Can Spot and Contract Markets Co-Exist in Agriculture?

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CARD Working Paper 02-WP 311

August 2002

Center for Agricultural and Rural Development
Working Paper Series

Ames, Iowa 50011

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Abstract

New production technologies, consumers who are more discriminating, and the need for improved coordination are among the forces driving the move from spot markets to contracts. Some worry that this tendency will result in the disappearance of spot markets, or at least that they will become too thin to be of help for an efficient price discovery process. Other authors point to the reduction in welfare of independent producers resulting from contracting in oligopsonistic industries. While a large body of literature is available tackling the contract versus spot market decision, much less is known about the reasons that lead to procurement in both markets. This paper provides a very simple model to study how fundamental economic factors influence the contracting behavior of farmers and processors. In the model, processors contract upstream into a competitive industry (farmers). We find that participation in both markets arises as a Nash equilibrium for a range of contract prices. We use numerical methods to examine the effects of fundamental economic factors on the relative size of the spot and contract markets and the effect of contract price on the relative profitability of farmers and processors.

Keywords: contracting in agriculture, spot markets, yield risk.
CAN SPOT AND CONTRACT MARKETS CO-EXIST IN AGRICULTURE?

Introduction

A growing proportion of agricultural production is being raised and sold under contract arrangements instead of being sold in spot markets. The particular forms of contracts used in agriculture embody a wide range of mechanisms (with differing degrees of complexity) that extend in a continuum from spot markets to vertically integrated firms (North 1990). The literature provides several reasons for the increasing use of contracts. Contracts can reduce the risk that processors will have insufficient quantity or quality of inputs to process (Drabenstott 1994; Dimitri and Jeaenicke 2001; Featherstone and Sherrick 1992; Murray 1995). Barkema (1993) notes that consumers are becoming more discriminating, which, together with new production technologies, allows the process of product engineering to begin on the farm itself. Sykuta and Parcell (2001) point out that producers’ introduction of new products and services tailored to satisfy consumer demand creates the need for much stricter coordination mechanisms than what can be accomplished with traditional spot markets. The need for improved protection of property rights and the push for identity preservation are also cited as incentives to move from spot markets to _ex ante_ contracts (Dimitri and Jeaenicke 2001; Sykuta and Parcell 2001). North (1990) and Milgrom and Roberts (1992) point to the same risk sharing and coordination benefits. Hennessy (1996) shows that spot markets might result in underinvestment in quality-enhancing technology with product heterogeneity and quality measurement error. Others suggest that the certainty that a contract brings is the only way to induce risk-averse farmers to move away from producing a commodity and toward producing a higher-valued product, for example, less commodity corn used for animal feed and more white corn used for direct human consumption. Farmers enter into contracts to reduce price risk (Hueth and Ligon 1999) and to increase their financial leverage. The latter reason was the most mentioned advantage of contracting according to a survey of 145 large U.S. hog production firms conducted by Hennessy and Lawrence (1999).
The move to production under contracts has some concerned about the viability of remaining spot markets and about the degree to which farmer welfare is negatively impacted by market power of processors. Hueth and Hennessy (2002) note that some contracts observed in the pork industry condition growers’ compensation on current spot market prices. Hence, the existence of such contracts is conditioned on the existence of a spot market. A function of the spot markets would be to enhance the price discovery process (Hueth and Hennessy 2002). While a lot of research has been devoted to the spot market versus contracts question (starting with the work of Coase [1937]), less attention has been paid to the motivations that lead to the co-existence of both spot and contract markets.

In this paper, we examine how fundamental economic factors (prices, production variability, and costs) influence the relative profitability of contract and spot production for farmers and processors. Our objective is to gain insight into how these factors help determine the incentives for moving from spot to contract production. The situation we examine is fairly typical in agriculture. Processors offer farmers contracts to purchase all production on a specified number of acres at a fixed price. The processor has a target amount of production to procure. Procurement in excess of this target—for example, when per-acre yields are very high—can be sold at some salvage price. When contract supply is low, the processor can turn to the spot market to make up the difference, but only if farmers have planted for the spot market. The price in the spot market clears the ex post (after random yields are realized) processor demand with the fixed supply of the product. Thus, stochastic yields lead to stochastic spot market prices. We account for such economic factors as the price the processor receives for output, salvage values of excess supply for the processor and the farmer, the farm-level cost of production, the number of processors, and the amount of yield variability. We find that purely financial reasons can explain the preference of contract or spot market procurement.

We begin by providing an overview of the problem to be addressed here, and then present a formal presentation of a model that captures the profit incentives of processors and farmers. Numerical simulations show how the Nash solution reacts to changes in key economic parameters. These simulation results allow us to determine the key factors affecting the preference of farmers and processors for contract and spot production.
Overview of the Problem

Suppose there are $M$ input buyers (processors) in a geographic region who can enter into grower contracts that specify that the processor will purchase at a guaranteed price all the production that comes off of contracted acres. There are $N$ growers. Each of the $M$ processors has a capacity constraint $Q$ that limits the amount of delivered input that can be used. Output technology for the processor is a fixed proportions technology

$$q = k(A_c \cdot y + x_s),$$

where $q$ is output, $A_c$ is acreage contracted by each processor, $x_s$ is the amount bought on the spot market, and $y$ is the per-acre yield on all spot and contract acres in a given year. A problem arises because per-acre yield on the contract acres is stochastic so that the total quantity of produced input from contracted acres will vary from the amount needed by the processor to achieve capacity production. Any excess production can be sold by the processor for a residual value at a constant price. Farmers have the option to plant additional acres of production without a contract. The amount of production depends on the expected spot price they will receive.

The ex post input price in the spot market equals the residual or salvage value if there is excess supply of the input. If, ex post, there is excess demand of the input in the spot market, then the spot price of the input (given that there are sufficient buyers) will be bid up to the point where profits for the processors equal zero. Under certain excess demand conditions, there is no equilibrium price that can be negotiated. That is, there is no general solution to the problem of a limited number of buyers bidding for a fixed supply of the input. To get around this problem, we assume, for now, that the spot price in such excess demand conditions is midway between the excess demand and the excess supply prices.

The processor offers to identical risk-neutral farmers a contract price, $r_c$, and a total number of contract acres, $A_c$. If this contract price exceeds the opportunity cost of land, then the processor will find an excess demand for the contract acres and will not have a problem finding takers for the contracts. The processor’s capacity constraint and the number of contract acres determines the probability that production will be less than that needed to run at capacity. This creates the possibility that the processor will find it profitable to buy in the spot market. If this probability is high enough, this creates an incentive for farmers to produce for the spot market.
The processor can control the profitability of farmers growing for the spot market through the choice of $A_c$. That is, increases in $A_c$ decreases the profitability because there will be less total spot demand. There is a spot market supply curve $\bar{A}_s = g(A_c)$ with $g_{A_c} < 0$ that captures farmers’ willingness to plant for the spot market.

The processor’s problem is to choose $r_c$ and $A_c$. For now, assume that $r_c$ is fixed. Then, the processor has a demand for contract acreage function that results from the profit-maximization problem that depends on the contract price and the number of spot acres: $A_c = h(r_c, \bar{A}_s)$. Presumably, $h_r < 0$; $h_{\bar{A}_s} < 0$ because an increase in the contract price increases the cost of excess production, and increases in spot market acreage decrease the payoff from additional contract acres. All of the above is common knowledge for both farmers and processors.

Farmers and processors face a two-stage optimization problem. In the first stage, both farmers and processors decide how many acres to plant for the spot market and how many to contract, respectively, faced with spot price uncertainty caused by supply uncertainty from random yields. In the second stage, processors compete to buy the input they need (if it turns out that the contracted input is not enough to work at capacity). The optimization problem is solved using backward induction. The next section formalizes the problem described so far. First, we analyze the optimization problem of processors; then, we analyze the problem farmers face.

The Model

The Processor Problem

The second stage of the processor problem occurs after harvest, so yield uncertainty has been resolved. Ex post, processors face a perfectly inelastic supply, given by the total acres planted for the spot market multiplied by its yield.

After observing yield $y$, processors decide whether or not to try to buy more input. The second stage (ex post) demand for each processor is $x = \left( \frac{Q}{k} - A_c y \right)$, which can be negative if it turns out that the processors get more input than they need.
The price in the spot market, and hence the allocation of the rent among farmers and processors, will depend on the *ex post* relative bargaining power. Spot price is determined by demand and supply in the spot market, both of which are determined by planting and contracting decisions made in the first period. In this stage, processors bid simultaneously and independently to purchase the amount of input they need (if any) to work at capacity. After seeing the bids of each processor, the farmers decide whether to sell their production to one of the processors (and to which one) or not to sell at all and obtain the salvage value for the production. For the allocation of the input, we assume that farmers will sell it first to the processor offering the highest price. This processor is able to buy all the input it needs (if less than the aggregate supply); then, farmers offer the input remaining to the second-highest bid, and so on. In case of a tie, the input is distributed evenly across processors.

*Ex post*, processors will find themselves in one of four possible situations regarding their demand for additional production. The first situation examined is when yields are high enough so that processors hit their capacity constraint with contracted production. This occurs when \( y \geq \frac{Q}{kA_c} \). Any surplus production on contract acres will be sold at price \( r_1 \) (the salvage value for the processor). In this case profits for the processor are

\[
\pi_1 = pQ - r_c A_c y - r_1 \left( \frac{Q}{k} - A_c y \right),
\]

or

\[
\pi_1 = (pk - r_c)A_c y + (pk - r_1) \left( \frac{Q}{k} - A_c y \right)
\]

where \( p \) is the price of output, and \( r_c \) is the per-unit price of contracted acreage. Here

\[
x_s = \left( \frac{Q}{k} - A_c y \right) \leq 0,
\]

and the firm is getting some money back for the excess input. This situation happens with probability \( \Pr(y \geq \frac{Q}{kA_c}) = \Pr(y \geq v) = 1 - F(v) \), where \( v = \frac{Q}{kA_c} \), and \( F(\cdot) \) is the cumulative distribution function of yield.
The second situation occurs when \( \text{ex post} \) demand by processors is positive but there is still excess supply in aggregate. That is, \( M \left( \frac{Q}{k} - A, y \right) \leq \bar{A}, y \), where \( \bar{A} \) is total acreage planted for the spot market by all farmers. Thus \( \bar{A}, y \) is \( \text{ex post} \) aggregate, fixed supply in the spot market. The range of yields for this situation is given by

\[
\frac{MQ}{k(\bar{A} + MA)} \leq y \leq \frac{Q}{kA}.
\]

Because there is still aggregate excess supply, an offer by processors of \( s = r_2 \) for spot production (where \( r_2 \) is the farmer’s salvage value for spot production) constitutes a pure strategy Nash equilibrium. This also what Sexton and Zhang (1996) observed in the market for California iceberg lettuce.³ Processors do not need to bid the price up to get all the input they need to work at capacity.

Again processors can work at capacity, and profits are

\[
\pi_2 = pQ - rA, y - r_2 \left( \frac{Q}{k} - A, y \right),
\]

or

\[
\pi_2 = (pk - r_2)A, y + (pk - r_2) \left( \frac{Q}{k} - A, y \right).
\]

This case happens with probability

\[
\Pr \left( \frac{MQ}{k(\bar{A} + MA)} \leq y \leq \frac{Q}{kA} \right) = \Pr (u \leq y \leq v) = F(v) - F(u),
\]

where \( v \) is defined as above, and \( u = \frac{MQ}{k(\bar{A} + MA)} \).

The third case is when there is excess demand \( \text{ex post} \) but only one processor would not have enough production to run at capacity. One would think that in an excess demand condition, processors would bid up the spot price to the point where all their rents are dissipated. But if only one processor does not have enough input to run at capacity, then there is an incentive for this processor to strategically underbid its rivals for the residual supply. This is the case where a Nash equilibrium in pure strategies fails, in general, to exist.⁴ Edgeworth cycles may arise in this case (following a loose dynamic argument). For example, the price may rise as the processors try to increase their share in the input
market, until the profit of doing so is lower than the one resulting from offering the reservation price of the farmers and keeping the residual supply. Once the price is at the reservation price of the farmers (or low enough), processors will find it profitable to bid \( \varepsilon \) above their rivals and work at capacity.

This case implies \( \frac{(M-1)Q}{k(\overline{A}_i + (M-1)A_y)} \leq y \leq \frac{MQ}{k(\overline{A}_i + MA_y)} \). Because we are assuming a symmetric solution in which processors get an even distribution of the fixed supply (i.e., each processor is able to buy \( \frac{\overline{A}_i y}{M} \) in the spot market), processors will not be able to work at capacity. Profits in this case are

\[
\pi_3 = pkA_y y + pk\frac{\overline{A}_i y}{M} - r_c A_y y - r_s \frac{\overline{A}_i y}{M},
\]

or

\[
\pi_3 = (pk - r_s)A_y y + (pk - r_c) \frac{\overline{A}_i}{M} y,
\]

where \( r_s \) is the resulting input price in the spot market.

In this case we cannot know what \( r_s \) will be. To close the model, we set it equal to the average of the marginal valuations of the processors and the farmers. This case happens with probability

\[
\Pr\left( \frac{(M-1)Q}{k(\overline{A}_i + (M-1)A_y)} \leq y \leq \frac{MQ}{k(\overline{A}_i + MA_y)} \right) = \Pr(s \leq y \leq u) = F(u) - F(s),
\]

where \( u \) is defined as above and \( s = \frac{(M-1)Q}{k(\overline{A}_i + (M-1)A_y)} \).

The last case is when yield is so low (given the areas contracted and planted for the spot) that at least one processor would be left out of the market if it tries to underbid its rivals. For example, the second-to-last processor finds some supply but it is not enough to work at capacity. In this case, there is no room for strategic underbidding by the last one. If the last one tries to underbid its rivals, it will be left out of the market. This will happen if \( (M-1) \left( \frac{Q}{k} - A_y y \right) \geq \overline{A}_i y \), or, equivalently, \( y \leq \frac{(M-1)Q}{k(\overline{A}_i + (M-1)A_y)} \). The pure strategy Nash equilibrium here is for processors to offer their marginal valuation \( (pk) \) for the
input. Processors will again split the input evenly and will not be able to work at capacity. Profits in this case are

$$\pi_4 = pkA_c y + pk \frac{A_c y}{M} - r_c A_c y - pk \frac{A_c y}{M},$$

or, equivalently,

$$\pi_4 = (pk - r_c)A_c y.$$

This will occur with probability

$$\Pr \left( y \leq \frac{(M - 1)Q}{k(A_c + (M - 1)A_c)} \right) = \Pr(y \leq s) = F(s)$$

where $s$ is defined as above.

**Processor Expected Profits**

Now we are ready to write the first-stage objective function of a representative processor. Bearing the rules that will arise in the second stage in mind, processors make the decision of how many contracts to offer in order to maximize their expected payoff. That is, each processor chooses $A_c$ to maximize its expected profits, which are defined as

$$E(\pi) = \left( (pk - r_c)A_c y + (pk - r_2) \left( \frac{Q}{k} - A_c y \right) \right) \Pr \left( y \geq \frac{Q}{kA_c} \right)$$

$$+ \left( (pk - r_c)A_c y + (pk - r_2) \left( \frac{Q}{k} - A_c y \right) \right) \Pr \left( \frac{MQ}{k(A_c + (M - 1)A_c)} \leq y \leq \frac{Q}{kA_c} \right)$$

$$+ \left( (pk - r_c)A_c y + (pk - r_2) \left( \frac{Q}{k} - A_c y \right) \right) \Pr \left( \frac{(M - 1)Q}{k(A_c + (M - 1)A_c)} \leq y \leq \frac{MQ}{k(A_c + (M - 1)A_c)} \right)$$

$$+ \left((pk - r_c)A_c y\right) \Pr \left( y \leq \frac{(M - 1)Q}{k(A_c + (M - 1)A_c)} \right).$$

This can be rewritten as follows:

$$E(\pi) = \left( (pk - r_c)A_c y + (pk - r_2) \left( \frac{Q}{k} - A_c y \right) \right) \Pr \left( y \geq v \right)$$

$$+ \left( (pk - r_c)A_c y + (pk - r_2) \left( \frac{Q}{k} - A_c y \right) \right) \Pr \left( u \leq y \leq v \right)$$

$$+ \left( (pk - r_c)A_c y + (pk - r_2) \left( \frac{Q}{k} - A_c y \right) \right) \Pr \left( s \leq y \leq u \right) + \left((pk - r_c)A_c y\right) \Pr \left( y \leq s \right),$$
or more clearly as

\[ E(\pi) = (pk - r_c)A_c E(y) + \int_v^u (pk - r_1) \left( \frac{O_k}{k} - A_c y \right) dF(y) + \int_u^v (pk - r_2) \left( \frac{O_k}{k} - A_c y \right) dF(y) \]

\[ + \int_s^v (pk - r_s) \frac{A_c y}{M} dF(y). \]

We can rearrange this to get

\[ E(\pi) = (pk - r_c)A_c E(y) + (pk - r_1) \frac{O_k}{k} (1 - F(v)) - (pk - r_1) A_c \int_v^u y f(y) dy \]

\[ + (pk - r_2) \frac{O_k}{k} (F(v) - F(u)) - (pk - r_2) A_c \int_u^v y f(y) dy + \frac{A_c}{M} \int_u^v y f(y) dy, \tag{1} \]

where \( f(y), 0 \leq y \leq y_m \) is the density function for the random yield and \( E(y) \) is the expected value of yield.

The first-order condition for a maximum is found by differentiating equation (1) with respect to \( A_c \). After some algebra we get

\[ \frac{\partial E(\pi)}{\partial A_c} = (pk - r_c) \int_0^u y f(y) dy + (r_1 - r_c) \int_v^u y f(y) dy + (r_2 - r_c) \int_u^v y f(y) dy \]

\[ + \frac{kA_c}{MQ} \left( (r_2 - r_1) f(u) u^3 + (pk - r_1) f(s) s^3 \right) \leq 0, \tag{2} \]

with equality if \( A_c > 0 \).

Processors take into account that they can affect the probability of being in each of the cases described. They realize, for example, that if they increase the contracted area, it is less likely that they will have to buy in the spot market. Of course, the magnitude of the marginal effect each processor has decreases as the number of processors increase.

Before exploring the ramifications of this assumption, we will first examine the problem of farmers.

**Farmer Decisions**

Farmers take contract area and contract price as given. They decide whether to take a processor’s offer of acreage and price and whether to plant for the spot market. Assume a large number \( N \) of identical farmers. The large number assumption is not crucial. The important assumption is that farmers take aggregate acreages as given. Thus, they do not
act as if they can affect the probability of ex post spot market demand. Here, \( a_i \) is the acreage contracted by farmer \( i \), \( a_s \) is the area planted for the spot market by farmer \( i \), and \( C(a_i + a_s) \) is the cost of production. Because it is assumed that farmers are identical, the subscript \( i \) will be dropped. The expected profit of a farmer can be written as

\[
E(\pi) = E(r_c a_c y + r_s a_s y - C(a_c + a_s)).
\]

Again, four different scenarios may arise in the spot market. These situations are the same as the ones described in the processor problem and for that reason we will only state here what farmers expect, that is, their payoff function and their probability of being in each case. In the first case, there is no demand in the spot market. Here, farmers do not receive any bid for their output and therefore the value of the crop equals their reservation price. Farmers use the input for the next-best destination. The profit function for this case is

\[
\pi_1 = r_c a_c y + r_s a_s y - C(a_c + a_s),
\]

which, as before, occurs with probability

\[
\Pr\left( y \geq \frac{Q}{kA_c} \right) = \Pr(y \geq v) = 1 - F(v).
\]

In the second case, there is demand for the input in the spot market. However, aggregate supply exceeds aggregate demand. Profits and probability of occurrence for this case are

\[
\pi_2 = r_c a_c y + r_s a_s y - C(a_c + a_s)
\]

and

\[
\Pr\left( \frac{MQ}{k(\bar{A}_s + MA_c)} \leq y \leq \frac{Q}{kA_c} \right) = \Pr(u \leq y \leq v) = F(v) - F(u).
\]

The third situation corresponds to the case where only \((M - 1)\) processors can work at capacity. Farmer profits and probability of occurrence for this case are

\[
\pi_3 = r_c a_c y + r_s a_s y - C(a_c + a_s)
\]

and

\[
\Pr\left( \frac{(M - 1)Q}{k(\bar{A}_s + (M - 1)A_c)} \leq y \leq \frac{MQ}{k(\bar{A}_s + MA_c)} \right) = \Pr(s \leq y \leq u) = F(u) - F(s).
\]
And the last case is as before, where at least one processor would be left out of the market if it tries to underbid its rivals so that the spot price is bid up to \((pk)\), the processor’s marginal valuation for the input. Profits and probability of occurrence for this case are

\[
\pi_a = r_c a_c y + pk a_y - C(a_c + a_s)
\]

and

\[
\Pr \left( y \leq \frac{(M-1)Q}{k(A_c + (M-1)A_t)} \right) = \Pr(y \leq s) = F(s).
\]

Now we can write the expected profit of the farmers as

\[
E(\pi) = \int_a^\infty \left( r_c a_c y + r_c a_y - C(a_c + a_s) \right) f(y) dy + \int_0^a \left( r_c a_c y + r_c a_y - C(a_c + a_s) \right) f(y) dy
\]

\[
+ \int_a^\infty \left( r_c a_c y + r_c a_y - C(a_c + a_s) \right) f(y) dy + \int_0^a \left( r_c a_c y + pk a_y - C(a_c + a_s) \right) f(y) dy
\]

or, rearranging, as

\[
E(\pi) = r_c a_c E(y) + \int_a^\infty r_c a_y f(y) dy + \int_0^a r_s a_y f(y) dy + \int_0^a pk a_y dy - C(a_c + a_s).
\] (3)

The first-order condition for a maximum is obtained by differentiating equation (3) with respect to \(a_s\):

\[
\frac{\partial E(\pi)}{\partial a_s} = r_c \int_a^\infty y f(y) dy + r_s \int_0^a y f(y) dy + pk \int_0^a y f(y) dy - C'(a_c + a_s) \leq 0,
\]

with equality if \(a_s > 0\). \(\text{(4)}\)

Note that farmers take the aggregate amount planted for the spot as given. Hence, we do not use the Leibnitz rule here. Farmers do not realize they can change the probability of being in the cases described (they take \(Na_s\) as given). This is because individual farmer output is assumed to be too small to affect the aggregate spot supply. This assumption gives the problem a rational expectations flavor, in that farmers believe the aggregate acreage for the spot will be \(\bar{A}_s\), and in equilibrium their expectations are realized.

**Equilibrium**

In this section, we characterize the equilibrium acreages for both the farmer and processor problems. A pure strategy Nash equilibrium for this model is a number of acres planted for the spot market and a number of contracted acres \((A^*_c, a^*_s)\), such that neither
farmers nor processors can benefit from unilateral deviations. That is to say that in equilibrium, a processor cannot increase its profit by unilaterally choosing to contract a different number of acres. The same applies for a farmer with regard to the number of acres to plant for the spot market. Three additional conditions have to hold in equilibrium. First, beliefs regarding aggregate spot acreage are confirmed: \( \bar{A}_s = Na_s \).

Second, aggregate demand and supply for contracts are equalized: \( Na_c = MA_c \). Lastly, farmers’ profits are nonnegative. In short, any equilibrium has to satisfy equations (5) and (6), which obtain from imposing the equilibrium conditions to equations (2) and (4), respectively.

\[
\begin{align*}
(pk - r_c) \int_0^v yf(y)dy + (r_1 - r_c) \int_v^y yf(y)dy + (r_2 - r_c) \int_0^u yf(y)dy \\
+ \frac{kNa_c}{MQ} \left[ (r_3 - r_2) f(u)u^3 + (pk - r_3) f(s)s^3 \right] & \leq 0, \text{ with equality if } A_c > 0 \\
(5)
\end{align*}
\]

\[
\begin{align*}
N_c \int_0^v yf(y)dy + r_i \int_v^y yf(y)dy + pk \int_0^u yf(y)dy - C^* \left( A_c \frac{M}{N} + a_s \right) & \leq 0, \\
\text{with equality if } a_s > 0, \text{ for } v = \frac{Q}{kA_c}, u = \frac{MQ}{k(Na_s + MA_c)}, s = \frac{(M - 1)Q}{k(Na_s + (M - 1)A_c)}, (6)
\end{align*}
\]

and equation (3) \( \geq 0 \).

The Nash equilibrium for a particular environment can be predicted by use of the best response functions of processors \( A_c = h(r_c, \bar{A}_s) \) and farmers \( a_{si} = z(A_c) \) (for \( z(A_c) = g(A_c) / N \) ) previously introduced in the problem overview section. These functions are defined implicitly by equations (5) and (6), respectively. The equilibrium is determined by the intersection of the best response functions or, equivalently, by solving equations (5) and (6) simultaneously for \( A_c^*(r_c) \) and \( a_{si}^*(r_c) \). Unfortunately, we cannot obtain closed solutions for this model. However, numeric techniques can be used to characterize the predicted Nash equilibrium as well as responses to changes in the environment. In what follows, we provide a characterization of the equilibrium that results for a particular calibration of the model (hereupon referred to as the benchmark case). The parameter values assumed for the benchmark case are shown in Table 1. In
TABLE 1. Parameterization for the benchmark case

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p )</td>
<td>4</td>
</tr>
<tr>
<td>( N )</td>
<td>20</td>
</tr>
<tr>
<td>( M )</td>
<td>5</td>
</tr>
<tr>
<td>( r_1 )</td>
<td>0.5</td>
</tr>
<tr>
<td>( r_2 )</td>
<td>1.0</td>
</tr>
<tr>
<td>( k )</td>
<td>1.0</td>
</tr>
<tr>
<td>( Q )</td>
<td>5000</td>
</tr>
<tr>
<td>( y_m )</td>
<td>200</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>1.5</td>
</tr>
<tr>
<td>( \beta )</td>
<td>1.5</td>
</tr>
<tr>
<td>( b )</td>
<td>5</td>
</tr>
<tr>
<td>( d )</td>
<td>30</td>
</tr>
</tbody>
</table>

To solve the model, we need to assume a probability distribution for the random yield and a functional form for the farmer’s cost function. The probability distribution for the random yield is assumed to be a three-parameter beta distribution, which has the following density function:

\[
f(y) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} \cdot \frac{y^{\alpha-1}(y_m - y)^{\beta-1}}{y_m^{\alpha+\beta-1}}, \quad 0 \leq y \leq y_m.
\]

We will assume that the functional form of the cost function is the following:

\[
C(a_s + a_c) = d \cdot (a_s + a_c) + \frac{b}{2}(a_s + a_c)^2.
\]

The first term captures the factors of production that can be easily obtained to increase the area planted, whereas the quadratic term represents the increasing costs associated with less easily adjustable factors. An example of such a factor is the managerial capacity of the farmer.

Figure 1 shows aggregate best-response functions for farmers and processors for alternative contract prices. Since all processors are identical, the aggregate best-response function is obtained by multiplying the individual function by the number of processors. The same rule applies to the farmer best-response function.

As shown in Figure 1, the resulting equilibrium depends on the price at which the contracts are offered. Increases in the contract price result in inward shifts of the processor’s aggregate best-response function. That is, increases in the contract price decrease aggregate contract acreage offered given a level of spot acreage. The farmer’s best-response function is not influenced by the contract price because the first-order condition for the farmer’s problem is independent of the contract price. This independence means that the set of all Nash equilibrium coincides with the farmer’s
aggregate best-response function. Hence, increases in the contract prices result
unambiguously in equilibria with lower amounts of contracted acres and more activity in
the spot market. As the contract price increases, processors are willing to contract less
acreage. This is because the size of the loss in case of a high yield (that results in
overbuying of the crop) increases with the gap between the contract price and the salvage
value. Since there will be fewer contracts offered, farmers expect a stronger demand in
the spot market (and consequently a higher price), and hence it is in their interest to plant
more acres for that market. Reductions in the contract price lower the size of the loss in
when there is excess input. The area contracted will be higher, which results in a weaker
expected demand in the spot market.

The range of contract prices considered for this particular parameterization is from
0.818 to 1.893. The lower limit of the range is the contract price that leads to zero profits
for farmers in equilibrium. Farmers will not take any contract that offers less than this
price. The upper limit is the contract price where the equilibrium area contracted is zero.
Further increases in the contract price have no economic meaning, since there will be no
contracts offered by processors.
Figure 2 represents an alternative way of depicting the set of Nash equilibrium corresponding to different contract prices for our benchmark case. This representation will prove to be very useful in visualizing how the set of equilibrium responds to changes in the environment. Figure 2 makes clear the substitutability between contracted and spot acres. For example, for a contract price of $1.80, the processors will offer to contract a total of 69.3 acres, whereas farmers will plant for the spot market a total of 316 acres. If the contract price is lowered to $1.00, the processors will offer more contracts (327 acres) and the farmers will be less willing to plant for the spot (68.4 acres).

**Equilibrium Responses to Changes in Economic Parameters**

In this section, we examine how changes in production uncertainty, the price of the processed good, the number of processors, and the number of farmers affect the best-response functions of farmers and processors. The first variable we examine is production uncertainty. The effects of a mean-preserving spread in yields are illustrated by solving the model for two additional levels of the coefficient of variation of yields (0.3 and 0.7).
in addition to the benchmark value of 0.5. Figure 3 depicts the new sets of Nash equilibrium for 0.3 and 0.7. As Figure 3 makes clear, increasing (decreasing) the level of production uncertainty reduces (increases) the number of acres contracted for a given contract price. A mean-preserving spread in yield risk increases the expected excess supply of input, conditional on yields being high enough to cover all input demand. This effect tends to decrease contract acres because processors will want to reduce the amount of excess supply they have to sell at a loss. But, a mean-preserving spread in risk also increases the yield shortfall conditional on being low enough to generate positive ex post demand. This effect would tend to increase contract acres. However, processors know that if they marginally reduce their offer of contract acres, farmers will marginally increase their supply of spot acres. This substitution partly reduces the risk of a yield shortfall. Figure 3 shows that the net effect of an increase in yield risk is negative on contract acres being offered by processors for contract prices above $1.10. For low contract prices, the number of contracted acres offered by processors increases with an increase in

**Figure 3. Effects of yield uncertainty (cv) on the equilibrium acres in contract and spot markets**
their level of uncertainty. With low contract prices (and high contract acres), the probability of yields being low enough to trigger *ex post* purchases on the spot market are low enough that processors cannot rely on the positive supply response of farmers to a reduction in contract acres. Thus, contract acres actually may increase with an increase in yield risk.

At contract prices greater than $1.10, an increase in yield risk reduces the number of contract acres offered by processors. Because there are fewer outstanding contracts, the probability that *ex post* demand will be positive increases, thereby increasing the expected return to spot production. Thus, farmers are willing to plant more acres without a contract. For very low contract prices, processors do not reduce their offer of contract acreage, so farmers do not tend to increase their spot acreage in response to an increase in yield risk.

Overall, for most situations, an increase in yield uncertainty will tend to increase spot market activity as processors try to avoid situations of excess supply from contracted acres. This seems to provide some evidence that spot markets will be more prevalent in situations where yield risk is relatively large.

Figure 4 shows how output profitability (measured by the per-unit margin) affects the set of Nash equilibrium. Higher profit margins result in more contracts offered and less area planted for the spot. This result is very intuitive for processors—higher

![Figure 4. Effect of output price (p) on equilibrium acreage](image-url)
margins imply less willingness to operate at less than capacity; however, this result is less obvious for farmers. For the farmer, there are two forces acting in opposite directions. On the one hand, increases in the output price result in a higher return in the spot market in the case where processors have to bid their marginal valuation to get the input. On the other hand, this situation of excess demand is less likely to arise because of the reduced expected demand in the spot market. Figure 4 shows that the latter force dominates farmers’ decisions.

Analogous results apply for changes in processors’ salvage value. Here the loss associated with overbuys of the inputs are reduced for any given contract price. This makes processors less reluctant about being in the situation of having to sell any excess input. However, the difference tends to disappear as the contract price increases. Because processors are contracting more acres, farmers anticipate a weaker demand in the spot market and hence decide to reduce the area to plant for that destination.

Figure 5 shows the effects of an increase in the number of processors, holding per-processor demand constant. This situation simulates the effects of an increase in total

**Figure 5.** Equilibrium responses to changes in the number of processors (M) (per-processor capacity held constant)
demand for the input holding the number of farmers constant. An increase in demand increases the probability that processors will have to purchase in the spot market, thereby raising the input price. This induces processors to offer more contract acres. Farmers respond by planting more total acres, which, with increasing marginal costs, makes them more reluctant to plant in the spot market. Thus, an increase in demand holding the number of suppliers constant results in more contract acres.

Similar results are found by holding total market demand constant but increasing the number of farmers. The effect of such an increase depends on the form of the farm-level marginal cost function. If marginal costs increase with output (as in our benchmark case), then an increase in the number of (identical) farmers will increase farmers’ willingness to plant for the spot market, because marginal cost for each farmer is lower due to each farmer having a smaller share of total production. As shown in Figure 6, an increase in the number of farmers does indeed decrease contract acres. Thus, higher farmer margins will tend to increase spot acreage whereas higher processor margins will tend to increase contract acres, as shown in Figure 4.

**Figure 6. Equilibrium responses to changes in the number of farmers**
Processor and Farmer Preferences

Although this paper does not develop a formal bargaining model that allows analysis of the formation of contract prices, some insight into the problem can be obtained by examining the effect of contract price on the payoffs of farmers and processors. For any contract price, there is an associated equilibrium payoff for both parties. Substituting the equilibrium acreages in equations (1) and (3) gives the equilibrium payoffs for processors and farmers, respectively. This allows us to construct (point-wise) equilibrium payoff functions. These functions are depicted in Figure 7.

Figure 7 makes clear that farmers and processors have conflicting preferences with regard to the contract price. Processors prefer a contract price such that they only participate in one market—preferably the market for contracts. Farmers’ payoff, on the other hand, is highest when the equilibrium entails the existence of both markets. Furthermore, the contract price that one party would choose if allowed to do so freely is the least preferred by the other party. Figure 7 shows that farmers’ profits are maximized

![Graph showing the effect of contract price on aggregate profits for farmers and processors.](image)
at the contract price where processors’ payoffs attain a minimum. The contract price that in equilibrium minimizes (maximizes) the payoff of processors (farmers) is $1.71.

The shape and location of the equilibrium payoff functions have both efficiency and distributional implications. For our benchmark case, if processors can choose freely the contract price that best fits their interest, they will capture all the rents generated in the industry. At this point, there will be no production for the spot market. As the contract price increases, there is a transfer of profits from processors to farmers, as well as an increase in the total rents generated. For high contract prices the tendency is reversed, as processors cut back their acreage offers and the market for contract production disappears. Wealth transfers from independent producers to firms have been found as a result of vertical integration (Perry 1978 and Murray 1995 among other authors) and as a result of captive supplies (Azzam 1998).

**Concluding Remarks**

In this study we develop a simple theoretical model that, suitably parameterized, allows for the co-existence of contracts and spot markets in agriculture. An attractive feature of the model is its generality, in the sense that it was developed without having any particular production activity in mind. Therefore, suitable parameterizations allow for the study of a wide variety of production processes. Numerical simulations were conducted for one particular arbitrary parameterization, and the impacts of fundamental economic factors on the equilibrium outcomes were studied. Our results suggest that for a range of contract prices, participation in both markets constitutes a Nash equilibrium for the model. Because the model assumes that all the market participants are risk neutral, the equilibrium outcomes are the results of purely financial considerations.

Changing the contract price makes clear the substitutability between both systems of production. This implies, as expected, that the contract price is a key variable in the determination of the relative size of each market. Although we do not model the contract price determination, the model demonstrates the antagonistic interests between farmers and processors concerning the contract price. Processors prefer a contract price such that in equilibrium they procure all the input they need under contracts. Farmers prefer a higher contract price but not so high as to drive the offer of contracts to zero. To predict
which contract price will arise as an equilibrium, it is necessary to impose more structure on the problem and introduce suitable bargaining rules between farmers and processors. This is an area of the model that could be improved.

Although the model presented here may not capture many aspects of contracting decisions in agriculture, it is a reasonable starting point. The model could be modified in several directions to tackle different complexities that commonly arise in the economic activity. For example, informational problems could be introduced by assuming that the acreage contracted by rival processors is private information. Other extensions could be the introduction of quality issues, or transaction costs related to the contracted versus spot market procurement.
**Endnotes**

1. Note that we are not making any claim about the efficiency of this contract relative to other mechanisms.

2. We are not modeling the equilibrium choice of $r_c$ in this paper. To determine the equilibrium contract price, we would have to model explicitly the bargaining process at planting time, which would unnecessarily complicate our analysis. We want the simplest model possible that still yields information about the coexistence of contracts and production for the spot market.

3. This paper differs from that of Sexton and Zhang in that the latter treated *ex post* supply and demand as being exogenous.

4. Kreps and Scheinkman (1983) and Levitan and Shubik (1972) addressed this problem for capacity-constrained duopolists. Weninger and Reinhorn, studying a more closely related problem (oligopsonists facing a fixed supply), concluded that a pure strategy Nash equilibrium for this problem exists if the number of buyers is sufficiently large and the price space is discrete.
References


