should be 936098 acre-feet if the water price is $57$ per acre-foot and the water-use efficiency is 0.7135. Given this assumption, we specify three functions of the benefit of water release satisfying, respectively, that 1) the derived water demand (or marginal benefit of water release) is isoelastic and has an elasticity of -1.21, 2) the derived water demand is isoelastic and has an elasticity of -0.79, and 3) the derived water demand is linear and has an elasticity of -0.79 when the demand is 936098 acre-feet. We also assume free disposal of water so that the marginal benefit of water will never be negative. The three functions of the benefit of water release are then shown as in Table 1.

In water project evaluations, the annual discount rate recommended by the California Department of Water Resources (2008) is 0.06. The discount factor is then $\left(1 + 0.06\right)^{-1} = 0.9434$. We then finish specifying the empirical example, as shown in Table 2.

tomatoes for processing, tomatoes for market, cucurbits, onions and garlics, potatoes, other truck crops, almonds and pistachios, other deciduous fruit crops, subtropical fruits, and vines.

19Note that only the relative price but not the absolute price matters, so the choice does not matter for the qualitative results that we illustrate.
Table 1: Specifications of the benefit of water release in the empirical example

<table>
<thead>
<tr>
<th>Benefit of water release</th>
<th>Demand for water release</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B(w, \alpha) = 3.0 \times 10^7 \cdot (\alpha w)^{1 - \frac{1}{\gamma_1}}$</td>
<td>Isoelastic, elastic, $\mu = -1.21$</td>
</tr>
<tr>
<td>$B(w, \alpha) = -7.1 \times 10^9 \cdot (\alpha w)^{1 - \frac{1}{\gamma_2}}$</td>
<td>Isoelastic, inelastic, $\mu = -0.79$</td>
</tr>
<tr>
<td>$B(w, \alpha) = 181.0 \cdot \alpha w - \frac{1.5 \times 10^{-4}}{181.0} \cdot (\alpha w)^2$, where $x \equiv \min {w, \frac{181.0}{1.5 \times 10^{-4}}}$</td>
<td>Linear, equivalent to $\mu = -0.79$</td>
</tr>
</tbody>
</table>

The price elasticity of the water demand is denoted as $\mu$.

Table 2: Specification of the empirical example

<table>
<thead>
<tr>
<th>Inflow in acre-feet</th>
<th>$e_t \sim$ Adjusted, estimated historical inflows, i.i.d.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Evaporation-loss rate</td>
<td>$d = 0.04$</td>
</tr>
<tr>
<td>Discount factor</td>
<td>$\rho = 0.9434$</td>
</tr>
<tr>
<td>Benefit of water release in $</td>
<td>$ $B(w, \alpha) = 3.0 \times 10^7 \cdot (\alpha w)^{1 - \frac{1}{\gamma_1}}$</td>
</tr>
<tr>
<td>(one in each illustration)</td>
<td>$B(w, \alpha) = -7.1 \times 10^9 \cdot (\alpha w)^{1 - \frac{1}{\gamma_2}}$</td>
</tr>
<tr>
<td></td>
<td>$B(w, \alpha) = 181.0 \cdot \alpha x - \frac{1.5 \times 10^{-4}}{181.0} \cdot (\alpha x)^2$, where $x \equiv \min {w, \frac{181.0}{1.5 \times 10^{-4}}}$</td>
</tr>
<tr>
<td>Baseline water-use efficiency</td>
<td>$\alpha = 0.7135$</td>
</tr>
<tr>
<td>Baseline dam capacity in acre-feet</td>
<td>$\bar{a} = 2025335$</td>
</tr>
</tbody>
</table>

Table 3: The empirical example: Responses to a 1% increase in water-use efficiency or water-storage capacities

<table>
<thead>
<tr>
<th>Variable</th>
<th>Elasticity w.r.t. $\alpha$</th>
<th>Elasticity w.r.t. $\bar{a}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Isoelastic, elastic demand</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Marginal benefit of water release with a full dam, $B_1(\bar{a} - \bar{s}, \alpha)$</td>
<td>0.17</td>
<td></td>
</tr>
<tr>
<td>Net present frequency of a full dam, $\sum_{t=0}^{\infty} \rho^t P_0 [a^*_t &gt; \bar{a}]$</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>Marginal benefit of dam capacities, $V_1^* (\bar{a}, a_0, \alpha)$</td>
<td>0.17</td>
<td>-20.27</td>
</tr>
<tr>
<td>Optimal dam capacity, $\bar{a}^*$</td>
<td>(0, 0.01)</td>
<td></td>
</tr>
<tr>
<td>Marginal contribution of water use efficiency, $V_3^* (\bar{a}, a_0, \alpha)$</td>
<td>-0.82</td>
<td>0.01</td>
</tr>
<tr>
<td>Optimal water-use efficiency, $\alpha^*$</td>
<td>(0, 0.01)</td>
<td></td>
</tr>
<tr>
<td><strong>Panel B: Isoelastic, inelastic demand</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Marginal benefit of water release with a full dam, $B_1(\bar{a} - \bar{s}, \alpha)$</td>
<td>-0.26</td>
<td></td>
</tr>
<tr>
<td>Net present frequency of a full dam, $\sum_{t=0}^{\infty} \rho^t P_0 [a^*_t &gt; \bar{a}]$</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>Marginal benefit of dam capacities, $V_1^* (\bar{a}, a_0, \alpha)$</td>
<td>-0.26</td>
<td>-13.88</td>
</tr>
<tr>
<td>Optimal dam capacity, $\bar{a}^*$</td>
<td>(-0.02, 0)</td>
<td></td>
</tr>
<tr>
<td>Marginal contribution of water use efficiency, $V_3^* (\bar{a}, a_0, \alpha)$</td>
<td>-1.25</td>
<td>-0.01</td>
</tr>
<tr>
<td>Optimal water-use efficiency, $\alpha^*$</td>
<td>(-0.01, 0)</td>
<td></td>
</tr>
<tr>
<td><strong>Panel C: Linear demand, inelastic at the mean of water deliveries</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Marginal benefit of water release with a full dam, $B_1(\bar{a} - \bar{s}, \alpha)$</td>
<td>-3.02</td>
<td></td>
</tr>
<tr>
<td>Net present frequency of a full dam, $\sum_{t=0}^{\infty} \rho^t P_0 [a^*_t &gt; \bar{a}]$</td>
<td>4.13</td>
<td></td>
</tr>
<tr>
<td>Marginal benefit of dam capacities, $V_1^* (\bar{a}, a_0, \alpha)$</td>
<td>0.99</td>
<td>-5.86</td>
</tr>
<tr>
<td>Optimal dam capacity, $\bar{a}^*$</td>
<td>(0, 0.17)</td>
<td></td>
</tr>
<tr>
<td>Marginal contribution of water use efficiency, $V_3^* (\bar{a}, a_0, \alpha)$</td>
<td>-1.72</td>
<td>0.05</td>
</tr>
<tr>
<td>Optimal water-use efficiency, $\alpha^*$</td>
<td>(0, 0.03)</td>
<td></td>
</tr>
</tbody>
</table>

Initial conditions: $\bar{a} = 2025335$, $a_0 = 0$, and $\alpha = 0.7135$. The optimal water storage when the dam reaches the full capacity is denoted by $\bar{s}$. Specification follows Tables 1 and 2.
Table 4: The empirical example: Responses to a 5% increase in water-use efficiency or water-storage capacities

<table>
<thead>
<tr>
<th>Variable</th>
<th>Response to $\Delta \alpha$ (%)</th>
<th>Response to $\Delta \bar{\alpha}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Isoelastic, elastic demand</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Marginal benefit of water release with a full dam, $B_1(\bar{\alpha} - \bar{s}, \alpha)$</td>
<td>0.85</td>
<td></td>
</tr>
<tr>
<td>Net present frequency of a full dam, $\sum_{t=0}^{\infty} \rho^t P_0 [a_t^* &gt; \bar{\alpha}]$</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>Marginal benefit of dam capacities, $V_1^*(\bar{\alpha}, a_0, \alpha)$</td>
<td>0.85</td>
<td>-43.86</td>
</tr>
<tr>
<td>Optimal dam capacity, $\bar{\alpha}^*$</td>
<td>(0, 0.10)</td>
<td></td>
</tr>
<tr>
<td>Marginal contribution of water use efficiency, $V_3^*(\bar{\alpha}, a_0, \alpha)$</td>
<td>-3.95</td>
<td>0.03</td>
</tr>
<tr>
<td>Optimal water-use efficiency, $\alpha^*$</td>
<td>(0, 0.04)</td>
<td></td>
</tr>
<tr>
<td><strong>Panel B: Isoelastic, inelastic demand</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Marginal benefit of water release with a full dam, $B_1(\bar{\alpha} - \bar{s}, \alpha)$</td>
<td>-1.29</td>
<td></td>
</tr>
<tr>
<td>Net present frequency of a full dam, $\sum_{t=0}^{\infty} \rho^t P_0 [a_t^* &gt; \bar{\alpha}]$</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>Marginal benefit of dam capacities, $V_1^*(\bar{\alpha}, a_0, \alpha)$</td>
<td>-1.29</td>
<td>-46.56</td>
</tr>
<tr>
<td>Optimal dam capacity, $\bar{\alpha}^*$</td>
<td>[-0.14, 0)</td>
<td></td>
</tr>
<tr>
<td>Marginal contribution of water use efficiency, $V_3^*(\bar{\alpha}, a_0, \alpha)$</td>
<td>-5.99</td>
<td>-0.06</td>
</tr>
<tr>
<td>Optimal water-use efficiency, $\alpha^*$</td>
<td>[-0.05, 0)</td>
<td></td>
</tr>
<tr>
<td><strong>Panel C: Linear demand, inelastic at the mean of water deliveries</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Marginal benefit of water release with a full dam, $B_1(\bar{\alpha} - \bar{s}, \alpha)$</td>
<td>-7.07</td>
<td></td>
</tr>
<tr>
<td>Net present frequency of a full dam, $\sum_{t=0}^{\infty} \rho^t P_0 [a_t^* &gt; \bar{\alpha}]$</td>
<td>12.84</td>
<td></td>
</tr>
<tr>
<td>Marginal benefit of dam capacities, $V_1^*(\bar{\alpha}, a_0, \alpha)$</td>
<td>4.87</td>
<td>-25.18</td>
</tr>
<tr>
<td>Optimal dam capacity, $\bar{\alpha}^*$</td>
<td>(0, 0.97)</td>
<td></td>
</tr>
<tr>
<td>Marginal contribution of water use efficiency, $V_3^*(\bar{\alpha}, a_0, \alpha)$</td>
<td>-8.57</td>
<td>0.31</td>
</tr>
<tr>
<td>Optimal water-use efficiency, $\alpha^*$</td>
<td>(0, 0.18)</td>
<td></td>
</tr>
</tbody>
</table>

Initial conditions: $\bar{\alpha} = 2025335$, $a_0 = 0$, and $\alpha = 0.7135$. The optimal water storage when the dam reaches the full capacity is denoted by $\bar{s}$. Specification follows Tables 1 and 2.
Period 0

Wet season: Given water availability, \( a_0 \), and the dam capacity, \( a \), the dam captures water of min\( \{a_0, a\} \).

Dry season: Given water use efficiency, \( \alpha \), the dam releases water of \( w_0 = \min \{a_0, \alpha\} \), which generates benefit of \( B(w_0, \alpha) \).

Figure 1: Operation of the dam in the single-period, two-season, deterministic model

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Period 0

Wet season: Given water availability, \( a_0 \), and the dam capacity, \( a \), the dam captures water of min\( \{a_0, a\} \).

Dry season: Given water use efficiency, \( \alpha \), the dam releases water of \( w_0 = \min \{a_0, \alpha\} \), which generates benefit of \( B(w_0, \alpha) \).

Figure 2: Operation of the dam in the two-period, stochastic model

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Period 1

Wet season: The dam receives a stochastic inflow, \( e_t \). Given the evaporation loss rate, \( d \), water availability is \( a_t = (1-d)x_0 + e_t \). The dam captures water of min\( \{a_t, \overline{a}\} \).

Dry season: The dam releases water of \( w_1 = \min \{a_t, \overline{a}\} \), which generates benefit of \( B(w_1, \alpha) \).
With \( B_{12}(w, \alpha) \leq 0, B_{121}(w, \alpha) \leq 0, B_{111}(w, \alpha) \leq 0, \) and \( B_{1211}(w, \alpha) \geq 0, \) water-use efficiency improvement will increase the optimal water storage, \( s_0^* \), and the likelihood of the dam reaching the full capacity in the future, \( P[(1 - d)s_0^* + e_1 \geq \tilde{a}] \). Specification: \( B(w, \alpha) = 181.0 \cdot \alpha x - \frac{1.5 \times 10^{-4}}{2} \cdot (\alpha x)^2 \), where \( x = \min \left\{ w, \frac{181.0}{1.5 \times 10^{-4} \cdot \alpha} \right\} \), low \( \alpha = 0.6 \), high \( \alpha = 0.8 \), \( \tilde{a} = 2038052 \), \( a_0 = 0.8\tilde{a} \), \( d = 0.04 \), \( \rho = 0.9434 \), the probability of \( e_1 = 975785 \) is 0.8, and the probability of \( e_1 = 1536597 \) is 0.2

Figure 3: An example of water-use efficiency increasing the optimal water storage
The marginal benefit of water release when the dam reaches the full capacity is $p \equiv B_1(\bar{a} - \bar{s}, \alpha)$, where $\bar{s}$ is the optimal water storage when $a_t \geq \bar{a}$. Specification follows Table 2 in Section 5.

Figure 4: Example of the solution to the water-inventory management problem in the extended model