Reputations, Market Structure, and the Choice of Quality Assurance Systems in the Food Industry

by

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Abstract

A repeated-purchases model is developed to explore the fundamental economic factors that lie behind the choice of different quality assurance systems and their associated degrees of stringency by firms. Differences in the quality discoverability of a sought-after attribute, market structure, attractiveness of a market, nature of reputations, and the value placed in the future are among the factors contributing to the implementation of widely diverse systems across participants in different markets. Close attention is paid to the role of reputations in providing the incentives for firms to deliver high-quality goods. We model three different scenarios—monopoly, duopoly with firm-specific reputations, and duopoly with industry-wide reputations—and compare the resulting welfare of processors and their customers. We also provide a rationale for the branding efforts of many firms to distinguish their products along the supply chain.

Keywords: quality assurance, reputations, repeated purchases, product quality, supply chain, value-added agriculture, imperfect information.
Reputations, Market Structure, and the Choice of Quality Assurance Systems in the Food Industry

Food manufacturers increasingly use quality assurance systems (QASs) to provide information about product attributes to consumers and/or downstream processors. The types of QASs include certification marks, traceability programs, third-party auditing programs, and producer-signed affidavits. Firms use QASs to obtain a market advantage and to build their reputation as a provider of products with claimed attributes. However, given the inherent quality heterogeneity of agricultural output (Ligon), QASs can only increase the probability that a product has a claimed attribute. We are interested here in providing insight into how optimizing firms choose the optimal degree of “stringency” or assurance in their QAS. We model stringency as the probability that a product has a claimed attribute.

Firms choose among QASs knowing that their competition also has the opportunity to choose a QAS. Thus, the optimal choice of stringency will generally depend on the level of stringency that competing firms choose so that firms compete in both output and reputation. Klein and Leffler, and Shapiro have studied the role of reputation as a deception-preventing device, examining a situation where quality is completely determined by a producer’s investments. We build on these previous studies by linking firm reputation to the choice of stringency in a QAS and modeling this choice as a function of the degree to which consumers can discover whether the sought-after attribute is actually present; the potential price premium paid for the attribute; the market structure in which firms compete; and the nature of firm reputation.

Modeling Quality Assurance

We model a situation that is becoming pervasive in the food industry, whereby an input buyer requires its suppliers to implement a QAS (Caswell, Bredahl, and Hooker; Reardon and
The information obtained through the QAS may allow the processing firm to better sort the input it buys, to gain a better idea of the actual quality of the inputs, and to be able to convey assurance to its customers about the quality of its product.

The topic of this article is relevant in an environment in which quality is variable and difficult to verify. Many food attributes can be thus classified (see, e.g., Caswell and Mojduszka; Antle, 1996; and Unnevehr and Jensen). Clearly, if quality is readily observable by both input buyers and consumers, there is no need for a QAS in the procurement process.

We define a high-quality product as one that is certified as meeting an agreed-upon standard. The stringency of a selected QAS informs the processor about the proportion of the purchased input that actually meets the standard. Whether a particular unit of input meets the standard is unknown. We assume that the quality of the processed output has a direct relationship to its input counterpart, an assumption that is equivalent to claiming that the processing technology cannot be used as a substitute for input quality, or that it does so only at prohibitively high costs. We assume also that there is a one-to-one correspondence between the amount of input bought and output sold by a processor. Hence, the production technology works in a Leontief fashion, and the decision on the output rate essentially determines how much of the agricultural input is needed.

Let the random vector $Q$ denote the vector of imperfectly observable quality attributes. For tractability we assume that only one quality attribute is of interest. The unconditional cumulative distribution function of $Q$ that is available in the market is $F_Q(q) = \Pr_Q(Q \leq q)$ for all $q$. This approach accommodates both the case in which the quality attribute or trait is the production method itself and the case in which the process alters the probability
distribution of quality. The former has an analog to a discrete attribute (the good was produced using a desired process or it was not), whereas quality in the latter case is a continuous random variable whose distribution is altered by the process followed. In the continuous case, let \( q^M \) be the minimum quality standard. Hence, \( F_{\bar{Q}}(q^M) \) is the unconditional probability that the product is inferior or unacceptable.

In the discrete case, the input has or fails to have a particular attribute or was or was not produced following a value-adding (cost-increasing) production process. For example, eggs can be produced using animal welfare enhancing techniques (such as free-range production) or by conventional means. The processor buys from producers who have the capabilities needed to produce, and are believed to produce, following the desired processes. Having the capabilities does not necessarily mean that the process will be strictly followed under conditions of imperfect information. Because production of the high-quality input is costlier than production for a commodity market, and there is a strictly positive probability that deviant behavior will not be discovered and penalized, suppliers will find it rational to deviate from perfect compliance.\(^1\)\(^2\) Hence, there is a strictly positive fraction of the output that will not be produced under the desired cost-increasing conditions. This fraction is again represented by \( F_{\bar{Q}}(q^M) \).

Let \( S = \{s \in \mathcal{R} : s^O \leq s \leq s^U \} \) be the set of alternative QASs where \( s = s^O \) represents the absence of quality verification and \( s = s^U \) represents perfect revelation of quality. The processor that procures raw materials from certified suppliers using the QAS indexed by \( s \) expects to certify a fraction of good-quality input, denoted by \( \lambda(s) = 1 - F(q^M | s) \) and a fraction \( 1 - \lambda(s) = F_{\bar{Q}}(q^M | s) \) of the inferior input. All certified input purchased will be
processed and sold to downstream customers as possessing the desired trait.

Implementation of different levels of stringency switches the relevant distributions for quality as follows. For any \( s', s' \in S \) there is an associated conditional distribution for quality, namely, \( F_q(q|s') \) and \( F_q(q|s') \). Increasing the level of stringency involves moving from \( s' \) to \( s' \) where \( s' \leq s' \) leads to a first-order stochastically dominating shift on the distribution of quality. Therefore, \( F_q(q|s') \geq F_q(q|s') \) for all \( q \in Q \). In particular, this implies that

\[
\lambda(s') = 1 - F(q^M|s') \geq 1 - F(q^M|s') = \lambda(s').
\]

Increasing \( s \) reduces the probability of incurring both type I errors (rejecting an input that is of good quality) and type II errors (certifying a product that is of low quality). We assume that \( F_q(q|s) \) is differentiable with respect to \( s \).

Adoption of a QAS by processors incurs a cost, which can include compensation for sellers’ implementation costs and the costs of monitoring. We capture such costs for firm \( i \) with a cost function \( C(s, y_i) \), with \( \partial C(y_i, s)/\partial s > 0 \) and \( \partial C(y_i, s)/\partial y_i > 0 \), where \( y_i \) is output.

Participation in the certified market for high-quality goods yields a per-period profit of

\[
\pi^{ir}(y, s; a) = R(y; a) - C(y_i, s),
\]

where the revenue function \( R(y; a) \) potentially depends on the vector of firms’ output, \( y = (y_i, y_{-i}) \) and an indicator of the strength or size of consumer preference for high-quality goods, \( a \). Clearly, \( \partial R(y; a)/\partial a > 0 \). The superscript in the profit function represents the state of the world, where processor \( i \) has reputation \( r \).

We could append a term to the profit function, representing the economic loss due to certifying a product that is of low quality. This would require specification of a damage function due to the discovery of false certification. We capture punishment to a processor that
is discovered as falsely certifying a product by assuming that the firm loses its reputation, which forces it out of the certified market. Note however that the “reputable” processor obtains the price for the certified commodity no matter what the actual quality might be, because customers cannot assert a priori whether the claims made by the processor are false. In other words, processors will be trusted until proven wrong. Consumers’ trust is what defines the states of the world in this model. For a given processor, demand is state contingent, where the states of the world reflect whether it is trusted by consumers or not. For this sort of punishment mechanism to have an impact on a firm’s decisions, modeling more than one period is required (Klein and Leffler).

We introduce $\omega \in [0,1]$ to measure the degree to which consumers can ascertain the actual quality of the good, where $\omega = 1$ implies that quality is perfectly observable after consumption, or the sought-after characteristic is an experience attribute. Credence attributes are represented by $\omega = 0$. Intermediate values of $\omega$ may be interpreted as attributes that are occasionally detected, or detected only by a proportion of consumers. For example, only a proportion of consumers will be able to discern whether a steak was produced from cattle fed exclusively grain.

We are now in a position to examine how fundamental characteristics of the economic environment influence decisions about the implementation of a QAS paying special attention to market structure and different forms of punishment. Clearly, QASs will be observed if $E\left(\prod^{i^*}(s, y)\right) \geq \pi^*$ for some $s \in S$, and $y > 0$, where $\pi^*$ is the profit level available in an alternative market. That is, if there is a combination of output rate and QAS that makes the expected return of the value-added market positive, then the firm has an incentive to adopt the QAS and supply the high-price market. Throughout the analysis, we assume for mathematical
convenience that there is a continuum of stringency levels from which to choose.

Consumers are assumed to be homogeneous in their tastes and willingness to pay for the value-added product. We also assume that consumers can observe whether a QAS is in place, but are not able to infer the actual quality of the product from the particular QASs used. This implies that the QAS implemented cannot be used as a signal of quality by processors to differentiate themselves from other certifying suppliers.

**Monopolist Processor**

We begin our analysis by examining the case where there is only one processor in the market and the processor is trusted until proven wrong. Therefore, there are only two possible states of the world, denoted by \( r = 1, 2 \). The first state denotes the periods in which the processor has a good reputation and hence faces a positive demand. In state two, the demand for the high-quality product is zero. Since there is only one processor, and profits are zero in the second state of the world, the superscript of the per-period profit function will be dropped.

Let \( T \) denote the time when reputation is lost because consumers discover that they purchased a product that does not meet the promised standards. A processor that moves from state 1 to state 2 in period \( T \) has profits given by

\[
\Pi(s, y) = \sum_{t=1}^{T} \beta^{t-1} \pi(y, s; a) = \pi(y, s; a) \sum_{t=1}^{T} \beta^{t-1} = \pi(y, s; a) \frac{1 - \beta^T}{1 - \beta}
\]

where \( \pi(\cdot) \) represents the per-period profits of a processor that has a good reputation and \( \beta \) is the relevant discount factor. However, quality is random so the processor cannot exert perfect control over it. The processor’s expected profits are

\[
E(\Pi(y, s)) = E\left( \pi(y, s; a) \frac{1 - \beta^T}{1 - \beta} | m(s, \omega) \right) = \pi(y, s; a) \frac{1 - E\left( \beta^T | m(s, \omega) \right)}{1 - \beta}
\]
where \( m(s, \omega) \) denotes the probability that a processor with a QAS \( s \) in place will stay in the value-added market for a trait with discoverability \( \omega \). In particular, note that the probability of staying in the market for two successive periods, \( m(s, \omega) = \lambda(s) + (1 - \lambda(s))(1 - \omega) \), combines the probability that the resulting quality is high with the probability of type II error weighted by the consumers’ level of awareness.\(^4\) A processor will face a zero demand in the second period with probability \( 1 - m(s, \omega) \).

We now need an expression for \( E\left( \beta^T \mid m(s, \omega) \right) \). Note that \( T \) is just counting the number of periods until the first notorious (discovered) failure. Since the outcome in a given period is independent of the outcome of other periods, \( T \) is the number of Bernoulli trials required to get the first failure. This is just the description of a geometric random variable with “success” probability \( 1 - m(s, \omega) \). The previous observation allows us to obtain the required expression:

\[
E\left( \beta^T \mid m(s, \omega) \right) = \sum_{t=1}^{\infty} \beta^t \Pr(T = t) = \sum_{t=1}^{\infty} \beta^t m(s, \omega)^{t-1} (1 - m(s, \omega)) = \frac{1 - m(s, \omega)}{1 - \beta m(s, \omega)}
\]

so that \( E(\Pi(y, s)) = \frac{\pi(y, s; a)}{1 - \beta m(s, \omega)} \).

Antle (2001) classifies quality-control technologies for producing quality-differentiated goods as process control, inspection, testing, and identity preservation. He argues that all these technologies except testing affect the variable costs of production. However, the costs of the testing technologies are not independent of the rate of output, since testing typically involves sampling a small proportion of the product. This discussion reveals that the choices of
stringency of the QAS to implement and the output rate are usually interrelated, specifically through the cost function. The processor’s problem is

$$\max_{y \geq 0, s \geq 0} E\left(\Pi(y, s)\right) = \max_{y \geq 0, s \geq 0} \frac{\pi(y, s; a)}{1 - \beta m(s, \omega)}$$

and the corresponding first-order conditions are given by

$$\frac{\partial E\left(\Pi(y, s)\right)}{\partial y} = \frac{\partial \pi(y, s; a)}{\partial y} \leq 0, \quad y \geq 0$$

$$\frac{\partial E\left(\Pi(y, s)\right)}{\partial s} = \frac{\partial \pi(y, s; a)}{\partial s} + \pi(y, s; a) \frac{\beta \frac{\partial m(s, \omega)}{\partial s}}{1 - \beta m(s, \omega)} \leq 0, \quad s \geq 0.$$  

The Hessian for this problem (which needs to be negative semidefinite for a maximum) is the 2x2 matrix in system (5) below. Equations (3) and (4) have the usual interpretation. Equation (3) is the standard necessary condition for the monopolist profit-maximization problem and will not be discussed further. Equation (4) states that the level of stringency should be increased until the marginal benefits of increased stringency equal the marginal costs. Marginal benefits of an increase in $s$ equal the change in the proportion of purchases that are of high quality multiplied by the probability that low-quality output will be discovered, the per-period profit rate, and a factor that takes into account the multi-period nature of the problem at hand. The marginal benefits of increased assurance rise as the quality of the good is more readily observable by the processor’s customers and as the potential punishments for false certification become more severe. Switching to the second state of the world is a harsher punishment when per-period profits are high and when the future is important to the processor. The marginal cost of an increase in $s$ is simply the increase in costs that must be incurred to implement a more stringent QAS. The first-order conditions can in principle be solved to
obtain the optimal choices for output and stringency of controls represented by $y^*(\omega, \beta, a)$ and $s^*(\omega, \beta, a)$, respectively. We are interested in signing the following: $\frac{\partial y^*}{\partial \omega}$, $\frac{\partial s^*}{\partial \omega}$, $\frac{\partial y^*}{\partial \beta}$, $\frac{\partial s^*}{\partial \beta}$, $\frac{\partial y^*}{\partial a}$, and $\frac{\partial s^*}{\partial a}$. The first four of these can be signed under reasonable assumptions. Stronger assumptions are needed, however, to sign the last two derivatives.

We next show that the ability of consumers to perceive quality increases the optimal level of stringency of the QAS and is likely to decrease the output rate. Differentiating the system of equations (3) and (4) with respect to $\omega$ (at an interior solution) and using the chain rule, we get (after some rearrangement and omitting arguments for brevity)

$$
\begin{pmatrix}
\frac{\partial^2 \pi}{\partial y^2} & \frac{\partial^2 \pi}{\partial y \partial s} & \frac{\partial^2 \pi}{\partial s^2} + \pi \beta \frac{\partial^2 m}{\partial s^2} \left(1 - \beta m\right)

\frac{\partial y^*}{\partial \omega} \\
\frac{\partial s^*}{\partial \omega}
\end{pmatrix}
= 
\begin{pmatrix}
0 \\
- \frac{\pi \beta}{(1 - \beta m)^2} \left(\frac{\partial^2 m}{\partial s \partial \omega} (1 - \beta m) + \frac{\partial m}{\partial s} \frac{\partial m}{\partial \omega}\right)
\end{pmatrix}.
$$

Because the parameter $\omega$ enters by itself only in equation (4) of the necessary conditions, Samuelson’s conjugate pairs theorem immediately asserts that

$$
\text{sgn}\left(\frac{\partial s^*}{\partial \omega}\right) = \text{sgn}\left(\frac{\pi \beta}{(1 - \beta m)^2} \left(\frac{\partial^2 m}{\partial s \partial \omega} (1 - \beta m) + \frac{\partial m}{\partial s} \frac{\partial m}{\partial \omega}\right)\right).$$

Recalling that $m(s, \omega) = \lambda(s) + (1 - \lambda(s))(1 - \omega)$, the previous expression simplifies to

$$\text{sgn}\left(\frac{\partial s^*}{\partial \omega}\right) = \text{sgn}\left(\frac{\pi \beta}{(1 - \beta m)^2} \frac{\partial \lambda}{\partial s} (1 - \beta)\right) > 0.$$ 

As consumers become more able to discern quality, processors will find it optimal to adopt more stringent controls. This result is similar in a sense to one of the findings of Darby and Karni. These authors argued that it is very likely (albeit not necessarily true) that as consumers become more knowledgeable, the optimal
amount of fraud is reduced. In our paper, firms would have incentives to reduce the number of mistakes they make as consumers become increasingly able to discern qualities. There exists a key trade-off between the benefits and costs of information acquisition on the part of processors. Having a more precise QAS, though costly, decreases the probability that firms will lose consumers’ trust. Furthermore, as the expected losses derived from consumer distrust increase, the payoff from the processor becoming better informed about actual quality increases.

Using Cramer’s rule to solve for \( \frac{\partial y^*}{\partial \omega} \), we find that the sign is ambiguous without imposing further structure, since \( \omega \) enters by itself in equation (4). Moreover, system (5) (and the sign just uncovered) tells us that \( \text{sgn}\left(\frac{\partial y^*}{\partial \omega}\right) = \text{sgn}\left(\frac{\partial^2 \pi}{\partial y \partial s}\right) \), which is not implied by the maximization hypothesis alone. Since it is reasonable to assume that raising the levels of controls increases the marginal costs of production, and noting that \( \frac{\partial^2 \pi}{\partial y \partial s} = -\frac{\partial^2 C}{\partial y \partial s} \), we expect the optimal output rate to decrease as \( \omega \) increases.

The question of how the value that producers place on the future affects the optimal choices of QAS and output levels can be explored through a similar exercise. The results of differentiating equations (3) and (4) with respect to \( \beta \) and using Cramer’s rule (again omitting arguments) are as follows:

\[
\begin{bmatrix}
\frac{\partial y^*}{\partial \beta} \\
\frac{\partial s^*}{\partial \beta}
\end{bmatrix} = |H|^{-1} \begin{bmatrix}
\frac{\partial m}{\partial s} \left(1 - \beta m\right) \left(\frac{\beta m}{1 - \beta m} + \pi\right) & \frac{\partial^2 \pi}{\partial y \partial s} \\
-\frac{\partial^2 \pi}{\partial y \partial s} & \frac{\partial^2 \pi}{\partial y^2}
\end{bmatrix}
\]

where \( H \) is the Hessian matrix shown in (5). Thus we know that \( \text{sgn}\left(\frac{\partial y^*}{\partial \beta}\right) = \text{sgn}\left(\frac{\partial^2 \pi(y, s; a)}{\partial y \partial s}\right) \) and \( \frac{\partial s^*}{\partial \beta} \geq 0 \). As the future becomes more important, it is more valuable for processors to invest in quality assurance systems that give them a longer expected presence in the market. The sign of \( \frac{\partial y^*}{\partial \beta} \) is ambiguous as before (and due to the exact same reasons). However, the previous discussion suggests it is negative. Increasing the
expenses incurred to “learn” about the actual quality of the good increases variable costs of production, and hence it is optimal to cut back on the output rate.

As mentioned before, stronger assumptions are needed to sign the last two derivatives of interest. This is a direct result of the structure of the problem, where the strength of the demand for the valued attribute enters by itself in the two necessary conditions. The problem is that as $a$ increases, there is an incentive to increase both the rate of output and the stringency of the QAS, since it increases the marginal benefit part in equations (3) and (4). However, as long as the technology is not nonjoint in inputs (see Chambers), increasing either variable potentially has the effect of increasing the marginal-cost side of the other equation, thus making the net change ambiguous. The intuition is that when the output rate increases, so does the marginal costs associated with any given QAS. Also, increases in the profitability of the market for value-added products provide incentives to monitor product quality more closely to delay transition to the second state. However, this increases marginal costs of production. If it is appropriate to assume that the technology is nonjoint in inputs, then $\frac{\partial^2 \pi}{\partial y \partial s} = 0$ obtains, and both $\frac{\partial y^*}{\partial a}$ and $\frac{\partial s^*}{\partial a}$ are positive whenever revenues and marginal revenues increase with the strength of the demand parameter.

**Duopoly Processors with Public Reputation**

We consider first the situation where an entire industry can lose consumer trust as a result of the actions of one participant. Hennessy, Roosen, and Miranowski model a related issue and conclude that even when firms in the food industry may profit by increased investments in safety (assuming a leadership role), they may obtain higher levels of benefits by free riding on the efforts of other chain participants. In the first state of this case, all processors have a good reputation. In the second, all sellers are punished by consumers.
We model this situation as a potentially infinitely lived duopoly game, where the termination time is random. The uncertainty comes again from the fact that processors cannot exert absolute control on the quality of the good they procure and/or produce. Rob and Sekiguchi model a similar situation. However, in their model, firms compete in price in the second period, and only one firm is able to make sales until it loses its reputation and the other occupies its place. When the second firm loses the market, consumers switch back to the first firm, and so on.

In the first stage, the two processors decide once and for all, independently and in a non-cooperative fashion, what QAS to implement. After observing each other’s choice of QAS, processors compete a la Cournot. Production and QAS technologies are known by both firms. Consumers buy the product at the end of the first period and update their beliefs about the quality of the industry’s output. In the periods to follow, firms keep competing in output, and consumers keep updating their beliefs, until a failure occurs and is detected by consumers. In that period, confidence is lost by the entire industry forever.

We work backwards, first finding the equilibrium level of output after technologies have been chosen, and then solving the first-stage problem applying the second-period equilibrium rules. The second-stage problem is a standard Cournot game. Following the same argument used to construct the monopolist’s objective function in the previous section, we find that firm $i$’s expected profit is $E(\Pi^{i,1}(y,s)) = \frac{\pi^{i,1}(y_i,y_{-i},s_i)}{1 - \beta m_i(s_i)m_{-i}(s_{-i})}$, where $\pi^{i,1}$ denotes per-period profits of player $i$ in state 1, and $y = (y_i,y_{-i})$ represents a vector of output. Because per-period profits are zero if the firm has lost its reputation, we drop the superscript that indexes the state of the world. Since both $s_i$ and $s_{-i}$ are predetermined, the problem is to
choose the level of output that maximizes per-period profits in the standard way to find the Nash equilibrium levels \( y_i^* (s_i, s_{-i}), \ i = 1, 2 \). We now write the first-stage problem for firm \( i \) as

\[
\max_{s_i \in \mathcal{S}} \left( \frac{\pi_i^i (y_i^* (s_i, s_{-i}), y_{-i}^* (s_i, s_{-i}) , s_i)}{1 - \beta m_i (s_i) m_{-i} (s_{-i})} \right) = \max_{s_i \in \mathcal{S}} \left( \frac{\pi_i^i (s_i, s_{-i})}{1 - \beta m_i (s_i) m_{-i} (s_{-i})} \right),
\]

which has the first-order condition

\[
(6) \quad \frac{1}{1 - \beta m_i (s_i) m_{-i} (s_{-i})} \left( \frac{\partial \pi_i^i (s_i, s_{-i})}{\partial s_i} + \frac{\pi_i^i (s_i, s_{-i}) \beta m_{-i} (s_{-i})}{1 - \beta m_i (s_i) m_{-i} (s_{-i})} \frac{\partial m_i (s_i)}{\partial s_i} \right) \leq 0 \quad s_i \geq 0 \quad i = 1, 2
\]

and the corresponding complementary slackness conditions. Note that if it is difficult to detect quality deviations (i.e., when \( \omega \to 0 \)), the model predicts that processors will find it optimal not to invest in quality assurance (the solution will tend to the corner \( (s_i^* = 0, s_{-i}^* = 0) \)). The same result holds if \( \beta = 0 \) or when the probability that the processor’s rival is caught is close to one.

The second-order sufficient conditions are

\[
A = \begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix} \quad \frac{1}{(1 - \beta m_i m_{-i})^2} = \begin{bmatrix} \frac{\partial^2 E \Pi_1}{\partial s_i^2} & \frac{\partial^2 E \Pi_1}{\partial s_i \partial s_{-i}} \\ \frac{\partial^2 E \Pi_2}{\partial s_{-i} \partial s_i} & \frac{\partial^2 E \Pi_2}{\partial s_{-i}^2} \end{bmatrix}
\]

\[
a_{i,i} = \frac{\partial^2 \pi_i}{\partial s_i^2} (1 - \beta m_i m_{-i}) + \frac{\partial \pi_i}{\partial s_{-i}} \frac{\partial m_i}{\partial s_i} \beta m_{-i} \leq 0
\]

\[
a_{i,-i} = \frac{\partial^2 \pi_i}{\partial s_i \partial s_{-i}} (1 - \beta m_i m_{-i}) + \frac{\partial \pi_i}{\partial s_{-i}} \beta m_i \frac{\partial m_i}{\partial s_i} + \frac{\pi_i}{1 - \beta m_i m_{-i}} \frac{\partial m_i}{\partial s_i} \frac{\partial m_i}{\partial s_{-i}} \quad \text{and} \quad \Omega = a_{i,i} a_{-i,-i} - a_{i,-i} a_{-i,i} > 0, \quad \text{for} \quad i = 1, 2.
\]
The system of equations (6) implicitly defines the processor’s best response functions, $b_i(s_{-i})$, $i = 1, 2$. A vector of investments in safety $(s^*_1, s^*_2)$ is a Nash equilibrium for this model if and only if $s^*_i = b_i(s^*_{-i})$ for $i = 1, 2$. If an equilibrium for this stage exists, we can then compute the output quantities and prices using the equilibrium rules for the Cournot game that will be played at the second stage.

We now investigate the nature of the competition by studying a key property, the slope of the best-response functions. To that end, we substitute the best-response function of firm $i$ into its first-order condition and differentiate it (at an interior solution) with respect to the choice of its rival. After rearranging and omitting arguments for brevity, we get

$$\frac{\partial b_i(s_{-i})}{\partial s_{-i}} = -\frac{\frac{\partial^2 \pi^i}{\partial s_i \partial s_{-i}} + \frac{\partial m_i}{\partial s_i} \left(1 - \beta m_i m_{-i}\right) \left(\frac{\partial \pi^i}{\partial s_{-i}} m_{-i} + \pi_i \frac{\partial m_{-i}}{\partial s_{-i}} \left(1 - \beta m_i m_{-i}\right)\right)}{a_{i,i} \left(1 - \beta m_i m_{-i}\right)}.$$

The denominator of this expression is negative by the sufficient conditions for a maximum. Thus, the sign of the slope of the best-response function is the same as the sign of the numerator (see Dixit). The first term is negative, since as one firm increases investments in safety this reduces the benefits derived from increases in its rivals’ expenditures on QASs. By a similar argument, and the assumption that more stringent systems yield better products, the last two terms are positive. Overall, the sign is ambiguous without further structure, but close inspection of equation (7) reveals some insights into its sign.

First, if the probability of success for both processors is high, and the future is valued, the best-response function will likely slope upwards. Second, if the cross-partial terms are close to zero, for example, if the products are not close substitutes, the slope will be positive. But neither condition is necessary to obtain upward-sloping reaction functions. Downward-
sloping reaction functions may arise only when the cross effects are strong compared with the profitability of the industry.

Hence, the structure of the problem makes several types of interaction plausible. Free riding would be represented by downward-sloping best-response functions. Following the language of Bulow, Geanakoplos, and Klemperer, QASs are “strategic substitutes.” If one firm invests heavily in a QAS, it will be a weak competitor in the second stage. It then may be worthwhile for the other processor to free ride on the consumer trust obtained by the other firm’s investments. Though mathematically possible, it is hard to rationalize a situation in which a processor would choose stringent levels of assurance, believing that its rival will put a lax system in place. If the processor’s rival invests little in quality, the firm will face a tough Cournot competitor for the few periods the market is expected to last, which acts as a double incentive to implement less stringent QASs.

A more plausible and intuitively appealing scenario is that of QASs being “strategic complements.” That is, reaction functions have an upward slope. When a processor’s rival puts a lax QAS in place, the firm will find little incentive to invest in quality assurance. As the processor’s rival increases the stringency of the QAS, the firm will have two types of incentives to raise its own investments. First, the market will last longer. Second, quantity competition will be milder, which increases per period margins and makes more stringent systems worthwhile.

Now when the capacity of consumers to perceive quality increases, the direction of the response in the levels of assurance is ambiguous. Following Dixit, we totally differentiate the system of first-order conditions to get
where \( a_{i,i} \) and \( a_{i,-i} \) for \( i=1,2 \) are as previously defined. The system can in principle be solved to obtain

\[
\begin{bmatrix}
\frac{ds_1^*}{d\omega} \\
\frac{ds_2^*}{d\omega}
\end{bmatrix} = \frac{1}{\Omega} \begin{bmatrix}
-a_{2,2} & a_{1,2} \\
-a_{1,1} & a_{1,1}
\end{bmatrix} \begin{bmatrix}
\frac{\partial^2 E \Pi^1}{\partial s_1^2} & \frac{\partial^2 E \Pi^1}{\partial s_1 \partial s_2} \\
\frac{\partial^2 E \Pi^2}{\partial s_1 \partial s_2} & \frac{\partial^2 E \Pi^2}{\partial s_2^2}
\end{bmatrix} \begin{bmatrix}
ds_1 \\
ds_2
\end{bmatrix} = -\begin{bmatrix}
\frac{\partial^2 E \Pi^1}{\partial s_1 \partial s_2} \\
\frac{\partial^2 E \Pi^2}{\partial s_2^2}
\end{bmatrix} \frac{d\omega}{d\omega}
\]

where again the stability conditions are \( \Omega = (a_{1,1}a_{2,2} - a_{1,2}a_{2,1}) > 0 \) and \( a_{i,i} < 0 \) for \( i=1,2 \).

The decisions of both processors are equal, so it suffices to analyze the responses of processor 1. From equation (8), the change in the optimal stringency of assurance for firm 1 is given by

\[
\frac{ds_1^*}{d\omega} = \frac{1}{\Omega} \left( -a_{2,2} \frac{\partial^2 E \Pi^1}{\partial s_1^2} + a_{1,2} \frac{\partial^2 E \Pi^1}{\partial s_1 \partial s_2} \right).
\]

The sign of expression (9) is ambiguous in general. However, we can further analyze some cases. From the stability conditions, we know that \( \Omega \) is positive and \( a_{2,2} \) is negative. Nothing more can be ascertained without imposing further structure. If symmetry is assumed (as it is here) the cross-partial terms are equal and hence we need to sign only one of them.

We study the case in which QASs are strategic complements. From the previous analysis we know that this case arises when \( a_{1,2} > 0 \). Using the stability conditions, we immediately see that \( \text{sgn} \left( \frac{ds_1^*}{d\omega} \right) = \text{sgn} \left( \frac{\partial^2 E \Pi}{\partial s_1 \partial s_2} \right) \). Cross-partial differentiation of the objective
function and some tedious rearranging yields $\text{sgn} \left( \frac{ds_1^*}{d\omega} \right) = \text{sgn} \left( m_2(s_2)(2 - \beta m_2(s_2)) - 1 \right)$. The expression within parentheses is a quadratic equation with negative coefficient $-\beta \in (-1,0)$. It is straightforward to check that the expression is positive if and only if $m_2 \in [m^0,1]$, where $m^0$ is the only root (in the unit interval) of the quadratic equation. This shows that if assurance systems are complementary, processors will increase their investments in quality only if their rival’s probability of success is above a certain threshold given by $m^0$. Note also that this condition is conducive to the property of strategic complementarity. For small values of $m_2$, processor 1 will find it optimal to reduce investments in quality as consumers become more knowledgeable.

In summary, when reputations are public, we show that QASs can potentially be strategic complements or substitutes. However, we argue that the former is more plausible. For this case, we showed that firms will implement more stringent QASs as the ability of consumers to perceive quality increases (provided that their rivals’ probability of maintaining consumers’ approval is high enough).

**Duopoly When Reputation Is Private**

We now turn to the case in which reputation is a private good. The structure of the problem is similar to the previous analysis in that firms have to choose the optimal level of investment in QASs that affects quality stochastically and then compete in quantities for the random number of periods in which they have consumers’ approval. The key difference with the previous section is that in this scenario when a processor fails, the rival benefits because it increases its market share. We still maintain the assumption that firms will be trusted until
proven wrong. Unlike the situation in which reputation is a public good, when a processor’s reputation is lost, the other processor fills the market and behaves as a monopolist until its product fails and is discovered by consumers.

To accommodate this sequencing we need to expand the number of states of the world from 2 to 4. In state 1, both processors are trusted. State 2 arises if processor 2 loses its reputation. In state 3, processor 2 acts as a monopolist. The market disappears when state 4 is reached (both processors lose reputation). Reputations can then be modeled as following a stochastic process, where the probabilities of reaching the different states of the world are affected by the choices made by the market participants. Since only the immediate past determines the state of the world in the following period, the stochastic process exhibits the Markov property. Therefore, it is natural to use the concept of a Markov process to model the dynamics of this market.

A Markov process is defined by the possible states of the system, the transition matrix, and a vector that records the initial state (Ljungqvist and Sargent). The states of the world are described in the previous paragraph, and the assumption that processors will be trusted until proven wrong is equivalent to assuming that the system starts at the first state with probability one (and hence the remaining three entries of the initial state vector are assigned zero probability). The transition matrix $M$ is as follows:

$$
M = \begin{pmatrix}
\frac{m_1m_2}{m_1 (1-m_2)} & m_2 (1-m_1) & (1-m_1)(1-m_2) \\
0 & m_1 & 0 \\
0 & 0 & m_2 \\
0 & 0 & 1
\end{pmatrix}.
$$

The probabilities of success $m_1(s_1)$ and $m_2(s_2)$ are defined as before. Then, entry $M_{i,j}$ denotes the probability that the system will be in state $j$ in the next period, given that the
current state is $i$. Note that any state can follow state 1, but neither state 1 nor state 3 can be reached after state 2. In the jargon of Markov chains, state 4 is absorbing, that is, once it is reached, the system stays in it forever, or, alternatively, it will go to other states with probability zero. Note that as required in a Markov matrix, all the entries are nonnegative and \[ \sum_{j=1}^{4} M_{i,j} = 1 \text{ for } i = 1, \ldots, 4. \]

With this in place and noting that processor 1 makes zero profits in states 3 and 4, we can write the firm’s first-stage problem as

\[
(10) \quad \max_{s_1 \in S} \mathbb{E} \prod_{i=1}^{k} \left( \pi_1^i \left( s_1, s_2 \right) \sum_{j=0}^{\infty} \beta^j e_1 M^j e_1^T + \pi_1^i \left( s_1, s_2 \right) \sum_{j=1}^{\infty} \beta^j e_2 M^j e_2^T \right)
\]

where $k = 1, 2, \ldots, \pi_1^k$, denotes per period profits for processor 1 when there are $k$ market participants; $e_k$ is a 4x1 vector that has zeros everywhere except for a 1 in the $k$th position; and $T$ is the transpose operator.

Equation (10) specifies that processors maximize the profits in the duopoly and monopoly situation, weighted by the likelihood of the different scenarios, which they can affect by the stringency of the QAS they choose. The first-order condition for this problem is

\[
(11) \quad \frac{\partial \pi_1}{\partial s_1} \sum_{i=0}^{\infty} \beta^i e_1 M^i e_1^T + \frac{\partial \pi_1}{\partial s_2} \sum_{i=0}^{\infty} \beta^i e_2 M^i e_2^T + \frac{\partial \pi_1}{\partial s_1} \sum_{j=1}^{\infty} \beta^j e_2 M^j e_2^T + \frac{\partial \pi_1}{\partial s_2} \sum_{j=1}^{\infty} \beta^j e_1 M^j e_1^T \leq 0,
\]

with equality if $s_i > 0$. Of course, a symmetric equation exists for the other processor.

Equation (11) has the usual interpretation. Processors will equate marginal benefits (given by the second and fourth terms in (11)) against marginal costs (given by the first and third terms in (11)) in their choice of QAS. The new terms are associated with the introduction of the possibility of being a monopolist if firm 2 loses its reputation first. An equilibrium for
this model is again a pair of QASs \((s_1, s_2)\) such that \(s_i^* = b_i(s_{-i}^*)\), \(i = 1, 2\), where \(b_i(\cdot)\) represents the best-response function of player \(i\). Given the complexity of the model, further analysis requires a bit more specification and the use of numerical simulations. We conduct such simulations next using linear demands and constant marginal costs to give additional insight into how the nature of competition and reputations are likely to influence the choice of QASs.

**Numerical Simulations**

Assume that consumers’ valuation of the homogenous final product in each period can be represented by a utility function of the form

\[
U(y_1, y_2) = a(y_1 I_1 + y_2 I_2) - \frac{b}{2}(y_1^2 + y_2^2 + 2y_1 y_2),
\]

where \(a\) and \(b\) are parameters, and \((I_1, I_2)\) are indicator functions denoting the state of the world. The specific form of the indicators will depend on the structure of the market and the nature of the reputations. Consumers choose quantities to maximize \(U(y_1, y_2) - p_1 y_1 - p_2 y_2\).

The link between the stringency of the QAS and the probability of obtaining a product of good quality is given by the monotonic and concave function \(\lambda(s) = s/(s + 1)\). Therefore, we assume that if investments in quality are zero, processors will be obtaining a high-quality product with probability zero. This sort of link is more likely to occur when quality is given by cost-increasing practices. When the variability is natural and processors just need to select what input to buy, it may be more reasonable to employ a link that assigns equal probability to obtaining a good product and incurring a statistical type II error when investments in quality are zero.\(^{11}\) We begin by specifying the objective functions for each situation studied.
**Monopolist processor**

The monopoly situation can be obtained by setting $I_2 = 0$. $I_1 = 1$ if the processor is trusted and zero otherwise. This yields an inverse demand function for good 1 given by $p_1(y_1) = aI_1 - by_1$, and hence per-period profits (after substitution of the equilibrium output levels) are

$$\pi_1^* = \frac{(a - s_1)^2}{4b}.$$ 

Plugging this back into the monopolist problem, we obtain

$$\max_{s_i \in S} \frac{(a - s_1)^2}{4b(1 - \beta(1 - \omega + \omega\lambda(s_1)))}.$$ 

**Duopoly situation when reputation is a public good**

This case is obtained by letting $I_1$ and $I_2$ equal one if no failure has been detected and zero otherwise. The inverse demand for processor $i$ is given by $p_i(y_i, y_{-i}) = aI_i - b(y_i + y_{-i})$. Then, equilibrium quantities for the second stage and per-period profits are easily found to be

$$y_i^* = \frac{(a - 2s_i + s_{-i})}{3b}$$

and

$$\pi_2^* (s_i, s_{-i}) = \frac{(a - 2s_i + s_{-i})^2}{9b}, \quad i = 1, 2.$$ 

Plugging this into the first-stage problem, we find that processor’s 1 objective is given by

$$\max_{s_i \in S} \frac{(a - 2s_1 + s_2)^2}{9b(1 - \beta(1 - \omega + \omega\lambda(s_1))(1 - \omega + \omega\lambda(s_2)))}.$$ 

**Duopoly situation when reputation is a private good**

For this case, $I_1$ equals one in states 1 and 2 and zero otherwise. Also, $I_2$ equals one in states 1 and 3, and zero otherwise. As long as the stochastic process stays in the initial state (both processors have good reputations), per-period profits are as in the duopoly situation previously presented. When the system reaches states 2 or 3, the monopolist’s per-period profit (also previously presented) becomes relevant. Here, the problem of processor 1 is
Results and Discussion

In this section, we examine the solutions to the problems posed and study how changes in the economic environment affect the equilibrium levels of stringency of the QAS implemented, associated expected profits for firms, and utility for consumers. Specifically, we study how the structure of the industry, nature of reputations, and ability of consumers to detect quality deviations affect the equilibrium outcomes.

Figure 1 presents the best-response functions for the two duopoly scenarios. Under the specific parameterization and if reputation is public, reaction curves slope upwards. Processors find few incentives to invest in QASs when the market is not going to last long (i.e., when their rivals invest little in quality) and when they expect to face a tough (low-cost) Cournot competitor in the following periods. However, as previously discussed, processors find it worthwhile to put more stringent systems in place if they anticipate their rivals will do the same thing. For the market to last more than a few periods, and compensate the investments in quality assurance, both players need to invest in high levels of $s$ in this scenario.

When reputations are private goods, reaction curves have a negative slope. That is, when reputations are private goods, as processors anticipate their rivals will put a lax system in place, it is worthwhile to invest in a more stringent QAS. The driving force is that processors find it beneficial to give up some of the duopoly profits while increasing the likelihood of
outlasting the rival and capturing the entire market. But when a processor anticipates its rival will invest heavily in quality assurance (and will become harder to outlast), the best the firm can do is to reduce its own expenses and try to capture a higher per-period duopoly profit.

Comparing the stringency of assurance across the different types of reputation, Figure 1 reveals that, as expected, the equilibrium investments in QASs are lower when reputation is a public good. When firms do not capture the full returns from their QASs, they will under-invest in quality and try to free ride on other firms’ investments. Branding for example, could be seen as an effort to convert reputations into a private good and hence capture a higher share of the returns to investments in QASs. Economic theory predicts that if branding allows processors to capture the returns to their investments, it will result in higher levels of quality assurance. Also, though costlier QASs result in lower per-period outputs, the expected total production (and hence consumption) is higher when reputation is a private good. This is driven by the fact that more stringent controls result in a longer duopoly stage (in addition to any output produced during the later monopoly periods).

Results (not shown, but see footnote 13) indicate that monopoly total output levels are higher than under duopoly when reputations are public in nature but not when reputations are
private. This illustrates that both firms and consumers can benefit when firms have large incentives to protect their reputations.

Figure 2. Equilibrium responses of the stringency of controls to changes in quality observability \((a = 50, \ b = 1, \ \beta = 0.9)\)

Figure 2 shows that as consumers can detect quality deviations more easily, the equilibrium levels of stringency increase for all market structures and natures of reputation considered.\(^14\) As discussed before, when quality is a credence attribute, firms will find it optimal not to invest in QASs. Firms do not have incentives to avoid punishments that have zero probability of occurrence. The level of stringency is always greater when reputation is a private good. Furthermore, it reveals that the monopolist will provide the highest quality (in expectations). It is straightforward to show analytically, for the proposed function specifications, that monopolist will employ more stringent QASs and obtain higher profits than duopolist with public reputations. Monopolists are the ones that will lose the most if a quality deviation occurs and is detected.

Figure 3 shows that expected profits decline as consumers can more readily discern quality. The reason is the increased cost of the optimal QAS and the increased probability that a firm will lose its reputation. This (envelope) result is not sensitive to the particular
parameterization. Figure 3 also suggests that firms have an incentive to privatize their reputations. For example, many food retailers sell products labeled with their own brand as high quality products (Noelke and Caswell). The drawback of this strategy is that when a problem occurs, the firms are identified with these brands and are clear targets in the marketplace (Hennessy, Roosen, and Jensen).

**Figure 3. Equilibrium responses of expected profits to changes in quality observability**

(a = 50, b = 1, β = 0.9)

It is not possible to state which situation yields higher welfare in general. Processors would prefer, as expected, to be the only market participants and for reputations to be private rather than public goods. It is also clear from Figure 4 that consumers prefer the scenario of a duopoly with private reputations to that of a monopoly with collective reputations. However, consumers’ choice between monopoly and duopoly with public reputations depends on the level of quality observability, and the relevant discount factor. In short, we can only say that the duopoly situation with collective reputations is Pareto dominated by the other duopoly scenario considered. This discussion provides support for some of the policies proposed by Hennessy, Roosen, and Jensen to overcome some causes of systemic risk in the food industry. The authors advocate for strategies such as improving traceability, testing, mandated labeling,
and interpreting policies on mergers more leniently for the food industry. Our results suggest that market concentration and/or identification of firms producing a given unit of output may result in higher overall welfare.

![Figure 4. Equilibrium expected utility for the three scenarios considered for different levels of quality observability (\(a = 50\), \(b = 1\), \(\beta = 0.9\))](image)

**Conclusions**

There exists a great disparity in existing QASs concerning the degree of stringency (and associated costs) of the systems employed. We provide a rationale for those differences based on market structure, the nature of the reputation mechanisms, and the size of the markets or strength in demand for value-added products and whether the sought-after attributes are credence, experience, or a mixture of both. We argue that QASs can be seen as efforts made by firms to position themselves strategically in a marketplace where consumers can differentiate between firms that deliver quality goods and those that deliver substandard quality.

Three models are developed to accommodate the different scenarios (monopoly, duopoly with collective reputations, and duopoly with private reputations), and predictions are obtained through comparative statics and numerical simulations. The later are performed under the widely used assumptions of linear demands and constant marginal costs.
Our results suggest that monopolists will invest more heavily in quality assurance than will duopolists. In addition, being able to capitalize on the full returns (no free riding or externalities) of investments in quality provides further incentives to employ more stringent QASs. That is, under the collective reputations scenario, duopolists will reduce their expected quality. Also, as the ability of consumers to detect the actual quality of the good increases, it is likely that the stringency of the quality controls will increase. Perhaps not surprisingly, our numerical simulations show that the size of the market (and hence the potential premiums) has a positive impact on the level of investments in quality.

In terms of welfare, we can only say that the duopoly with private reputations Pareto dominates the collective reputation scenario. However, it is less straightforward to compare the results of the monopoly situation with that of the other scenarios. Processors of course prefer the monopoly over any of the duopoly situations. Consumers prefer the duopoly with private reputations over the other two scenarios considered. However, under some conditions (when the level of quality observability is high, or if consumers value the future highly), consumers will prefer the monopoly over the duopoly with public reputations. Hence, our model suggests that it is not clear that market concentration hurts consumers when there is a trade-off between quantity and quality, and the latter is imperfectly observable. Additionally, if there is no way that the producer of a given unit of output can be identified among several firms (if it is not possible to develop private reputations), overall welfare may be increased by promoting the existence of a monopoly (under the conditions just noted).
Footnotes

1 There is a large body of literature showing that when certification is imperfect, some producers of low quality will apply for and obtain certification. See De and Nabar, and Mason and Sterbenz.

2 Hennessy, and Chalfant et al. showed that imperfect testing and grading lead to under-investment in quality-enhancing techniques by farmers. This is because producers of low quality impose an externality on producers of high quality.

3 This assumption is consistent with a large body of work in the marketing and psychology literature (see for example Oliver, Rust and Oliver, Kahneman and Tversky), where negative events generate stronger and more rapid reactions than positive events.

4 Note that \( m(s, \omega) = \lambda(s) + (1 - \lambda(s))(1 - \omega) = (1 - \omega) + \omega \lambda(s) \) is the convex combination between the true probability of having a product of high quality and one. We see that as \( \omega \to 0 \), \( m(s, \omega) \to 1 \), and the processors are expected to stay in state 1 for a large number of periods, even if they are offering a product that does not meet the promised standards.

5 Recall that \( \frac{\partial m(s, \omega)}{\partial s} = \omega \frac{\partial \lambda(s)}{\partial s} \).

6 Note that in Darby and Karni’s paper, supplying firms knew the actual quality of the product (repair services) they were offering.

7 To see this, recall that \( m(s_i) = 1 - \omega + \omega \lambda(s_i) \), and \( \lim_{\omega \to 0} \frac{\partial m_i(s_i)}{\partial s_i} = \lim_{\omega \to 0} \omega \frac{\partial \lambda(s_i)}{\partial s_i} = 0. \)

8 By increasing its first-period investments in QASs, a firm is raising its own costs for the second-stage competition, which benefits its rivals. However, these benefits are lower if the rivals also increase their costs.
9 For strategic substitutes, the optimal adjustment will depend not only on the sign of the cross partial studied below, but also on the relative magnitude of \( a_{i, 1} \) and \( a_{i, 2} \). To see this, impose symmetry and rewrite 
\[
\frac{ds_i^*}{d\omega} = \frac{1}{\Omega} \left( -a_{2,2} + a_{i,2} \right) \frac{\partial^2 E \Pi^1}{\partial s_i \partial \omega}.
\]

10 Note that the case where reputations is a public good, is also a Markov process, but with only two possible states (corresponding to the first and fourth states of the current setting).

11 An example would be 
\[
\lambda(s) = \frac{e^s}{e^s + 1}.
\]

12 However, as Hennessy, Roosen, and Jensen note, branding also provides a target to consumers in the marketplace when a problem occurs (more on this to follow).

13 A figure illustrating this point is available from the authors upon request.

14 Though not presented, as the demand for high quality products, (parameterized by \( a \)) rises, the stringency of the QAS under all scenarios increases.

15 A necessary condition for consumers to weakly prefer the monopoly situation over the duopoly with public reputations is 
\[
\beta \left( 16m_{M} - 9m_{D} \right) \geq 7,
\]
where \( m_j, \ j = M, D \) represents the equilibrium probabilities that reputations are maintained in two successive periods by the monopolist and a duopolist respectively. A sufficient (not necessary) condition, is given by 
\[
\beta m_{D} (16 - 9m_{D}) \geq 7,
\]
which is much harder to satisfy.

16 Note that if consumers value the future more than producers, or beta is close to 1 (higher than 0.98), the expected utility from the monopoly scenario exceeds that of the duopoly with public reputations for all \( \omega \). In those cases, the monopoly scenario would also Pareto dominate the duopoly with public reputations.
References


http://www.kellogg.nwu.edu/academic/deptprog/meds_dep/decent/%5Crob.pdf.

