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OLIGOPOLY, UNCERTAIN DEMAND AND FORWARD MARKETS

by

Rafael Eldor and Itzhak Zilcha*

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*Department of Economics, Tel-Aviv University, Ramat Aviv, ISRAEL.

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FOERDER INSTITUTE FOR ECONOMIC RESEARCH
Faculty of Social Sciences,
Ramat Aviv, Israel.

1. INTRODUCTION

The behavior of a competitive firm facing price uncertainty with or without forward markets has been studied extensively in recent years [for example, Sandmo (1971), Danthine (1978), Holthausen (1979), Katz and Paroush (1979), Feder, Just and Schmitz (1980)]. The behavior of a monopoly and a price discriminating firm facing uncertain demand have also been studied (for example, Leland (1972), Katz, Paroush and Kahana (1982), Katz (1984), Eldor and Zilcha (1985)). However, the behavior of an oligopolistic firm facing uncertain demand with or without forward markets has not been addressed.

This paper considers an oligopolistic market with uncertain demand where firms are risk-averse. We analyze the impact of demand uncertainty upon the Nash Equilibrium (N.E.) output and the firm's profits. It has been indicated, at least implicitly, that for a competitive firm and monopoly, introducing forward markets where the forward price is unbiased (i.e. equal to the expected future spot price), increases production and the profits of the firms (for example, Feder, Just and Schmitz (1980); Eldor and Zilcha (1985)). We show that in the oligopoly case the Nash equilibrium output increases in the presence of unbiased forward market, thus forward markets enhance competition in this case as well. However, it is not necessarily true that firms are better off. The introduction of unbiased forward markets have two opposing effects upon the profits of the firms. First, it provides the firm with actuarially fair risk-sharing; thus, since the firm is risk averse, it renders its profit riskless, and improves its position. Secondly, due to the introduction of unbiased forward markets, the N.E. total output rises, resulting in lower profits; hence this effect lowers the profits of the firms. We show that, although firms are risk-averse, in some cases such unbiased markets make all the firms worse off in the N.E. when forward markets are utilized. Moreover, the N.E. without unbiased forward markets is not a N.E. in the presence of these markets.

Such cases can also occur when we consider oligopoly with asymmetric information; namely, when some firms are "informed" (know the demand curve before production takes place) and some are "uninformed", but can observe the actions of the informed firms. In equilibrium, if the informed firms use their information all firms are worse off in such cases. Palfrey (1982) has shown that private information can be disadvantageous to the informed firm (which is the less risk averse firm in his examples), but the uninformed firm, which observes the output of the informed firm, is better off.

The paper is organized as follows. We present in section 2 the model and derive some results when forward markets are not available. In section 3 we analyze Nash equilibria with forward market. Section 4 contains an example showing that unbiased forward markets may not be desirable by any risk-averse firm. We reinterpret this example in an asymmetric information model in section 5. In section 6 we consider competitive industry and show that the industry's output, in a long-run equilibrium, increases when an unbiased forward market is introduced.

2. THE MODEL

Consider an oligopoly with n identical firms. Each firm produces a homogeneous good with a cost function $C(q)$ where $C' > 0$, $C'' \geq 0$. The market demand is random and is given by $\tilde{P} = P(Q,\tilde{r})$ where $Q = \sum_{i=1}^{n} q_i$ is the total output of the producing firms and $\tilde{r}$ is a random variable with a known distribution function. We assume that P is continuously differentiable, $\frac{\partial P}{\partial Q} < 0$, $\frac{\partial P}{\partial r} \leqq 0$ and $\frac{\partial^2 P(Q,r)}{\partial Q \partial r} \leqq 0$. Each firm maximizes expected utility of profits where its utility function U satisfies: $U' > 0$ and $U'' < 0$, i.e. it is a risk-averse firm.

Let us consider first the case where no forward market or any risk-sharing mechanism, are available. In this case an output profile $(\bar{q}_1,\ldots,\bar{q}_n)$ is a Nash Equilibrium if for all i the maximum of

$$(1) \qquad EU[q_i P(\textstyle\sum_{j\neq i}\bar{q}_j + q_i,\tilde{r}) - C(q_i)]$$

is attained at $\bar{q}_i$, $i = 1,\ldots,n$. Since all functions are continuously differentiable a symmetric Nash Equilibrium exists under a very mild assumption about the distribution of $\tilde{r}$. Since all firms are identical assuming that the N.E. output is positive implies that $\bar{q}_i = \bar{q} > 0$ for all i; this equilibrium is unique and $\bar{q}$ can be determined from the following necessary and sufficient condition:

$$(2) \qquad E[P(n\bar{q},\tilde{r}) + \bar{q}\,\frac{\partial P}{\partial Q}(n\bar{q},\tilde{r}) - C'(\bar{q})]U'(\bar{\pi}) = 0$$

where $\bar{\pi} = \bar{q}P(n\bar{q},\tilde{r}) - C(\bar{q})$. Under our assumptions there exists a unique solution $\bar{q}$ to (2).

Let us compare the output in this industry under uncertain demand with the output in the "certainty-equivalent" case, i.e. when demand is $P(Q,\bar{r})$,

$\bar{r} = E\tilde{r}$. Denote by $MR(\hat{q},\tilde{r}) = \frac{\partial}{\partial q}[qP((n-1)\hat{q}+q,\tilde{r})]\Big|_{q=\hat{q}}$ i.e. the marginal revenue function when all firms produce the same outputs. We assume that $MR(q,r)$ is non-increasing in q i.e. $\frac{\partial MR}{\partial q}(q,r) \leqq 0$ for all (q,r). By our assumptions about the demand $MR(q,r)$ is nonincreasing in r. Now we prove:

<u>Theorem 1</u>: (a) If $MR(q,r)$ is concave in r then the Nash equilibrium output under uncertain demand is lower than the output under the certainty-equivalent demand.

(b) If $MR(q,r)$ is strictly convex in r, in some cases the N.E. output under uncertain demand is higher than the certainty-equivalent case.

All proofs are relegated to the Appendix. Let us show how increasing risk aversion affects the production in N.E.,

<u>Theorem 2</u>: Assume that $QP(Q,r)$ is a concave function of Q (this is the total revenue since Q is the total output). As a firm becomes more risk-averse its production in N.E. decreases (keeping the output of all other firms fixed).

Note that the assumption in Theorem 2 holds when $P(Q,r)$ is linear in Q. Theorem 2 implies that when all firms become more risk averse the N.E. production declines.

It is usually well accepted in economic theory that for risk-averse economic agents, removing the uncertainty, e.g. by considering the certainty equivalent case, results in a <u>better</u> position, namely higher utility. For a competitive firm or a monopoly replacing the random price $P(Q,\tilde{r})$ by $P(Q,\bar{r})$ will certainly not make this risk averse firm worse off (see, for example, Sandmo (1971), Leland (1972)). In section 4 we bring an example which shows that in the case of an oligopoly, with risk-averse firms, this is not necessarily the case. Namely, in some cases removing the uncertainty, i.e. replacing $P(Q,\tilde{r})$ by the demand function $P(Q,\bar{r})$, may make <u>all</u> the firms worse off in the Nash equilibrium.

3. NASH EQUILIBRIUM IN THE PRESENCE OF FORWARD MARKETS

We introduce now forward markets for the good produced by this oligopoly. The firms may buy or sell output in the forward market at a forward price P_f. Modern financial markets theory argues that the forward price, i.e. the price that equilibrate demand and supply for forward contracts should not be studied in isolation. Richard and Sundaresan (1981) argue that the relationship between the forward price and the expected spot price depends on the covariance of the marginal utility of wealth of the forward market participants and the spot price; i.e. if this covariance is zero (positive) (negative) then the forward price incorporates zero (negative) (positive) risk premium.

Let us denote by X_i the forward sale (or purchase if X_i is negative) of the commodity. We assume that production takes time and that the delivery date, i.e. the maturity of the contract, occurs on the date where the production process is completed. Assuming the existence of many other agents taking long and short positions in this forward market, it follows that each firm's position in the market X_i has no effect upon the forward price P_f. These firms can affect the forward price only through their total production. Hence the future spot price can be written as a function of the total output produced, i.e. $P_f=P_f(\sum_{j=1}^{n}q_j)$. Clearly, P_f may also depend upon the distribution of the random variable $\tilde{r}$. We assume that P_f is a decreasing and continuously differentiable function of Q. The optimal output and forward sale of firm i, given the output levels of the other firms $(q_j^*)_{j\neq i}$ is given by:

$$\max_{q_i, X_i} EU[q_i P(\sum_{j\neq i} q_j^* + q_i, \tilde{r}) - C(q_i) + (P_f(\sum_{j\neq i} q_j^* + q_i) - P(\sum_{j\neq i} q_j^* + q_i, \tilde{r}))X_i]$$

The first-order conditions (which are sufficient for optimum) are:

(3) $$E[MR(q_i^*, \tilde{r}) - C'(q_i^*) + (\frac{\partial P_f}{\partial Q}(\sum q_j^*) - \frac{\partial P}{\partial Q}(\sum q_j^*, \tilde{r}))X_i^*]U'[\pi_i^*] = 0$$

(4) $$E[P_f - P(\sum q_j^*, \tilde{r})]U'(\pi_i^*) = 0$$

where

(5) $$\pi_i^* = (q_i^* - X_i^*)P(\sum_{j=1}^n q_j^*, \tilde{r}) + P_f X_i^* - C(q_i^*).$$

We first analyze the oligopolistic firms' optimal hedging policies in various cases of forward markets and then the impact of a forward market on the firm's production decisions.

<u>Theorem 3:</u> When forward markets are available with forward price P_f, Firm i's optimal hedge X_i^* in the Nash Equilibrium will satisfy:

$$X_i^* \lesseqqgtr q_i^* \quad \text{if and only if} \quad P_f \lesseqqgtr EP(\sum_{j=1}^n q_j^*, \tilde{r}) .$$

Theorem 3 states that when forward markets for the firm's product is unbiased, i.e. $P_f \equiv EP(\sum_{j=1}^n q_j^*, \tilde{r})$ then each risk averse firm will sell forward all its planned output. When the risk premium is positive, i.e. $P_f < EP(Q^*, \tilde{r})$, the normal backwardation case, the firm sells forward only part of its output. Under our assumptions there exists a unique symmetric Nash equilibrium with $q_i^* = X_i^* = q^*$ for all i. This clearly means that in the unbiased case the profit at equilibrium Π_i^* is <u>nonrandom</u> (see (5)). Thus (3) can be rewritten as follows:

$$(6)\qquad E[MR(q^*,\tilde{r}) - C'(q^*)]U'[P_f q^* - C(q^*)] = 0$$

which clearly implies that

$$(7)\qquad E\,MR(q^*,\tilde{r}) = C'(q^*)$$

Now we state

<u>Theorem 4</u>: When an unbiased forward market is made available to the firms, the production of each firm in the Nash Equilibrium will rise. Moreover, the production in this case is independent of the firm's degree of risk aversion.

Theorem 4 demonstrates the role of unbiased forward markets in increasing competitiveness and economic efficiency due to the higher production of this oligopoly. Condition (7) clearly shows that the output depends upon the distribution of $\tilde{r}$, thus the Separation property does not hold for oligopoly market (i.e. that the output is independent of the preferences <u>and</u> the distribution of $\tilde{r}$); however, (7) demonstrates that the output in N.E. <u>does not</u> depend on the firm's utility function (only on its monotonicity property). Let us consider now a special case.

<u>Corollary 1</u>: Assume unbiasedness, i.e. $P_f \equiv EP(\sum q_j^*,\tilde{r})$, and that $E\frac{\partial P(Q,\tilde{r})}{\partial Q}$ is independent of the distribution of $\tilde{r}$. Then the optimal production of each firm in Nash equilibrium is independent of the distribution of $\tilde{r}$.

Under the assumptions of the Corollary the separation result holds. Note that the second assumption of this Corollary holds when the stochastic demand function is linear in Q and linear in r, or in the case where $P(Q,r)$ is additive.

The question we pose now is: In this N.E. are the oligopolistic firms better off as a result of introducing unbiased forward markets Does this fair risk-sharing mechanism always improve the positions of these firms in the Nash equilibrium even though the expected profits decline? The next section shows that the answer may be negative in some cases.

4. ARE UNBIASED FORWARD MARKETS ALWAYS DESIRABLE

In the previous section it is shown that unbiased forward markets increase economic efficiency, by eliminating the uncertainty faced by the oligopoly, thus increasing their output. Now we show that all firms may end up with a lower utility, compared to the expected utility in N.E. without the fair risk-sharing device, i.e. the forward market.

Consider a duopoly with two identical firms. For simplicity let us take the cost functions to be identically zero, i.e. $C(q) \equiv 0$. Let the demand function be linear,

$$(8) \qquad P(Q) = \alpha - \tilde{r}Q \qquad \alpha > 0,\ \tilde{r} > 0,\ \text{a.s.}\ .$$

The Nash equilibrium output $\bar{q}$ of each firm can in this case be computed from

$$(9) \qquad E\{(\alpha - 3\tilde{r}\bar{q})U'[\alpha\bar{q} - 2\tilde{r}\bar{q}^2]\} = 0$$

Introducing unbiased forward market with forward price $P_f = \alpha - \bar{r}(q_1 + q_2)$, the Nash equilibrium will now be with outputs (q^*, q^*) where

$$\alpha - 3\bar{r}q^* = 0 \quad \text{i.e.} \quad q^* = \frac{\alpha}{3\bar{r}} .$$

The profits of each firm in this case are

(10) $$\pi^* = \frac{\alpha^2}{9\bar{r}}$$

Note that these outputs correspond to N.E. with "certain" demand curve $P(Q) = \alpha - \bar{r}Q$.

Now let us choose the following utility functions and demand:

$$U(x) = x^{0.9} \qquad \tilde{r} = \begin{cases} 4.5 & \text{in prob. } 0.5 \\ 1.5 & \text{in prob. } 0.5 \end{cases}$$

and $\alpha = 10$. Since $\bar{r} = 3$ the profit of each firm in the N.E. with the unbiased forward market is given by (10), namely

(11) $$\Pi^* = \frac{100}{27} \quad , \quad q^* = \frac{10}{9} .$$

The production in N.E. which takes place under uncertain demand can be computed from (9), i.e.

(12) $$(10-13.5\bar{q})[10\bar{q}-9\bar{q}^2]^{-0.1} + (10-4.5\bar{q})[10\bar{q}-3\bar{q}^2]^{-0.1} = 0$$

or

(13) $$\frac{10-9\bar{q}}{10-3\bar{q}} = \left[\frac{10 - 13.5\bar{q}}{10 - 4.5\bar{q}}\right]^{10}$$

It can be verified that the solution to (13) satisfies:

$$1.0465 < \bar{q} < 1.047$$

Let us check the profits and expected utility when $q = 1.047$:

$$\Pi_1 = 0.604119 \qquad \Pi_2 = 7.18137$$

Now it can be shown that

$$\frac{1}{2}\cdot\pi_1^{0.9} + \frac{1}{2}\pi_2^{0.9} = 3.2659 > (\frac{100}{27})^{0.9} = 3.249$$

Now when $q = 1.0465$ we obtain:

$$\pi_1 = 0.60854 \quad \text{and} \quad \pi_2 = 7.1795$$

$$\frac{1}{2}[\pi_1^{0.9} + \pi_2^{0.9}] = 3.2672 > (\pi^*)^{0.9} = 3.249$$

which proves that in the N.E. $(\bar{q},\bar{q})$ each firm has higher expected utility than in the N.E. (q^*,q^*) where the unbiased forward markets are introduced into this economy and utilized by each firm. To complete this example let us show now that when the forward market is available to the dupoly, $(\bar{q},\bar{q})$ is not a Nash equilibrium any more. Assume that firm 2 produces $\bar{q}$, say 1.047. Firm 1's profit π_1, when the forward market is being used by firm 1 can be computed directly, taking into account that $q_1 = X_1$ (condition (12) still holds),

$$\pi_1 = 10q_1 - 3q_1^2 - 3\cdot 1.047q_1$$

From the maximization problem for Firm 1, given $q_2 = 1{,}047$ and that π_1 is nonstochastic since the forward market is being used by this firm, we obtain

$$E[10 - \tilde{r}\,1.047 - \tilde{r}a_1 - 3q_1]U'(\pi_1) = 0$$

Thus $q_1 = 1.143$ and $\pi_1 = 3.920$. Since $(\pi_1)^{0.9} = 3.41$ is higher than the expected utility when the firm produces $\bar{q}$, it is going to deviate from $\bar{q}$ i.e. $(\bar{q},\bar{q})$ is not a Nash equilibrium in the presence of unbiased forward market.