# Getting Away with Robbery? Patenting Behavior with the Threat of Infringement

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## 1. Introduction

Patents have been used over the last 500 years as means of protecting intellectual property. The level of protection enabled by a patent depends on two interdependent elements: patent shape (i.e., patent length and patent breadth) and the innovator's ability to enforce and/or defend her patent rights when her patent is infringed and/or its validity is directly challenged. While the maximum patent length is predetermined by law and thus exogenous to the innovator, patent breadth is determined, to a large extent, by the innovator through the claims that he makes in the patent application.

The innovator's patent breadth decision is critical in determining whether and when competing innovations will enter the market as well as whether the patent will stay active after grant (i.e., the effective patent life). While a large patent breadth makes it harder for competitors to enter the market with non infringing innovations, empirical evidence suggests that the greater is patent breadth, the greater is the probability that the patent will be infringed, its validity challenged and that the courts will invalidate the patent or narrow its scope during an infringement trial (Merges and Nelson 1990, Lerner 1994, Lanjouw and Schankerman 2001).

The analysis of the innovator's patenting behavior in the existing patent literature has focused on either the decision to patent the innovation or to keep it a secret (Waterson 1990, Horstmann et al. 1985), or on the optimal patent breadth decision when an infringer always faces an infringement trial (Yiannaka and Fulton 2003). While the above decisions have been studied in isolation, they are clearly related; in addition other possibilities exist, such as the case where an infringer does not necessarily face an infringement trial.

The purpose of this paper is to examine how the decision to patent and the optimal patent breadth decision affect, and are affected by, the innovator's ability to enforce his patent rights. Specifically, the paper develops a game theoretic model to examine the optimal patenting behavior of an innovator who has generated a patentable product innovation and who is faced with potential entry by another firm. To keep the analysis simple, we assume that the innovator patents his innovation and outline the conditions under which patenting is not profitable. When patenting is profitable, the innovator has to decide what patent breadth should be claimed. If his patent is infringed, the innovator also has to decide whether he should invoke a trial. One of the features of the model is that the entrant may be able, by her choice of location in product space, to affect the innovator's decision to invoke a trial.

Analytical results show that the innovator's monopoly profits and trial costs, along with the potential entrant's R&D effectiveness and trial costs, are key factors in determining the innovator's decision to patent, her patent breadth choice and her decision to invoke a trial. In general, the greater is the entrant's R&D effectiveness, the smaller is the innovator's incentive to patent her product. If patenting occurs, however, the greater is R&D effectiveness, the greater is the patent breadth that could be chosen without triggering infringement. This result occurs because the greater is the entrant's R&D effectiveness, the further away from the patentee the entrant can locate in the product space. The outcome is increased product differentiation, less competition and thus higher profits for both players. The paper also shows that the greater are the innovator's trial costs, the smaller is the profit maximizing patent breadth and the smaller is her incentive to invoke a trial.

One of the key results of the paper is that if the innovator decides to patent his innovation he will choose a patent breadth that makes it optimal for him to take the entrant to trial when his patent is infringed. Other factors affecting the innovator's optimal patenting behavior are the probability that the patent will be found valid at trial and the duopoly profits earned when market entry cannot be deterred.

The rest of the paper is organized as follows. Section two describes the theoretical development of the strategic patent breadth model; it describes the market conditions, defines patent breadth and models the choice of patent breadth as a sequential game of complete information. Section three provides the analytical solution of the model. Finally, section four concludes the paper.

## 2. The Patent Breadth Model

### 2.1 Model Assumptions

The model builds upon the model developed by Yiannaka and Fulton (2003) to study the optimal patent breadth decision when under infringement a trial always takes place. In our model the optimal patent breadth strategy is determined in a sequential game of complete information. The agents in the game are an incumbent/patentee who, having invented a patentable drastic product innovation and having decided to seek patent protection, decides on the patent breadth claimed and a potential entrant who decides on whether to enter the patentee's market and, if entry occurs, where to locate in a vertically differentiated product space. To keep the analysis simple we assume that the innovator chooses to patent and outline the conditions under which patenting is not optimal. Both the incumbent and the entrant are risk neutral and maximize profits. It is assumed that the regulator (e.g., Patent Office) always grants the patent as claimed; thus, the regulator is not explicitly modeled.

The patentee's investment decision that led to the development of a new product is not examined – this decision is treated as exogenous to the game. In addition, it is assumed that the patentee and the entrant each produce at most one product and that the entrant does not patent her product since further entry is not anticipated. The production process for the entrant is assumed to be deterministic, so that once the entrant chooses a location she can produce the chosen product with certainty. It is also assumed that there is no time lag between making and realizing a decision.

The patentee and the entrant, if she enters, operate in a vertically differentiated product market. To keep the analysis tractable, it is assumed that no substitute exists for the products produced by the patentee and the entrant. Consumers differ according to some attribute $\lambda$, uniformly distributed with unit density $f(\lambda) = 1$ in the interval $\lambda \in [0,1]$, each buying one unit of either the patentee's or the entrant's product but not both. The patentee is assumed to have developed a product that provides consumers with utility $U_p = V + \lambda q_p - p_p$, where V is a base level of utility, $q_p$ is the quality of the patentee's product $p_p$ is the price of the product produced by the patentee. The entrant's product has quality $q_e > q_p$, $q_e \in (0,1]$, that provides consumers with utility $U_e = V + \lambda q_e - p_e$, where $p_e$ is the price of the entrant's product. Without affecting the qualitative nature of the model, the quality of the patentee's product $q_p$ is set equal to zero (i.e., $q_p = 0$). As a result, the entrant's quality $q_e$ is interpreted as the difference in quality between her product and that of the patentee, or more generally as the distance the entrant has located away from the patentee.[1]

Product $i$ $(i = p, e)$ is consumed as long as $U_i \geq 0$ and $U_i > U_j$. It is assumed that V is large enough so that $V \geq p_i \; \forall i = p, e$ and the market is always served by at least one product. The consumer who is indifferent between the two products has a λ denoted by $\lambda^*$, where $\lambda^*$ is determined as follows: $V \geq p_i \; \forall i = p, e$

(1) $$U_p = U_e \Rightarrow \lambda^* = \frac{p_e - p_p}{q_e}$$

[1] With $q_p \neq 0$, equation (1) becomes $\lambda^* = \frac{(p_e - p_p)}{q_e - q_p}$. Since the quality difference, $q_e - q_p$, in the denominator is the relevant parameter of interest in the subsequent analysis, the assumption that $q_p = 0$ can be made to ease the notation without affecting the qualitative nature of the model.

Since each consumer consumes one unit of the product of her choice, the demand for the products produced by the patentee and the entrant are given by $y_p = \lambda^*$ and $y_e = 1 - \lambda^*$, respectively.

The patentee has already incurred the development costs associated with the product quality that he has patented. Thus, the R&D costs for the patentee are sunk. For the entrant, however, market entry can only occur if she develops a higher quality product. To do so, she incurs R&D costs $F_e(q_e)$, where $F_e = \beta \frac{q_e^2}{2}$ and $\beta \geq \frac{4}{9}$. The restriction on the parameter $\beta$ ensures that the quality chosen by the entrant, $q_e$, is bounded between zero and one. Note that with this formulation, $F_e'(q_e) > 0$ and $F_e''(q_e) > 0$, thus, it is increasingly costly for the entrant to locate away from the patentee in the one-dimensional product space (i.e., to produce the better quality product). In addition, since $q_e$ represents the quality difference between the patentee's and the entrant's product the filing of a patent by the patentee provides the entrant with knowledge of how to produce the patentee's product (i.e., $F_e(q_p) = 0$ – the assumption of perfect information disclosure by the patent is made). In the absence of the patent it is assumed that reserves engineering of the product innovation is possible and costless. The R&D costs are assumed sunk once they have been incurred and neither the patentee nor the entrant find it optimal to relocate once they have chosen their respective qualities. Once the R&D costs are incurred, production of the products by both the patentee and the entrant occur at zero marginal cost.

The patent breadth claimed and granted to the patentee's product is denoted by $b$ and it defines the area in the one-dimensional product space that the patent protects, thus, $b \in (0,1]$. Patent breadth values close to zero indicate protection of the patented innovation only against duplication. When the entrant locates at a distance $q_e < b$ away from $q_p$ the patent is infringed and the patentee must decide whether to invoke an infringement trial or not. It is assumed that the filing of an infringement lawsuit by the patentee is always met with a counterclaim by the accused infringer that the patent is invalid.[2] The costs incurred during the infringement trial/validity attack by the patentee and the entrant are denoted by $C_p^T$ and $C_e^T$, respectively. These costs are assumed to be independent of the breadth of protection and of the entrant's location. The trial costs will only be incurred if $q_e < b$ and they are assumed to be sunk – once made they cannot be recovered by either party.[3]

The patent system being modeled is assumed to be that of the fencepost type, in which patent claims define an exact border of protection. Under the fencepost system, infringement will always be found when an entrant locates within the patentee's claims, unless the entrant proves that the patent is invalid (Cornish 1989).[4] In the fencepost system the probability that infringement is found does not depend on how close the entrant has located to the patentee. The implication of assuming a fencepost patent system is that the probability that infringement will be found (given that the entrant has located at $q_e < b$ distance away from $q_p$) is equal to the probability that the validity of the patent will be upheld. Thus, the fencepost patent system implies that the events that the patent is found to be infringed and that the patent is found to be invalid can be treated as mutually exclusive and exhaustive.[5]

Patent validity is directly linked to patent breadth. In general, the broader is the patent protection, the harder it is to show novelty, nonobviousness and enablement (Miller and Davis 1990). Thus, the broader is patent protection, the harder it is to establish validity. In addition, evidence from the literature shows that courts tend to uphold narrow patents and invalidate broad

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[2] This is a standard defence adopted by accused infringers (Cornish 1989, Merges and Nelson 1990).

[3] With this assumption we exclude the possibility of the court awarding lawyers' fees to either party.

[4] In contrast, a signpost patent system implies that claims provide an indication of protection and the claims are interpreted using the doctrines of equivalents and reverse equivalents. Under a signpost system the closer the entrant locates to the patentee the easier it is to prove infringement using the doctrine of equivalents. In addition, infringement may be found even when the entrant locates outside the patentee's claims using the doctrine of reverse equivalents.

[5] Note that, our analysis and results are not affected by whether only certain claims are invalidated during the infringement/validity trial or the entire patent; that is, when patent breadth is narrowed rather than the entire patent revoked. This occurs because further entry is not anticipated in our model.

ones (Waterson 1990, Cornish 1989, Merges and Nelson 1990). To capture these observations, the probability $\mu(b)$ that the patent will be found to be valid, or equivalently that infringement will be found, is assumed to be inversely related to patent breadth – i.e., $\mu'(b) < 0$. Specifically, $\mu(b) = 1 - \alpha b$.[6] Thus, $1 - \mu(b) = \alpha b$ is the probability that the patent will be found to be invalid. The validity parameter $\alpha$, $\alpha \in (0,1)$, reflects the degree that patent breadth affects patent validity. For any given patent breadth, the greater is the validity parameter $\alpha$, the greater is the probability that the patent will be found invalid.

## 2.2 The Game

The patent breadth game consists of four stages. In the first stage of the game, the patentee applies for a patent, claiming a patent breadth, $b$. In the second stage of the game, a potential entrant observes the patentee's product and the breadth of protection granted to it and chooses whether or not to enter the market. If the entrant does not enter she earns zero profits while the patentee operates as a monopolist in the last stage of the game and earns monopoly profits $\Pi_p^M$. If the entrant enters, she does so by choosing the quality $q_e$ of her product relative to that of the patentee. This decision determines whether the entrant infringes the patent or not, as well as whether the patentee will invoke a trial in the case the patent is infringed.

If the entrant chooses a quality greater than or equal to the patent breadth claimed by the patentee (i.e., $q_e \geq b$), then no infringement occurs, and she and the patentee compete in prices in the last stage of the game and earn duopoly profits $\Pi_e^{NI}$ and $\Pi_p^{NI}$, respectively. If the entrant locates inside the patent breadth claimed by the patentee (i.e., $q_e < b$), the patent is infringed and

[6] Patent breadth is not the only factor affecting the validity of the patent. A patent may also be invalidated because of unallowable amendments during patent examination and because the innovation is not regarded as an invention under the patent law (Cornish 1989). By assuming that the innovator has generated a patentable innovation, the latter case is excluded. To keep the analysis simple, it is also assumed that the probability of patent invalidation due to unallowable amendments is negligible.

the patentee need to decide whether to invoke a trial or not. This decision is made in the third stage of the game. The payoffs for the patentee and the entrant when the entrant chooses $q_e < b$ and the patentee chooses not to invoke a trial are $(\Pi_e^I)^{NT}$ and $(\Pi_p^I)^{NT}$, respectively. If the patentee invokes a trial then the validity of the patent is examined. With probability $\mu(b)$, the patent is found to be valid (i.e., infringement is found), the entrant is not allowed to market her product and the patentee operates as a monopolist in the last stage of the game. With probability $1-\mu(b)$, the patent is found to be invalid, and the entrant and the patentee compete in prices. The payoffs for the patentee and the entrant when the entrant chooses $q_e < b$ and the patentee invokes a trial are $E(\Pi_e^I)^T$ and $E(\Pi_p^I)^T$, respectively. Figure 1 illustrates the extensive form of the game outlined above.

The solution to this game is found by backward induction. The fourth stage of the game in which the patentee and the entrant – when applicable – compete in prices is examined first, followed by the third stage in which the patentee makes his trial decision, the second stage in which the entrant makes her entry decision, and then the first stage in which the patentee makes his decision regarding patent breadth.

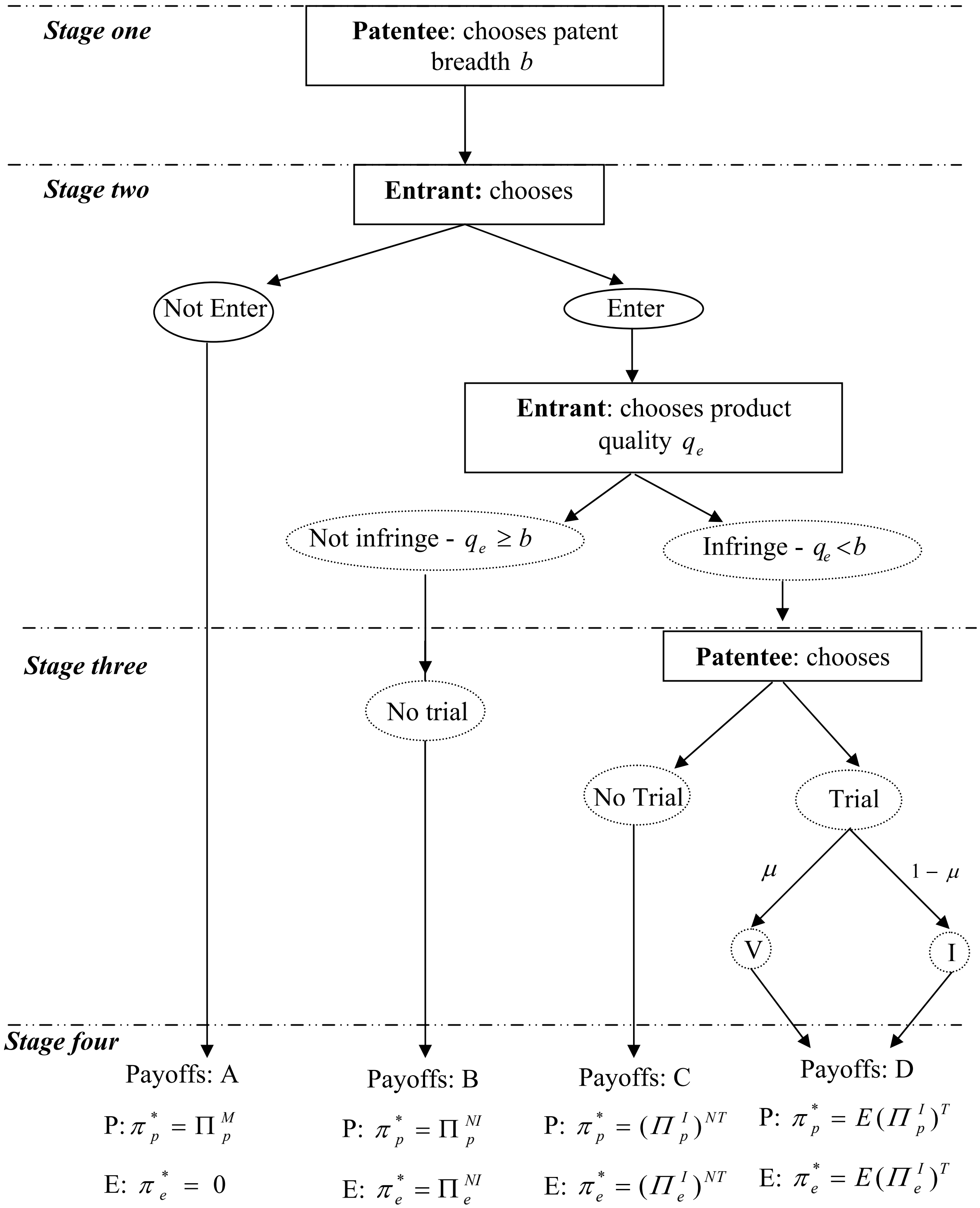



**Figure 1.** The Patenting Game in Extensive Form

## 3. Analytical Solution of the Game

### *3.1 Stage 4 – The Pricing Decisions*

In the fourth stage of the game, two cases must be considered – the case where the entrant has entered and the case where the entrant has not entered. Considering the last case first, in the absence of entry by the entrant, the patentee will charge $p_p = V$ and earn monopoly profits $\Pi_p^M = V - F_p$.

If entry occurs, the problem facing duopolist *i* is to choose price $p_i$ to maximize profit $\pi_i = p_i y_i - F_i$ ($i = p, e$), where $y_p = \dfrac{p_e - p_p}{q_e}$ and $y_e = \dfrac{q_e + p_p - p_e}{q_e}$. Recall that the R&D costs, $F_p$ and $F_e$ for the patentee and the entrant, respectively, are assumed to be sunk at this stage in the game. The Nash equilibrium in prices, as well as the resulting outputs and profits, are given by:

(2) Patentee: $$p_p^* = \frac{q_e}{3},\ y_p^* = \frac{1}{3},\ \pi_p^* = \frac{q_e}{9}$$

(3) Entrant: $$p_e^* = \frac{2q_e}{3},\ y_e^* = \frac{2}{3},\ \pi_e^* = \frac{4q_e}{9}$$

Since the entrant has the higher quality product, she charges the higher price. Profits are increasing in the distance $q_e$ between the patentee's and the entrant's location. The greater is the difference in quality between the two products, the less intense is competition at the final stage of the game and the greater are the profits for both the incumbent and the entrant.[7]

### *3.2 Stage 3 – The Trial Decision*

As illustrated in Figure 1, the entrant's location decision (her quality choice $q_e$) will determine whether the patent will be infringed and whether in the case of infringement a trial will take place.

[7] This is a well-established result in the product differentiation literature in simultaneous games. When competitors first simultaneously choose their locations in the product space and then compete in prices they choose maximum differentiation to relax competition in the pricing stage that would curtail their profits (Lane 1980, Motta 1993, Shaked and Sutton 1982).

When the entrant infringes the patent, the patentee needs to decide whether to invoke an infringement trial or not. Given the quality chosen by the entrant, the patentee will invoke a trial when the patent is infringed as long as his expected profits when a trial takes place, $E(\Pi_p^I)^T$, are greater than his profits when a trial does not take place, $(\Pi_p^I)^{NT}$, i.e., $E(\Pi_p^I)^T > (\Pi_p^I)^{NT}$.

When the patentee invokes a trial his expected profits are given by:

(4) $$E(\Pi_p^I)^T = \mu\Pi_p^M + (1-\mu)\pi_p - C_p^T = (1-ab)\Pi_p^M + ab\frac{q_e}{9} - C_p^T$$

Equation (4) demonstrates that with probability $\mu$ infringement will be found (or equivalently that the validity of the patent will be upheld) at trial and the entrant will not be allowed in the market while the patentee will have a monopoly position and with probability $1-\mu$ infringement will not be found, the entrant will be allowed to market her product and the patentee and the entrant will operate as duopolists.

When the patentee does not invoke a trial his profits are given by:

(5) $$(\Pi_p^I)^{NT} = \pi_p^* = \frac{q_e}{9}$$

Equation (5) shows that when the patentee does not invoke a trial when infringement occurs he shares the market with the entrant realizing duopoly profits which depend on the entrant's choice of location in the quality product space.

Given the above the patentee will invoke a trial when his patent is infringed if:

(6) $$E(\Pi_p^I)^T > (\Pi_p^I)^{NT} \Rightarrow q_e < 9(\Pi_p^M - \frac{C_p^T}{1-ab})$$

Equation (6) shows that the patentee's decision on whether to invoke a trial when his patent is infringed may be affected by the entrant's location decision. We denote the quality that makes the patentee indifferent between invoking and not invoking a trial by $\overline{q}_e$, i.e., $\overline{q}_e = 9(\Pi_p^M - \frac{C_p^T}{1-ab})$,

$\overline{q}_e \in (0,1)$ and assume that when the patentee is indifferent he will choose to not invoke a trial. The quality $\overline{q}_e$ depends among other things on the patent breadth chosen by the patentee. The greater is the patent breadth chosen, the smaller is the quality chosen by the entrant that will infringe the patent without invoking a trial. Figure 2 below illustrates the relationship between the quality chosen by the entrant, $q_e$, and the patentee's decision to invoke a trial or not for any patent breadth choice, $b$.

As depicted in Figure 2, as long as the entrant chooses a product quality $q_e \geq b$ the patent is not infringed. When the entrant chooses a product quality $q_e$ such that $\overline{q}_e < q_e < b$ (i.e., a quality to the right of locus $\overline{q}_e$ and below the locus $b = q_e$) the patent will be infringed but the patentee will not invoke a trial. This outcome is depicted by the dotted area in Figure 2. When the entrant chooses a product quality $q_e$ such that $q_e < b$ and $q_e < \overline{q}_e$ (i.e., a quality to the left of locus $\overline{q}_e$ and below the locus $b = q_e$) the patent will be infringed and the patentee will invoke a trial. This outcome is depicted by the horizontally hatched area in Figure 2.

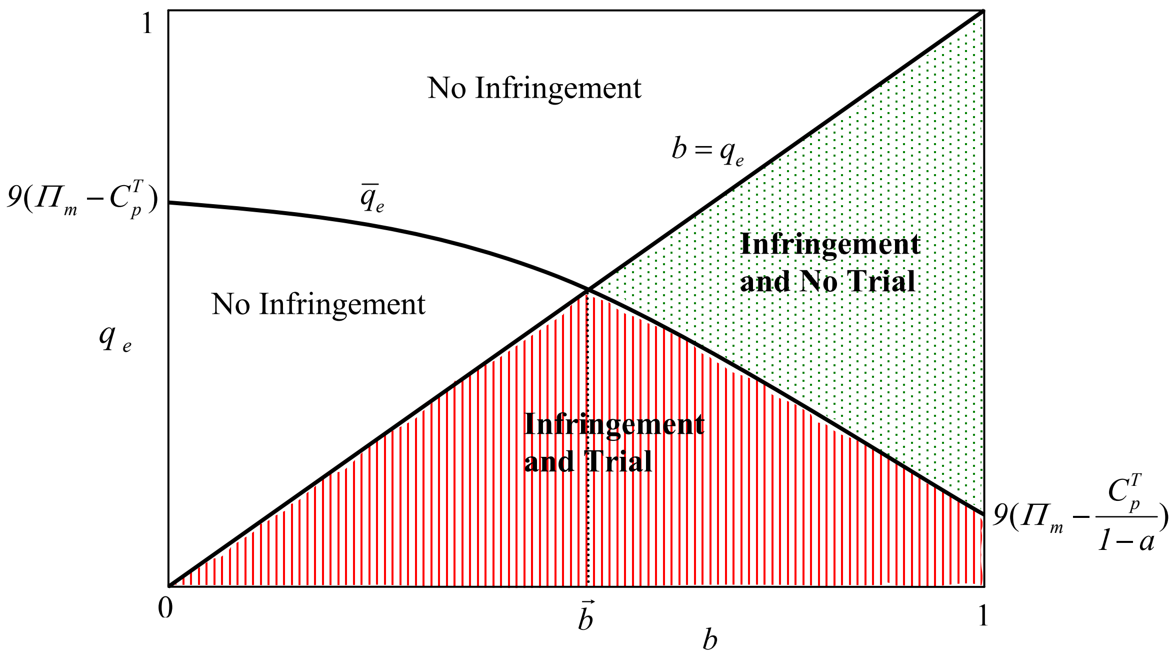



Figure 2. The Entrant's Quality Choice, $q_e$, and the Patentee's Trial Decision

Note that, when $q_e \leq \overline{q}_e \ \forall \, q_e \in (0,1]$, invoking a trial when the patent is infringed ($q_e < b$) is always an optimal strategy for the patentee, regardless of the quality chosen by the entrant. This case occurs when $9(\Pi_m - \frac{C_p^T}{1-a}) \geq 1 \Rightarrow \Pi_m \geq \frac{1}{9} + \frac{C_p^T}{1-\alpha} \ \forall \, \Pi_m \geq 0, C_p^T \geq 0$ and $\alpha \in (0,1)$ (i.e., the locus $\overline{q}_e$ is above the locus $q_e = b$ in Figure 2 $\forall q_e \in (0,1)$ and $b \in (0,1]$. The case where under infringement a trial always occurs regardless of the entrant's product quality choice, $q_e$, has been examined by Yiannaka and Fulton (2003) and will not be considered here.

Also note that, when $q_e > \overline{q}_e \ \forall \, q_e \in (0,1]$, invoking a trial when the patent is infringed ($q_e < b$) is never an optimal strategy for the patentee, regardless of the quality chosen by the entrant. This case occurs when $9(\Pi_m - C_p^T) \leq 0 \Rightarrow \Pi_m \leq C_p^T \ \forall \, \Pi_m \geq 0, C_p^T \geq 0$ (i.e., the locus $\overline{q}_e$ is below the locus $q_e = b$ in Figure 2 $\forall q_e \in (0,1)$ *and* $b \in (0,1]$. In this case, however, as long as the patenting costs are positive, the patentee will not have an incentive to take a patent. This is so because, given our model assumptions of complete information and costless and possible reverse engineering, if the entrant knows that irrespective of her quality choice a trial will never take place, she will always find it optimal to locate at her most preferred location, $q_e^*$, regardless of the patent breadth chosen.[8]

As mentioned above, we are interested in examining the case under which the patentee seeks patent protection and the entrant by her choice of location in the quality product space, $q_e$, affects the patentee's trial and patent breadth decisions. This case occurs when the following conditions are satisfied. First, the monopoly profits realized by the patentee when the entrant does not enter or when the validity of the patent is upheld during an infringement trial, $\Pi_m$, must satisfy the condition $C_p < \Pi_m < \frac{1}{9} + \frac{C_p}{1-\alpha}$ (i.e., the locuses $\overline{q}_e$ and $b = q_e$ in Figure 2 cross for $q_e \in (0,1)$ and $b \in (0,1]$). Thus, our analysis focuses on the case where a patent breadth $\vec{b} \in (0,1)$ that satisfies the condition $b = \overline{q}_e$ exists. It can be easily shown (see Appendix for a proof) that for $\Pi_m \in (C_p^T, \frac{1}{9} + \frac{C_p^T}{1-\alpha})$ the patent breadth $\vec{b} = \frac{1 + 9\alpha\Pi_m - \sqrt{1 + 36\alpha C_p^T - 18\alpha\Pi_m + 81\alpha^2\Pi_m^2}}{2\alpha}$ satisfies the $b = \overline{q}_e$ condition.

Second, if we denote the entrant's most preferred location when patent breadth is not binding by $q_e^*$ then the condition $q_e^* < \vec{b}$ must also be satisfied. Note that, when $q_e^* \geq \vec{b}$ the entrant will always choose $q_e^*$ and the patentee will not invoke a trial. However, knowing that when $q_e^* \geq \vec{b}$, regardless of the patent breadth chosen he won't be able to enforce/defend his patent rights, the patentee will not seek patent protection. Thus, for positive patenting costs when $q_e^* \geq \vec{b}$ a patent will

[8] Note that this is not necessarily true when reverse engineering is possible and costly because the entrant's optimal location choice $q_e^*$ will be different under patenting where the information about the patentee's product is public knowledge and under no patenting where the entrant has to incur a cost to obtain the information.

not be sought by the patentee. The condition $q_e^* < \vec{b}$ is satisfied for R&D cost values, $\beta$ such that

$\beta > \beta_0 = \dfrac{8\alpha}{9(1+9\alpha\Pi_m - \sqrt{1+36\alpha C_p^T - 18\alpha\Pi_m + 81\alpha^2\Pi_m^2})}$. The greater is the validity parameter, $\alpha$,

and the patentee's trial costs, $C_p^T$, and the smaller are the monopoly profits $\Pi_m$, the greater should be the entrant's R&D costs to satisfy the condition $q_e^* < \vec{b}$ (for a proof see the Appendix).

### 3.3 Stage 3 – The Location Decision

Anticipating the patentee's behavior concerning trial given $q_e$, the entrant must choose one of four options – Not Enter, Enter and Not Infringe the Patent, Enter, Infringe the Patent and Induce a Trial or Enter, Infringe the Patent and Not Induce a Trial. For any given patent breadth, $b$, the entrant will choose the option that generates the greatest profit.

The outcome of the Not Enter option is straightforward – the entrant earns zero profits. The outcomes of the other three options depend on a number of factors, including patent breadth, R&D costs and trial costs. The benefits and costs associated with the Enter and Not Infringe option are examined below, followed by an examination of the benefits and costs associated with the Enter and Infringe option. The examination of the Enter and Infringe option consists of the examination of the Enter, Infringe and Not Induce a Trial and the Enter Infringe and Induce a Trial options. Once the net benefits of each option are formulated, the most desirable option for the entrant is determined for any given patent breadth.

#### 3.3.1 Entry with No Infringement ($q_e \geq b$)

For the entrant to enter without infringing the patent, the entrant must choose a quality location that is greater than or equal to the patent breadth – i.e., $q_e \geq b$. Let $q_e^*$ be the optimal quality the entrant would choose when the patent breadth is not binding, where $q_e^*$ solves the following problem:

(7) $$\max_{q_e} \Pi_e = \pi_e - F_e = \frac{4q_e}{9} - \beta \frac{q_e^2}{2}$$

Optimization of equation (7) yields the optimal quality $q_e^*$:

(8) $$q_e^* = \frac{4}{9\beta}$$

Equation (8) gives the entrant's most preferred location and indicates that the less costly it is to produce the better quality product (i.e., the smaller is $\beta$), the further away from the incumbent the entrant locates. Note that, when the entrant's R&D cost parameter takes its minimum value ($\beta = \frac{4}{9}$) the entrant never infringes the patent as $q_e^* = 1$.

As long as $q_e^* \geq b$, the patent breadth does not affect the location chosen by the entrant, since the entrant can choose her optimal quality without fear of infringement. Thus, patent breadth will only be binding if $q_e^* < b$. Since an increase in quality beyond $q_e^*$ results in a reduction in profits, the entrant's profit is decreasing in $q_e$ for all $q_e > q_e^*$. As a result, the entrant, when faced with a binding patent breadth, will always choose a quality equal to the patent breadth chosen by the patentee (i.e., $q_e = b$).

Thus, a profit-maximizing entrant that wishes to not infringe the patent will choose her entry location $q_e^{NI}$ as follows:

(9) $$q_e^{NI} = \begin{cases} \frac{4}{9\beta} & \text{if } b < \frac{4}{9\beta} \\ \text{b} & \text{if } b \geq \frac{4}{9\beta} \end{cases}$$

while the profits earned by the entrant are:

(10) $$\Pi_e^{NI} = \begin{cases} \dfrac{8}{81\beta} & \text{if } b < \dfrac{4}{9\beta} \\ \dfrac{4}{9}b - \dfrac{\beta}{2}b^2 & \text{if } b \geq \dfrac{4}{9\beta} \end{cases}$$

### *3.3.2 Entry with Infringement ($q_e < b$)*

When the entrant decides to infringe the patent she must determine whether to induce the patentee to invoke a trial or not. The entrant's profits under infringement and trial are determined below followed by her profits when she infringes and does not induce the patentee to invoke a trial.

*The Entrant's Profits under Infringement and Trial*

Recall that during an infringement trial there is a probability $\mu = 1 - \alpha b$ that the validity of the patent will be upheld (or equivalently that infringement will be found) and a probability $1 - \mu = \alpha b$ that the patent will be revoked. If the patent is found to be valid during trial, the entrant cannot enter and the patentee has a monopoly position in the market. If the patent is found to be invalid, the entrant is allowed to market her product and the patentee and the entrant operate as duopolists. With this background, the quality chosen by the entrant is determined by solving:

(11) $$\max_{q_e} E\left(\Pi_e^I\right)^T = (1-\mu)\cdot \pi_e - F_e - C_e^T = \alpha b \frac{4q_e}{9} - \beta \frac{q_e^2}{2} - C_e^T$$

The optimal quality chosen is given by:

(12) $$(q_e^I)^T = \frac{4\alpha b}{9\beta}$$

From Equation (12) it follows that the optimal quality under infringement satisfies the condition $(q_e^I)^T < b \Rightarrow \alpha < \frac{9}{4}\beta \;\; \forall \alpha \in (0,1)$ and $\beta > \beta_0 > \frac{4}{9}$. Equation (12) shows that when the entrant infringes the patent she finds it optimal to locate at a distance proportional to the breadth of the patent. Because there is uncertainty with respect to whether the entrant will be able to continue in the market, she 'underlocates'; to reduce the R&D costs, which are incurred with certainty, the entrant locates closer to the patentee than she would have done had infringement not been a possibility.

The expected profits for the entrant are given by equation (13):

$$(13) \qquad E\left(\Pi_e^I\right)^T = \frac{8\alpha^2 b^2}{81\beta} - C_e^T$$

When patent breadth is negligible (i.e., $b$ approaches zero), the expected profits from infringement approach $-C_e$, since the probability of the patent being found valid approaches one. As patent breadth increases, expected profits from infringement also increase, a reflection of the rising probability that the patent will be found invalid.

*The Entrant's Profits under Infringement and No Trial*

When the choice of the entrant's most preferred quality $q_e^*$ results in infringement and trial and the entrant wishes to infringe but not induce a trial, she maximizes her profits by choosing the lowest $q_e$ associated with ensuring that the patentee does not invoke a trial. Thus, to maximize her profits under the infringement and no trial outcome the entrant will choose the quality $(q_e^I)^{NT} = \overline{q}_e$ (recall that when the patentee is indifferent between invoking and not invoking a trial he will choose to not invoke a trial). Given the above, the entrant's profits under infringement and no trial are given by equation (14):

$$(14) \qquad (\Pi_e^I)^{NT} = \pi_e - F_e = \frac{4\overline{q}_e}{9} - \beta\frac{\overline{q}_e^2}{2} = 4(\Pi_m - \frac{C_p^T}{1-\alpha b}) - \frac{81\beta}{2}(\Pi_m - \frac{C_p^T}{1-\alpha b})^2$$

The entrant's profits under infringement and no trial $(\Pi_e^I)^{NT}$ are non decreasing in $b$, $\forall b \in [\vec{b}, \overline{b})$, $\beta > \beta_0$ and $\Pi_m \in (C_p^T, \frac{1}{9} + \frac{C_p^T}{1-\alpha b})$ (for a proof see Appendix). Thus, the greater is patent breadth, $b$, the smaller is $\overline{q}_e$ and thus the closer to $q_e^*$ the entrant can locate without inducing the patentee to invoke a trial.

Figure 3 illustrates the entrant's quality choices and the patentee's trial decision for $q_e^* < \vec{b}$ as well as the entrant's profits under no infringement, infringement and trial and infringement and no trial. As illustrated in Figure 3, for patent breadth values $b \in (0, b_0] \vee b \in [\bar{b}, 1]$ where

$b_o = q_e^* = \dfrac{4}{9\beta}$, $b_0 \in (0, \vec{b})$ and $\bar{b} = \bar{q}_e \Rightarrow \bar{b} = \dfrac{81\beta\Pi_m - 81C_p^T\beta - 4}{\alpha(\beta\Pi_m - 4)}$, $\bar{b} \in (\vec{b}, 1]$, the entrant will always

find it optimal to enter the market and locate at her most preferred location, $q_e^*$, without invoking a trial; patent breadth is not binding for $b \in (0, b_0]$ (i.e., the entrant will locate at $q_e^*$ without infringing the patent) while for $b \in [\bar{b}, 1]$ the patentee's optimal strategy when the patent is infringed is to not invoke a trial. Note, however, that the patent breadth values $b \in (0, b_0]$ and $b \in [\bar{b}, 1]$ will never be chosen by the patentee. Since in this case the patentee cannot deter the entrant from entering the market and locating at her most preferred location, he is better off not taking a patent as long as patenting costs are positive.

When the patent breadth chosen is such that $b \in (b_0, \vec{b})$, the entrant cannot locate at her most preferred location, $q_e^*$ without infringing the patent while the patentee will always find it profitable to invoke a trial when the patent is infringed (i.e., $q_e < b$). In this case, the entrant will have to decide whether to enter and if entry occurs whether to infringe or not the patent knowing that if she infringes a trial will always take place.

Finally, when the patent breadth chosen is such that $b \in [\vec{b}, \bar{b})$, the entrant cannot locate at her most preferred location, $q_e^*$ without infringing the patent but she can by her choice of location on the quality product space, $q_e$, affect whether the patentee will invoke a trial or not when the patent is infringed. For the profit curves depicted in Figure 3, if the patentee chooses patent breadth $b_1$ the entrant will find it optimal to choose a product quality, $(q_e^1)^T$, that infringes the patent and induces the patentee to invoke a trial while if the patentee chooses patent breadth $b_2$ the entrant will find it optimal to choose a product quality $(q_e^I)^{NT}$, that infringes the patent and induces the patentee to not invoke a trial. Thus, under this scenario the entrant has to decide whether to enter and if entry occurs whether to induce the patentee to invoke a trial or not. Note that when $b \in [\vec{b}, \bar{b})$ the entrant will never choose to not infringe the patent since the non infringement strategy is always dominated by the infringement and no trial strategy (i.e., the entrant's profits under no infringement and under infringement and no trial are equal at $\vec{b}$, $\Pi_e^{NI}(\vec{b}) = E(\Pi_e^I)^{NT}(\vec{b})$ and $\Pi_e^{NI}$ and $E(\Pi_e^I)^{NT}$ are decreasing and increasing at $b$, respectively, for any $b \in (\vec{b}, 1]$).

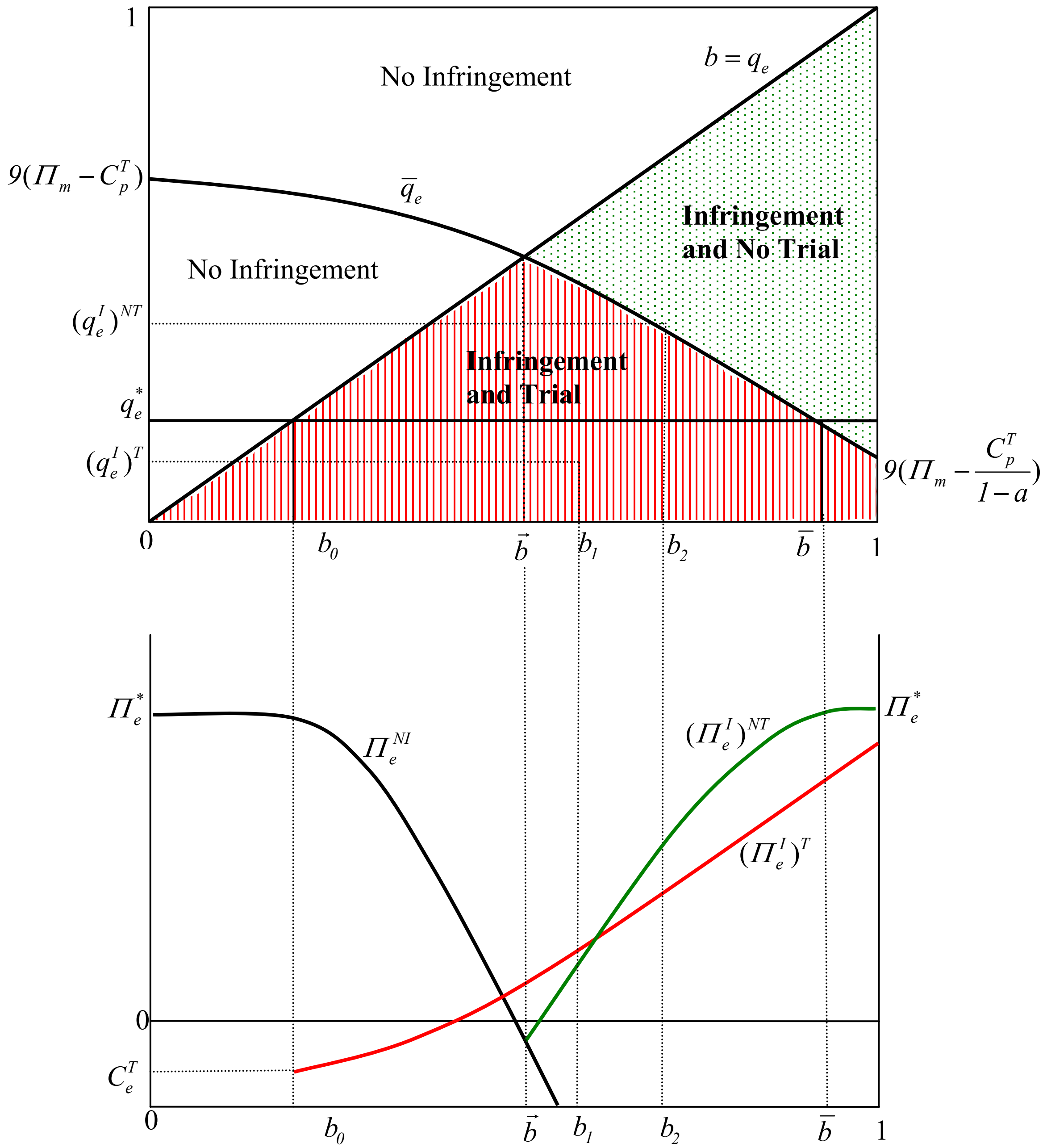



Figure 3. The Entrant's Quality Choice and the Patentee's Trial Decision for $q_e^* < \vec{b}$ and the Entrant's profits under No Infringement, Infringement and Trial and Infringement and No Trial.

**The Entry/Infringement Decision**

To determine the entrant's optimal strategy we must first determine whether there exists a patent breadth $\hat{b} \in (b_o, \vec{b})$ that can deter the entrant from entering in the market, i.e.,

$\Pi_e^{NI}(\hat{b}) \le 0 \wedge E(\Pi_e^I)^T(\hat{b}) \le 0 \wedge E(\Pi_e^I)^{NT}(\hat{b}) \le 0$. Define $b_I^T$ as the patent breadth that makes the entrant indifferent between entering the market, infringing the patent and inducing a trial and not entering the market. Thus, $b_I^T$ solves: $E(\Pi_e^I)^T(b_I^T) = 0$ where $b_I^T \in (b_0, 1]$.[9] Also, define $b_I^{NT}$ as the patent breadth that makes the entrant indifferent between entering the market, infringing the patent and not inducing a trial and not entering the market. Thus, $b_I^{NT}$ solves: $E(\Pi_e^I)^{NT}(b_I^{NT}) = 0$ where $b_I^{NT} \in (b_0, 1]$. Finally, define $b_{NI}$ as the patent breadth that makes the entrant indifferent between entering the market without infringing the patent and not entering the market – i.e., $b_{NI}$ satisfies $\Pi_e^{NI}(b_{NI}) = 0$, where $b_{NI} \in (b_0, 1]$. It is straightforward to show that $b_I^T = \sqrt{\frac{81\beta C_e^T}{8\alpha^2}}$, $b_I^{NT} = \frac{81\beta(\Pi_m - C_e^T) - 8}{81\beta\alpha\Pi_m - 8\alpha}$ and $b_{NI} = \frac{8}{9\beta}$; since $b_{NI} \in (b_0, 1]$, $b_{NI}$ exists only for $\beta$ values such that $\beta \ge \frac{8}{9}$ or $\beta \ge \beta_0$ whichever is greater. Given the above, any $b \in (b_0, 1]$ such that $b \le b_I^T$ makes entry under infringement and trial unprofitable for the entrant, any $b \in (b_0, 1]$ such that $b \le b_I^{NT}$ makes entry under infringement and no trial unprofitable for the entrant, while any $b \in (b_0, 1]$ such that $b \ge b_{NI}$ makes entry under no infringement unprofitable for the entrant.

*Scenario A: Entry Deterrence*

The entrant will not find it profitable to enter the market if there exists a $\hat{b} \in (b_0, 1]$ such that $b_{NI} \le \hat{b} \le b_I^T$ for $b_I^T < b_I^{NT}$ or $b_{NI} \le \hat{b} \le b_I^{NT}$ for $b_I^T > b_I^{NT}$ and $\vec{b} \ge b_{NI}$. The entry deterrence outcome is illustrated in Figure 4. The larger is the R&D cost parameter $\beta$, the easier it is to deter entry, *ceteris paribus,* because as $\beta$ increases $b_{NI}$ becomes smaller, $b_I^T$ becomes larger while $\vec{b}$ is

[9] The assumption is made that the entrant will not enter when she is indifferent between entering and infringing the patent and not entering.

unaffected, making it more likely that the entry deterrence condition $b_{NI} \le \hat{b} \le b_I^T$ and $\vec{b} \ge b_{NI}$ will be satisfied.

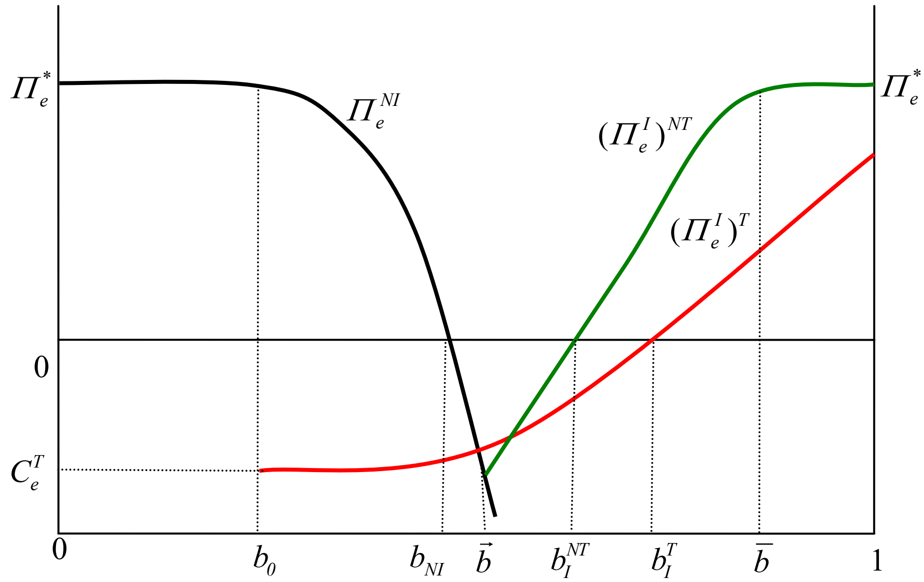



Figure 4. The Entrant's Profits under No Infringement, Infringement and Trial and Infringement and No Trial when Entry can be Deterred.

*Scenario B: Entry Cannot be Deterred*

There are a number of different cases where entry cannot be deterred leading to different optimal strategies for both the patentee and the entrant. Generally, entry cannot be deterred when either a patent breadth $b_{NI}$ that makes the entrant indifferent between entering without infringing and not entering does not exist, when it exists and $\vec{b} < b_{NI}$ and/or a patent breadth, $\tilde{b}$, exists that makes the entrant indifferent between infringing the patent and inducing a trial and not infringing the patent, while still generating positive profits for the entrant – i.e., $\tilde{b}$ solves $\Pi_e^{NI}(\tilde{b}) = E((\Pi_e^I)^T(\tilde{b})) > 0$ where $\tilde{b} \in (b_0, 1]$. The expression for $\tilde{b}$ is derived in the Appendix. To examine the different cases that can emerge when entry cannot be deterred, let $\tilde{\tilde{b}}$ be the patent breadth that makes the entrant indifferent between infringing the patent and inducing a trial and infringing the patent and not inducing a trial, while still generating positive profits for the entrant – i.e., $\tilde{\tilde{b}}$ solves $(\Pi_e^I)^T(\tilde{\tilde{b}}) = E(\Pi_e^I)^{NT}(\tilde{\tilde{b}})) > 0$ where $\tilde{\tilde{b}} \in (b_0, 1]$. The expression for $\tilde{\tilde{b}}$ is derived in the Appendix. As will become evident in the cases below, the optimal strategy for the entrant when entry cannot be deterred (scenario B) depends on the relationship between $\vec{b}$, $b_{NI}$ and $\tilde{b}$ as well as on the existence of $\bar{b}$. Cases I and II examine the entrant's profits when $\bar{b}$ exists which implies that at $b = 1$ the entrant's profits under infringement and no trial will always be greater than her profits under infringement and trial, $(\Pi_e^I)^{NT}(b = 1) > (\Pi_e^I)^T(b = 1)$. Cases III and IV examine the entrant's profits when $\bar{b}$ does not exist, which implies that at $b = 1$ the entrant's profits under infringement and no trial can be greater, equal to or smaller than her profits under infringement and trial. Cases III and IV consider the situation where $(\Pi_e^I)^{NT}(b = 1) < (\Pi_e^I)^T(b = 1)$. When $\bar{b}$ does not exist and $(\Pi_e^I)^{NT}(b = 1) \geq (\Pi_e^I)^T(b = 1)$ cases III and IV are equivalent to cases I and II, respectively.

- Case I: $\exists \bar{b} \wedge \tilde{b} \;\wedge\; \vec{b} \leq \tilde{b}$

Under this case, the entrant's optimal strategy is to either not infringe the patent for relatively low patent breadth values ($b \in (b_0, \vec{b})$) or to infringe the patent and not induce a trial for relatively high patent breadth values ($b \in [\vec{b}, 1)$); the infringement and trial strategy is a dominated strategy for all patent breadth values. This case is most likely to emerge when the entrant's trial costs, $C_e^T$, and R&D cost parameter, $\beta$, are relatively high and low, respectively, (making infringement and trial less attractive to the entrant) and the patentee's monopoly profits, $\Pi_m$, and trial costs, $C_p^T$ are relatively, low and high, respectively (making $\bar{q}_e$ low and thus the infringe and no trial strategy attractive to the entrant). Case I is depicted in panel (i) in Figure 5.

- Case II: $\exists \bar{b} \wedge \tilde{b} \;\wedge\; \vec{b} > \tilde{b}$

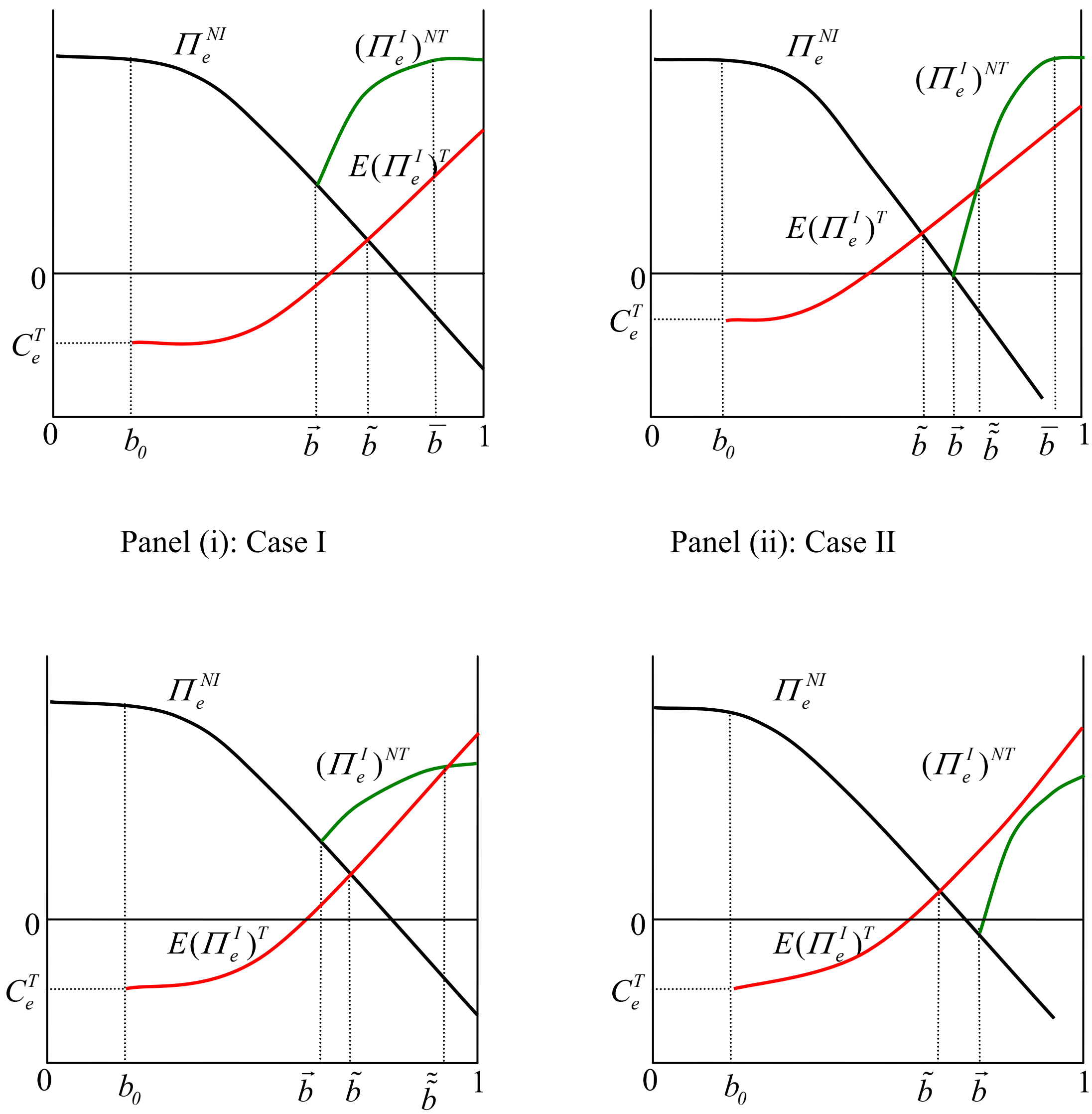
Under this case, the entrant's optimal strategy is to not infringe the patent for relatively low patent breadth values ($b \in (b_0, \tilde{b})$), infringe the patent and induce a trial for intermediate patent breadth values ($b \in [\tilde{b}, \tilde{\tilde{b}})$) and infringe the patent and not induce a trial for relatively large patent breadth values ($b \in [\tilde{\tilde{b}}, \bar{b})$). This case is most likely to emerge when the entrant's trial costs, $C_e^T$, and the validity parameter, $\alpha$, are relatively low, and the R&D cost parameter, $\beta$, is relatively high, (making infringement with trial attractive to the entrant) and when the patentee's monopoly profits, $\Pi_m$, and trial costs, $C_p^T$, are relatively high and low, respectively (making infringement and no trial attractive only for large values of patent breadth). Case II is depicted in panel (ii) in Figure 5.

- Case III: $\exists \tilde{b}$, $\nexists \bar{b}$ $\wedge$ $\vec{b} \leq \tilde{b}$

Under this case, the entrant will find it optimal to not infringe the patent for relatively low patent breadth values ($b \in (b_0, \vec{b}]$), she will infringe the patent and not induce a trial for intermediate patent breadth values, $b \in (\vec{b}, \tilde{\tilde{b}}]$ and she will infringe the patent and induce a trial for relatively high patent breadth values $b \in (\tilde{\tilde{b}}, 1]$. This case is most likely to emerge when the entrant's trial costs, $C_e^T$, R&D cost parameter, $\beta$, and the validity parameter, $\alpha$, are high (making infringement and trial attractive only for high patent breadth values) and the patentee's monopoly profits $\Pi_m$, and trial costs, $C_p^T$, are low and high, respectively (making infringement and no trial attractive even for relatively low patent breadth values). Case III is depicted in panel (iii) in Figure 5.

- Case IV: $\exists \tilde{b}$, $\nexists \bar{b}$ $\wedge$ $\vec{b} > \tilde{b}$

Under this case, the entrant will find it optimal to not infringe the patent for relatively low patent breadth values ($b \in (b_0, \tilde{b}]$) and infringe the patent inducing the patentee to invoke a trial for relatively large patent breadth values ($b \in (\tilde{b}, 1]$); the infringe-and-not-induce-a-trial strategy is dominated by the other two strategies for all patent breadth values. This case is most likely to occur when the entrant's trial costs, $C_e^T$, and R&D cost parameter, $\beta$, are relatively low and high, respectively, the validity parameter, $\alpha$, is high (making infringement and trial attractive) and the patentee's monopoly profits $\Pi_m$, and trial costs, $C_p^T$, are relatively high and low, respectively (making infringement and no trial less attractive). Case IV is depicted in panel (iv) in Figure 5.




Panel (i): Case I

Panel (ii): Case II

Panel (iii): Case III

Panel (iv): Case IV

Figure 5. The Entrant's Profits under No Infringement, under Infringement and Trial and under Infringement and No Trial when Entry cannot be Deterred and $\exists \bar{b}$ – panels (a) and (b) and when $\nexists \bar{b}$ – panels (c) and (d).

### 3.3 Stage 1 – The Patent Breadth Decision

In stage 1 of the game, the patentee chooses the patent breadth $b$ that maximizes profits, given his knowledge of the entrant's behavior in the second stage of the game. Since the entrant's behavior depends on the values of $\Pi_m$, $C_p^T$, $C_e^T$, $\alpha$ and $\beta$, the patent breadth chosen by the patentee also depends on these parameters. Specifically, the following situations are possible, each one corresponding to one of the scenarios and cases outlined above.

*Scenario A: Choose Patent Breadth to Deter Entry*

If there exists a patent breadth $\hat{b} \in (b_0, 1]$ such that $b_{NI} \leq \hat{b} \leq b_I^T$ for $b_I^T < b_I^{NT}$ or $b_{NI} \leq \hat{b} \leq b_I^{NT}$ for $b_I^T > b_I^{NT}$ and $\vec{b} \geq b_{NI}$ then the patentee will choose this patent breadth and deter entry. By deterring entry, the patentee earns monopoly profits, $\Pi_m$. Since these profits are higher than what can be earned under a duopoly, the patentee always finds it optimal to deter entry.

*Scenario B: Entry Cannot be Deterred*

- Case I: $\exists \bar{b} \wedge \tilde{b} \;\; \wedge \; \vec{b} \leq \tilde{b}$

Under this case, it is never optimal for the entrant to infringe the patent and induce the patentee to invoke a trial. The patentee has to decide whether to choose a patent breadth $b \in (b_0, \vec{b})$ that will induce the entrant not to infringe the patent or to choose a patent breadth $b \in [\vec{b}, \bar{b})$ that will induce the entrant to infringe the patent without inducing a trial. Note that the entrant is indifferent between not infringing the patent and infringing the patent without inducing a trial when $b = \vec{b}$, i.e., $\Pi_e^{NI}(\vec{b}) = (\Pi_e^I)^{NT}(\vec{b})$. In both cases, the patentee's profits are increasing in the entrant's quality choice $q_e$, i.e., $\Pi_p^{NI} = (\Pi_p^I)^{NT} = \pi_p^* = \frac{q_e}{9}$, thus, the patentee maximizes his profits by forcing the entrant to locate the furthest away in the quality product space. Since under no infringement the entrant will choose $q_e^{NI} = b$ while under infringement and no trial she will choose $(q_e^I)^{NT} = \bar{q}_e$ and $\bar{q}_e = b$ for $b = \vec{b}$ while $\bar{q}_e < b \; \forall \; b \in (\vec{b}, 1]$ the patentee's profit maximizing strategy under case I is to choose the patent breadth $b = \vec{b}$.

- Case IV: $\exists \tilde{b}, \nexists \bar{b} \;\; \wedge \; \vec{b} > \tilde{b}$

Under this case, it is never optimal for the entrant to infringe the patent without inducing a trial. The patentee has to decide whether to choose a patent breadth $b \in (b_0, \tilde{b}]$ and induce the entrant to not infringe the patent or to choose a patent breadth $b \in (\tilde{b}, 1]$ and induce the entrant to infringe the patent and induce a trial. This case has been examined by Yiannaka and Fulton (2003) who find that the patentee will induce non infringement by claiming $b = \tilde{b}$ or induce infringement and trial by claiming either $b = \tilde{b} + e$ where $e \to 0$ or $b = 1$. The optimal strategy for the patentee depends in a complex way on the values of the parameters $\Pi_m$, $C_p^T$, $C_e^T$, $\alpha$ and $\beta$.

- Case II: $\exists \bar{b} \wedge \tilde{b} \;\; \wedge \; \vec{b} > \tilde{b}$

Under this case, the patentee has to decide whether to choose a patent breadth $b \in (b_0, \tilde{b}]$ and induce the entrant to not infringe the patent, choose a patent breadth $b \in (\tilde{b}, \tilde{\tilde{b}})$ and induce the entrant to infringe the patent and induce a trial or choose a patent breadth $b \in [\tilde{\tilde{b}}, \bar{b})$ and induce the entrant to infringe the patent and not induce a trial. The optimal strategy for the patentee depends on his profits under no infringement, $\Pi_p^{NI}$, infringement and trial, $(\Pi_p^I)^T$ and infringement and no trial, $(\Pi_p^I)^{NT}$. It is straightforward to show that the infringement and no trial strategy is always dominated by the non infringement strategy. The reasoning is as follows. If the patentee were to choose to induce non infringement the optimal strategy would be to choose the patent breadth $\tilde{b}$ since this is the patent breadth that forces the entrant to locate the furthest away possible in the quality space without infringing the patent. If the patentee were to choose to induce infringement and no trial then the optimal strategy would be to choose patent breadth $\tilde{\tilde{b}}$ since this is the patent breadth that induces the entrant to locate the furthest away possible under infringement and no trial (for any $b > \tilde{\tilde{b}}$ the entrant locates closer to the patentee – note that in panel (ii) in Figure 5 the entrant's profits under infringement and no trial are increasing in patent breadth). Moreover, as it can be seen in panel (ii) in Figure 5, the entrant's profits at $\tilde{\tilde{b}}$ are greater than her profits at $\tilde{b}$ which implies that the quality chosen by the entrant when $\tilde{b}$ is chosen by the patentee, $q_e = \tilde{b}$ is greater than the quality chosen by the entrant when $\tilde{\tilde{b}}$ is chosen by the patentee, $\overline{q}_e = 9(\Pi_m - \frac{C_p^T}{1-\alpha\tilde{\tilde{b}}})$. Since the patentee's profits under non infringement and under infringement and no trial are both increasing in the quality chosen by the entrant, $q_e$, the patentee is better off choosing $\tilde{b}$ rather than $\tilde{\tilde{b}}$, i.e., $\Pi_p^{NI}(\tilde{b}) > (\Pi_p^I)^{NT}(\tilde{\tilde{b}})$. Given the above, in this case the patentee's choice is between inducing non infringement by claiming $\tilde{b}$ and inducing infringement and trial by claiming either $\tilde{b} + e$ or $\tilde{\tilde{b}} - e$ where $e \to 0$.

The choice that maximizes the patentee's profits depends in a complex way on the relative values of the parameters, $\Pi_m$, $C_p^T$, $C_e^T$, $\alpha$ and $\beta$. In general, the greater are the patentee's monopoly profits, the greater is the patentee's incentive to induce infringement since the only opportunity the patentee has to realize monopoly profits (when entry cannot be deterred) is when his patent is infringed and its validity is upheld during the infringement trial. The larger are the patentee's monopoly profits and the validity parameter and the smaller are the entrant's R&D costs the more likely it is that the patentee will find it optimal to induce non infringement. It is important to note that, in this case, the patentee never finds it optimal to claim the maximum breadth of patent protection or choose a patent breadth that allows the entrant to infringe the patent without facing an infringement trial.

- Case III: $\exists \tilde{b}, \nexists \overline{b} \wedge \vec{b} \leq \tilde{b}$

In this case, the patentee has to decide whether to choose a patent breadth $b \in (b_0, \vec{b}]$ and induce the entrant to not infringe the patent, choose a patent breadth $b \in (\vec{b}, \tilde{\tilde{b}}]$ and induce the entrant to infringe the patent and not induce a trial or choose a patent breadth $b \in (\tilde{\tilde{b}}, 1]$ and induce the entrant to infringe the patent and induce a trial. When deciding between inducing non infringement and infringement and no trial the patentee's optimal strategy is to choose the patent breadth $\vec{b}$ as demonstrated in case I. Thus, under case III the patentee will either choose the patent breadth $\vec{b}$ which makes him indifferent between inducing non infringement and inducing infringement and no trial or he will choose to induce infringement by choosing either $\tilde{\tilde{b}} + e$ or $b = 1$. The choice that maximizes the patentee's profits depends in a complex way on the relative values of the parameters, $\Pi_m$, $C_p^T$, $C_e^T$, $\alpha$ and $\beta$; their effect on the patentee's optimal decision is as described in case II.

## 4. Concluding Remarks

A simple game theoretic model was developed to examine how an innovator's decision to seek patent protection and his optimal patent breadth decision affect and are affected by his ability to enforce his patent rights. The innovator in our model seeks patent protection for a product innovation under potential entry by a firm producing a better quality product. The innovator must decide on the breadth of patent protection claimed, the entrant observes the patent breadth granted and decides whether to enter and if entry occurs where to locate in the quality product space and the innovator observes the entrant's quality choice and in the case of infringement decides on whether to invoke a trial or not. A key feature of the model is that the entrant can, by her choice of product quality, affect the innovator's trial decision when the patent is infringed.

The paper determines the innovator's optimal patent breadth and trial choices when patenting is profitable and examines the conditions under which patenting is not an optimal strategy for the innovator. Analytical results show that the innovator will not find it profitable to patent his innovation when the monopoly profits that he can earn under no entry or when the validity of the patent is upheld during an infringement trial are small, his trial costs and the validity parameter are large and the entrant's R&D costs are very low. When patenting is profitable, the innovator may be able to choose a patent breadth that deters the entrant from entering the market. Entry deterrence is achieved by claiming a patent breadth that is less than the maximum breadth possible. The greater are the entrant's R&D and trial costs, the larger are the patentee's monopoly profits and the smaller is the validity parameter and the patentee's trial costs, the greater is the likelihood that entry can be deterred.

When entry cannot be deterred, the patentee will never find it profitable to allow the entrant to infringe the patent and not invoke a trial. The optimal patent breadth when entry cannot be deterred is in most cases smaller than the maximum breadth possible. This is so because as patent breadth increases, the closer the entrant can locate to her most preferred location without inducing a trial and the smaller are the profits earned by the patentee. In general, the entrant's ability to affect the innovator's trial decision by her choice of product quality results in a smaller patent breadth claimed by the patentee.

The above results hold under our model assumptions of complete and perfect information, single entry, a deterministic R&D process and possible and costless reverse engineering of the innovator's product. Relaxing the above assumptions is the focus of future research.

**References**


Cornish, W.R. *Intellectual Property*: *Patents, Copyright, Trademarks and Allied Rights*. New York: Matthew Bender & Company, 1989.

Horstmann, I., G. M. MacDonald and A. Slivinski. "Patents as Information Transfer Mechanisms: To Patent or (Maybe) Not to Patent." Journal of Political Economy 93(5, October 1985): 837-858.

Lanjouw, J.O. and M. Schankerman. "Characteristics of Patent Litigation: A window on Competition." *RAND Journal of Economics* 32(1, Spring 2001): 129-151.

Lerner, J. "The Importance of Patent Scope: An Empirical Analysis." *RAND Journal of Economics* 25(2, Summer 1994): 319-333.

Merges, R.P. and R. R. Nelson. "On the Complex Economics of Patent Scope." Columbia Law Review 90(4, May 1990): 839-916.

Waterson, M. "The Economics of Product Patents." The American Economic Review 80(4, September 1990): 860-869.

Yiannaka, A. and M. Fulton. "Strategic Patent Breadth and Entry Deterrence with Drastic Product Innovations." Paper presented at the First Biennial Conference of the Food System Research Group, Madison-Wisconsin, June 2003.

## Appendix

- **The existence of $\vec{b} \in (0,1]$**

The solution of the condition $b = \overline{q}_e$ in terms of $b$ yields the following two roots:

$$b_1 = \frac{1+9\alpha\Pi_m + \sqrt{1+36\alpha C_p^T - 18\alpha\Pi_m + 81\alpha^2\Pi_m^2}}{2\alpha} \text{ and } b_2 = \frac{1+9\alpha\Pi_m - \sqrt{1+36\alpha C_p^T - 18\alpha\Pi_m + 81\alpha^2\Pi_m^2}}{2\alpha}.$$

The root $b_1$ is rejected as a possible solution since $b_1 > 1\ \forall \Pi_m \in (C_p^T, \frac{1}{9} + \frac{C_p^T}{1-\alpha}), \Pi_m > 0, C_p^T \geq 0, \alpha \in (0,1)$.

The root $b_2$ is accepted as a possible solution as $b_2 \in (0,1)$ for

$\Pi_m \in (C_p^T, \frac{1}{9} + \frac{C_p^T}{1-\alpha}), \Pi_m > 0, C_p^T \geq 0, \alpha \in (0,1)$. Given the above $b_2 = \vec{b}$.

- **The conditions for $q_e^* < \vec{b}$**

Given that $q_e^* = \frac{4}{9\beta}$ and $\vec{b} = \frac{1+9\alpha\Pi_m - \sqrt{1+36\alpha C_p^T - 18\alpha\Pi_m + 81\alpha^2\Pi_m^2}}{2\alpha}$ the condition $q_e^* < \vec{b}$ can be

written as $\beta > \frac{8\alpha}{9(1+9\alpha\Pi_m - \sqrt{1+36\alpha C_p^T - 18\alpha\Pi_m + 81\alpha^2\Pi_m^2})} = \beta_0$.

- The Effect of $\alpha$, $C_p^T$ and $\Pi_m$ on $\beta_0$.

$$\frac{\partial\beta_0}{\partial\alpha} = \frac{-8\alpha(9\Pi_m - \frac{162\alpha\Pi_m^2 - 18\Pi_m + 36C_p^T}{2\sqrt{1+36\alpha C_p^T - 18\alpha\Pi_m + 81\alpha^2\Pi_m^2}}) + 8(1+9\alpha\Pi_m - \sqrt{1+36\alpha C_p^T - 18\alpha\Pi_m + 81\alpha^2\Pi_m^2})}{9(1+9\alpha\Pi_m - \sqrt{1+36\alpha C_p^T - 18\alpha\Pi_m + 81\alpha^2\Pi_m^2})^2} \geq 0$$

$\forall \alpha \in (0,1), C_p^T \geq 0 \wedge \Pi_m \in (C_p^T, \frac{1}{9} + \frac{C_p^T}{1-\alpha})$.

$$\frac{\partial\beta_0}{\partial C_p^T} = \frac{16\alpha^2}{(\sqrt{1+36\alpha C_p^T - 18\alpha\Pi_m + 81\alpha^2\Pi_m^2})(-1-9\alpha\Pi_m + \sqrt{1+36\alpha C_p^T - 18\alpha\Pi_m + 81\alpha^2\Pi_m^2})^2} \geq 0$$

$\forall \alpha \in (0,1), C_p^T \geq 0 \wedge \Pi_m \in (C_p^T, \frac{1}{9} + \frac{C_p^T}{1-\alpha})$.

$$\frac{\partial \beta_0}{\partial \Pi_m} = \frac{8\alpha\left(\dfrac{162\alpha^2 \Pi_m - 18\alpha}{2\sqrt{1+36\alpha C_p^T - 18\alpha\Pi_m + 81\alpha^2 \Pi_m^2}} - 9\alpha\right)}{9\left(\sqrt{1+36\alpha C_p^T - 18\alpha\Pi_m + 81\alpha^2\Pi_m^2} - 1 - 9\alpha\Pi_m\right)^2} \le 0 \;\; \forall \alpha \in (0,1), C_p^T \ge 0 \;\wedge\; \Pi_m \in (C_p^T, \frac{1}{9} + \frac{C_p^T}{1-\alpha}).$$

$$\frac{\partial^2 \beta_0}{\partial {\Pi_m}^2} = -\frac{16\alpha\left(\dfrac{162\alpha^2 \Pi_m - 18\alpha}{2\sqrt{1+36\alpha C_p^T - 18\alpha\Pi_m + 81\alpha^2 \Pi_m^2}} - 9\alpha\right)^2}{9\left(\sqrt{1+36\alpha C_p^T - 18\alpha\Pi_m + 81\alpha^2\Pi_m^2} - 1 - 9\alpha\Pi_m -\right)^3} +$$

$$\frac{8\alpha\left(\dfrac{(162\alpha^2 \Pi_m - 18\alpha)^2}{4(1+36\alpha C_p^T - 18\alpha\Pi_m + 81\alpha^2\Pi_m^2)^{3/2}} + \dfrac{18\alpha^2}{\sqrt{1+36\alpha C_p^T - 18\alpha\Pi_m + 81\alpha^2\Pi_m^2}}\right)}{9\left(\sqrt{1+36\alpha C_p^T - 18\alpha\Pi_m + 81\alpha^2\Pi_m^2} - 1 - 9\alpha\Pi_m\right)^2} \ge 0$$

$$\forall \alpha \in (0,1), C_p^T \ge 0 \;\wedge\; \Pi_m \in (C_p^T, \frac{1}{9} + \frac{C_p^T}{1-\alpha}).$$

Figure A.1 depicts the combinations of $\beta$ and $\Pi_m$ values for given $C_p^T$ and $\alpha$ values for which the condition $q_e^* < \vec{b}$ is satisfied. The shaded area in Figure A.1 included all combinations of $\beta$ and $\Pi_m$ values for which the $q_e^* < \vec{b}$ condition is satisfied.

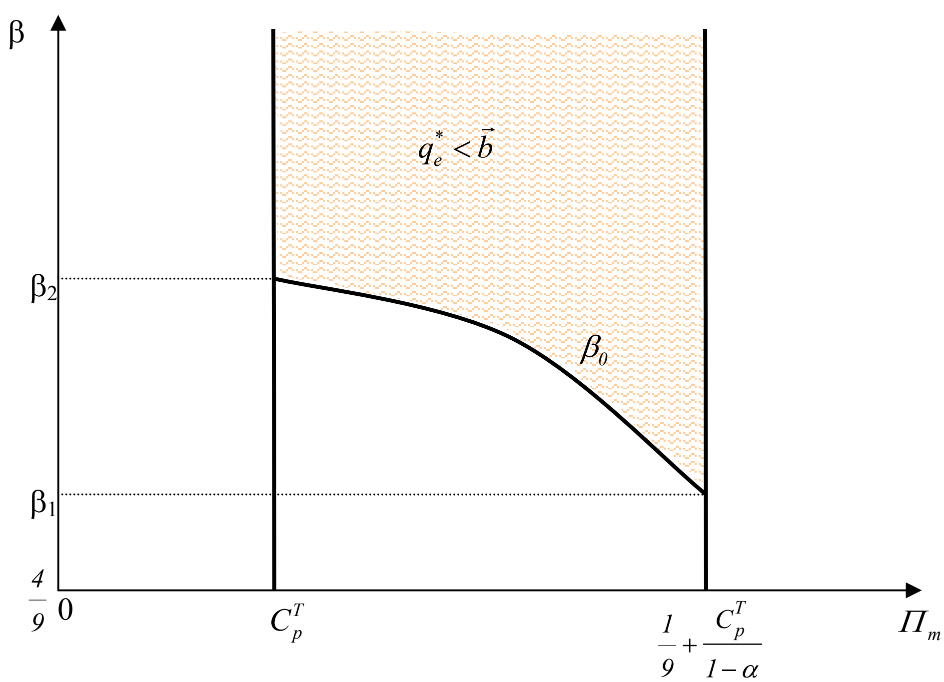



Figure A.1. Combinations of β and $\Pi_m$ for which $q_e^* < \vec{b}$

- **The Entrant's Profits under Infringement and No Trial**

The entrant's profits under infringement and no trial are given by:

$$(\Pi_e^I)^{NT} = 4(\Pi_m - \frac{C_p^T}{1-\alpha b}) - \frac{81\beta}{2}(\Pi_m - \frac{C_p^T}{1-\alpha b})^2$$

$$\frac{\partial(\Pi_e^I)^{NT}}{\partial b} = -\frac{4\alpha C_p^T}{(1-\alpha b)^2} + \frac{81\alpha C_p^T \beta(\Pi_m - \frac{C_p^T}{1-\alpha b})}{(1-\alpha b)^2} \geq 0 \quad \forall b \in [\vec{b}, \bar{b}),\ \beta > \beta_0 \text{ and } \Pi_m \in (C_p^T, \frac{1}{9} + \frac{C_p^T}{1-\alpha b}).$$

$$\frac{\partial^2(\Pi_e^I)^{NT}}{\partial b^2} = -\frac{8\alpha^2 C_p^T}{(1-\alpha b)^3} - \frac{81\alpha^2 C_p^{T2}\beta}{(1-\alpha b)^4} + \frac{162\alpha^2 C_p^T \beta(\Pi_m - \frac{C_p^T}{1-\alpha b})}{(1-\alpha b)^3} \leq 0 \quad \forall b \in [\vec{b}, \bar{b}),\ \beta > \beta_0 \text{ and}$$

$$\Pi_m \in (C_p^T, \frac{1}{9} + \frac{C_p^T}{1-\alpha b}).$$

- **The Existence of $\tilde{b} \in (b_0, 1)$**

If a patent breadth $\tilde{b}$ that makes the entrant indifferent between infringing and not infringing the patent, while still generating positive profits for the entrant, exists it should satisfy the conditions $\tilde{b} \in (b_0, 1]$ and $\Pi_e^{NI}(\tilde{b}) = E(\Pi_e^I)^T(\tilde{b})) > 0$. The solution of $\Pi_e^{NI}(\tilde{b}) = E(\Pi_e^I)^T(\tilde{b})) \Rightarrow (\frac{8\alpha^2}{81\beta} + \frac{\beta}{2})\tilde{b}^2 - \frac{4}{9}\tilde{b} - C_e = 0$ in terms of $\tilde{b}$ yields the following two roots: $\tilde{b}_{1,2} = \frac{9(4\beta \pm \sqrt{2}\sqrt{\beta}\sqrt{16C_e^T\alpha^2 + 8\beta + 81C_e^T\beta^2})}{16\alpha^2 + 81\beta^2}$.The root $\tilde{b}_1 = \frac{9(4\beta - \sqrt{2}\sqrt{\beta}\sqrt{16C_e^T\alpha^2 + 8\beta + 81C_e^T\beta^2})}{16\alpha^2 + 81\beta^2} \leq 0 \quad \forall\ \beta > \frac{4}{9},\ \alpha \in (0,1) \wedge C_e^T \geq 0$ and it is thus rejected since $b_0 < \tilde{b} \leq 1$. The root $\tilde{b}_1 = \frac{9(4\beta + \sqrt{2}\sqrt{\beta}\sqrt{16C_e^T\alpha^2 + 8\beta + 81C_e^T\beta^2})}{16\alpha^2 + 81\beta^2} \geq 0 \quad \forall\ \beta > \frac{4}{9},\ \alpha \in (0,1) \wedge$ $C_e^T \geq 0$ and it is accepted as a possible solution. If $\tilde{b} = \frac{9(4\beta + \sqrt{2}\sqrt{\beta}\sqrt{16C_e^T\alpha^2 + 8\beta + 81C_e^T\beta^2})}{16\alpha^2 + 81\beta^2}$ exists it should also satisfy the conditions $b_0 < \tilde{b} \leq 1$, $\Pi_e^{NI}(\tilde{b}) > 0$ and $E(\Pi_e^I)^T(\tilde{b})) > 0$. The condition $\tilde{b} > b_0$ is satisfied since $\tilde{b} - b_0 = \dfrac{9(4\beta + \sqrt{2}\sqrt{\beta}\sqrt{16C_e^T\alpha^2 + 8\beta + 81C_e^T\beta^2}\,)}{16\alpha^2 + 81\beta^2} - \dfrac{4}{9\beta} > 0 \;\; \forall \;\; \beta > \dfrac{4}{9},\; \alpha \in (0,1) \wedge C_e^T \geq 0$.

The condition $\tilde{b} \leq 1$ is satisfied for certain combinations of $\beta$, $\alpha$ and $C_e^T$. To determine the combinations of $\beta$, $\alpha$ and $C_e^T$ values which satisfy the condition $\tilde{b} \leq 1$, the pairs of $\beta$, $\alpha$ and $C_e^T$ values that satisfy the above constraint as an equality ($\tilde{b} = 1$) are determined first. The solution of $\tilde{b} = 1$ with respect to $C_e^T$ yields $C_e^T = \dfrac{16\alpha^2 - 72\beta + 81\beta^2}{162\beta}$. The area to the right of the locus $\tilde{b} = 1$ represents all combinations of $\beta$ and $C_e^T$ values, for a given $\alpha$ value, for which $\tilde{b} < 1$. If $\tilde{b}$ exists it must also satisfy the conditions $\Pi_e^{NI}(\tilde{b}) > 0$ and $E(\Pi_e^I)^T(\tilde{b})) > 0$. Thus, $\tilde{b}$ must violate the entry deterrence condition – $\tilde{b}$ must take values in the interval $b_I^T < \tilde{b} < b_{NI}$ when $\beta \geq \dfrac{8}{9}$ (i.e., when $b_{NI}$ exists) or in the interval $b_I^T < \tilde{b}$ when $\dfrac{4}{9} < \beta < \dfrac{8}{9}$ (i.e., when $b_{NI}$ does not exist). To determine the combination of $\beta$, $\alpha$ and $C_e^T$ values for which $b_I^T < \tilde{b} < b_{NI}$ the locus $b_I^T = b_{NI}$ must first be determined. The locus $b_I^T = b_{NI}$ refers to the pairs of $\beta$, $\alpha$ and $C_e$ values for which $\dfrac{8}{9\beta} = \sqrt{\dfrac{81C_e^T\beta}{8\alpha^2}}$ holds true. Solution of the above condition with respect to $C_e^T$ yields: $C_e^T = \dfrac{512\alpha^2}{6561\beta^3}$. All combinations of $\beta$ and $C_e^T$ values, for a given $\alpha$ value, below the locus $b_I^T = b_{NI}$ and to the right of locus $\beta = \dfrac{8}{9}$ are such that $b_I^T < \tilde{b} < b_{NI}$ while all combinations of $\beta$ and $C_e^T$ values, for a given $\alpha$ value, below the locus $\tilde{b} = 1$ and to the left of locus $\beta = \dfrac{8}{9}$ are such that $b_I^T < \tilde{b}$.