

# THE AUSTRALIAN JOURNAL OF AGRICULTURAL ECONOMICS

VOL. 22

DECEMBER 1978

NO. 3

## DECOMPOSING THE VARIANCE OF GROSS REVENUE INTO DEMAND AND SUPPLY COMPONENTS

R. R. PIGGOTT\*

*University of Newcastle*

**Some recent analyses relating to gross revenue instability have used a procedure for decomposing the variance of gross revenue into components attributable to price variability, quantity variability and interaction between these. This paper offers some criticisms of the procedure and outlines an alternative procedure which divides the variance of gross revenue into components due to demand variability, supply variability and interaction between these. The procedure is used to investigate the causes of instability in quarterly beef revenues in the Australian beef industry. Since demand and supply variability are both important contributors to beef revenue instability, direct market intervention to stabilise beef industry revenues would be a complex and risky task.**

### *Introduction*

Considerable research has been devoted to the topic of income fluctuations in Australian agriculture—their causes, consequences and cures. The measure of 'income' used in these studies has varied with respect to level of aggregation (farm versus industry, single enterprise versus combined enterprises) and the extent to which costs are taken into account.

In this paper, the focus of attention is on the variance of industry level gross revenue.<sup>1</sup> A valuable paper by Houck (1973) also focuses on this topic. Using a technique suggested by Burt and Finley (1968) he tried to apportion the variance of industry-level gross revenue (which he refers to as 'income') for the wool, wheat and beef industries into separate components attributable to price variability, quantity variability and inseparable interaction between price and quantity variability. Following Houck's analysis, the authors of the rural policy 'Green Paper' (Harris et al. 1974) decomposed the sources of gross revenue instability for 24 agricultural commodities and used this as part

\* Lecturer in Economics, University of Newcastle. Without implicating them in any way, the author wishes to acknowledge useful comments from A. Dobson, R. Irish, K. McLaren, W. Tomek and the anonymous referees. Thanks are due to the Bureau of Agricultural Economics—particularly J. Longmire—for information relating to the econometric model of the beef industry.

<sup>1</sup> The causes of fluctuations in industry-level revenue may not be a good guide to the causes of fluctuations in individual farm revenues, and the extent of revenue fluctuations may not be a good guide to the extent of income fluctuations. However, in the author's view, studying the causes of industry-level revenue fluctuations remains a legitimate research exercise. The macroeconomic impacts of these fluctuations can be severe.

of the basis for discussing the relative desirability of various types of stabilisation schemes.<sup>2</sup>

The general question examined in this paper is: given that one is interested in analysing the components of the variance of industry-level gross revenue, is the procedure suggested by Burt and Finley the most useful technique to adopt? More particularly, some shortcomings of the Burt and Finley procedure are discussed and an alternative procedure is proposed. The procedure is used to decompose the variance of quarterly saleyard-level gross revenue for the beef industry.

#### *The Burt and Finley Decomposition Procedure*

Gross revenue ( $GR$ ) is the product of price ( $P$ ) and quantity sold ( $Q$ ). The variance of  $GR$  is a complicated expression involving the variances of  $P$  and  $Q$ , their covariance and higher-order interaction terms.<sup>3</sup> However, Burt and Finley suggest that, by omitting higher-order interactions, this variance is approximated as:

$$(1) \quad \text{var}(GR) = E^2(Q) \text{var} P + E^2(P) \text{var} Q \\ + 2 E(Q)E(P) \text{cov}(PQ),$$

where 'var' denotes variance, 'cov' denotes covariance,  $E$  denotes mean value and  $E^2(\cdot)$  denotes the square of the mean of ( $\cdot$ ). The first two terms in (1) are the direct effects of variability in  $P$  and  $Q$ . The third term is the first-order interaction which cannot be decomposed into separate effects due to variability in  $P$  and  $Q$ . Burt and Finley claim that this term should approximate the full interaction, so that the remaining terms in the exact var ( $GR$ ) formula can be omitted.

For easier interpretation, Burt and Finley suggest that the three terms in (1) be divided by the sum of the first two. The positive direct price and quantity effects sum to 1.0. The third term can be positive or negative.

To demonstrate, consider Houck's analysis of revenue variability for Australian wool, wheat and beef (Table 1).

He wrote as follows (Houck 1973, pp. 212-14):

The evidence . . . indicates that most of the variation in annual wool income can be attributed to price fluctuation. . . . Producer price stability for wool, however achieved, would do much to stabilise aggregate wool income in Australia.

. . .

An industry income stabilisation scheme for wheat would have to do more than stabilise producer prices. . . . It would have to take into account and allow for substantial year-to-year changes in plantings

<sup>2</sup> The Industries Assistance Commission's report on rural income fluctuations draws on these studies (IAC 1978, p. 5):

As regards income stability, analyses have shown that for many rural commodities, especially crops, output fluctuations account for a larger degree of gross income variability than fluctuations in price. Further, these analyses showed that for most commodities changes in prices and production were predominantly in opposite directions, tending to offset each other to some degree. Where this tendency is marked, price stabilisation (say through a buffer fund) could destabilise industry revenue from that commodity.

<sup>3</sup> See Bohrnstedt and Goldberger (1969, p. 1439).

TABLE 1

*Separation of Gross Income Variation for Australian Wool,  
Wheat and Beef*

Item	Income variation attributable to:		
	Price	Quantity	Linear interaction
Wool	98.0	2.0	-1.0
Wheat	8.4	91.6	-2.5
Beef	60.8	39.2	-65.5

Source: Houck (1973, p. 212).

and sizeable yield fluctuations. . . . An income stabilisation scheme might indeed *add* instability to wheat prices realised by producers.

. . .

. . . it proved quite difficult to separate beef income variation clearly into price and quantity components. . . . The large negative interaction term makes any such attribution quite tenuous. Movement along stable demand curves is surely quite important here.

Goldberger (1970) criticised the Burt and Finley procedure, pointing out that sometimes equation (1) is a poor approximation of the *GR* variance. In response, Burt and Finley (1970) claimed that the approximation error tends to be less when the data are detrended. Applications to Australian data have used detrended data (Houck 1973 and Harris et al. 1974).

Some additional criticisms and questions not mentioned by Goldberger are outlined below.

(a) Is the approximation formula really necessary? Why not compute the interaction effect as the difference between the true value of the variance of *GR* and the sum of the first two R.H.S. terms of equation (1)? If the sum of the omitted terms is insignificant compared to the individual terms in (1), then it is immaterial whether or not they are included.

(b) Is it valid to call the first R.H.S. term of (1) the 'direct' effect of price variability? After all, it contains the square of the mean quantity. Similar reasoning applies to the second R.H.S. term.

(c) Assume that the covariance term accurately reflects all interaction in a particular application. How does one interpret empirical results when it is large relative to the direct effects as in Houck's analysis of beef incomes (Table 1)? Such results could occur with a static, negatively-sloped demand and a volatile supply. Houck correctly ignored the relative sizes of the direct effects but not all researchers have adopted this approach. Motha, Sheales and Saad (1975, p. 42) concluded that, '. . . for mutton and lamb and pigmeats, price changes are the relatively more important variable in contributing to income variation'. For these products, the interaction term was well in excess of 50 per cent of the sum of the direct effects. While they emphasised the strong interaction effect, the conclusion quoted above should not have been drawn.

(d) Consider a situation in which the interaction term is small relative to the direct effects, as with Houck's results for wool in Table 1. Houck concluded that producer price stability for wool would be an effective way to stabilise wool income. The cause of income variability seems quite apparent.

How useful is this conclusion from the policy-maker's viewpoint? Consider a commodity for which the demand function is stationary and highly inelastic, while the supply function is perfectly inelastic and volatile. Results from the decomposition procedure would suggest that price instability is the main cause of income instability—in fact, if the shifts in supply are small, then the results might approach those obtained by Houck for wool. While Houck concluded that a price stabilisation scheme was appropriate, the income instability is really due to shifts in supply or quantity sold! A program aimed at stabilising quantity would stabilise prices and incomes.

There are great dangers in implementing price stabilisation policies when the root-cause of the fluctuating prices remains unknown. Price fluctuations are due to demand fluctuations, supply fluctuations or both. The effects of price stabilisation schemes on revenue and quantity stability, stock holdings by buffer authorities etc., will depend upon the cause of the fluctuating prices.

To sum up, the Burt-Finley procedure provides descriptive statistics requiring subjective interpretation. Any one pattern of price-quantity variability could be the result of a number of different patterns of supply and demand shifts. The procedure leaves much unknown about the root-causes of revenue instability.

#### *An Alternative Procedure*

The remainder of the paper outlines an alternative procedure aimed at uncovering the historical pattern of *supply and demand variability* underlying a particular pattern of revenue instability.<sup>4</sup> Insofar as the procedure goes beyond descriptive statistics and is based on an underlying analytical framework, it may be more appealing to researchers.

#### *Outline of the procedure*

Two assumptions are that the demand and supply functions for a particular commodity are linear and that the slopes of these functions remain constant over time—not uncommon assumptions in applied econometric work. That is:

$$(2) \quad Q_t^d = a_t + k_1 P_t \quad (\text{demand}),$$

$$(3) \quad Q_t^s = c_t + k_2 P_t \quad (\text{supply}),$$

where  $Q_t^d$  = quantity demanded in time period  $t$ ,  
 $Q_t^s$  = quantity supplied in time period  $t$ ,  
 $a_t$  = net demand intercept for time period  $t$ ,  
 $c_t$  = net supply intercept for time period  $t$  ( $c_t < a_t$ ),

<sup>4</sup> The danger of market intervention without knowledge of the underlying pattern of supply and demand shifts has already been mentioned. Another motive for studying the pattern of supply and demand variability results from recent work (e.g. Turnovsky 1974) showing that the gainers and losers from stabilisation depend on whether instability is generated mainly by demand or supply volatility.

$k_1$  = demand slope (fixed over time and negative),  
 $k_2$  = supply slope (fixed over time and positive),  
 $P_t$  = price in period  $t$ .

The terms  $a_t$  and  $c_t$  are interpreted as net intercepts in that they embody the influence of a number of demand and supply shifters, respectively. In the case of  $a_t$ , these might include income and substitute good prices and in the case of  $c_t$ , key input prices and technology.

It follows that:

$$\begin{aligned} Q_t^e &= (a_t k_2 - c_t k_1) / (k_2 - k_1), \\ P_t^e &= (a_t - c_t) / (k_2 - k_1), \end{aligned}$$

and

$$(4) \quad GR_t = \Pi_1 a_t^2 + \Pi_2 c_t^2 - (\Pi_1 + \Pi_2) a_t c_t,$$

where  $Q_t^e$  = equilibrium quantity in time period  $t$ ,

$P_t^e$  = equilibrium price in time period  $t$ ,

$GR_t$  = gross revenue in time period  $t$ ,

$\Pi_1 = k_2 / (k_2 - k_1)^2$ ,

and  $\Pi_2 = k_1 / (k_2 - k_1)^2$ .

The variance of  $GR$  over time is given by equation (5) below:

$$(5) \quad \begin{aligned} \text{var}(GR) &= \Pi_1^2 \text{var}(a^2) && \text{(component 1)} \\ &+ \Pi_2^2 \text{var}(c^2) && \text{(component 2)} \\ &+ (\Pi_1 + \Pi_2)^2 \text{var}(ac) && \text{(component 3)} \\ &+ 2\Pi_1\Pi_2 \text{cov}(a^2, c^2) && \text{(component 4)} \\ &- 2\Pi_1(\Pi_1 + \Pi_2) \text{cov}(a^2, ac) && \text{(component 5)} \\ &- 2\Pi_2(\Pi_1 + \Pi_2) \text{cov}(c^2, ac) && \text{(component 6)} \end{aligned}$$

The terms on the R.H.S. of (5) can be written in terms of  $a$  and  $c$  using the formula for the variance of a product variable (which can be applied to the first three terms in (5)) and the formula for the covariance between two product variables (which can be applied to the last three terms in (5)). Both formulae appear in Bohrnstedt and Goldberger (1969). Letting  $\Delta a$  and  $\Delta c$  denote  $[a - E(a)]$  and  $[c - E(c)]$  respectively, the following relationships hold:

$$(6) \quad \text{var}(a^2) = 4E^2(a) \text{var}(a) - \text{var}^2(a) + 4E(a) E(\Delta a)^3 + E(\Delta a)^4,$$

$$(7) \quad \text{var}(c^2) = \text{analogous to (6) above},$$

$$(8) \quad \begin{aligned} \text{var}(ac) &= E^2(c) \text{var}(a) + E^2(a) \text{var}(c) + 2E(a) E(c) \text{cov}(a, c) \\ &+ E[(\Delta a)(\Delta c) - \text{cov}(a, c)]^2 + 2E(a) E[(\Delta a)(\Delta c)^2] \\ &+ 2E(c) E[(\Delta a)^2(\Delta c)], \end{aligned}$$

$$(9) \quad \begin{aligned} \text{cov}(a^2, c^2) &= 4E(a) E(c) \text{cov}(a, c) + E[(\Delta a)^2(\Delta c)^2] \\ &+ 2E(a) E[(\Delta a)(\Delta c)^2] + 2E(c) E[(\Delta a)^2(\Delta c)] \\ &- \text{var}(a) \text{var}(c), \end{aligned}$$

$$(10) \quad \begin{aligned} \text{cov}(a^2, ac) &= 2E^2(a) \text{cov}(a, c) + 2E(a) E(c) \text{var}(a) \\ &+ 3E(a) E[(\Delta a)^2(\Delta c)] + E(c) E(\Delta a)^3 \\ &+ E[(\Delta a)^3(\Delta c)] - \text{var}(a) \text{cov}(a, c), \end{aligned}$$

$$(11) \quad \text{cov}(c^2, ac) = \text{analogous to (10) above}.$$

The terms in (6) through (11) directly attributable to variability in the demand intercept alone are the terms in (6), the first term in (8) and the second and fourth terms in (10). Analogously, the terms directly attributable to variability in the supply intercept alone are the terms in

(7), the second term in (8) and the second and fourth terms in (11). The remaining terms are attributable to inseparable interaction between variability in the demand and supply intercepts.

Hence, the variance of  $GR$  is attributable to a demand effect ( $DE$ ), a supply effect ( $SE$ ) and an interaction effect between demand and supply ( $I$ ). In particular:

$$(12) \quad \text{var}(GR) = DE + SE + I.$$

Furthermore,  $DE$  and  $SE$  can be expressed as the sum of three components:

$$(13) \quad DE = D_1 + D_2 + D_3,$$

$$\begin{aligned} \text{where } D_1 &= \text{var}(a)[4E^2(a)\Pi_1^2 + (\Pi_1 + \Pi_2)^2E^2(c) \\ &\quad - 4\Pi_1(\Pi_1 + \Pi_2)E(a)E(c)] - \text{var}^2(a)\Pi_1^2, \\ D_2 &= E(\Delta a)^3[4E(a)\Pi_1^2 - 2E(c)\Pi_1(\Pi_1 + \Pi_2)], \\ D_3 &= E(\Delta a)^4\Pi_1^2, \end{aligned}$$

and  $SE = S_1 + S_2 + S_3$  (defined analogously with  $D_1$ ,  $D_2$  and  $D_3$ ). The interaction term,  $I$  in (12), is defined as:

$$(14) \quad I = \text{var}(GR) - DE - SE.$$

The rationale for expressing  $DE$  as the sum of three components lies in the fact that  $D_1$  is attributable to the variance of the distribution of the historical set of demand intercepts,  $D_2$  to the skewness and  $D_3$  to the kurtosis. The same reasoning applies to the supply effect. In the context of linear models, the variances of  $a$  and  $c$  will be functions of the variances and covariances of the various shift variables they incorporate.

#### *Special cases*

There are some special cases in which the formula for the variance of  $GR$ , and hence the expressions for the demand, supply and interaction effects, simplify. The substance of these special cases is reported here. Interested readers can determine the revised expressions for the demand, supply and interaction terms by proceeding in a manner similar to that shown in the previous section.

*Perfectly inelastic supply.* This case is of obvious importance in the context of agriculture. Since  $k_2$  in equation (3) is zero in this case,  $\Pi_1$  equals zero and components 1, 4 and 5 in equation (5) vanish. Therefore,

$$\text{var}(GR) = \Pi_2^2 \text{var}(c^2) + \Pi_2^2 \text{var}(ac) - 2\Pi_2^2 \text{cov}(c^2, ac),$$

where  $\Pi_2 = 1/k_1$  (since  $k_2 = 0$ ).

*Perfectly elastic demand.* This is also important in the context of Australian agriculture—the export demand for most products is taken to be perfectly elastic. In this case the price can be taken as pre-determined ( $P^*$ ). That is:

$$\begin{aligned} P_t &= P_t^* && \text{(demand),} \\ Q_t^s &= c + k_2 P_t && \text{(supply),} \end{aligned}$$

$$GR_t = cP_t^* + k_2P_t^{*2},$$

and  $\text{var}(GR) = \text{var}(cP^*) + k_2^2 \text{var}(P^{*2}) + 2k_2 \text{cov}(cP^*, P^{*2}).$

This expression can be rewritten in terms of the variances of  $c$  and  $P^*$ , their covariance and higher order interaction terms. In this case demand variability could be called price variability.

*Perfectly elastic demand—perfectly inelastic supply.* This would correspond to the situation in which a predetermined quantity,  $Q^*$ , is sold at a predetermined price,  $P^*$ . Hence:

$$GR_t = P_t^* Q_t^*,$$

and  $\text{var}(GR) = \text{var}(P^* Q^*).$

This variance can be written in terms of the variances of  $P^*$  and  $Q^*$ , their covariance and higher order interaction terms. It is precisely the variance one would analyse using the Burt and Finley procedure. That is, one can view their procedure as partitioning the variance of  $GR$  into components due to supply ( $Q^*$ ) and demand ( $P^*$ ) shifts, if one assumes that demand is perfectly elastic and supply perfectly inelastic. Alternatively, if one made the less realistic assumptions of perfectly inelastic demand and perfectly elastic supply, their procedure, in effect, partitions the variance of  $GR$  into demand variability ( $Q^*$ ) and supply variability ( $P^*$ ).

*Several markets.* The aggregate market for many commodities can be divided into submarkets, such as domestic and export. One could partition the variance of  $GR$  from the aggregate market in two steps. First, the variance of the aggregate market  $GR$  could be partitioned into components due to the variances and covariances of the  $GR$  received from the individual submarkets. Second, the variances of the  $GR$  received from the submarkets could be partitioned into demand, supply and interaction components.

### *Shortcomings*

Implementation of the procedure requires estimates of the demand and supply functions—save for the special cases of perfectly elastic or perfectly inelastic specifications.<sup>5</sup> Burt and Finley's procedure does not entail econometric estimation—save for the fitting of trend lines if one opts for detrended data. The additional computational effort is the cost of using an analytically-based procedure.

Linear demand and supply functions are assumed. No more justification for this can be offered than is offered by many other authors of applied econometric studies. The assumption of constant slopes over time could probably be relaxed using dummy variables, but the additional complexity this might add to the formulae has not yet been determined.

In so far as the procedure is applied to estimated demand and supply functions, it provides an *approximation* to the components of the vari-

<sup>5</sup> For simplicity, 'net' intercept terms were included in equations (2) and (3). In practice, one would ensure that the econometric specification of these functions satisfied identification requirements.

ance of *GR*. Viewing actual *GR* as consisting of a component due to systematic forces (represented by the estimated functions) and a random component, the procedure partitions the variance of the former.

The procedure might seem to involve a conceptual problem akin to one mentioned in relation to Burt and Finley's procedure. The composite term labelled 'demand effect' depends upon supply parameters ( $E(c)$  and  $k_2$ ). The same applies to the term labelled 'supply effect'. On reflection, however, this is how things ought to be. The influence of demand (supply) shifts on the variance of *GR* depends on the nature of the supply (demand) function. A simple case is that of constant unitary elastic demand—if such a function remained stable, the variance of *GR* would be zero irrespective of the nature of supply shifts.

#### *Comparison with Powell's analysis*

Conceptually, the procedure outlined here is similar to a procedure used by Powell (1960) in analysing income variability in the wool industry. In essence, Powell traced income variability back to production instability and demand instability. His procedure was based on assumptions of a constant-elasticity demand function and a perfectly inelastic supply function. There seems to be little difference between the two techniques with respect to computational effort. The basis for selecting between the two procedures would presumably be on the basis of the assumptions regarding the nature of the supply and demand functions. While Powell assumed a perfectly inelastic short-run supply function, such a function constitutes a 'special case' in the procedure outlined here.

#### *Application*

The Bureau of Agricultural Economics is developing a quarterly econometric model of the Australian beef industry for the simulation of policy proposals, including stabilisation policies (Longmire and Main 1977, p. 14). In view of recent debates concerning stabilisation for the beef industry, it is appropriate to demonstrate the procedure with reference to this industry. The BAE kindly allowed the author to use the model for this purpose.<sup>6</sup>

#### *Summary of demand and supply model*

The focus of the model is on the quarterly deflated saleyard price of beef, 1962-76. It has a simultaneous demand component encompassing various identities and behavioural equations for saleyard price, the marketing margin, per caput beef consumption, exports of beef to the United States and the price of exports in markets other than the United States. The supply component is recursive and contains inventory and slaughter equations for cows and heifers, calves, and steers and bulls. The quarterly supply function is assumed to be perfectly inelastic, quarterly supply being determined by inventory and slaughter decisions in

<sup>6</sup> It should be emphasised that the BAE officers regard the model as being still in the development stage. The author regarded it as sufficient for the present purpose.



previous quarters. In other words, the model specification fits the first 'special case' discussed in the subsection above relating to perfectly inelastic supply.<sup>7</sup>

The solved reduced forms corresponding to the estimated (two-stage least squares) demand structure were used to 'estimate' the historical values of the endogenous variables.<sup>8</sup> Some simple tests of the degree of correspondence between 'estimated' and actual historical values are shown in Table 2.

The results for the variables measuring exports to the United States and exports to other markets are disappointing. In the case of the

TABLE 2  
*Tests of 'Estimation Ability' for Simultaneous Demand Submodel using Solved Reduced Forms<sup>a</sup>*

Endogenous variables	Simple correlation: actual vs 'estimated'	Average percentage error <sup>b</sup>	Number of errors exceeding 10 per cent <sup>c</sup>
1. Saleyard price of beef	0.96	7.3	15
2. Marketing margin	0.85	4.4	1
3. Retail price of beef <sup>d</sup>	0.98	1.8	0
4. Per caput beef consumption	0.91	6.8	13
5. Total beef consumption <sup>d</sup>	0.94	6.8	13
6. Exports to the United States	0.64	20.9	38
7. Exports to other markets <sup>d</sup>	0.74	163.2	54
8. Price of beef in other markets	0.68	13.7	30
9. Saleyard revenue <sup>d</sup>	0.97	7.3	15

<sup>a</sup> Unlike the ordinary least squares reduced forms, the solved reduced forms do not minimise the error sum of squares between actual and estimated values. Rather, they are derived from the estimated structure.

<sup>b</sup> Shows the average value of the percentage errors ignoring the signs of the errors.

<sup>c</sup> Maximum number possible was 57 (the number of observations used in estimation).

<sup>d</sup> Estimated from an identity in the econometric model.

<sup>7</sup> It may seem odd that a 'special case' example is used to demonstrate the procedure. However, although referred to as a 'special case', the perfectly inelastic supply specification has wide applicability in the context of agriculture. For applications of the procedure to cases where supply is not perfectly inelastic, see Piggott (1977).

<sup>8</sup> Longmire and Main (1977) presented 'preliminary' estimates of the model in the sense that the model specification was tentative and the estimates of the simultaneous demand submodel were obtained using ordinary least squares. The author obtained two-stage least squares estimates of the demand submodel. Generally speaking, these differed only marginally from the ordinary least squares estimates.

latter, this may be due in large part to the fact that the model 'estimates' this quantity as a residual. It should also be remembered that the test results in Table 2 relate to the solved reduced forms (see footnote (a), Table 2). The model performs reasonably well with respect to the estimation of saleyard revenue (Table 2) and the estimates of price flexibility of demand (saleyard) and elasticities of retail demand.<sup>9</sup>

The two-stage least squares estimate of the crucial saleyard-level demand equation is as follows (ratios of regression coefficients to their standard errors in parentheses).<sup>10</sup>

$$\begin{aligned}
 PBS_t = & -13.648 - 0.042QSB_t + 0.331PBUS_t \\
 & (-2.5) \quad (-6.5) \quad (6.0) \\
 & + 0.324PBOM_t + 0.189PBR_t, \\
 & (5.9) \quad (3.0)
 \end{aligned}$$

where  $PBS_t$  = deflated quarterly saleyard price of beef (c/kg carcass weight),

$QSB_t$  = domestic production of beef and veal (kt carcass weight),

$PBUS_t$  = deflated price of Australian beef in the United States (cA/kg),

$PBOM_t$  = deflated average price of exports to other markets (cA/kg f.o.b.),

$PBR_t$  = deflated domestic retail price of beef (c/kg carcass weight).

#### *Decomposition results*<sup>11</sup>

The variance of actual deflated saleyard revenue can be written as:

$$(15) \quad \text{var}(GR) = \text{var}(GRE) + \text{var}(U) + 2\text{cov}(GRE, U),$$

where  $GR$  = actual gross revenue,

$GRE$  = gross revenue estimated from econometric model,

$U$  =  $GR - GRE$ .

The three components on the R.H.S. of (15) account for 78 per cent, 7 per cent and 15 per cent, respectively, of  $\text{var}(GR)$ . In other words, reliance on the econometric model entails ignoring 22 per cent of the variance of actual gross revenue and directing attention to that component (78 per cent) attributable to the systematic forces captured by the model.

The components of the variance of estimated gross revenue, expressed as a percentage of  $\text{var}(GRE)$ , are as follows: demand effect ( $DE$ ), 75 per cent; supply effect ( $SE$ ), 47 per cent; and negative interaction ( $I$ ), 22 per cent. Current supply is predetermined by (among other things)

<sup>9</sup> The estimated price flexibility of demand at the saleyard level was  $-0.27$  while the various elasticities of demand at retail were  $-1.53$  (own price),  $0.39$  (price of shipment) and  $0.49$  (income). These are very close to the estimates obtained by Longmire and Main (1977).

<sup>10</sup> Estimates of the complete model (omitted to conserve space) are available from the author.

<sup>11</sup> The author will make available on request the various data involved in the calculations underlying these results.

previous quarterly prices. In turn, these prices are determined, in part, by demand in the same quarter. Hence, supply fluctuations are linked with demand fluctuations in previous quarters. In this sense the supply effect may be somewhat overstated and the demand effect somewhat understated.

Nevertheless, the results suggest that, over the period concerned, variability in quarterly demand and in quarterly supply were *both* important contributors to the variance of quarterly saleyard revenues. Even though the demand effect is numerically the largest, the supply effect is certainly not insignificant. Furthermore, there was some tendency for demand and supply shifts to have offsetting effects upon  $\text{var}(GRE)$ .

By virtue of the model specification, the demand effect is solely attributable to the *variance* of the net demand intercept. ( $D_2$  and  $D_3$  in equation (13) are zero because  $\Pi_1$  is zero.) Three components stand out as important contributors to this variance (Table 3): the variance in the price of Australian beef in the United States and the covariances of this price with the price of beef in other export markets and with the domestic retail beef price. The variance of the domestic retail beef price makes only a relatively small contribution.

The three components of the supply effect, expressed as percentages of  $\text{var}(GRE)$ , are 56.8 per cent ( $S_1$ ), -11.3 per cent ( $S_2$ ) and 1.7 per cent ( $S_3$ ). In other words, the supply effect is dominated by the *variance* of the net supply intercept (which, because of the perfectly inelastic supply assumption, corresponds to the variance of the domestic production of beef and veal). The relatively small 'skewness' effect of the distribution of values for the supply intercept served to reduce the overall supply effect.

Domestic production of beef and veal can be separated into production of veal, production of beef from cows and heifers and production of beef from steers and bulls. The variance in the net supply intercept

TABLE 3  
*Percentage Components of the Variance of the Net Intercept of the  
Saleyard-level Demand Function*

Component <sup>a</sup>	Percentage of net intercept variance
var ( <i>PBUS</i> )	22.5
var ( <i>PBOM</i> )	11.4
var ( <i>PBR</i> )	6.4
2 cov ( <i>PBUS</i> , <i>PBOM</i> )	27.1
2 cov ( <i>PBUS</i> , <i>PBR</i> )	20.0
2 cov ( <i>PBOM</i> , <i>PBR</i> )	12.5
Total	100.0

<sup>a</sup> *PBUS* denotes the price of Australian beef in the United States; *PBOM* denotes average price of exports to other markets; *PBR* denotes domestic retail beef price; var denotes variance and cov denotes covariance.

can be decomposed into components due to the variances and covariances of these components. The variance in the production of beef from steers and bulls emerges as the dominant source of variance in total domestic production (46 per cent), followed by the covariance between the production of beef from steers and bulls and the production of beef from cows and heifers (32 per cent). Other components of the variance of total domestic production are relatively insignificant. The causes of variability in each of the components of total domestic production could be traced using the estimated inventory and slaughter equations in the BAE model.

Viewing the results overall, they are consistent with the author's prior expectations concerning the causes of variability in industry-level beef revenues—namely, volatile demand *and* volatile supply. Export demand has generally been regarded as an important source of instability and this shows through in the results. Recall that Houck had difficulty in sorting out the relative importance of price and quantity instability in causing income instability (although he was concerned with annual incomes). Presumably, these results would lead Houck to argue against market interference to stabilise beef prices. The results reported here lead the author to a similar conclusion: with volatile supply and demand, the intervention authority would surely require substantial foresight and an atypical run of good fortune for longevity.

### References

- Bohrnstedt, G. W. and Goldberger, A. S. (1969), 'On the exact covariance of products of random variables', *Journal of the American Statistical Association* 64 (328), 1439-42.
- Burt, O. M. and Finley, R. M. (1968), 'Statistical analysis of identities in random variables', *American Journal of Agricultural Economics* 50(3), 734-44.
- , ——— (1970), 'On the statistical analysis of identities: Reply', *American Journal of Agricultural Economics* 52(1), 155-6.
- Goldberger, A. S. (1970), 'On the statistical analysis of identities: Comment', *American Journal of Agricultural Economics* 52(1), 154-5.
- Goodman, L. A. (1960), 'On the exact variance of products', *Journal of the American Statistical Association* 55(292), 708-13.
- Harris et al. (1974), *Rural Policy in Australia—Report to the Prime Minister by a Working Party*, AGPS, Canberra.
- Houck, J. P. (1973), 'Some aspects of income stabilization for primary producers', *Australian Journal of Agricultural Economics* 17(3), 200-15.
- Industries Assistance Commission (1978), *Rural Income Fluctuations*, Report No. 161, AGPS, Canberra.
- Kendall, M. G. and Stuart, A. (1963), *The Advanced Theory of Statistics*, Vol. 1, 2nd edn, Charles Griffin, London.
- Longmire, J. L. and Main, G. W. (1977), An econometric model of the Australian beef industry. Paper contributed to the 22nd Annual Conference, Australian Agricultural Economics Society, Sydney.
- Motha, G., Sheales, T. C. and Saad, M. M. (1975), 'Fluctuations in Australian rural production and prices—some implications for support policies', *Quarterly Review of Agricultural Economics* 28(1), 38-48.
- Piggott, R. R. (1977), Decomposing the variance of gross income: a possible alternative to the Burt-Finley procedure. Paper contributed to the 22nd Annual Conference, Australian Agricultural Economics Society, Sydney.
- Powell, A. (1960), 'Production and income uncertainty in the wool industry: an aggregative approach', *Australian Journal of Agricultural Economics* 4(1), 86-96.

- Tomek, W. G. (1969), *Stability for Primary Products: Means to What Ends?*, Occasional Paper No. 28, USAID Prices Research Project, Department of Agricultural Economics, Cornell University.
- Turnovsky, S. J. (1974), 'Price expectations and the welfare gains from price stabilization', *American Journal of Agricultural Economics* 56(4), 706-16.