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THE OUTPUT-COST RELATIONSHIP FOR RETAIL FERTILIZER PLANTS: AN EMPIRICAL APPLICATION OF MULTIPRODUCT FIRM THEORY**

by

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Abstract

Retail fertilizer plants produce a number of products and services. To analyze the relationship between cost and output for these multiproduct firms, a short-run, translog cost function is estimated using pooled data. Results indicate plants can lower average cost by increasing output and by diversifying into anhydrous ammonia. Furthermore, preliminary evidence indicates that firms in the sample are over-invested in plant and equipment.

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Volume-cost relationships developed from statistical analysis of accounting data have long provided information for managerial decision making (French). By estimating a cost function to examine volume-cost questions, these studies attempt to provide managers with operating relationships useful in evaluating firm efficiency. Descriptive and statistical analyses of accounting data as well as economic-engineering cost studies have been available to retail fertilizer firms in the past. These previous cost studies have focused on a single product line (Anderson and Miller; Raikes and Heubrock; Williams, et al.), service (O'Rourke) or have simply summarized industry accounting data (Akridge and Downey).

The problems associated with using accounting data to estimate statistical cost functions have been recognized since the technique's inception (French; Johnson; Johnston; Stollsteimer, et al.). Two criticisms in particular stand out: the failure to handle multiproduct firm cost relationships in a satisfactory manner and the dependence of the results on the choice of functional form used in the estimation. Recent cost studies have addressed both issues (Brown, et al.; Christensen and Greene; Cowing and Holtman). Combining advances in multiproduct firm theory with a flexible functional form, statistical cost functions can now be estimated for industries where firms produce more than one output. This richer model captures the cost implications of changing output mix (in addition to the level of output) without imposing arbitrary restrictions on the cost function. The research reported in this paper attempts to determine the relationship between output level/mix and cost for retail fertilizer plants.

Modern retail fertilizer plants market a diverse array of products and services, ranging from dry bag fertilizer, which the retailer simply warehouses until needed by the farmer, to fluid fertilizer which the retailer blends to customer specification. The fluid product may then be custom-applied by the dealer for the farmer. This custom-application service represents yet another product which the firm may or may not provide. Faced with this complex, multiproduct problem, retail fertilizer plant managers have considerable difficulty comparing their firm's performance to that of others. In addition, they lack an adequate framework for sorting out the effects which changes in product mix are likely to have on cost. Such information is important to managers making pricing and promotion decisions.

In this paper, we briefly review the theoretical measures of output-cost relationships for multiproduct firms. We then present the methods and data used to estimate a short-run, translog variable cost function based on time-series, cross-section data from 24 retail fertilizer plants over an eight-year period. The estimated model is then presented along with measures of scope and scale economies. We find that the average plant in the sample exhibits economies of scale and could lower average cost by increasing output. Furthermore, the observed scale economies are primarily the result of cost savings achieved through economies of diversification which exist between product categories. After developing these results in some detail, we discuss the implications of this research for retail fertilizer plant managers.

THEORY

Retail fertilizer firms are assumed to minimize short-run variable cost given vectors of outputs (Y), variable input prices (W), and fixed inputs (F). Solving the firm's short-run cost minimization problem yields the variable cost function:

(1) $C_v(Y,W,F)$

where C_v is the minimal level of cost, conditional on available fixed factors. We examine the short-run problem since retail fertilizer plants experience a high degree of year-to-year variation in operating season length and output level. It is unlikely plants operating under such conditions are in long-run equilibrium. Variable inputs include labor, gasoline, electricity, mechanic services, repair parts, and advertising. Retail fertilizer firms are price-takers in the markets for these types of products, thus input prices are exogenous. Since the investment required to add a new product is substantial, managers have little control over the mix of products marketed in the short-run. Actual quantities of each product are determined by factors affecting the firm's farmer/customers: crop prices, interest rates, government programs, weather, etc. The assumption of exogenous output levels in the short-run appears reasonable.

There are three distinct ways output mix and level can influence cost in the multiproduct firm (Baumol, et al.). Economies of scope (EOS), the first measure of cost-output relationship, are said to exist if it is cheaper for the firm to produce some group of products jointly than it is to produce the same group of products individually. More formally, economies of scope exist if:

(2) $\sum_{i=1}^{K} C_v(Y_{Ti}) > C_v(Y_S)$

where the Y_{Ti} are orthogonal non-negative output vectors, Y_S is an output vector containing all of the Y_{Ti} vectors, and W and F have been supressed for notational convenience.[1] Dividing (2) by $C_v(Y_S)$ provides a scale-free measure of scope economies:

$$(3)\quad EOS = \frac{\sum_{i=1}^{K} C_v(Y_{Ti}) - C_v(Y_S)}{C_v(Y_S)}$$

Economies of scope exist if EOS is greater than zero.

Product-specific economies of scale (PSE) is the second cost-output concept we examine. PSE measures the impact on cost of increased production in a single product line holding all other output levels, input prices, and fixed input quantities constant. The measure is based on the notion of product-specific incremental cost, $IC(y_i)$, which is defined as:

$$(4)\quad IC(y_i) = C_v(\overline{Y}) - C_v(\overline{Y}-y_i)$$

where $C_v(\overline{Y}-y_i)$ is the cost of producing every product at level $\overline{Y}$, except the i^{th} product, y_i, which is not produced. Average incremental cost, $AIC(y_i)$, is then:

$$(5)\quad AIC(y_i) = IC(y_i)/y_i.$$

Finally, product-specific scale economies are given by:

$$(6)\quad PSE = AIC(y_i)/(\partial C_v(\overline{Y})/\partial y_i).$$

Hence, PSE is the average incremental cost of producing the i^{th} output divided by the marginal cost of producing the i^{th} output. If $PSE > 1$, then product-specific scale economies exist.

The final measure of output-cost relationship corresponds closely to the traditional concept of economies of scale for a single product firm. Multiproduct scale economies (MSE) exist if simultaneously increasing the production of all outputs lowers ray average cost. Ray average cost is the multiproduct equivalent of "average cost" as defined for the single product firm. Formally, ray average cost is expressed as:

$$(7)\quad RAC_v(\overline{Y}) = C_v(\overline{Y}) / \sum_{i=1}^{L} y_i$$

where $RAC_v(\overline{Y})$ is the ray average cost associated with the output vector $\overline{Y}$ and y_i is the quantity of the i^{th} output produced. Multiproduct scale economies exist at $\overline{Y}$ if:

$$(8)\quad dRAC_v(t\overline{Y})/dt < 0$$

where t is evaluated at 1. Writing (8) in elasticity form provides an expression for the elasticity of ray average cost along the output ray defined by $\overline{Y}$. Multiplying this cost elasticity by -1 provides a measure of multiproduct scale economies:

$$(9)\quad MSE = 1 - \sum_{i=1}^{L} \partial \ln C_v(\overline{Y}) / \partial \ln y_i .$$

Hence, multiproduct scale economies exist if MSE is greater than 0 (Cowing and Holtman).

Since MSE measures the cost implications of varying all outputs simultaneously, it is a function of the cost response associated with product-line diversification (EOS) and with changing the production level of individual products (PSE). Thus (9) may be rewritten as:

$$(10)\quad MSE = 1 - \frac{(1 - EOS)}{\alpha_{T1} PSE_{T1} + (1 - \alpha_{T1}) PSE_{T2}}$$

where $\alpha_{T1} = (\Sigma_{j \varepsilon T1} Y_j MC_j)/(\Sigma_{j \varepsilon S} Y_j MC_j)$ and PSE_{T1} and PSE_{T2} are the product-specific scale economies associated with output vectors Y_{T1} and Y_{T2} respectively. This expression links the three output-cost measures (Baumol, et al.). It is clear from (10) that scope economies serve to magnify the effects of scale economies associated with individual products. If EOS is zero, implying there are no gains to diversification, then MSE is simply a function of the weighted average product-specific scale economies associated with Y_{T1} and Y_{T2}. The weights can be roughly interpreted as the proportion of total variable cost expended on production of the respective output vector.

PROCEDURE

We assume a multiproduct translog variable cost function:

$$(11)\quad \ln C_v = \alpha_0 + \sum_r \alpha_r \ln Y_r + \sum_i \beta_i \ln W_i + \sum_k \gamma_k \ln F_k$$
$$+ 1/2\Big[\sum_r \sum_s \alpha_{rs} \ln Y_r \ln Y_s + \sum_i \sum_j \beta_{ij} \ln W_i \ln W_j + \sum_k \sum_l \gamma_{kl} \ln F_k \ln F_l$$
$$+ \sum_r \sum_i \delta_{ri} \ln Y_r \ln W_i + \sum_r \sum_k \Psi_{rk} \ln Y_r \ln F_k + \sum_i \sum_k \Theta_{ik} \ln W_i \ln F_k$$
$$+ \sum_i \sum_r \delta_{ir} \ln W_i \ln Y_r + \sum_k \sum_r \Psi_{kr} \ln F_k \ln Y_r + \sum_k \sum_i \Theta_{ki} \ln F_k \ln W_i\Big]$$

where C_v is (minimal) total variable cost; Y represents a set of six outputs -- dry fertilizer, fluid fertilizer, anhydrous ammonia, chemicals, services, and other farm supplies; W is a set of three input prices for labor, energy, and other variable inputs; and F represents a set of fixed inputs -- management, plant and equipment, and other fixed inputs.[2]

Neoclassical theory suggests the matrix of second-order terms will be symmetric. In addition, the cost function is expected to be homogeneous of degree one in input prices. These restrictions are imposed on the model for estimation (Brown, et al.; Cowing and Holtman). Logarithmic differentiation of the cost function and use of Shephard's lemma yields cost share equations for each variable input:

$$(12)\quad S_i = \partial \ln C_v / \partial \ln W_i = \beta_i + \sum_j \beta_{ij} \ln W_j + \sum_r \delta_{ir} \ln Y_r + \sum_k \Theta_{ik} \ln F_k$$
$$i = 1,2,3$$

where S_i is the proportion of total variable cost expended on the i^{th}

variable input. One of the share equations is dropped for estimation since only two of the three equations are linearly independent (Christensen and Greene).

Imposing homogeneity forces one of the input prices to be defined as a numeraire price. Hence, labor and energy prices are expressed in terms of the other variable input price and the share equation for other variable inputs is dropped. The estimating form of the model consists of equation (11) plus two share equations defined in (12) with symmetry and homogeneity imposed. We assume intercept and slope parameters are invariant across time and plants, and that the disturbance term is contemporaneously correlated between plants within a given year. The 78 parameters in the three equation system were estimated using Zellner's seemingly unrelated regression technique.

DATA

Data for the estimation were collected from 24 Indiana and Illinois retail fertilizer plants over the 1975-1982 period. Annual data for output quantities, expenses, and fixed input levels were obtained from accounting information submitted by the firms to the Purdue Fertilizer Retail Efficiency Data (FRED) Project (Akridge and Downey). Input price proxies were constructed from state and county price data. Summary statistics for the data set are contained in Table 1.

Variable Costs: Total variable costs were defined as those expenditures directly controllable by the firm during the firm's fiscal year. Total variable costs included outlays for labor, repair and maintenance, utilities, fuel and oil, advertising, and miscellaneous operating expenses. Bad debt loss, depreciation, and interest expense were not included in total

Table 1. Descriptive Statistics.

Variable	Sample Mean	Std. Dev.	Minimum Value	Maximum Value	Geometric Mean
VC-Variable Cost ($/yr)	60450	17752	31469	127624	58117
Y1-Dry Fertilizer (Tn/yr)	2435	1082	550	6147	2223
Y2-Fluid Fertilizer (Tn/yr)	1066	514	112	2957	941
Y3-Anhydrous Ammonia (Tn/yr)	464	361	0	1788	127
Y4-Chemicals ($/yr)	140087	81197	24811	570754	122300
Y5-Services (Ac/yr)	15134	8171	3811	47286	13319
Y6-Other Farm Supplies (Ac/yr)	2620	1676	39	8770	2007
W1-Labor Price ($/wk)	98.40	3.65	90.91	102.00	98.33
W2-Energy Price (¢/btu)	.46	.07	.34	.58	.46
F1-Management ($)	16702	8275	7951	53262	15159
F2-Other Fixed Inputs ($)	17942	7564	5077	43783	16422
F3-Plant & Equipment ($)	34108	17268	8556	97002	30301
S1-Labor Share (% of Variable Cost)	63.99	5.82	45.40	77.10	63.72
S2-Energy Share (% of Variable Cost)	11.01	2.31	5.88	16.59	10.75
S3-Other Variable Inputs Share (% of Variable Cost)	25.00	6.42	11.78	46.52	24.19