The Effects of Tenancy and Risk on Cropping Patterns: A Mathematical Programming Analysis

By Donald Baron

Abstract

Most analyses of allocative efficiency under different forms of agricultural tenure—share tenancy, fixed cash tenancy, and owner cultivation—employ single product models of production. These models show that risk sharing encourages share tenants to produce as much as or more than equally risk-averse owner-operators and cash tenants. However, when risk and risk aversion are introduced into multiproduct linear programming models, relative allocative efficiency under share tenancy may decline. The result depends on the relative production costs and the relative risk premiums of the different products.

Keywords

Farm tenancy, risk, allocative efficiency

The 1978 Census of Agriculture reports that nearly 40 percent of all land in farms in the United States is rented and that approximately 86 percent of this rented acreage is owned by nonoperator landlords. Studies by Reinsel and Johnson (28), Johnson (21), and Reiss (29, 30, 31) suggest that the contributions landlords make to the management of the land they rent out has declined in recent years. An increasing percentage of tenants simultaneously rents several tracts of land from several landlords rather than a single tract from a single landlord. Contributions made by individual landlords to total rented acreage operated by tenants have declined, as have opportunities and incentives for landlords to contribute entrepreneurial skills and to help finance production costs (28, 29, 30, 31). Tenants have expressed greater willingness to operate without assistance from landlords. Many have indicated they prefer arrangements that provide them primary managerial control over farming operations so that they can coordinate their owned and rented resources effectively.

The increasing frequency of rental arrangements that allocate most managerial responsibilities to tenants has renewed interest in an issue that Alfred Marshall first raised: Is the efficiency of resource allocation lower on tenant-operated than on owner-operated farmland? Many economists have argued that tenants who operate with little managerial or cost-sharing assistance from nonoperator landlords, but who pay landlords fixed cash rents, will farm just as efficiently as will owner-operators. However, if these tenants pay share rents, they will farm less efficiently than will either owners or cash tenants.

Most analyses of farm tenancy have employed single-input, single-product models of production. Owner-operators and fixed cash tenants receive the entire value of the marginal product from each unit of input, and they employ inputs up to an amount at which marginal value product equals marginal input cost. Share tenants pay a share of the marginal value product as rent. Share tenants maximize their own net revenues according to these models, by producing greater amounts of lower cost products and smaller amounts of higher cost products than owner-operators and cash tenants produce. Because output combinations chosen by owners and cash tenants always maximize total net revenues for the entire farm, the discrepancy measures the loss of efficiency attributable to the share rental arrangement within the linear programming model.

Other studies of farm tenancy have applied multi-input, multiproduct, linear programming models. Within a range of output-price ratios, share tenants maximize their own net revenues, according to these models, by producing greater amounts of lower cost products and smaller amounts of higher cost products than owner-operators and cash tenants produce. Because output combinations chosen by owners and cash tenants always maximize total net revenues for the entire farm, the discrepancy measures the loss of efficiency attributable to the share rental arrangement within the linear programming model.

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1 Many economists recommend that landlords share variable costs with their share tenants in the same proportion that they share output. As the marginal value received by the tenants is greater than the share of input cost they must pay, they will be encouraged to employ as much of the input as owners and cash tenants employ.
Incorporating Risk and Risk Aversion into the Single- and Multiproduct Models

Previous multiproduct programming models of farm tenancy have either ignored production risks or have assumed that all farm operators are risk neutral. They have not considered how risk and risk aversion might affect predictions of tenancy-related differences in farm planning.

A few presentations of single-input, single-product models have addressed this question. Sutinen (34, 35) and Baron (5) showed that, although production by risk-averse operators declines as risk levels increase, this reduction is smaller under share tenancy than under owner operation and cash tenancy. Risk-sharing increases share tenants' production relative to owner-operator and cash tenants' production. The excess of owner-operator/cash tenants' production over share tenants' production that occurs under risk neutrality is, therefore, always reduced, or possibly eliminated, under risk and risk aversion (5, 19, p 24). If the negative effect of marginal product sharing under share tenancy is overcome through landlord supervision or through contractual specification of tenants' obligations to employ variable inputs, risk sharing may encourage share tenants to produce even more than owners and cash tenants produce (35, pp 617-19).

However, risk sharing may not have the same unequivocally positive impact on the efficiency of share tenancy relative to owner operation and to cash tenancy when it is examined within a multi-input, multiproduct programming model. To introduce risk and risk aversion into such a model, several assumptions need to be made about the attitudes of farm operators toward risk and toward changes in risk levels associated with changes in production levels. First, assume operators measure risk as a dispersion of gross revenue per acre of each crop around expected gross crop revenue per acre. Moreover, operators assign a cost, or risk premium, to this risk level. The premium equals some constant percentage of expected gross crop revenue per acre. Thus, the marginal rate of increase in the risk premium that results from an increase in production will be constant at all levels of production. Risk premiums for each crop will be included in the objective function as constant costs per acre of output, so that the function will remain linear.

Price maps for alternative output combinations can display the results of parametric programming of the objective function. Two sets of maps are presented in this study. The first is derived for a risk-neutral owner-operator and cash tenant and for a risk-neutral share tenant through maximization of the standard risk-neutral objective function presented in previous studies. This set is compared with a second set of price maps derived for risk-averse owner-operators and cash tenants, and for risk-averse share tenants through maximization of the risk-adjusted or certainty-equivalent objective function defined here. The comparison shows that the introduction of risk and risk aversion into the programming model may either increase or decrease the likelihood that an inefficient share tenant plan will be chosen, depending on the relative prices, relative input costs, and relative risk premiums per acre of soybeans and corn. That is, contrary to the conclusion of other studies reviewed above, risk and risk aversion may either augment or offset the negative effect of product-sharing on the efficiency of share tenancy relative to owner operation and cash tenancy.

Price Mapping Under Risk Neutrality

The programming model considers alternative production plans on a 200-acre farm in Illinois that produces soybeans or corn. Cost estimates for this farm are available from the Firm Enterprise Data System (FEDS) developed by the Economic Research Service and Oklahoma State University (12).

The most recent FEDS estimates, based on data from the 1977 crop year, suggest that variable costs per acre of corn are approximately twice as high as those for soybeans. These estimates are derived from a production function which utilized approximately 30 inputs. For convenience, the programming model used here assumes that land is the only resource in limited supply.

A simplified linear programming model of farm planning by a risk-neutral operator can be represented by the equation below.

Maximize

\[ I = (rP_1 - c) a_1 + (rP_2 - hc) a_2 \]

subject to

\[ a_1 + a_2 = 200 \text{ acres} \]

where \( I \) is net farm income, \( P_1 \) and \( P_2 \) are gross revenues per acre of soybeans and corn, respectively, \( r \) is the share rental rate, which equals 1 for owner-operators and cash tenants and some fraction less than 1 for share tenants, \( a_1 \) and \( a_2 \) are total soybean and corn acreages, \( c \) and \( hc \) are costs per acre of soybeans and corn, and \( h \) is the ratio of per acre costs of corn to soybeans. The constraint equation indicates that 200 acres of land are available.

Parametric variation of the coefficients, \( P_1 - c \) and \( P_2 - hc \), defines price maps which indicate the combinations of soybean and corn acreages that yield maximum profits for farm operators at any given ratio of gross soybean revenue to gross corn revenue. As land is the sole resource constraint, only two plans may be optimal at any given ratio: "soybeans only" or "corn only." The price map for the risk-neutral owner operator and cash tenant is represented by AB in figure 1 in this situation.
Price Map for Risk-Neutral Farmers

Gross revenue per acre of soybeans, $p_1$

Cash tenants and owners produce corn

All operators produce corn

Share tenants produce soybeans

$P_1 = P_2 - c(h - 1)/r$

$P_1 - c = P_2 - hc$

$r = 1$ The axes measure gross revenues per acre of corn and soybeans. For all per-acre revenue combinations which fall to the left of AB, owner-operators and cash tenants receive higher net revenues per acre from soybeans than from corn. Thus, they produce soybeans only. For all combinations to the right of AB, they receive more from corn than from soybeans, and they produce corn only.

For risk-neutral share tenants, assume that $r$ equals 0.6 and that the tenants receive no cost sharing from landlords, so that their variable costs remain at $c$ per acre of soybeans and at $hc$ per acre of corn. The share tenants’ price map is defined by line CD, which is the new boundary between the soybeans-only and corn-only farm plans. Revenue combinations within the area bounded by AB and CD define a different optimal plan for share tenants than for owner-operators and cash tenants. Combinations falling to the left of AB and to the right of CD result in the adoption of the same plan by all operators.

Maps AB and CD define ratios of gross corn to gross soybean revenue at which net soybean and corn revenues are equal. CD is further right than AB, indicating that share tenants require a higher ratio of corn revenue to soybean revenue before they will switch from soybeans to corn. To determine why CD is further right, observe that the equality between net soybean and corn revenues per acre is

$$P_1 - c = P_2 - hc$$

which implies that

$$P_1 = P_2 - c(h - 1)/r$$

Under owner operation and cash tenancy, $r = 1$, equation (1) indicates that gross revenue per acre of corn ($P_2$) must exceed gross revenue per acre of soybeans ($P_1$) at all points on AB by exactly the amount that cost per acre of corn exceeds cost per acre of soybeans ($c(h - 1)$). Under share tenancy, $r = 0.6$, equation (1) indicates that gross corn revenues per acre must exceed gross soybean revenues per acre at all points on CD by approximately $1/0.6$ (or 1.67) times the cost differential.

The price maps also indicate why allocative efficiency is lower under share tenancy than under owner operation and cash tenancy. Net soybean and corn revenues received by owner-operators and cash tenants, $P_1 - c$ and $P_2 - hc$, are identical to corresponding net crop revenues generated for the whole farm. Therefore, the farm plan that owner-operators and cash tenants select always maximizes total net farm revenues. Net revenues received by share tenants, $0.6P_1 - c$ and $0.6P_2 - hc$, are less than the net crop revenues received by the entire farm. Where the share tenant’s farm plan differs from the owner-operator and cash tenant’s farm plan, as is the case within region ABCD, total net farm revenues generated by the share tenant’s plan will necessarily be less than the maximum possible. As an example, consider point F. Here, $P_1 = 230$, $P_2 = 310$, $c = 60$, and $h = 2$. The share tenant’s plan—200 acres of soybeans—yields total net farm revenues of $34,000. A maximum possible net revenue of $38,000 is generated by the owner-operator and cash tenant’s plan—200 acres of corn.

Price Mapping Under Risk and Risk Aversion

The introduction of risk and risk aversion into the linear programming model requires adjustments in the objective function. Gross revenues per acre are uncertain, expected values. The operator’s reaction to risk is measured as reductions in expected gross soybean and corn revenues per acre. Assume that these reductions, or risk premiums, equal...
constant percentages of expected gross revenues. Thus, the
risk premium from 1 acre of soybeans equals the given per­
centage rate multiplied by the expected gross soybean re­
venue per acre.

With the risk premiums, the objective function becomes

\[ I = rP_1 - c - r\theta_1P_1 + rP_2 - hc - r\theta_2P_2 \]  

(2)

Risk premiums per acre of soybeans and corn are \( \theta_1 P_1 \) and \( \theta_2 P_2 \), respectively. \( \theta_1 \) and \( \theta_2 \) are the risk premium coefficients, they indicate the percentages of expected gross soy­
bean and corn revenues deducted to account for risk.

The assumption that risk premiums per acre remain constant
as gross crop revenues per acre and total acreages increase may
appear restrictive. Risk premiums per acre may in fact be
highly correlated with gross crop revenues. Moreover, if covar­
iance between gross soybean and gross corn revenue is nonzero,
risk premiums per acre may vary as output ratios vary.

However, if risk premiums per acre are expressed as func­
tions of gross revenues and as functions of output ratios,
the objective function will be nonlinear. This nonlinearity
will make the analysis more complicated, but little will be
 gained conceptually. The linear objective function of equa­
tion (2) is sufficiently complex to establish the fundamental
point of the analysis—namely, that share tenancy may com­
pare even less favorably with owner operation and cash ten­
ancy under risk and risk aversion than under risk neutrality.

Parametric variation of certainty-equivalent net soybean and
corn revenues per acre, \( rP_1 - c - r\theta_1P_1 \) and \( rP_2 - hc - r\theta_2P_2 \), defines price maps for risk-averse farm operators. Two sets of
these maps are derived. In the first, \( \theta_1 \) is assumed to be
greater than \( \theta_2 \), indicating that operators consider soybeans
riskier than corn. In the second, \( \theta_2 \) is greater than \( \theta_1 \),
indicating that corn is riskier than soybeans.

**Case 1: \( \theta_1 > \theta_2 \)**

Whereas maps AB and CD (fig 1) equate expected net soy­
bean and corn revenues per acre, price maps for risk-averse
farm operators (fig 2) equate certainty-equivalent net soy­
bean and corn revenues per acre. When \( \theta_1 > \theta_2 \), equa­

cency is satisfied on map A'B' for owner-operators and cash tenants
and on map C'D' for share tenants. For purposes of com­
parison, maps AB and CD are also shown in figure 2. To the
right of A'B', risk-averse owners and cash tenants receive
higher certainty-equivalent net revenues from corn than from soybeans, therefore, they produce only corn. To the left
of A'B', they receive more from soybeans than from corn
and produce only soybeans. Similarly, to the right (left)
of C'D', share tenants receive higher certainty-equivalent
revenues from corn (soybeans) than from soybeans (corn),
therefore, they produce only corn (soybeans) at all points.

**Figure 2**

**Price Map for Risk-Averse Farmers When Soybeans
are Considered Riskier than Corn**

Gross revenue per acre of soybeans, \( P_1 \)

\[ p' = \frac{p_2(1-\theta_2)}{(1-\theta_1)} - \frac{c(h-1)}{r(1-\theta_1)} \]  

(3)

This compares with a value of

\[ p_1 = p_2 - \frac{c(h-1)}{r} \]  

(4)

on maps AB and CD. Note that when \( \theta_1 > \theta_2 \), the slope of
A'B' and C'D' (1 - \( \theta_2 /1 - \theta_1 \)) is slightly greater than the slope
of AB and CD, which equals 1. However, the vertical inter­
ccept of A'B' and C'D' (\( -c(h-1)/r(1-\theta_1) \)) is smaller than
that of AB and CD (\( -c(h-1)/r \)). It follows that \( p'_1 \) in equa­
tion (3) is greater than, equal to, or less than \( p_1 \) in equa­
tion (4) and that A'B' and C'D' therefore lie above, intersect
with, or fall below AB and CD, depending on whether

\[ \frac{p_2(1-\theta_2)}{(1-\theta_1)} - \frac{c(h-1)}{r(1-\theta_1)} > c(h-1)/r \]  

(5)

or, to simplify, on whether

\[ p_2 > \frac{(\theta_1 - \theta_2)(c(h-1)/r)}{\theta_1 - \theta_2} \]  

(6)
The more $\theta_1$ exceeds $\theta_2$ and the smaller $h$ is, the greater is the value of $p_2$ relative to $(\theta_1/\theta_1 - \theta_2)(c(h - 1)/r)$ and the greater are the values of $p_1'$ on A'B' and C'D' relative to $p_1$ on AB and CD, respectively.

Suppose the terms in equation (6) are limited to a range of values within which $p_2 > \theta_1(p_2 - c(h - 1)/r)$ and, thus, $p_1' > p_1$. For example, assume that input costs per acre, $c$ and $hc$, approximate the recent FED estimates $60$ for soybeans and $120$ for corn. Also assume that $\theta_1$ exceeds $\theta_2$ significantly. Reasonable values might be $\theta_1 = 0.04$ per $\$1$ of expected gross soybean revenue, and $\theta_2 = 0.01$ per $\$1$ of expected gross corn revenue. It follows from equation (6) that $p_1' > p_1$ for all $p_2 > (4/3)(60/4)$. Under owner operation and cash tenancy, $p_1$ on A'B' will exceed $p_1$ on AB, and thus A'B' will lie above AB (fig. 2) for all cases where $p_2 > \$80$. Under share tenancy, with $r = 0.6$, $p_1'$ on C'D' will exceed $p_1$ on CD, and thus C'D' will lie above CD, for all cases where $p_2 > \$133.33$. Furthermore, given that during the 1974-80 period, the minimum reported gross corn revenue in Illinois was about $\$220$ per acre, $p_1' > p_1$ is the only relevant case. Therefore, figure 2 shows only those portions of maps A'B' and C'D' that lie above AB and CD.

For an intuitive explanation of why A'B' and C'D' lie above AB and CD, consider the equality of expected net crop revenues that occurs on AB and CD. Recall that this equality implies that $p_1 = p_2 - c(h - 1)/r$. The risk premium per acre of soybeans on AB and CD is $\theta_1 p_1 = \theta_1(p_2 - c(h - 1)/r)$.

Suppose $c = \$60$, $hc = \$120$, $\theta_1 = 0.04$, and $\theta_2 = 0.01$. Then $\theta_1 p_1$ is greater than $\theta_2 p_2$ (the risk premium per acre of corn) at all points on AB where $p_2 > \$80$ and at all points on CD where $p_2 > \$133.33$. At all such points on AB and CD, certainty-equivalent net revenues are lower for soybeans than for corn. For certainty-equivalent net crop revenues to be equal, as is the case on maps A'B' and C'D', expected net soybean revenue per acre must exceed expected net corn revenue per acre to offset the higher risk premium per acre of soybeans. Thus, for each combination of expected gross soybean and expected gross corn revenues per acre defined by a point on AB or CD, the point on A'B' or C'D' that defines the same gross corn revenue per acre must have a higher gross soybean revenue per acre.

We can also show that $p_1'$ on A'B' exceeds $p_1$ on AB by more than the amount that $p_1$ on C'D' exceeds $p_1$ on CD. That is, for any $p_2$, the shift from AB to A'B' exceeds the shift from CD to C'D'. If $p_1$ is expressed as in equation (4), then

$$p_1' = p_1 - p_2(\theta_1 - \theta_2)/((1 - \theta_1) - \theta_1(c(h - 1)/r)(1 - \theta_1))$$

which is greater under owner operation and cash tenancy, with $r = 1$, than under share tenancy, with $0 < r < 1$.

Suppose gross corn revenue is $\$320$ per acre, $\theta_1 = 0.04$, $\theta_2 = 0.01$, $r = 0.6$, and input costs per acre are again $\$60$ for soybeans and $\$120$ for corn. Gross soybean revenue then increases from $p_1 = \$260$ per acre on map AB to $p_1' = \$267.50$ per acre on map A'B', but only from $\$220$ per acre on map CD to $\$225.83$ per acre on map C'D'. The risk-induced shift from AB to A'B' is $(\$267.50 - \$260) - (\$225.83 - \$220) = \$1.67$ greater than the shift from CD to C'D'.

Thus, when $p_2$, $c$, and $hc$ have reasonable values and when $\theta_1$ exceeds $\theta_2$ by enough to ensure that equal expected net corn and soybean revenues per acre imply higher risk premiums per acre for soybeans than for corn, risk and risk aversion will affect farm planning under owner operation and cash tenancy more than under share tenancy. Share tenancy's standing relative to owner operation and cash tenancy should also improve. Recall that, under risk neutrality, the crop plan chosen by owners and cash tenants is always optimal in the sense that it maximizes total expected net crop revenues for the entire farm. At all points to the left of AB, this optimal plan is $200$ acres of soybeans, at all points to the right, it is $200$ acres of corn.

To the left of A'B' and to the right of C'D' in figure 2, all risk-averse farmers do in fact choose optimal farm plans. Within region A'B'AB, however, only risk-averse share tenants choose the optimal plan—$200$ acres of soybeans, within ABC'D', only risk-averse owners and cash tenants choose the optimal plan—$200$ acres of corn.

Within A'B'AB, the risk-averse share tenant's plan is more profitable to the farm than the risk-averse owner and cash tenant's plan. This situation contrasts with the case of risk neutrality (fig. 1). Recall that the risk-neutral share tenant's plan can be equally profitable at best, but can never be more profitable than the owner and cash tenant's plan. Moreover, recall that within region ABCD, the risk-neutral share tenant's plan is less profitable to the farm than the risk-neutral owner and cash tenant's plan. Thus, ABCD is analogous to ABC'D', which encompasses an area where the risk-averse share tenant's plan is less profitable to the farm than the risk-averse owner and cash tenant's plan. Observe that ABC'D' is smaller than ABCD. That is, the number of soybean-corn revenue combinations that result in a suboptimal share tenant's plan is smaller under risk neutrality.

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* The objective function for risk-averse share tenants who receive proportionate cost sharing payments from landlords is $0.06(p_1a_1 - ca_1 - \theta_1 p_1a_1 + p_2a_2 - hca_2 - \theta_2 p_2a_2)$. Maximization of this function is equivalent to maximization of the risk-averse owner operator and cash tenant's objective function at all crop revenue combinations. Within ABA'B', the crop plan chosen by risk-averse share tenants who receive proportionate cost sharing payments as well as by risk-averse owners and cash tenants is less profitable to the farm than the plan chosen by risk-averse share tenants who receive no cost sharing payments. This result contradicts the claim of many economists (discussed in footnote 2) that proportionate cost sharing should always increase allocative efficiency under share tenancy.
and risk aversion than under risk neutrality. The probability that the revenue combination in any given year will result in a suboptimal share tenant's plan is also smaller under risk and risk aversion than under risk neutrality.

Therefore, when risk and risk aversion are incorporated into the programming model under the assumption that soybeans are riskier than corn ($\theta_1 > \theta_2$), share tenancy should compare more favorably with owner operation and with cash tenancy than it does in the absence of risk and risk aversion. The probability that the share tenant's plan will be less profitable to the farm than the owner-operator and cash tenant's plan should decline. The probability that the share tenant's plan will be more profitable should increase from zero to some positive value.

**Case 2: $\theta_2 > \theta_1$**

The results are much different if operators consider corn riskier than soybeans. For $\theta_2 > \theta_1$, parametric variation of certainty-equivalent net soybean and corn revenues per acre, $r_{p1} = c - t\theta_1 p_1$ and $r_{p2} = h(c - t\theta_2 p_2)$, defines price maps $A'B''$ and $C'D''$ (fig 3). On these maps, as on maps $A'B'$ and $C'D'$, certainty-equivalent net crop revenues are equal, therefore, the value of $p_1$ on $A'B''$ and $C'D''$, hereinafter $p_1''$, is

$$p_1'' = p_2(1 - \theta_2)/(1 - \theta_1) - c(h - 1)/r(1 - \theta_1) \quad (8)$$

**Figure 3**

*Price Map for Risk-Averse Farmers When Corn is Considered Riskier than Soybeans*

Gross revenue per acre of soybeans, $p_1$

<table>
<thead>
<tr>
<th>$p_1$</th>
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<td>$C'$</td>
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<td>$D'$</td>
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Note that with $\theta_2 > \theta_1$, the slope of $A'B''$ and $C'D''$, $(1 - \theta_2)/(1 - \theta_1)$, is slightly less than 1, the slope of $A'B$ and $CD$. Furthermore, the vertical intercept of $A'B''$ and $C'D''$ ($c(h - 1)/r(1 - \theta_1)$) is less than the intercept of $A'B$ and $CD$. Thus, $p_1''$ will fall below $p_1$, and $A'B''$ and $C'D''$ will fall below $AB$ and $CD$, for all $p_2$. Note also that the more $\theta_2$ exceeds $\theta_1$ and the larger $h$ is, the more $p_1$ exceeds $p_1''$ and the greater the gap is between the price maps, for any $p_2$. Furthermore, $p_1$ on $CD$ always exceeds $p_1''$ on $C'D''$ by more than $p_1$ on $AB$ exceeds $p_1''$ on $A'B''$. That is, the shift from $CD$ to $C'D''$ everywhere exceeds the shift from $AB$ to $A'B''$. If $p_1$ is expressed as in equation (4), then

$$p_1'' = p_2(1 - \theta_2)/(1 - \theta_1) + \theta_1 c(h - 1)/r(1 - \theta_1) \quad (9)$$

which is higher under share tenancy than under owner operation and cash tenancy.

For example, suppose gross corn revenue again equals $320 per acre, while $\theta_1 = 0.01$, $\theta_2 = 0.04$, and input costs per acre again equal $80$ for soybeans and $120$ for corn. Gross soybean revenue must then decline from $320$ on map CD to $209.29$ on map $C'D''$, but only from $260$ on map AB to $249.70$ on map $A'B''$. The risk-induced shift from CD to $C'D''$ is $0.41$ greater than the shift from AB to $A'B''$.

Thus, given any values of $p_1$, $p_2$, $c$, $h$, $\theta_1$, and $\theta_2$, if $\theta_2 > \theta_1$, risk and risk aversion will always affect farm planning more under share tenancy than under owner operation and cash tenancy. Moreover, the impact of risk and risk aversion on the comparative efficiency analysis, if $\theta_2 > \theta_1$, is quite different than the impact if $\theta_1 > \theta_2$. When $\theta_2 > \theta_1$, we compare $A'B''C'D''$ and $ABCD$ (fig 3). $A'B''C'D''$ defines combinations of expected gross soybean and corn revenues per acre where risk-averse owner-operators and cash tenants (but not risk-averse share tenants) choose the optimal, profit-maximizing plan—200 acres of corn. Region $ABCD$ defines gross revenue combinations where risk-neutral owners and cash tenants (but not risk-neutral share tenants) choose the optimal, profit-maximizing plan—200 acres of corn. Region $ABCD$ defines gross revenue combinations where risk-neutral owners and cash tenants (but not risk-neutral share tenants) choose the same optimal plan. Because the shift from CD to $C'D''$ exceeds the shift from AB to $A'B''$, the number of soybean-corn revenue combinations falling within $A'B''C'D''$ exceeds the number falling within $ABCD$. That is, the share tenant's farm plan is less profitable to the farm than the owner and cash tenant's plan at more revenue combinations under risk and risk aversion than under risk neutrality. Everything else being equal, the probability that the revenue combination that occurs in any given year will fall within $A'B''C'D''$ will exceed the probability that it will fall within $ABCD$.

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Note that for any $\theta_2 > \theta_1$, the excess of $\theta_2 P_2$ over $\theta_1 P_1$ will decline, and the gap between $A'B''$ and $AB$ and between $C'D''$ and $CD$ will narrow as $p_1$ and $p_2$ decline.
Thus, the probability that the share tenant will actually choose a suboptimal plan will also be higher under risk and risk aversion than under risk neutrality.

### Comparison of Crop Risks

This analysis has shown that the impact of risk and risk aversion on the comparative efficiency analysis presented by the programming model depends on the values of the risk premium coefficients assigned to the alternative crops. This result raises the question of which of the two sets of values assumed in this article better represents the attitudes of farm operators toward the riskiness of soybeans and corn.

One answer is suggested by estimates of the variability of gross soybean and corn revenue that can be derived from recent average annual yield and price data provided by USDA's Crop Reporting Board (37, 38, 39, 40, 41, 42, 43, 44). Let us take Illinois as an example. A sample of seven gross corn and soybean revenues per acre of Illinois cropland from the 1974-80 period indicates recent variations. The sample standard deviation was approximately $39 per acre for corn and $33 per acre for soybeans. Corn, therefore, exhibited greater variability than soybeans. Thus, a typical Illinois farm operator would likely consider corn as the riskier crop. The assumption that $\theta_2 > \theta_1$ appears to be more appropriate than the assumption that $\theta_1 > \theta_2$.

If so, the conclusion for case 2 applies. Share tenancy will likely compare less favorably with owner operation and cash tenancy under risk and risk aversion than under risk neutrality.

### Conclusion

Previous analyses of the comparative efficiency of different farm tenancies under risk and risk aversion have relied exclusively on single-input, single-output models. When risk and risk aversion are introduced into these models, the efficiency of share tenancy relative to owner operation and cash tenancy always increases.

When the comparative efficiency analysis is extended to multi-product programming models, however, the results can be quite different. Under risk neutrality, share tenancy compares unfavorably with owner operation and cash tenancy. Within a certain range of crop revenue combinations, share tenants choose a crop plan which is less profitable to the farm as a whole than the plan owner-operators and cash tenants choose.

To determine the impact of risk and risk aversion on this comparative efficiency analysis, we examined the implications of two different assumptions concerning reactions of farm operators to income variability. Case 1 assumed that farm operators assign a higher risk premium to $1 of gross soybean revenue than they assign to $1 of gross corn revenue, even though the variable production cost per acre of corn is higher than the cost of soybeans. The number of soybean-corn revenue combinations that call for a suboptimal share tenant's plan is then lower under risk and risk aversion than under risk neutrality. Some revenue combinations even result in a risk-averse share tenant's plan that is more profitable than the risk-averse owner-operator and cash tenant's plan. The probability that the share tenant's plan will be less profitable than the owner and cash tenant's plan is unequivocally lower under risk and risk aversion than under risk neutrality.

In case 2, risk and risk aversion were introduced under the assumption that the risk premium per $1 of gross corn revenue is higher than the risk premium per $1 of gross soybean revenue. The number of soybean-corn revenue combinations that result in a suboptimal share tenant's plan is now greater under risk and risk aversion than under risk neutrality. Therefore, the probability that the share tenant's plan in any year will be less profitable than the owner and cash tenant's plan is also greater under risk and risk aversion than under risk neutrality.

An empirical review of crop and yield data for Illinois suggests that the second of the two assumptions—that is, that the risk premium per $1 of corn is greater than the risk premium per $1 of soybeans—more accurately represents Illinois agriculture. As the analysis in case 2 appears unfavorable to share tenancy, a contrast between the single-input, single-output model and the multiproduct programming model becomes obvious. Although the presence of risk and risk aversion always improves share tenancy's standing relative to owner operation and cash tenancy under the first model, it probably weakened share tenancy's relative standing under the second model, at least for Illinois agriculture during the late seventies.

Mutiproduct models may generate different results for other regions and other time periods. Where the analysis in case 1 applies, share tenancy will compare more favorably with owner operation and cash tenancy under risk and risk aversion than under risk neutrality. When risk and risk aversion are relevant, the comparative efficiency of alternative farm tenancies can no longer be determined by theoretical considerations alone. Comparative efficiency can be determined only after one has assigned observed or reasonable values to the model parameters.

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4 It is conceivable, if improbable, that despite the greater number of revenue combinations within A' B' C' D' than within ABCD, the probability that the combination in any given year will fall within ABCD still exceeds the probability that it will fall within A' B' C' D'. The probability that the share tenant will choose an inefficient plan will then be lower under risk and risk aversion than under risk neutrality. The introduction of risk and risk aversion into the model will offset, rather than augment, the distortive effect of product-sharing on the relative efficiency of share tenancy. 
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