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# DETERMINATION OF THE PREDOMINANCE OF VARIOUS EXPECTATIONS PATTERNS IN COMMODITY FUTURES AND SPOT MARKETS

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## I. Introduction

Most empirical models of storable commodities skirt issues of specifying internally consistent dynamic representations for intertemporal markets. Gardner, in an excellent treatment of public and private stocks, argues that the demand function for stocks ". . . cannot (to the author's knowledge) be derived analytically even under simple specifications of other equations." Subotnik and Houck model the determination of one-quarter ahead futures prices within the same model that produces current quarter spot prices. Their formulation excludes altogether any consideration of expectations; future prices in their model are determined exclusively on the basis of current period supply-demand conditions. Both of these models and numerous other models that have been advanced in the literature are based on Working's "theory of storage." This framework provides a simple specification of intertemporal price spreads based upon current stocks.

In a dynamic world of uncertainty, however, the Working formulation is in essence a self-contained but static theory of intertemporal price relationships. The conceptual inconsistency in Working's hypothesis was demonstrated first by Weymar who used the Muth rational expectation hypothesis to show that the spread between future prices for two different dates of delivery should depend upon expected stocks rather than stocks already in existence. In contrast, Working has stated that "it is only supplies already in existence which have any significant bearing . . . on current intertemporal price relationships. . . ." Only a static theory would support such a statement.

Available empirical evidence on the relevance of the Working static framework versus an internally consistent rational expectation formulation is indeed unclear. Nevertheless, some studies (e.g., Pearson and Houck) have found that information concerning future supply-demand conditions and future expected stock levels do influence current period spot prices. Hence, models which failed to properly conceptualize and measure this influence are expected to generate inferior forecasts. In other words, if tractable dynamic representations of these influences can be captured, it is expected that their forecasting accuracy will dominate models that are currently available in the literature.

The focus of this paper is on dynamic representations of intertemporal markets for storable commodities. Our purpose is to develop a general theoretical framework that will allow us to determine estimatable dynamic equations which can be used to distinguish between various static and dynamic representations which are imparted by alternative conditional expectation formation patterns. Much of dynamic economic modeling suffers from the lack of sufficiently rich data sets to discriminate across alternative expectation formation patterns. Generally, economists impose the expectation formation pattern as part of their maintained hypothesis. However, given the rich data sets that are available for both spot and future commodity markets, this is perhaps the most likely area of application where real empirical progress can be made in discriminating across expectation formation patterns.

Another motivation for the structure of the theoretical model advanced in this paper relates to the notion of "rationally" expected prices. In the original formulation of rational expectations by Muth and its subsequent use by economists, rationality has been defined only in terms of benefits. That is, the cost of collecting information to formulate rationally expected prices has been neglected. It can be shown theoretically that, for some economic environments, naive expectations are, in fact, rational. This apparent paradox results from the failure of rational expectations as defined in the economic literature to incorporate the costs of collecting information on critical random variables.

The theoretical model developed in this paper for storable commodities presumes active futures and spot markets. Uncertainty, risk aversion, and basis risk are formally incorporated in the model representation. Numerous authors have dealt with risk aversion and uncertainty in future and spot market prices, but most all authors neglect basis risk and production uncertainty (e.g., Turnovsky, Sarris, Feder, et al.). As usual, speculators are presumed to transact only in the futures market while hedgers are assumed to transact in both the futures and spot markets.

For the above model, dynamic representations of both the futures and spot prices are derived. Each of these representations is based on expected spot prices for period t conditional on information available at t - 1. Six different formulations of the conditional expectations can be investigated by the formulations. The six expectation-formation patterns are:

1. Rational expectations
2. Adaptive expectations
3. Naive expectations
4. Future market prices
5. Normal expectations
6. Various convex combinations of 1-5.

For each expectation formulation, the price dynamics for both the futures and spot markets are compared and contrasted. An econometric test is developed to discriminate among these formations.

## II. The Microeconomic Framework

In terms of individual agents, the behavior of four separate trading groups are identified and investigated in this section: producer/hedger, storer/hedger, forward-contracting hedger, and speculator. Some of these groups are involved in both the current spot and future markets, some in future markets, and some in future markets and forward-contracting markets for export or processed goods. Behavior of other groups is summarized by the spot-market demand and forward-contracting demand for export or processed goods. Interaction of these demands with the behavior of the four explicit groups then gives rise to three markets for which equilibrium conditions must be satisfied:

1. The futures market
2. The spot market
3. The forward-contracting market.

Individuals are assumed not to migrate among groups as perceived short-run profitability changes due to asset fixities associated with all groups except speculators. Each decision-maker explicitly included in the model faces a two-stage decision problem in which, first, any spot-market plans for time period t and futures market positions with delivery date t are decided in time period t - 1 and, second, at time period t any futures position with delivery date t can be closed out or not depending on spot and futures prices at the delivery date. The first-stage decision is assumed to maximize expected utility of income. In each case, expected utility is approximated locally by a linear mean-variance relationship following the arguments of Just and Zilberman. The second-stage decision simply maximizes income since all random elements become known at time t. The consideration of the latter decision is not usually found in papers of this nature and suggests distinctly different results as shown below. The reason for the difference is that if basis risk is small relative to overall price risk, then the risk faced by the decision-maker can be made relatively inconsequential if actual delivery/acceptance on the futures market is considered as an alternative at contract termination. That is, some profit or loss can be locked in at the initial decision stage (except for speculators) so the decision-maker only faces the smaller risk related to the basis at contract termination. The existance of a certain outcome in portfolio selection models has been shown in the finance literature to distinctly alter the role of risky alternatives.

For notational purposes, let

$p_t$ = spot-market price at time t

$p_t^c$ = forward contracting price at time t - 1 for delivery at time t (this may be a raw product equivalent price for a processed commodity or a price at which the commodity will be exported)

$p_t^f$ = futures price at time t - 1 for contracts with maturity at time t

$\tilde{p}_t^f$ = futures price at time t for contracts with maturity at time t

$p_{ti}$ = spot-market price for time t expected by decision-maker i at time t - 1 except in the case of speculators where $p_{ti}$ is the decision-maker's expectation for $\tilde{p}_t^f$.

$f_{ti}$ = futures market position taken at time t - 1 in contracts with maturity at time t (positive for sales, negative for purchases)

$\tilde{f}_{ti}$ = futures market transactions at time t in contracts with maturity at time t (negative for sales, positive for purchases)

$\sigma_i$ = variance of $p_{ti}$ with respect to $p_t$, i.e., $E_{t-1}(p_{ti} - p_t)^2$ where $E_{t-1}$ is the expectation operator at time t - 1 except in the case of speculators where $\sigma_i = E_{t-1}(p_{ti} - \tilde{p}_t^f)^2$

$\phi_i$ = absolute risk aversion of decision-maker i

$Q_{ti}$ = production planned by producer i at time t - 1 for time t

$I_{t-1,i}$ = inventory held by storer i out of supply at time t - 1 for release at time t

$X_{t-1,i}$ = raw product quantity required by processor/exporter i at time t to honor commitments made at time t - 1.

The Producer/Hedger.--Consider first the case of a producer/hedger i who uses the futures market to hedge against price declines during the production period. Suppose his cost of production is quadratic and given by $\alpha_{0i} Q_{ti}$ + $(1/2)\alpha_{1i} Q_{ti}^2$. The associated utility of income is

$$U_i(\pi_{ti}) = U_i[p_t(Q_{ti} + e_{ti} - f_{ti} + \tilde{f}_{ti}) - \alpha_{0i} Q_{ti} - \frac{1}{2\alpha_{1i}} Q_{ti}^2 + p_t^f f_{ti} - \tilde{p}_t^f \tilde{f}_{ti}]$$

where $e_{ti}$ is a random disturbance in production unknown at time t - 1 but known at time t, $E_{t-1}(e_t) = 0$. Also, consistent with competition, producers are assumed not to perceive the effect of their own production on price or correlation of their production with price, $E_{t-1}(p_t e_{ti}) = 0$. Suppose also that, due to basis risk,

(1) $$\tilde{p}_t^f - p_t \sim N(0, 2\tilde{\sigma})$$

where $E[(\tilde{p}_t^f - p_t) e_{ti}] = 0$ and $\tilde{\sigma} > 0$. Then the producer has a two-stage decision problem where at time t - 1 he chooses expected production $Q_{ti}$ and an initial futures market position $f_{ti}$. At time t he then decides how much of his futures position to close out.

Using the optimality principle of dynamic programming, the problem can first be solved at the second stage given the first stage decisions and then at the first stage after substituting second-stage decision functions. At the second stage (time t), all random forces become known so the problem is one of certainty or simple profit maximization where profit is

(2) $$\pi_{ti} = \pi^*_{ti} + \Delta\pi_{ti}$$

where

(3) $$\pi^*_{ti} = p_t (Q_{ti} + e_{ti} - f_{ti}) - \alpha_{0i}Q_{ti} - \frac{1}{2\alpha_{1i}} Q^2_{ti} + p^f_t f_{ti}$$

(4) $$\Delta\pi_{ti} = (p_t - \tilde{p}^f_t) \tilde{f}_{ti}.$$

Since $\pi^*_{ti}$ is completely determined at time t, the decision problem is to maximize $\Delta\pi_{ti}$ subject to $0 \leq \tilde{f}_{ti} \leq f_{ti}$ assuming $f_{ti} \geq 0$; the solution is

(5) $$\tilde{f}_{ti} = \begin{cases} f_{ti} & \text{if } p_t > \tilde{p}^f_t \\ 0 & \text{if } p_t \leq \tilde{p}^f_t. \end{cases}$$

Next, substituting (5) into (4) and using (1) to take expectations obtains

(6) $$E_{t-1}(\Delta\pi_{ti}) = \sigma^* f_{ti}$$

(7) $$V_{t-1}(\Delta\pi_{ti}) = c\tilde{\sigma}\ f_{ti}^2$$

where

$$\sigma* = \sqrt{\frac{\tilde{\sigma}}{\pi}},$$

$$c = 1 - \frac{1}{\pi},$$

and $V_{t-1}$ is the variance operator at time t - 1 (see Patel and Read for moments of the half normal distribution which support these results). Thus, using (2)-(4) and approximating expected utility at time t - 1 with a mean-variance function obtains

$$EU_{ti} \equiv E_{t-1}[U_i(\pi_{ti})] \doteq p_{ti}(Q_{ti} - f_{ti}) - \alpha_{0i}\,Q_{ti} - \frac{1}{2}\alpha_{1i}\,Q_{ti}^2 + (p_t^f + \sigma*)\ f_{ti}$$

$$- \frac{\phi_i}{2}[\sigma_i(Q_{ti} - f_{ti})^2 + (\sigma_i + p_{ti}^2)\ V_{t-1}\ (e_t) + c\tilde{\sigma}\ f_{ti}^2].$$

First-order conditions for expected utility maximization yield

(8) $$Q_{ti} = \frac{p_{ti} - \alpha_{0i} + \phi_i\ \sigma_i\ f_{ti}}{\alpha_{1i} + \phi_i\ \sigma_i} = \frac{p_{ti} - \bar{\alpha}_{0i} + \phi_i\ \sigma_i f_{ti}}{\alpha_{1i} + \phi_i \sigma_i} + \varepsilon_{ti}^{\alpha}$$

(9) $$f_{ti} = \frac{p_t^f - p_{ti} + \sigma* + \phi_i\ \sigma_i\ Q_{ti}}{\phi_i\ (\sigma_i + c\tilde{\sigma})},$$

where $\alpha_{0i} = \bar{\alpha}_{0i} + \tilde{\varepsilon}_{ti}^{\alpha}$, $\varepsilon_{ti}^{\alpha} = \tilde{\varepsilon}_{ti}^{\alpha}/(\alpha_{1i} + \phi_i\ \sigma_i)$, and $\tilde{\varepsilon}_{ti}^{\alpha}$ represents random variation in production costs from time to time which are anticipated at production planning time, $E(\tilde{\varepsilon}_{ti}^{\alpha}) = E(\varepsilon_{ti}^{\alpha}) = 0$. Second-order conditions for a maximum can be shown to hold if $\alpha_{1i} > 0$ and $\phi_i > 0$, i.e., if the production cost curve is upward bending and the decision-maker is risk averse.

The Store/Hedger.--Consider next the case of a storer of the commodity who also has the option of hedging against price declines during the period of storage. Suppose his cost of storage is quadratic and is given by $\beta_{0i}I_{t-1,i}$ + (1/2) $\beta_{1i}I_{t-1,i}$. The associated utility of income is

$$(10) \quad U_i(\pi_{ti}) = U_i[p_t(I_{t-1,i} - f_{ti} + \tilde{f}_{ti}) - p_{t-1}\, I_{t-1,i} - \beta_{0i}\, I_{t-1,i}$$

$$- \frac{1}{2}\beta_{1i}\, I^2_{t-1,i} + p^f_t\, f_{ti} - \tilde{p}^f_t\, \tilde{f}_{ti}].$$

Considering this case as a two-stage decision problem as for the producer case, the storer decides at time t how much of his futures position to close out after observing $\tilde{p}^f_t$ and $p_t$ and given initial decisions $I_{t-1,i}$ and $f_{ti}$. Representing profit as in (2) where

$$\overset{*}{\pi}_{ti} = p_t(I_{t-1,i} - f_{ti}) - p_{t-1}I_{t-1,i} - \beta_{0i}I_{t-1,i} - \frac{1}{2}\beta_{1i}\, I^2_{t-1,i} + p^f_t\, f_{ti}$$

and $\Delta\pi_{ti}$ is given by (4) makes this second-stage problem mathematically equivalent to the producer case so that close out decisions follow (5) and the mean and variance of $\Delta\pi_{ti}$ follow (6) and (7).

Substituting this decision function in (10) and approximating expected utility at time t - 1 with a mean-variance function obtains

$$EU_{ti} \equiv E_{t-1}[U_i(\pi_{ti})] \doteq p_{ti}(I_{t-1,i} - f_{ti}) - p_{t-1}\, I_{t-1,i} - \beta_{0i}\, I_{t-1,i}$$

$$- \frac{1}{2}\beta_{1t}\, I^2_{t-1,i} + (p^f_t + \sigma^*)\, f_{ti} - \frac{\phi_i}{2}\left[\sigma_i(I_{t-1,i} - f_{ti})^2 + c\tilde{\sigma}\, f^2_{ti}\right].$$

First-order conditions for expected utility maximization yield

$$(11) \qquad I_{t-1,i} = \frac{p_{ti} - p_{t-1} - \beta_{0i} + \phi_i \sigma_i f_{ti}}{\beta_{1i} + \phi_i \sigma_i} = \frac{p_{ti} - p_{t-1} - \bar{\beta}_{0i} + \phi_i \sigma_i f_{ti}}{\beta_{1i} + \phi_i \sigma_i} + \varepsilon_{ti}^{\beta}$$

$$(12) \qquad f_{ti} = \frac{p_t^f - p_{ti} + \sigma^* + \phi_i \sigma_i I_{t-1,i}}{\phi_i(\sigma_i + c\tilde{\sigma})},$$

where $\beta_{0i} = \bar{\beta}_{0i} + \tilde{\varepsilon}_{ti}^{\beta}$, $\varepsilon_{ti}^{\beta} = \tilde{\varepsilon}_{ti}^{\beta}/(\beta_{1i} + \phi_i \sigma_i)$, and $\tilde{\varepsilon}_{ti}^{\beta}$ represents random changes in storage costs from time to time which are anticipated at the time of storage decisions, $E(\tilde{\varepsilon}_{ti}^{\beta}) = E(\varepsilon_{ti}^{\beta}) = 0$. Second-order conditions can be shown to hold if $\beta_{1i} > 0$ and $\phi_i > 0$, i.e., the storage cost curve is upward bending and the storer is risk averse.

The Exporter-Processor/Hedger.--A third distinctly different group of decision-makers is the one that forward contracts a delivery of commodity possibly in processed form and then uses the futures market to hedge against price increases before the commodity is actually purchased to prepare for contracted delivery. Suppose the cost of processing is quadratic and given by $\gamma_{0i} X_{t-1,i} + (1/2) \gamma_{1i} X_{t-1,i}$. Alternatively, these costs can represent an effect on revenue due to quadratic demand for the product or a loss rate incurred in handling. The utility of income is

$$U_i(\pi_{ti}) = U_i[p_t^c X_{t-1,i} - p_t(X_{t-1,i} + f_{ti} - \tilde{f}_{ti}) + p_t^f f_{ti} - \gamma_{0i} X_{t-1,i} - \frac{1}{2} \gamma_{1i} X_{t-1,i}^2 - \tilde{p}_t^f \tilde{f}_{ti}]$$

(recall $f_{ti} < 0$ for purchases and $\tilde{f}_{ti} < 0$ for sales).