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Technical Change, Productivity and Welfare under Distorted Prices

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Abstract - Three conceptual scalar measures of technical change are change in consumer welfare, the rate of technical change and total factor productivity. The last two are biased measures of the first if commodities are subsidized or taxed. A general equilibrium analysis reveals the relationships among these three alternative measures of productivity.

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Productivity is measured in a number of ways, but standard indexing approaches measure aggregate productivity change as the “Solow residual”, *i.e.*, the share-weighted sum of changes in output minus the share-weighted sum of changes in inputs. Commonly, the weights are revenue shares for output and cost shares for inputs, averaged across beginning and ending periods. If prices are subsidized or otherwise distorted, it’s clear that the measured productivity will be different than if equilibrium prices had prevailed (a similar question was recently adressed for the case of unpriced public goods (Perrin and Fulginiti, AJAE, forthcoming). Which would be the “best” measure, if indeed either would be? That depends upon what it is, fundamentally, that one wishes to measure with productivity. The present paper asserts that the best measure is one that approximates the improvement in consumer welfare due to the technical change, and examines how price distortions cause this measure to differ from the standard indexing approach.

To demonstrate the ambiguity of the notion of productivity, let us introduce three different concepts of “progress” resulting from a technical change. First is the (dual) rate of technological change as measured by the rate of increase in profit at constant prices,

$$\begin{aligned} Rate\,of\,Technical\,Change\,(RTC) &= \frac{\Pi_\tau(w,\tau)}{\Pi(w,\tau)} \\ &= k\ \partial \ln y(w,\tau)/\partial\tau \\ &= k\ D_y^{-1}\ \Pi_{w,\tau}\, d\tau. \end{aligned} \qquad \textbf{(1)}$$

where $\Pi(\boldsymbol{w},\tau)$ is the profit function, Π_τ represents partial derivative with respect to τ, $\boldsymbol{w}$ is a vector of prices for a corresponding vector of netputs $\boldsymbol{y}$, τ is a scalar proxy for technical change,

$\boldsymbol{k}$ is a vector of shares of netputs in profit, and D_y is a matrix with vector $\boldsymbol{y}$ displayed on the diagonal. Consider the two-good world of figure 1, with y_1 as the numeraire good, in which a technical change has shifted the production possibilities curve outward and the economy has responded by moving from equilibrium bundle A to equilibrium bundle B. RTC as we define it is equivalent to the ratio of line segment *ad* to *Oa*.

The second concept of progress due to technical change is an index measure of total factor productivity change, the average-profit-share-weighted change in netputs (outputs minus inputs, since inputs are negative values in the netput vector):

$$\begin{aligned} &Total\,Factor\,Productivity\,(TFP) \\ &\equiv \bar{k}\;\frac{dln\,y(w,\tau)}{d\tau} \\ &= \bar{k}\;[\Sigma_{yw}\;\frac{dln\,w}{d\tau} + \frac{\partial \ln y}{\partial \tau}] \\ &= \;\delta + \bar{k}\,\Sigma_{yw}\,dlnp/d\tau. \end{aligned} \qquad \textbf{(2)}$$

where $\bar{\boldsymbol{k}}$ is the vector of average profit shares, $\boldsymbol{\Sigma}_{yw}$ is the supply elasticity matrix of output $\boldsymbol{y}$ with respect to prices $\boldsymbol{w}$. Total factor productivity measures the rate of technological change plus a price effect, but the price effect cannot be identified apart from an equilibrium model, a topic we will return to shortly.

The third concept relevant to progress from technical change is the Hicksian equivalent variation (EV), i.e., the minimum dollar amount that, if given to consumers facing initial prices, would allow them to realize an increase in welfare equivalent to what they would have gained from the introduction of the technology. In the two-good world of figure 1, EV is distance *cf* , which we normalize by dividing by the initial expenditure *Oc* as

$$\begin{aligned} Equivalent\ \ Variation\ \ (EV) &\equiv \frac{E(p,u')}{E(p,u)} = s\,dlnx(p,u) \\ &= s[\mathrm{H}_{xp}\frac{dlnp}{d\tau} + \eta\frac{dlnu}{d\tau}] \qquad \textbf{(3)} \\ &= \frac{dlnu}{d\tau}. \end{aligned}$$

where $E(\boldsymbol{p},u)$ is the minimum expenditure required, at prices $\boldsymbol{p}$, to achieve welfare level u, u' is welfare at the new equilibrium, $\boldsymbol{s}$ is a vector of expenditure shares, H_{xp} is the compensated demand elasticity matrix, and $\boldsymbol{\eta}$ is a vector of income elasiticities of demand. (The derivation of the third row from the second follows from the adding-up and homogeneity properties of the expenditure function.) It is clear that the EV measure of progress from technical change cannot be evaluated apart from an equilibrium system to evaluate the comparative statics of changes in u. This we take up in the next section.

We argue that EV is the more appropriate of these scalar measures of productivity. From the definitions above, it is evident that they differ by the set of weights used to evaluate changes in bundles of goods, and they differ by the reference bundle that is used to compare with the original bundle (in figure 1, RTC compares bundle C to A, TFP compares B to A and EV compares D to A). For weights on commodities, RTC uses initial prices facing firms, TFP uses the average of the two equilibrium sets of firm-level prices, while EV uses initial consumer prices. To extent that economists have attempted to adjust market prices, it has been toward estimation of producer prices (Cowing and Stevenson, Smith). But if these weights differ from one another, it seems clear, as Perrin and Fulginiti have argued, that consumer prices are the more appropriate, since firms have no purpose other than to increase the welfare of the consumers who either own the firms, are employed by the firms, or are customers of the firms. The thermodynamics metaphor is appropriate here: firm activity cannot increase or decrease the sum of matter and energy, only re-arrange it. We measure changes in productivity only because we do not account for all these inputs and outputs - we account only for those which are important to mankind. And it is surely the consumers' valuations of these inputs and outputs that is important, rather than prices faced by firms, if the two differ. Under price distortions created by subsidies or taxes, the two sets of prices certainly differ, and we now turn to a simple general equilibrium model that reveals how the distortions cause RTC and TFP to differ from EV.

General Equilibrium Models of Response to Technical Change

The foundation for the general equilibrium models consists of the expenditure and profit functions in equation (4). The expenditure function specifies the minimum cost of achieving welfare level *u*, given prices $\boldsymbol{p}$. The profit function can be

$$E(p,u) \equiv Min_x [px \| u(x) \geq u], \ and$$
$$\Pi(w,\tau) \equiv Max_y [wy \| (y,\tau) \in T], \qquad \textbf{(4)}$$

where: *x and y are nx1 vectors of quantities of goods chosen by consumers and producers, respectively, negative values indicating quantities supplie by consumers or demanded by producers,*
p is the nx1 vector of consumer prices for x,
w is the nx1 vector producer prices for y,
τ is an index of technological change,
T is the feasible technology set.

interpreted either as the maximum profit from an aggregate economy, or the aggregate of individual firm profit functions.

We first examine equilibria in a simple undistorted closed economy to show that the three measures of progress are equivalent in such a case. Then we examine the equilibrium adjustments in a distorted economy to observe how they diverge from one another. In the simple undistorted general equilibrium, the prices ***w*** and ***p*** are equal, and the equilibrium conditions are

$$a. \ E = \Pi$$
$$b. \ E_p = \Pi_w \ , \qquad \textbf{(5)}$$

the first equation representing the budget constraint, the second requiring equilibrium in the commodity markets. The logarithmic total differential of these equations can be expressed as

$$
\begin{aligned}
dlnu + s\,dlnp &= s\,dlnp + \delta\, d\tau \;, \quad or \\
dlnu &= \delta\, d\tau \;, \quad and
\end{aligned}
$$

$$\eta\, dlnu + (\mathrm{H}_{xp} - \Sigma_{xp})\, dlnp = (B + \iota\delta)\, d\tau.$$

$$
\begin{aligned}
where:\ & \delta = \Pi_\tau/\Pi = w\Pi_{w\tau} = s(dlnx/d\tau)\,,\ \textit{the rate of technological change}, \\
& \eta = uD_x^{-1}E_{pu}\,,\ \textit{nx1 vector of real income expenditure elasticities}, \\
& \mathrm{H}_{xp} = D_x^{-1}E_{pp}D_p\,,\ \ \textit{nxn compensated demand elasticity matrix}, \\
& \Sigma_{yp} = D_x^{-1}\Pi_{ww}D_p\,,\ \ \textit{nxn supply elasticity matrix}, \\
& B = D_x^{-1}\Pi_{w\tau} - \iota\delta\,,\ \ \textit{an nx1 vector of biases of technological change}, \\
& \iota\ \textit{is a unit vector of conformable length}\ .
\end{aligned}
\tag{6}
$$

Here it is obvious from the first rows that EV = dln$u$/dτ =δ =RTC. Solving the remaining equation for dln$\boldsymbol{p}$=$(H_{xp}$ - $\Sigma_{yw})^{-1}$ $(B + (\iota\text{-}\eta)\delta)d\tau$, it becomes clear that equilibrium prices will change unless the bias of technical change, $\boldsymbol{B}$, is zero and also the income elasticites of demand all equal unity so that ($\boldsymbol{\iota}$-$\boldsymbol{\eta}$) = 0. (In this special case, the shifts in the production possiblities curve and the iso-welfare curve of Fig. 1 are both radial expansions.) Given this price change, and given that the adding-up property assures that $\boldsymbol{s\Sigma}_{xw}$=0, TFP is evaluated from equation **2** as TFP = δ. In the case of undistorted equilibrium adjustments to technical change, then, RTC=TFP=EV=δ. The measures are equivalent.

Now distort these equilibria by introducing price wedges in the form of taxes or subsidies. The equilibrium conditions are now the three equations in **(7)**, in which consumer and producer prices are separated by a vector of wedges, $\boldsymbol{\rho}$, that we interpret as a vector of ad-valorem tax or subsidy rates, with tax proceeds being simple transfers to consumers, and subsidies (negative wedges) being negative transfers. The wedges might alternatively be interpreted as a vector of rental rates arising from imperfect competition or from quotas, that are also returned to consumers as the owners of the firms collecting the rents.

$$
\begin{aligned}
&a.\ E = \Pi + \rho\Pi_w \\
&b.\ E_p = \Pi_w \\
&c.\ p = w(I + D_\rho),
\end{aligned}
\qquad \textbf{(7)}
$$

where: ρ *is a vector of wedges between consumer and producer prices, and* D_z *denotes a matrix with vector z on the diagonal.*

Here, **7a** specifies the budget constraint, with tax receipts redistributed to consumers. Equation **7b** imposes commodity equilibrium. Equation **7c** specifies the nature of the wedges, ρ. With this specification, $\boldsymbol{p_i}/\boldsymbol{w_i} = 1 + \rho_i$, so that $\rho_i = 0$ indicates the absence of a wedge for good i, and $d\ln \boldsymbol{p}_i - d\ln \boldsymbol{w}_i = d\rho_i$.

Substituting **7c** into **7a** and **7b**, the total logarithmic differentials of **7a** and **7b** can be expressed as:

$$
\begin{aligned}
&a^{/}.\ dlnu - s_t\Sigma_{yw}dlnp = (\delta + s_t\mathrm{B})\,d\tau\ , \\
&b^{/}.\ \eta\, dlnu + (\mathrm{H}_{xp} - \Sigma_{xp})\,dlnp = (\iota\delta + \mathrm{B})\,d\tau .
\end{aligned}
\qquad \textbf{(8)}
$$

where: $s_t = D_\rho \Pi_w D_w / E$, *nx1 vector of tax receipts (subsidy payments) as shares of expenditures.*

The effect of a change in technology on this equilibrium can be represented by the solution of these comparative statics equations:

$$\begin{bmatrix} dlnu \\ dlnp \end{bmatrix} = \begin{bmatrix} 1 & -s_t\Sigma_{xw} \\ \eta & \mathrm{H}_{xp}-\Sigma_{xw} \end{bmatrix}^{-1} \begin{bmatrix} \delta + s_t\mathrm{B} \\ \iota\delta + \mathrm{B} \end{bmatrix} d\tau = \begin{bmatrix} 1 & -s_t\Sigma_{xw} \\ \eta & \mathrm{H}_{xp}-\Sigma_{xw} \end{bmatrix}^{-1} \begin{bmatrix} 1 & s_t \\ \iota & I \end{bmatrix} \begin{bmatrix} \delta \\ \mathrm{B} \end{bmatrix} d\tau$$

$$= \begin{bmatrix} 1-s_t\Sigma_{xw}\Delta^{-1}\eta & s_t\Sigma_{xw}\Delta^{-1} \\ -\Delta^{-1}\eta & \Delta^{-1} \end{bmatrix} \begin{bmatrix} 1 & s_t \\ \iota & I \end{bmatrix} \begin{bmatrix} \delta \\ \mathrm{B} \end{bmatrix} d\tau \qquad \textbf{(9)}$$

$$= \begin{bmatrix} 1+s_t\Sigma_{xw}\Delta^{-1}(\iota-\eta) \\ \Delta^{-1}(\iota-\eta) \end{bmatrix} \delta\, d\tau + \begin{bmatrix} s_t[I+1\Sigma_{xw}\Delta^{-1}(I-\eta s_t)] \\ -\Delta^{-1}(I-\eta s_t) \end{bmatrix} \mathrm{B}\, d\tau$$

where: $s_t = wD_\rho D_x/E$, *a vector of taxes as shares of expenditures,*
$\Delta = [\mathrm{H}_{xp} - \Sigma_{xw} + \eta s_t \Sigma_{xw}]$.

Here the effects of the technical change on welfare and prices are expressed in terms of the parametric characteristics of the change, the rate δ and the bias B.

The measures of technological progress can now be compared. From equation **(2)**, TFP is

$$TFP = \delta + \bar{k}\Sigma_{yw}\frac{dlnp}{d\tau}$$
$$= \delta + \bar{k}\Sigma_{yw}[\Delta^{-1}(\iota-\eta)\delta + \Delta^{-1}(I-\eta s_t)\mathrm{B}] \qquad \textbf{(10)}$$
$$=[1 + \bar{k}\Sigma_{yw}\Delta^{-1}(\iota-\eta)]\delta + \bar{k}\Sigma_{yw}\Delta^{-1}(I-\eta s_t)\mathrm{B}$$

For comparison, EV is evaluated from **(3)** as

$$EV = dln u = [1 + s_t \Sigma_{yw} \Delta^{-1}(\iota - \eta)]\delta \;+\; [s_t \;+\; s_t \Sigma_{yw} \Delta^{-1}(I - \eta s_t)]\mathrm{B} \qquad \textbf{(11)}$$

and the difference between the two is

$$EV - TFP \;=\; (s_t - \bar{k})\Sigma_{yw}\Delta^{-1}(\iota - \eta)\delta \;+\; [s_t \;+\; (s_t - \bar{k})\Sigma_{yw}\Delta^{-1}(I - \eta s_t)]\mathrm{B} \qquad \textbf{(12)}$$

Thus we see that when commodities are taxed or subsidized, neither RTC nor TFP provide unbiased measures of the gain in welfare from a technical change. Note from **(12)** however, that if income elasticities of demand are all unity (iso-welfare curves are homothetic), the first term in **(12)** disappears, and if the technology is unbiased, $B = 0$ and the second term also disappears. Only under these special circumstances will RTC and TFP be unbiased measures of the welfare gain from technical change.

Conclusions

This paper defines three scalar measures of technical change based on the rate of technological change (RTC), total factor productivity (TFP) and Hicksian equivalent variation (EV). A simple general equilibrium analysis for a closed economy demonstrates that in an undistorted economy the three measures are equivalent, so it wouldn't matter which is used. If some prices are subsidized or taxed, however, the three measures diverge (except in the special case in which the the technological change is an unbiased radial shift and the welfare function is homothetic). We argue that consumer prices are the appropriate weights to use for TFP index measurement in the case of taxed or subsidized goods, rather than the producer prices, as is general practice.

References

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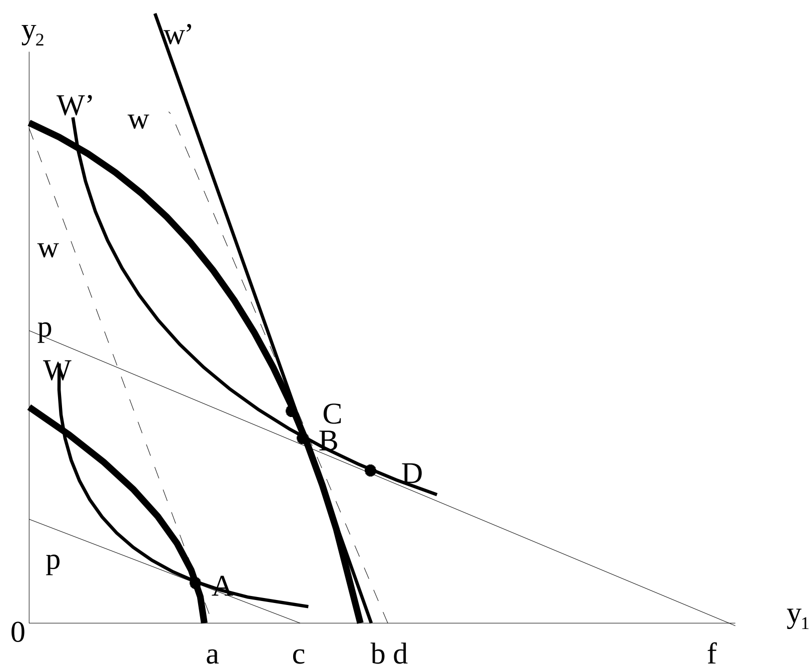

Figure 1. Effects of technical change on price-distorted equilibrium