"The Impact of International Trade on the American Income Distribution: A Cobb-Douglas Approach"

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THE AMERICAN INCOME DISTRIBUTION:
A COBB-DOUGLAS APPROACH

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I. Introduction

The Leontief (1953, 1956) Paradox implies that international trade tilts the American income distribution away from capital and toward labor but surprisingly, the magnitude of this impact has never been calculated. This paper calculates the changes in U.S. real income in the aggregate and to individual factors that would result from the prohibition of all American exports and competitive imports under the assumption that American tastes and production functions are Cobb-Douglas in form.

II. Expressing the Autarkic Utility Function in Terms of Factor Supplies

The simulations described in the introduction were actually performed using the data collected to calibrate the fixed proportions linear programming model, described in detail in Hartigan and Tower (1982). The data set is for 1967. Interindustry relationships are described by the U.S. 83 sector input-output table for that year. Product and factor prices are perfectly flexible within the U.S. Also, the rate of exploitation of natural resources is fixed at the base period level, productive factors are fixed in aggregate supply, are perfectly mobile internally, and are perfectly immobile externally.

Two simulations are involved. The "homogeneous" one assumes that both labor and capital are homogeneous, or that an infinite elasticity of substitution among various types of labor and among various categories of capital prevails, while the elasticity of substitution between capital and labor is one.
The second simulation which we label the "heterogeneous" one distinguishes between various types of labor. It postulates an elasticity of substitution of unity among 5 skill classes of labor and among 7 categories of capital. Labor is partitioned into scientists and professionals, nonfarm managers, skilled craftsmen, less skilled labor and farmers. Capital is subdivided into reproducible capital, land, and extractive resources. These resources are ferrous ores, nonferrous ores, coal, crude petroleum, and chemical fertilizer minerals.

The construction of the U.S. input-output table is such that all imports into final demand are defined as noncompetitive, whereas, imports for intermediate use are treated as competitive or noncompetitive, depending on whether or not similar goods are produced in the U.S. The noncompetitive intermediate imports are truly noncompetitive. That is, they are not produced in the U.S. However, imports into final demand generally have domestic counterparts. Given the assumption of Cobb-Douglas production functions, U.S. industries requiring noncompetitive imports as inputs would not be able to produce if a prohibitive tariff were enacted. Thus in simulating "autarky" we could not exclude noncompetitive intermediate imports and arrive at any interesting results. So we chose to treat noncompetitive intermediate imports as being just like other domestic factors of production: fixed in supply at their 1967 level, perhaps because they are assumed to be supplied perfectly inelastically on world markets. Moreover the income derived from their sale to the U.S. economy is taken to be spent in the U.S. economy in the same proportions as income accruing to domestic factors of production. While this treatment of noncompetitive imports is not ideal, it seems to be necessary in order to retain the simple
structure of the model which emerges. Moreover, truly noncompetitive imports constitute less than 1/2% of base period GNP. Therefore, their modeling is arguably unimportant quantitatively, as long as they continue to be supplied at roughly the same quantities and prices.

III. Simulating Autarky

In this section we derive expressions for social utility and the utility of each factor in autarky.

Social utility, \( U \), is taken to be a function of the goods consumed:

\[
U = \Pi C_j^j
\]

(1)

where \( c_j \) is the share of good \( j \) in final consumption, \( \Sigma c_j = 1 \), \( C_j \) is the amount of good \( j \) consumed and we identify consumption with the vector of domestic final demand from the input-output table. This means that

\[
F = cY
\]

(2)

where \( Y \) is a scaler denoting the value of final demand,

\( c \) is a (nx1) column vector of \( c_i \),

and

\( F \) is a (nx1) column vector with the \( i^{th} \) element, \( F_i \), being the value of final demand for the \( i^{th} \) industry.

Thus \( F_i = p_i C_i \), where \( p_i \) is the price of the \( i^{th} \) good.
Each good is produced according to a Cobb-Douglas production function of primary factors, \( L_i \) (consisting of capital, labor, non-competitive imports and in the heterogeneous case natural resources) and intermediate inputs, \( G_{ij} \) (consisting of goods, excluding noncompetitive imported inputs). Thus, the production function for each good is given by

\[
Q_j = \prod_i L_{ij}^{\Theta_{ij}} G_{ij} A_{ij}
\]

(3)

where \( Q_j \) is the quantity of good \( j \) produced; \( \Theta_{ij}, A_{ij} \) are shares of primary factors and intermediate inputs in production with \( \sum_i \Theta_{ij} + \sum_i A_{ij} = 1 \); and \( L_{ij} \) and \( G_{ij} \) are the amounts of factor and good \( i \) used in sector \( j \).

In autarky, the national income identity becomes

\[
X = F + AX
\]

(4)

where \( X \) is a \((nx1)\) column vector, the \( i^{th} \) element of which is the value of output of the \( i^{th} \) industry, \( X_j = p_j Q_j \), and \( A \) is the \((nxn)\) matrix of input output shares, \( A_{ij} \). Given that all production functions are Cobb-Douglas, their exponents are invariant with respect to changes in prices of either primary or intermediate factors of production. Finally, the identity in equation (4) simply states that in autarky, the output of each good must be either absorbed in final demand or reabsorbed in the production process as an intermediate input.

The vector of factor incomes can be written as

\[
W = eX
\]

(5)

where \( e \) is the \((fxn)\) matrix of primary factor shares

\( W \) is the \((fx1)\) column vector of factor incomes and
$W_i$ is the income of the $i$th factor, whose supply is $L_i$.

Since all incomes are assumed to be spent and the product is exhausted, the factor bill can also be written as

$$W = SY$$  \hspace{1cm} (6)$$

where $S$ is the $(fx1)$ column vector of factor shares, $S_j$, in national income, which because all income is spent is equal to final demand.

Substitute (2) into (4) to yield

$$X = (I-A)^{-1} cY.$$  \hspace{1cm} (7)$$

Then combine (5) and (6) to eliminate $W$ and substitute (7) into the result to yield

$$S = \odot (I-A)^{-1} c$$  \hspace{1cm} (8)$$

From this, we conclude that in autarky the primary factor shares depend only on the share coefficients given by tastes and technology ($\odot$, $A$ and $c$), and not at all on the physical availabilities of the factors.

Next, we need to derive a social utility function expressed in terms of the factor availabilities. The logarithmic derivative of (1) can be expressed as

$$\hat{U} = \sum_i c_i \hat{C}_i$$  \hspace{1cm} (9)$$

where a hat, "\^", denotes a proportional change.
Since factors are paid their marginal products, we also have

$$\sum_i p_i dC_i = \sum_j v_j dL_j$$  \hspace{1cm} (10)

where $v_j$ is the wage of the $j^{th}$ factor. Equation (10) can be rewritten as

$$\sum_i c_i \dot{C}_i = \sum_j S_j \dot{L}_j$$  \hspace{1cm} (11)

which when combined with (9) yields

$$\dot{U} = \sum_j S_j \dot{L}_j$$  \hspace{1cm} (12)

Recognizing the constancy of $S$ from (8), (12) can be integrated to yield

$$U = a \prod_j L_j^{S_j}$$  \hspace{1cm} (13)

where $a > 0$.

Thus, we have shown that for an autarkic country with Cobb-Douglas production and consumption functions, social utility can be written as a Cobb-Douglas function of factor supplies with the exponents given by (8).

III. The Utility Loss in Moving from Trade to Autarky

In calculating the utility change which results from the prohibition of trade in a flex price full employment model, we consider the movement in two steps. First, we adopt autarky while maintaining fixed factor and product
prices. Then we allow factor and product prices to adjust until full employment is restored. Why we adopt this strategy will become apparent shortly.

Let us suppose that from an initial position of balanced trade, prohibitive trade barriers are erected, while product and factor prices are maintained at their former levels, and factor employment is allowed to vary. Then the utility of those factors that continue to be employed will be unchanged except that they lose any tariff revenues which were distributed to them in the trading equilibrium. Hence, in the fixed price autarky equilibrium, the new level of aggregate utility will be given by

\[ \begin{align*}
U^* &= U_T \cdot \frac{U*}{Y_T} \\
&= U_T \left( \sum_i \frac{v_i^T L_i^*}{\{ \sum_i v_i^T L_i \} + T} \right)
\end{align*} \]

where

- \( U_T \) and \( U^* \) are utilities in trade and fixed price autarky respectively, \( L \) and \( L_i^* \) are the demands for the \( i \)th factor in trade and fixed price autarky, \( v_i^T \) is the wage to the \( i \)th factor in trade

and

- \( T \) is the tariff revenue under trade, all of which is assumed to be distributed to the factors of production as a proportional income subsidy.\(^4\)

Letting

\[ T = t \sum_i v_i^T L_i \]

and (14) is rewritten as

\[ U^* = U_T \frac{\sum_i v_i^T L_i^*}{\sum_i v_i^T L_i [1 + t]} \]

where

- \( U_T \) and \( U^* \) are utilities in trade and fixed price autarky respectively, \( L \) and \( L_i^* \) are the demands for the \( i \)th factor in trade and fixed price autarky, \( v_i^T \) is the wage to the \( i \)th factor in trade

and

- \( T \) is the tariff revenue under trade, all of which is assumed to be distributed to the factors of production as a proportional income subsidy.\(^4\)
Now, from (13)

\[ U^* = \prod_j L_j^* S_j \]  \hspace{1cm} (17)

and letting (13) and \( U_A \) denote the utility in autarky at full employment, where employment levels are identical to those under trade:

\[ U_A = \prod_j L_j^S_j \]  \hspace{1cm} (18)

Combining (17) and (18) yields

\[ \frac{\prod_j L_j^S_j}{\prod_j L_j^* S_j} U^* \]  \hspace{1cm} (19)

Thus from (16) and (19)

\[ \frac{\prod_j L_j^S_j \sum_i v_i^T L^*}{\prod_j L_j^* S_j (v_i^T L_i^*) (1 + t)} \]  \hspace{1cm} (20)

Define \( \sigma_j \) as the share of factor \( j \) in the trading equilibrium, and recognize that the share of factor \( j \) in the autarky fix price equilibrium must be identical to the corresponding income share in the autarky full employment equilibrium. Then (20) can be rewritten as

\[ \frac{U_A}{U_T} = \frac{\prod_j \sigma_j S_j}{\prod_j S_j [1 + t]} = \frac{\prod_j \sigma_j S_j}{1 + t} \]  \hspace{1cm} (21)

where \( \sigma_j = \frac{v_j^T L_j}{\sum_i v_i^T L_i} \),

\[ S_j = \frac{v_j^T L_j^*}{\sum_i v_i^T L_i^*} \]

and \( S_j \) is calculated from (8).
Finally, the utility accruing to a unit of the $j^{th}$ factor in flex price autarky is

$$U_{A_j} = U_A \frac{S_j}{L_j}.$$  \hspace{1cm} (22)

In trade, the same expression is

$$U_{T_j} = U_T \frac{\sigma_j}{L_j}.$$  \hspace{1cm} (23)

Thus, the utility accruing to a unit of the $j^{th}$ factor in autarky divided by that in trade is

$$\frac{U_{A_j}}{U_{T_j}} = \frac{U_A}{U_T} \frac{S_j}{\sigma_j} = \prod_i \frac{\sigma_i}{S_i} \frac{S_j}{\sigma_j}. \hspace{1cm} (24)$$

III. Results

Table I presents the results of two alternative simulations. The homogeneous model assumes only two productive factors, capital and labor, while the heterogeneous model recognizes the existence of 12 factors: 5 natural resources, land, reproducible capital and 5 categories of labor. Column II shows the income
distribution in the base period, and Column III shows the income distribution in autarky. Finally, Column IV shows the percentage change in real income that results from the elimination of trade.

For the homogeneous simulation, the Paradox is reaffirmed. The relative share of capital rises by 0.189 percent of the base year value, while that of labor falls by 0.101 percent of its base year solution. Therefore, capital is scarce relative to labor. It is interesting to note that both labor and capital lose in the movement from trade to autarky. This contrasts sharply with what the Stolper-Samuelson theorem would lead us to expect, the reason being that in our trading equilibrium the tariff revenue is distributed as a proportional income subsidy.

The heterogeneous results are much more dramatic. The five extractive resources constitute the U.S.'s two most abundant and three scarcest factors, as indicated in Table I. This illustrates the importance of disaggregating the productive factors, particularly extractive resources.

The Leontief method has handled this matter by deleting resource vectors from the input-output table and recomputing Leontief's \( \alpha \). Regression studies have focused on the manufacturing sector, or used a dummy variable for natural resources. Neither of these methods directly confronts the issue. However, the results in Table I indicate the extractive resources are the U.S.'s most scarce and most abundant factors. Hence, the explicit consideration of particular resources is imperative.

Another important observation is that all forms of labor gain from trade. The decrease in the relative shares of scientists and professionals, nonfarm managers, and skilled craftsmen in the autarkic solution is noteworthy in view of the degree of effort that has been directed towards demonstrating that the
inclusion of human in addition to physical capital in the definition of capital establishes that the U.S. is capital abundant. Reproducible capital also proves relatively scarce in the sense that its share in autarky is higher than in trade. This also confirms Leontief's results with respect to physical capital.

At the extremes of the rankings, this corresponds well with the regression results of Harkness (1978). However, there is little similarity in the intermediate range. A distinct discrepancy occurs in Harkness' finding that scientists and engineers are very abundant, but are only moderately abundant here. Finally, when autarky is established, social utility falls by .140% in the homogeneous case and by .577% in the heterogeneous case.

IV. Conclusion

The appropriate way to explore the consequences of the Leontief Paradox is to simulate a general equilibrium model that provides the information that Leontief had attempted to deduce, namely what happens to the distribution of U.S. national income when international trade ceases. This paper provides the first answer to that question, for a disaggregated model. In the case of homogeneous capital and labor, the Paradox is reaffirmed. However, the disaggregation decision is crucial. It must be emphasized that natural resources cannot be treated as a catch-all term. This rubric contains the U.S.'s most abundant, as well as, its most scarce productive factors.

In addition, the paper demonstrates that it is not necessary to simulate a model with both variable input coefficients, and product and factor prices to be able to draw Cobb-Douglas inferences. All that is necessary is a matrix multiplication with variable levels of employment.
Table I
The Ranking of U.S. Factor Scarcity for Autarky Relative to the 1967 Base Period.

<table>
<thead>
<tr>
<th>Simulation</th>
<th>Productive Factor</th>
<th>I ( \sigma_i )</th>
<th>II ( \sigma_i )</th>
<th>III ( S_i )</th>
<th>IV ( S_i - \sigma_i ) ( \times 100 )</th>
<th>V % Change in Utility in Movement from the Base Period to Autarky</th>
</tr>
</thead>
<tbody>
<tr>
<td>Homogeneous</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All Factors Except M</td>
<td></td>
<td>99.583</td>
<td>99.582</td>
<td>-.140</td>
<td>- .140</td>
<td></td>
</tr>
<tr>
<td>K Capital</td>
<td></td>
<td>34.785</td>
<td>34.849</td>
<td>.189</td>
<td>-.056</td>
<td></td>
</tr>
<tr>
<td>Labor</td>
<td></td>
<td>64.798</td>
<td>64.733</td>
<td>-.101</td>
<td>-.340</td>
<td></td>
</tr>
<tr>
<td>Non Competitive Imports</td>
<td></td>
<td>.417</td>
<td>.418</td>
<td>-.001</td>
<td>.001</td>
<td></td>
</tr>
<tr>
<td>Heterogeneous</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All Factors Except M</td>
<td></td>
<td>99.583</td>
<td>99.582</td>
<td>-.577</td>
<td>- .577</td>
<td></td>
</tr>
<tr>
<td>N1 Ferrous Ores</td>
<td></td>
<td>.009</td>
<td>.014</td>
<td>55.556</td>
<td>54.504</td>
<td></td>
</tr>
<tr>
<td>N2 Nonferrous Ores</td>
<td></td>
<td>.011</td>
<td>.015</td>
<td>36.364</td>
<td>35.442</td>
<td></td>
</tr>
<tr>
<td>N3 Crude Petroleum</td>
<td></td>
<td>.148</td>
<td>.169</td>
<td>14.094</td>
<td>13.417</td>
<td></td>
</tr>
<tr>
<td>L1 Less Skilled Labor</td>
<td></td>
<td>27.870</td>
<td>27.943</td>
<td>.261</td>
<td>-.417</td>
<td></td>
</tr>
<tr>
<td>R Reproducible Capital</td>
<td></td>
<td>33.903</td>
<td>33.963</td>
<td>.179</td>
<td>-.500</td>
<td></td>
</tr>
<tr>
<td>L2 Nonfarm Managers</td>
<td></td>
<td>13.137</td>
<td>13.100</td>
<td>-.180</td>
<td>-.955</td>
<td></td>
</tr>
<tr>
<td>L3 Skilled Craftsmen</td>
<td></td>
<td>13.905</td>
<td>13.865</td>
<td>-.294</td>
<td>-.962</td>
<td></td>
</tr>
<tr>
<td>L4 Scientists and Professionals</td>
<td></td>
<td>9.480</td>
<td>9.440</td>
<td>-.420</td>
<td>-1.095</td>
<td></td>
</tr>
<tr>
<td>T Land</td>
<td></td>
<td>.672</td>
<td>.650</td>
<td>-3.259</td>
<td>-3.928</td>
<td></td>
</tr>
<tr>
<td>L5 Farmers</td>
<td></td>
<td>.406</td>
<td>.385</td>
<td>-5.147</td>
<td>-5.813</td>
<td></td>
</tr>
<tr>
<td>N4 Coal</td>
<td></td>
<td>.032</td>
<td>.029</td>
<td>-9.375</td>
<td>-9.988</td>
<td></td>
</tr>
<tr>
<td>N5 Chemical and Fertilizer Minerals</td>
<td></td>
<td>.010</td>
<td>.009</td>
<td>-10.000</td>
<td>-10.608</td>
<td></td>
</tr>
<tr>
<td>M Non Competitive Imports</td>
<td></td>
<td>.417</td>
<td>.418</td>
<td>-.775</td>
<td>-10.608</td>
<td></td>
</tr>
</tbody>
</table>
Footnotes

1 Leontief attempted to ascertain the factor requirements of replacing 1 million dollars of competitive imports with domestic production, and compared it with the factor requirements of 1 million dollars of U.S. exports. Thus, he tried to discern how factor requirements would change if the U.S. ceased exporting and importing. See Hartigan (1981).

2 The sectoral skill disaggregation was determined as follows. The U.S. Bureau of Labor Statistics (1977) wage tape was used to determine employment in 1973 in each of the five skill groups, mentioned above, for industries according to the BLS classification. We then scaled these figures up or down by a given percentage in each industry so that total employment in each industry would add up to the employment figures for 1967 in Walderhaug (1978). We then regressed wages and salaries by industry from this same publication on the employment figures to obtain average wage rates for each of the skill groups. From these wages and employment figures we determined the shares of each of the skill groups in labor income for each industry. We then calculated disaggregated labor coefficients by separating the aggregate labor coefficient for each industry into the coefficients for these various groups according to these same ratios. The supply of each skill type of labor was then calculated by summing for each skill the products of the base period outputs with the corresponding coefficients for that particular skill group.

3 Splitting the capital vector into land and reproducible capital was attained by assuming the ratio of the value of land to the value of depreciable capital for forestry, fishing, and agriculture given in the U.S. Corporate
Income Tax (1967) returns was identical to the ratio of land to reproducible capital for input-output sectors 1, 2, and 3. Assuming that competition assures equal rates of return on both assets in the base period, we determine the coefficients for reproducible capital and land in these sectors by dividing the coefficient for property-type income in the input output table for these sectors according to that same ratio, and we determined the supplies of the two factors in these sectors by dividing total property-type income accordingly.

The partitioning for crude petroleum and the four mining industries was obtained by dividing total property type income in these sectors into return to depletable capital and reproduceable capital in the proportions .125, PTI -.125 where V is value added and PTI is property type income. This is done because leases in petroleum and all types of mining typically specify that 12 1/2% of value added in the activity be distributed to the owner of the primary resource. We derived the depletable resource coefficients by splitting the coefficients for property type income in the same ratio.

This approach is a result of suggestions by R. Conrad and V. Treml.

In the base period trade was not balanced. But our goal was to assess in some sense the impact of moving from balanced trade to "autarky." Thus, we pretend that any expenditure in excess of income (including tariff revenue) was for government consumption, and that this component of expenditure is eliminated when "autarky" is achieved.

The Leontief α is calculated as

\[ \alpha = \frac{(K/L)_M}{(K/L)_X} \]

where \((K/L)_M\) and \((K/L)_X\) refer to the capital labor ratios in competitive import replacements and exports, respectively.
6 Harkness found that coal was the U.S. most abundant factor, while ferrous and nonferrous ores were the U.S. scarcest factor. Chemical and fertilizer minerals, depending on his regression specification, were second or third most abundant. A discrepancy between Harkness' and these results is his finding that scientists and engineers were the other second or third most abundant. Part of the problem in comparing results is a result of different factor compositions. For instance, land is homogeneous here, but Harkness divided it into crop, pasture, and forest land.

7 Throughout the paper we have ignored the fact that indirect business taxes are not uniform across sectors. It is not difficult to show that even with such non-uniform ad valorem taxes relative factor shares will be fixed in autarky, regardless of the mix of factor supplies. Moreover, social utility will continue to be a Cobb-Douglas function of factor supplies. However, the exponents of this Cobb-Douglas function are no longer the factor shares.

As discussant of this paper at the meetings of the Econometric Society in December 1981, Gene Grossman noted that he felt the assumption of unitary elasticity of substitution to be much too high for this degree of aggregation, which means that the benefits of trade and the impact of trade on income distribution are both understated by this paper. We concur, but have picked what seemed to us to be an interesting point on the trade off between photographic reality and simplicity of analysis. Finally, as is readily seen from our calculations, the results do depend strongly on the level of aggregation assumed, and we picked an arbitrary degree of aggregation to work with. Also, we chose to work with perfect factor mobility within the U.S. economy, but could have postulated certain factors of production to be industry specific to portray the short run, and the basic analytical tools would have continued to apply.
References


