This paper discusses plant location models applied to a honey processing-marketing firm with emphasis on sensitivity of findings to some alternative objectives and specifications.

**Objectives and Methodology**

The first objective was to determine the number, size, and location of honey processing plants which would minimize total assembly, processing, and product shipment cost. In pursuit of the first objective, two of several questions receiving consideration were: (1) In comparing an optimizing least-cost result to an actual operating cost result, what is a valid specification of "actual" cost? \(^1\) (2) Is there a simple way of checking on the validity of a minimum cost solution?

The second objective was to compare the minimum cost solution to alternative net revenue maximization analyses.

The major analytical methodology was a computerized adaptation of the transshipment spatial equilibrium model with economies-of-scale in plant processing as presented by King and Logan \(^5\). Technical details concerning the transshipment model are thoroughly documented elsewhere \(^3,5,8\).

\(^1\) In a review of this study, F. E. Bender emphasized the importance of this question.

In this study, the United States was divided into 54 regions. The firm had markets in all of these regions and raw honey supply in 30 of these same regions.

Findings and Their Sensitivity

Regarding the first objective, major findings were: (1) A current network operating with 5 processing plants could be reduced to a 2 plant network. One 3 plant network could be nearly as efficient as a 2 plant network. (2) One plant could increase its share of total volume from 40 percent to about 67 percent. (3) Annual costs of combined processing and transportation could be reduced 8 percent.

Comparing Model Costs and Actual Costs

Conceptually, a comparison of actual operating costs to model optimal costs could provide some useful quantifiable indication of whether or not an actual network tends to operate at minimum cost.

In practice, comparisons usually must be made with less than perfect information conditions. For example, a comparison of "model" processing costs based on "synthetic" economic-engineering equations and "actual" costs based on accounting statements could occur. However, accounting statements' cost data have several limitations with respect to conventions, judgment and insufficient detail regarding the effects of scale, excess capacity, work methods, delays and idle work time [97]. With such insufficiency of detail, it would be an impossible task to determine whether differences between model and accounting costs were due to differences in costing methods or reality. Thus, any claim of substantial
potential saving or efficiency based on this approach, while not conclusively disproved, would be questionable. The main difficulty with this approach is that two different costing methods prevent true comparability.

Alternatively, the costs of "actual" processing plants could be estimated by substituting current plant volumes into their respective processing synthetic unit cost equations to get estimated unit costs for each plant. Unit costs multiplied by the respective plant volumes would yield estimated "actual" total processing costs. In this manner, the same costing method would be used for both the optimizing model costs and the current operation and a better comparability base would be established.

A better comparability base reduces the risk of producing illusions regarding potential cost savings.

A cost savings illusion can be one of three types: (1) Overstating or understating a valid savings possibility (correct as to direction but seriously wrong as to extent), (2) claiming that a savings would occur when in reality none exists and (3) claiming that no savings potential exists, when in reality it could occur.

Cases of cost savings illusion can be explored in more detail.

Let

\[ A = \text{accounting cost of "actual" operation} \]
\[ S = \text{synthetic budget cost of "actual" operation} \]
\[ M = \text{least-cost (optimal) of an operation using synthetic costs in an optimizing model} \]
\[ D_A = \text{accounting cost saving} = A - M \]
\[ D_S = \text{synthetic cost saving} = S - M \]
\[ I = \text{cost savings illusion}. \]
First, assume that M< either A or S. Also, assume A,S,D and M > 0.

Some of the conditions which may occur are:

1. If A=S, then \( D_A=D_S \) and no illusion exists.
2. If A < S, then \( D_A < D_S \) and \( D_A \) understates by an amount \( D_S - D_A = S - A = I \).
3. If A > S, then \( D_A > D_S \) and \( D_A \) overstates by an amount \( D_A - D_S = A - S = I \).

Next, assume M=S but < A. Also, assume D > 0. Then

4. \( S - M = D_S = 0 \) but \( A - M = D_A > 0 \).

Thus, a decision-maker relying on (5) would be misled into thinking that a potential savings may exist and that the network is not optimal. However, (4) shows no savings existing as the network is optimal.

Next, assume M=A but < S.

Then

6. \( A - M = D_A = 0 \) but \( S - M = D_S > 0 \).

A decision-maker relying on (6) would be misled into thinking that no savings exists and that the network is optimal. However, (7) shows that savings exists and that the network is not optimal.

In the honey packer study, condition (3) occurred. Table 1 shows the numerical details for an optimal 2 plant network.

**Table 1.--Cost Savings Illusion**

<table>
<thead>
<tr>
<th>&quot;Actual&quot; Operation Costs:</th>
<th>Model Accounting:</th>
<th>Synthetic Least-Cost:</th>
<th>Savings:</th>
<th>Savings Illusion Dollars:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accounting: Synthetic</td>
<td>Least-Cost:</td>
<td>Savings:</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
A substantial synthetic savings will remain after disregarding a large illusion of 155 thousand dollars. This overstatement is more than half of the synthetic cost savings. The synthetic cost savings are 8.23 percent and the accounting cost savings are 12.59 percent of the synthetic "actual" costs. The accounting cost savings are 12.08 percent of the accounting "actual" costs.

A substantial cost savings illusion may seem to suggest poor synthetic modeling. This may or may not be so. Normally, synthetically derived costs will differ from accounting costs. Synthetic unit costs may be good approximations of accounting unit costs but large physical volume can magnify slight unit cost differences into total cost differences of some size. In this study, the savings illusion was about 4 percent of the accounting "actual" costs. That is, the synthetic "actual" costs were about 96 percent of the accounting "actual" costs.

Consistency of Minimum Cost with Marginal Rate of Substitution

A useful way to use the expected inverse relationship between plant processing costs and transportation costs (as the number of plants vary) is to apply the optimizing theory of marginal revenue equaling marginal cost. The cost savings from decreasing plant processing costs may be thought of as a marginal revenue or gain, $G$; the transportation cost increases may be thought of as a marginal cost, $C$. 
There are three relationships possible between \( G \) and \( C \),

\[
\begin{align*}
(8) & \quad G > C \\
(9) & \quad G = C \\
(10) & \quad G < C
\end{align*}
\]

Expressions (8), (9), and (10) can be arranged so that (8) becomes

\[
(11) \quad \frac{G}{C} > 1 \text{ (or } \frac{C}{G} < 1) .
\]

and

(9) becomes

\[
(12) \quad \frac{G}{C} = \frac{C}{G} = 1
\]

and

(10) becomes

\[
(13) \quad \frac{G}{C} < 1 \text{ (or } \frac{C}{G} > 1) .
\]

Table 2 compares the extra savings, \( G \), in processing costs to the extra costs of transportation, \( C \), for each of several successive pairs of plant number reduction alternatives in the honey packer study.\(^2\)

Table 2.--Processing and Transportation Cost Changes for Successive Pairs of Plant Number Alternatives

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Processing Savings, ( G )</th>
<th>Extra Transportation Costs, ( C )</th>
<th>Relationship Between ( G ) and ( C )</th>
<th>Ratio of ( G ) to ( C )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 to 4 plants</td>
<td>36,801</td>
<td>15,416</td>
<td>( G &gt; C )</td>
<td>2.39</td>
</tr>
<tr>
<td>4 to 3 plants</td>
<td>161,583</td>
<td>67,854</td>
<td>( G &gt; C )</td>
<td>2.38</td>
</tr>
<tr>
<td>3 to 2 plants</td>
<td>109,823</td>
<td>104,188</td>
<td>( G = C )</td>
<td>1.05</td>
</tr>
<tr>
<td>2 to 1 plant</td>
<td>143,832</td>
<td>206,689</td>
<td>( G &lt; C )</td>
<td>.70</td>
</tr>
</tbody>
</table>

\(^2\) For each number of plants, the least-cost alternative of several alternatives was chosen.
The ratios of $\frac{G}{C}$ in Table 2 may be considered as crude or rough measures of marginal rates of substitution of transportation dollars for processing dollars \[4\]. For example, going from 5 to 4 or from 4 to 3 plants, each additional dollar of transportation cost produces more than two dollars in processing savings and $\frac{G}{C} > 1$. However, changing from a 3 to a 2 plant alternative, each additional dollar of transportation cost produces only about one dollar in processing savings and $\frac{G}{C}$ is practically equal to unity which supports the optimality of a 2 plant alternative. If a change were made from a 2 to a one plant alternative, an additional dollar of transportation cost would yield only 70 cents in processing savings and $\frac{G}{C} < 1$.

Net Revenue Maximization Allocations Compared to Least-Cost Allocations

The three net revenue maximization models used in the study were distinguished by the following features: Model I used weighted gross market prices (horizontal demand curve in each market) which were added to the objective function of the otherwise unchanged cost matrix; Model II modified Model I by replacing given fixed final market demands by initially unknown final market demands which could vary up to a moderate growth target amount; and Model III was a modified Model II in which each initially unknown final market demand could vary up to an estimated amount of market demand available locally to all competitors. Aggregate demand remained constant in all cost and net revenue models.

Findings were based on allocations at three processing plants located in California, Iowa and Florida. Thus, the number and location of plants were given. However, the size of each plant was not given and, therefore, was to be determined by the analyses.
Lack of sufficient detail regarding demand functions and industry demand in each of the various market regions precluded any attempts to make reasonably accurate statements concerning comparisons of net revenues or market shares. Therefore, comparisons focus on changes in plant sizes and costs.

Table 3 compares the results with respect to plant sizes and plant shares of total volume.

Table 3.--Comparison of plant sizes and shares of total volume: Least-cost model and three net revenue maximization models

<table>
<thead>
<tr>
<th>Type</th>
<th>Plant Location</th>
<th>Iowa</th>
<th>California</th>
<th>Florida</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Least-Cost</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Plant Size:</td>
<td>100 Pounds</td>
<td>279,970</td>
<td>101,917</td>
<td>38,264</td>
<td>420,151</td>
</tr>
<tr>
<td>Plant Share:</td>
<td>Percent</td>
<td>66.63</td>
<td>24.26</td>
<td>9.11</td>
<td>100.00</td>
</tr>
<tr>
<td>Model I</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Plant Size:</td>
<td>100 Pounds</td>
<td>277,574</td>
<td>90,700</td>
<td>51,877</td>
<td>420,151</td>
</tr>
<tr>
<td>Plant Share:</td>
<td>Percent</td>
<td>66.06</td>
<td>21.59</td>
<td>12.35</td>
<td>100.00</td>
</tr>
<tr>
<td>Model II</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Plant Size:</td>
<td>100 Pounds</td>
<td>266,994</td>
<td>101,321</td>
<td>51,836</td>
<td>420,151</td>
</tr>
<tr>
<td>Plant Share:</td>
<td>Percent</td>
<td>63.54</td>
<td>24.12</td>
<td>12.34</td>
<td>100.00</td>
</tr>
<tr>
<td>Model III</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Plant Size:</td>
<td>100 Pounds</td>
<td>325,465</td>
<td>40,922</td>
<td>53,764</td>
<td>420,151</td>
</tr>
<tr>
<td>Plant Share:</td>
<td>Percent</td>
<td>77.46</td>
<td>9.74</td>
<td>12.80</td>
<td>100.00</td>
</tr>
</tbody>
</table>

All three net revenue maximizing models caused changes in the minimum cost model sizes and shares of plant processing volume. In all three revenue models, the Florida plant volume increased approximately 1.5 million pounds and the California plant decreased by as much as 6 million pounds in Model III as compared to the least-cost solution. The Iowa plant size varied substantially in Models II and III as compared to the least-cost solution.
Table 4 compares the total cost differences between the least-cost model and the three net revenue models.

Table 4.--Cost differences between the least cost model and three net revenue maximization models

<table>
<thead>
<tr>
<th>Plant location</th>
<th>Model I</th>
<th>Model II</th>
<th>Model III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iowa</td>
<td>-11,427</td>
<td>-101,290</td>
<td>+305,568</td>
</tr>
<tr>
<td>California</td>
<td>-102,657</td>
<td>-19,590</td>
<td>-525,362</td>
</tr>
<tr>
<td>Florida</td>
<td>+123,234</td>
<td>+124,148</td>
<td>+150,895</td>
</tr>
<tr>
<td>Total</td>
<td>+9,150</td>
<td>+3,268</td>
<td>-68,899</td>
</tr>
</tbody>
</table>

Two of the three net revenue applications caused only slight changes in total costs. Within the totals, substantial cost changes occurred.

Dropping the least-cost model requirement that all 54 markets be satisfied, Model III showed lower total costs than the least-cost model. Satisfying the same fixed aggregate demand quantity required in the least-cost model, Model III excluded inefficient markets.

Conclusions

To reduce the risk of seriously misleading decision-makers with illusory cost savings possibilities, comparison of least-cost model results to actual operation results should be in terms of the same costing method. Substantial potential cost savings from fewer and larger honey processing plants as recommended by a synthetic cost minimizing model were 155 thousand dollars lower when compared to synthesized costs of the actual operation (cost savings
of 310 thousand dollars) rather than accounting costs of the actual operation (cost savings of 465 thousand dollars).

A method of supporting a mathematical programming optimal cost solution which recommends fewer plants is to require that the plant processing cost savings produced by N-1 rather than N plants (a marginal revenue or benefit) be equal to the increased costs of transportation (a marginal cost). Alternatively, the ratio of marginal processing savings to marginal transportation costs should equal unity. In the honey packer study, a decrease from 3 to 2 processing plants produced a ratio of 1.05.

Two of three net revenue maximization applications caused only slight changes in total costs compared to a least-cost solution. Within the totals, significant changes occurred. Individual plant sizes changed as much as 6 million pounds and costs varied as much as 500 thousand dollars.

Future Research

The findings of a study are partly predestined by characteristics inherent in the choice criteria and methods used. Since scarce resources prevent any one researcher from experimenting with all known economic choice criteria and methods, any particular finding must be stated cautiously. Other specifications which might have been applied to this study include: Alternative disaggregations of geographical regions, present value criteria, seasonality, Baumol's sales maximization with a profit constraint model, ecological and environmental constraints, separable and quadratic programming, stochastic models, and other specifications. The list could be endless. Much interesting research remains regarding sensitivity of plant location findings to alternative specifications.
References


