

**ECONOMIC DEVELOPMENT CENTER**



**LABOR SPECIALIZATION AND  
ENDOGENOUS GROWTH**

**Hamid Mohtadi  
Sunwoong Kim**

**ECONOMIC DEVELOPMENT CENTER  
Department of Economics, Minneapolis  
Department of Agricultural and Applied Economics, St. Paul**

**UNIVERSITY OF MINNESOTA**

## Labor Specialization and Endogenous Growth

Hamid Mohtadi and Sunwoong Kim\*

\* Invited paper at the American Economic Association, January, 1992.  
Forthcoming: American Economic Review May (1992). The authors are, respectively, at the Department of Applied and Agricultural Economics, University of Minnesota, St. Paul, MN 55108, and Department of Economics, University of Wisconsin-Milwaukee, Milwaukee, WI 53201.  
Comments were provided by our discussant, Heling Shi.

The University of Minnesota is committed to the policy that all persons shall have equal access to its programs, facilities, and employment without regard to race, color, creed, religion, national origin, sex, age, marital status, disability, public assistance status, veteran status, or sexual orientation.

The idea that steady-state growth rates may not converge over the long run has been the driving impetus in the proliferating literature on endogenous growth, and especially that segment of this literature that focuses on the role of human capital (Lucas, 1988 and Romer, 1990). Human capital, however, is an amorphous concept and little has been said about its features and the mechanism by which it contributes to production and thus growth. In this paper we focus on *specialization* as a crucial (and somewhat neglected) aspect of human capital accumulation, and study its impact on growth. Our effort differs from Yang and Borland's (1991) model in which the division of labor evolves via learning by doing and increasing returns to scale. In their work, division of labor is taken to mean different workers producing different goods. Consequently, worker productivity increases with the level of output as workers produce more of the same good. In our paper, productivity increases with specialization directly, rather than indirectly via the types of goods produced. The model indicates that economic growth is possible even with no technological change or learning by doing, but through increased specialization.

Our view of specialization is one based Kim's (1989), distinction between intensive human capital and extensive human capital. The former is a stock of specialized knowledge and skills that improves worker productivity in a given production activity; the latter is a stock of general knowledge that renders the workers more adaptable to a variety of activities. The key point is the existence of trade-off between two types of human capital. Loosely speaking, although a specialist will be more productive (thus command higher wages) than a generalist in a limited range of tasks that require specialist's skill,

the chance of such a good-matching job may be smaller for the specialist. Naturally, increased specialization means more of the intensive and less of the extensive human capital.

We assume that production, consumption, and labor market clearing takes no time whereas investment in human capital is both costly and time-consuming. As a result, our analysis consists of two parts; in the first part the firms and the labor market operate, assuming that households are endowed with a *given* level of human capital (see Kim 1989 for further detail). The contemporaneous link between human capital, specialization, and output is derived in this section. In the second part households maximize a utility function subject to an *intertemporal* choice between investment on intensive and extensive human capital, *versus* present consumption. This part therefore resembles the usual household dynamic maximization problem of choosing between consumption and investment in physical capital, but focuses on investment in human capital instead.

While the model is one of endogenous growth variety we abstract from the external spillover effects of human capital and yet are still able to generate non-convergence of growth rates in the long run. These turn out to depend on the "educational technology" that underlies the cost of acquiring intensive and extensive human capital, and on the growth of population. It is easy to extent this model to incorporate external spillover effects of specialization as well.

## I. Contemporaneous Labor Market Equilibrium

Consider a closed economy with a continuum of workers-cum-

consumers of aggregate size  $N(t)$  at given time  $t$ . Labor is assumed the only factor, producing a homogeneous output that is sold in a competitive market. The output price is normalized to one. Workers are indexed, by skill characteristic, on the circumference of a circle of unit length with uniform density. Because the circle has unit length, the density is also  $N(t)$ . Although workers are allowed to choose their optimal accumulation of intensive and extensive human capital,  $b(t)$  and  $G(t)$ , over time, their skill characteristics are assumed to be *exogenously given*. Dealing with a contemporaneous economy in this section, we shall drop the time subscript for now.

There are multiple of production technologies available to produce the homogeneous output. Each firm adopts a particular technology with specific skill requirements. If the worker's skill characteristic differs from the firm's job requirement, costly training of the worker is needed for what the job requires. As the job requirement represents the skill characteristic of the worker who does not require any on-the-job-training, firms also can be indexed on the same unit circle as workers. Thus, the productivity of a worker in a given firm depends on his skill characteristic, his intensive and extensive human capital, and the firm's job requirement. In particular, we shall assume that:

$$(1) \quad x(s) = b - s/G, \quad 0 \leq s \leq .5$$

where  $x$  is the worker's productivity (labor input in efficiency units) and  $s$  is the difference between the worker's skill characteristic and the firm's job requirement.

All technologies are assumed to require an identical minimum

efficient scale (M). Thus, a representative firm's production function can be written as:

$$(2) \quad Y = \begin{cases} 0, & \text{if } X < M \\ X - M & \text{if } X \geq M, \end{cases} \quad \text{where, } X = \int_{s \in S} (b - s/G) ds.$$

Y is output, X is the total labor input normalized to the equivalent labor unit with the firm's job requirement, and S is the set of workers who work for the firm.

The firm will hire a worker if his marginal value product with the firm exceeds the firm's training cost. The profit of the representative firm is:

$$(3) \quad \Pi(H) = 2N \int_0^H (b - s/G) ds - M - 2N \int_0^H w(s) ds$$

where H is the maximum acceptable skill difference of the representative firm. We limit our analysis to the zero profit symmetric bargaining equilibrium. By symmetry we mean that: 1) All workers have the same (b, G); 2) The wage functions are identical for all firms; and 3) Firms are equally spaced on the circle. Hence, the number of firms in the market (n) is equal to 1/2H, since the job requirement difference between any two neighboring firms is 2H.

The wage is assumed to be determined according to an axiomatic bargaining solution between the worker and the firm. More specifically, each person engaged in the bargaining knows exactly what his gain will be from having the employment contract, and the bargaining outcome (i.e., wage) will occur at the mid-point, where the

worker's surplus of having the employment contract over his second best alternative (threat point) equals the firm's marginal profit from having the worker. We assume that the worker's threat point is the highest possible wage in the negotiation with the other firm.

To determine the wage function  $w(s)$ , consider a worker whose skill characteristic differs from the job requirement of the representative firm by  $s$  and therefore from the job requirement of the neighboring firm by  $2H - s$ . Given this wage determination rule, the equilibrium wage function of the competition case will be *independent* of  $s$ :

$$(4) \quad w(s) = \frac{(b - s/G) + (b - (2H-s)/G)}{2} = b - H/G,$$

In equilibrium, all firms earn zero profits, and the zero profit condition determines the equilibrium number of firms given the characteristic of the labor pool ( $b, G$ ). Substituting the wage function into the profit function and equating it to zero, we obtain:

$$(5) \quad H = (GM/N)^{1/2},$$

$$(6) \quad n = 1/2H = (1/2) (N/GM)^{1/2},$$

$$(7) \quad w = b - (M/GN)^{1/2}.$$

## II. Dynamic Equilibrium

Given the characteristics of the labor pool ( $b(t), G(t)$ ) at any time  $t$ , the wage bargaining rule and the zero profit condition determine the labor market equilibrium, summarized by  $w(s, t)$ , and  $H(t)$ . Here, the worker-consumer is assumed to decide on the *time path* of the

stock of intensive and extensive human capital,  $(b(t), G(t))$ , knowing that  $w(s, t)$ , and  $H(t)$  will prevail. Although  $H$  is a function of  $(b(t), G(t))$  in the aggregate, each worker views  $H$  as fixed. (In the case of the central planner problem,  $H$  will be allowed to vary). Further, since the equilibrium wage is independent of  $s$ ,  $s$  will be deleted from consideration. Assuming a discount rate of  $\rho$ , the worker's maximization problem is:

$$(8) \quad \text{Maximize } \int_0^{\infty} u(c(t))e^{-\rho t} dt \quad \text{subject to} \\ b_t, G_t$$

$$(9) \quad c = b - H/G - g(\dot{b}, \dot{G}),$$

where,  $g(\dot{b}, \dot{G})$  is the cost of the incremental accumulation of intensive and extensive human capital.<sup>1</sup> To simplify the analysis we assume that the cost function is separable in  $\dot{b}$  and  $\dot{G}$ . Using the utility function,  $u = [c^{1-\sigma} - 1]/(1-\sigma)$ , calculus of variations yields:

$$(10) \quad 1 - \rho g_{\dot{b}} - \sigma(\dot{c}/c)g_{\dot{b}} + g_{\dot{b}\dot{b}} \ddot{b} = 0.$$

$$(11) \quad H/G^2 - \rho g_{\dot{G}} - \sigma(\dot{c}/c)g_{\dot{G}} + g_{\dot{G}\dot{G}} \ddot{G} = 0$$

These equations describe the dynamics of the system. Let the separable cost function be  $g(\dot{b}, \dot{G}) = m\dot{b}^{\beta} + n\dot{G}^{\gamma}$  ( $m, n, \beta, \gamma > 0$ ). Further,  $g$  must be convex in  $\dot{b}$  ( $\beta > 1$ ), to rule out investing all of one's income on  $b$  (given the linearity of wage in  $b$ ) and thus zero consumption (note that

<sup>1</sup>Note that equilibrium implies  $nY = N[c + g(\dot{b}, \dot{G})]$ . Given the budget constraint, above, it is easy to show that this condition is satisfied.



$u'(0) = \infty$ ). However, convexity of  $g$  in both variables need not hold to ensure optimality (concavity of  $u$  suffices). Using this cost function, and eliminating  $\dot{c}/c$  between (10) and (11), we find:

$$(12) \quad \frac{1}{\beta m} \dot{b}^{1-\beta} + (\beta-1) \frac{\ddot{b}}{\dot{b}} = \frac{H}{\gamma n G^2} \dot{G}^{1-\gamma} + (\gamma-1) \frac{\ddot{G}}{\dot{G}}$$

Denote the steady-state growth of the two types of human capital by  $\dot{b}/b \equiv \theta$  and  $\dot{G}/G \equiv \phi$ ; not necessarily equal. We shall determine  $\theta$  and  $\phi$ , below. Using the fact that  $\ddot{b}/\dot{b} = \dot{\theta}/\theta$  and  $\ddot{G}/\dot{G} = \dot{\phi}/\phi$  in steady-state and substituting for  $H = (GM/N)^{1/2}$ , equation (12) becomes:

$$(13) \quad \frac{1}{\beta m} (\theta b(t))^{1-\beta} + (\beta-1)\theta = \frac{M^{1/2}}{\gamma n} \phi^{1-\gamma} G(t)^{-(1/2+\gamma)} N(t)^{-1/2} + (\gamma-1)\phi$$

This equation is valid for all  $t$ . Suppose that the population grows at an exogenously given constant rate,  $\dot{N}/N = \nu > 0$ . Differentiating (13) first to eliminate the constant terms, then using  $\dot{b} = \theta b$ ,  $\dot{G} = \phi G$ , and  $\dot{N} = \nu N$ , and finally log-differentiation of the result gives:

$$(14) \quad (\beta-1)\theta = \nu/2 + (1/2+\gamma)\phi$$

Equation (14) is one key equation of the model, relating steady-state growth rates of  $b$ ,  $G$  and  $N$ . It turns out, however, that while  $b$ ,  $G$  and  $N$  can grow at steady state rate at all  $t$ , the per capita rate of growth of consumption,  $\lambda \equiv \dot{c}/c$ , involves transitional dynamics for finite  $t$ , approaching a steady-state rate of growth *only* as  $t \rightarrow \infty$ . To see this,

first note that manipulating (10) gives:

$$(15) \quad \lambda = \frac{1}{\sigma} \left( \frac{1}{\beta m} (\theta b_0)^{1-\beta} e^{-(\beta-1)\theta t} + (\beta-1)\theta - \rho \right)$$

Similarly, from (11) one gets:

$$(16) \quad \lambda = \frac{1}{\sigma} \left( \frac{M^{1/2}}{\gamma n} (\phi^{1-\gamma} G_0^{-1/2-\gamma} N_0^{-1/2}) e^{-[(1/2+\gamma)\phi+\nu/2]t} + (\gamma-1)\phi - \rho \right)$$

where  $(.)_0$ 's represent initial values. Each of the equations (15) and (16) consists of a time-dependent and a time-independent component for  $\lambda$ . Since both equations describe the same  $\lambda$ , the long-run steady state component of  $\lambda$  must be identical in both, i.e.:

$$(17) \quad (\gamma-1)\phi = (\beta-1)\theta > 0$$

Solving (14) and (17) together for  $\theta$  and  $\phi$  we get:

$$(18) \quad \phi = -\nu/3$$

$$(19) \quad \theta = \frac{1-\gamma}{3(\beta-1)} \nu$$

Thus, extensive human capital *declines* over time. From this, we see that positive long-run growth implies  $\gamma < 1$  (eq. 16). By substituting  $\phi = -\nu/3$  into the time-dependent exponent of (15) the condition  $\gamma < 1$  turns out to be sufficient for the transitional component of  $\lambda$  to vanish in the long run. Further,  $\beta > 1$  means that  $\theta > 0$  (eq. 19).

This confirms the positive long run growth, now via equation (15). In short, the economy experiences positive sustained long-run growth, intensive human capital rises steadily and extensive human capital falls steadily, i.e., workers become more specialized over time.

The above range of parameters also implies that  $\lambda$  approaches steady-state from *above* since from equations (15) or (16) we find that  $\partial\lambda/\partial t < 0$ . In order to find the asymptotic features of  $\lambda$ , substitute from (18) into the long-run steady-state component of  $\lambda$  in (16):

$$(20) \quad \lambda|_{t \rightarrow \infty} = (\frac{1-\gamma}{3}\nu - \rho)/\sigma$$

As expected, the asymptotic value of  $\lambda$  falls with  $\gamma$ ,  $\sigma$  and  $\rho$ . To understand why asymptotic growth *increases* with population growth, first note, from the static analysis, that increasing  $N$  increases firm profits and thus the number of firms,  $n$ . With more firms the value of  $H$  (i.e. the "skills-market" segment per firm) will drop, as there will be more diverse job requirements available. From the dynamic analysis we see this drop by log-differentiating equation (5):

$$(21) \quad h \equiv \dot{H}/H = \frac{\phi - \nu}{2} = -\frac{2}{3}\nu < 0$$

Loosely speaking, this raises the chance of good matching jobs, lowering the importance of  $G$  relative to  $b$ . From equations (18) and (19) we see that  $G$  declines and  $b$  rises over time. This raises worker productivity, and thus per capita growth. From equations (15) and (16) we see that long-run steady-state value of  $\lambda$  increases with the rate of growth of  $b$  and the rate of decline in  $G$ . It is important to note that

this mechanism for specialization is *internal* and stems from the workers' optimal choice in response to worker-firm wage bargaining mechanism. This differs from analyses in which specialization occurs *externally* in the *process* of production (as the market expands), because of productivity gains that accrue via learning-by-doing and increasing returns [as in Yang and Borland (1991)].

Finally, with respect to human capital *scale* effects (Romer, 1990), *versus rate* or *intensity* of accumulation effects (Lucas, 1988), our model suggests that the Romerian scale effects are only *transitional*, vanishing over the long run ( $t \rightarrow \infty$ ), while the Lucas-like rate effects remain important even in the long-run.

Thus, in terms of the "convergence vs. divergence" debate (e.g. Baumol (1986), De Long(1988)), economies with similar population growth rates, tastes and educational technologies, but *different* initial human capital, will experience *same* growth rates in the very long run, but different growth rates in the short or intermediate run; growth rates differ, even in the long run, for different parameters values.

### III. Social Optimum

As was noted in Kim (1989), the contemporaneous economy in which  $(b(t), G(t))$  are fixed is socially optimal. The question now is whether the dynamic path of the competitive equilibrium is socially optimal, once the impact of  $b$  and  $G$  on  $H$  is considered. Endogenizing  $H$  from equation (5), the new the budget constraint becomes:

$$(9') \quad c = b - (M/GN)^{1/2} - g(b, \dot{G})$$

Note the smaller incremental contribution of  $G$ , here, compared to competitive economy (equation 9), as increasing  $G$  is now associated with increasing  $H$ . Following the earlier steps, equations (14) and (15) will stay the same, but equation (16) will be modified by a multiplicative factor of  $1/2$  in the first term. However, since the long-run steady-state component of (16) remains unchanged, equations (14) and (17) must stay the same. Thus, in this world, the central planner *cannot* affect the *long-run* steady state growth of the economy, although it may affect the transitional dynamics of the economy.

#### IV. Conclusion

With no external spillover effects of specialization, the competitive growth path is also socially optimal in the long-run. Existence of such effects (e.g., via an aggregate effect of intensive human capital on output, or via learning-by-doing, where perhaps the cost of acquiring new  $b$ ,  $g(b)$ , falls in  $b$ ), may cause socially optimal paths to differ from the decentralized path. We have not addressed these issues. In any case, we believe more work needs to be devoted to understand the link between specialization, human capital and growth.

## References

- Baumol, William J. "Productivity Growth, Convergence, and Welfare: What the Long-Run Data Show," American Economic Review, December 1986, 76, 1072-1085.
- De Long, J. Bradford, "Productivity Growth, Convergence, and Welfare: Comment," American Economic Review, December 1988, 78, 1138-1154.
- Kim, Sunwoong, "Labor Specialization and the Extent of the Market," Journal of Political Economy, June 1989, 97, 692-705.
- Lucas, Robert E., Jr., "On the Mechanics of Economic Development," Journal of Monetary Economics, July 1988, 22, 3-42.
- Romer, Paul, "Endogenous Technological Change," Journal of Political Economy, October 1990, 98, S71-S102.
- Yang, Xiaokai and Borland, Jeff, "A Microeconomic Mechanism for Economic Growth," Journal of Political Economy, June 1991, 99, 460-482

## RECENT BULLETINS

- 87-9 Pitt, Mark M. and Mark R. Rosenzweig. "Estimating the Intra-family Incidence of Health: Child Illness and Gender Inequality in Indonesian Households." July.
- 87-10 Hagen, James M. and Vernon W. Ruttan. "Development Policy Under Eisenhower and Kennedy." November.
- 88-1 Roe, Terry and Erinc Yeldan. "An Open Economy Model of Political Influence and Competition Among Rent Seeking Groups." February.
- 88-2 Mahe, L. P. and C. Tavera. "Bilateral Harmonization of EC and U.S. Agricultural Policies." August.
- 88-3 Pitt, Mark M. "The Determinants of Rice Variety Choice in Indonesia." September.
- 88-4 Sartorius, Rolf H. and Vernon W. Ruttan. "The Sources of the Basic Human Needs Mandate." November.
- 88-5 Coggins, Jay S., Theodore Graham-Tomasi and Terry L. Roe. "Existence of Equilibria in Lobbying Economies," December.
- 89-1 Pitt, Mark, Mark Rosenzweig, and Md. Nazmul Hassan. "Productivity, Health and Inequality in the Intrahousehold Distribution of Food in Low-Income Countries."
- 89-2 Johnson, Martin, Terry Roe and Louis Mahe, "The GATT Negotiations and US/EC Agricultural Policies Solutions to Noncooperative Games," March.
- 89-3 Fuglie, Keith, "Vent-For-Surplus" as a Source of Agricultural Growth in Northeast Thailand, 1950-1986," March.
- 89-4 Rosenzweig, Mark and Hans Binswanger, "Wealth, Weather Risk and the Composition and Profitability: of Agricultural Investments," June.
- 89-5 McGuire, Mark F. and Vernon W. Ruttan, "Lost Directions: U.S. Foreign Assistance Policy Since New Directions," August.
- 89-6 Coggins, Jay, "On the Welfare Consequences of Political Activity," August.
- 89-7 Ramaswami, Bharat and Terry Roe, "Incompleteness in Insurance: An Analysis of the Multiplicative Case," September.
- 89-8 Rosenzweig, Mark and Kenneth Wolpin, "Credit Market Constraints, Consumption Smoothing and the Accumulation of Durable Production Assets in Low-Income Countries: Investments in Bullocks in India," September.
- 89-9 Pitt, Mark and Mark Rosenzweig, "The Selectivity of Fertility and the Determinants of Human Capital Investments: Parametric and Semi-Parametric Estimates," October.
- 89-10 Ruttan, Vernon, "What Happened to Political Development," November.
- 90-1 Falconi, Cesar and Terry Roe, "Economics of Food Safety: Risk, Information, and the Demand and Supply of Health," July.
- 90-2 Roe, Terry and Theodore Graham-Tomasi, "Competition Among Rent Seeking Groups in General Equilibrium," September.
- 91-1 Mohtadi, Hamid and Terry Roe, "Political Economy of Endogenous Growth," January.
- 91-2 Ruttan, Vernon W., "The Future of U.S. Foreign Economic Assistance," February.

