MORE ON ESTIMATING ELASTICITIES USING LAGGED ENDOGENOUS VARIABLES

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Introduction

In an article appearing in this journal, Bopp and Pendley, hereafter abbreviated B-P, have considered the problem of constructing confidence intervals for long-term elasticities estimated from logarithmic equations containing lagged endogenous variables as explanatory variables. The estimated long-term elasticity for explanatory variable $i$ is

$$\hat{\eta}_i = \frac{\hat{B}_i}{1-\hat{\lambda}}$$

where $\hat{B}_i$ is the estimated regression coefficient for explanatory variable $i$, or equivalently, the short-term elasticity for explanatory variable $i$, and $\hat{\lambda}$ is the estimated coefficient for the lagged endogenous variable. Bopp and Pendley use an estimate of the asymptotic variance of $\hat{B}_i/(1-\hat{\lambda})$ to construct approximate confidence intervals for $\eta_i$. The purpose of this note is to present a simple technique for constructing exact confidence intervals for $\eta_i$. This technique is also illustrated by application to the data used by B-P.

**Exact Confidence Intervals for Long-Term Elasticities**

The ratio $\hat{B}_i/(1-\hat{\lambda})$ is, under the usual assumptions, a ratio of two normally distributed random variables with the numerator having an expected value of $B_i$ and variance $\sigma^2_{\hat{B}_i}$, and the denominator having expected value of $1-\lambda$ and variance $\sigma^2_{\hat{\lambda}}$. The covariance of the numerator and denominator is $-\sigma_{\hat{B}_i,\hat{\lambda}}$. Although Hinkley has derived an exact
expression for the exact cumulative distribution function (c.d.f.) for such a ratio, evaluation of that function involves use of tabulations of the bivariate normal distribution function. Since use of these tabulations usually requires trivariate interpolation, construction of exact confidence intervals for $\eta_i$ using this approach would be at best inconvenient.

However, a result due to Fieller\(^3\) allows construction of exact confidence intervals for $\eta_i$ and requires no more than solutions to quadratic equations. This result makes use of the fact that the ratio $R = \frac{\hat{B}_i}{(1-\lambda)}$ is equivalent to

$$\hat{B}_i - R(1-\lambda) = 0 \quad \text{(2)}$$

A $1-\alpha$ percent confidence interval for $R$ is given by those values of $R$ for which

$$\text{Prob} \left\{ \frac{(\hat{B}_i - R(1-\lambda))^2}{s^2_{(1-\lambda)} - 2R \hat{B}_i (1-\lambda) - \frac{t^2}{2} s_{B_i, (1-\lambda)}^2 + \frac{t^2}{2} s_{B_i}^2} \leq t^2_{\alpha/2} \right\} = 1-\alpha \quad \text{(3)}$$

where $s^2_{B_i}$ is the sample variance of $\hat{B}_i$; $s^2_{(1-\lambda)}$ is the sample variance of $(1-\lambda)$; $s_{B_i, (1-\lambda)}$ is the sample covariance of $\hat{B}_i$ and $(1-\lambda)$; and $t^2_{\alpha/2}$ is the squared value of the upper $\alpha/2$ percentage point of the t-distribution with appropriate degrees of freedom, or equivalently the upper $\alpha$ percentage point of the F-distribution with one degree of freedom in the numerator and the appropriate degrees of freedom in the denominator.

For the problem at hand, the appropriate degrees of freedom for the t-distribution or the denominator of the F-distribution are the error degrees of freedom of the regression used to estimate $\hat{B}_i$ and $\hat{\lambda}$. For the confidence interval to be closed, $(1-\lambda)$ must be significantly different.
from zero. The exact 1-\(\alpha\) confidence limits for \(\eta_i\) are those values of \(R\) for which

\[ R^2\left( (1-\lambda)^2 - \frac{t^2}{\alpha/2} s^2_{\lambda, B_i} (1-\lambda) \right) - 2R\left( \hat{B}_i (1-\lambda) - \frac{t^2}{\alpha/2} s^2_{\lambda, B_i} (1-\lambda) \right) + B_i^2 - \frac{t^2}{\alpha/2} s^2_{B_i} = 0. \] (4)

Note that the solution to this quadratic equation in \(R\) need not, and generally will not, be symmetric about \(\hat{\eta}_i\). Thus, the use of the asymptotic variance of \(\hat{\eta}_i\) to construct approximate confidence intervals for \(\eta_i\) centered upon \(\hat{\eta}_i\) can be misleading.

**An Application**

In order to illustrate the construction of exact confidence intervals for long-term elasticities, we make use of the data analyzed by B-P. They estimated a logarithmic equation expressing residential electrical consumption in Connecticut (\(Q_t\)) as a function of real disposable income (\(Y_t\)), own marginal price (\(P_t\)), and lagged consumption (\(Q_{t-1}\)) using annual observations from 1960 to 1975. The estimated short- and long-term elasticities and their respective variances are summarized in Table 1. The estimated coefficient of adjustment (\(1-\lambda\)) was 0.152, and the relevant covariance terms were estimated as follows: \(s_{\hat{Y}, (1-\lambda)} = -s_{\hat{Y}, \lambda} = 0.005\); and \(s_{\hat{P}, (1-\lambda)} = -s_{\hat{P}, \lambda} = 0.0008\). The error degree of freedom is 12. With this information, exact confidence intervals can be constructed for both the short- and long-term elasticities. These are also displayed in Table 1.

Note that the exact confidence intervals for the long-term elasticities are much wider than their respective short-term counterparts. Also, the exact confidence intervals for the long-term elasticities are not symmetric about their respective point estimates, a result not
Table 1. Exact Confidence Intervals for Short- and Long-Term Elasticities

<table>
<thead>
<tr>
<th>Variable</th>
<th>Estimated Elasticity&lt;sup&gt;a&lt;/sup&gt;</th>
<th>Exact 95% Confidence Intervals</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Short-term</td>
<td>Long-term</td>
</tr>
<tr>
<td>Income</td>
<td>0.17</td>
<td>1.13</td>
</tr>
<tr>
<td></td>
<td>(0.1221)</td>
<td>(0.474)</td>
</tr>
<tr>
<td>Price</td>
<td>-0.21</td>
<td>-1.4</td>
</tr>
<tr>
<td></td>
<td>(0.0480)</td>
<td>(0.587)</td>
</tr>
</tbody>
</table>

<sup>a</sup> Standard errors of estimated elasticities are shown in parentheses. The standard errors for long-term elasticities are asymptotic.

detected by the use of the asymptotic variance of $\hat{\eta}_i$ in constructing appropriate confidence intervals for $\eta_i$. Although the exact confidence interval for the short-term income elasticity is wider than the interval for short-term own price elasticity, the relative widths of the long-term elasticity intervals are reversed, a result agreeing with the findings of B-P.

Summary and Conclusions

The purpose of this note is to demonstrate a technique for constructing exact confidence intervals for long-term elasticities from logarithmic equations containing lagged endogenous variables as regressors. This technique involves only simple computations based upon results generated in the course of parameter estimation.
Notes


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