Decomposing Input Adjustments Under Price and Production Uncertainty

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Abstract

A decomposition of input adjustments for stochastic technologies is developed and applied to the case of actuarially fair production insurance. The decomposition consists of a pure-risk effect and an expansion effect which are analogous to the Hicks-Allen decomposition familiar from consumer theory.

Keywords: uncertainty, duality, state-contingent technology, input use

Duality applies under uncertainty. In particular, Chambers and Quiggin (1998) have shown that dual cost structures exist for the continuous, stochastic technologies most familiar to agricultural economists. Beyond merely demonstrating existence, however, this finding has important implications for the analysis of stochastic decisionmaking. Agricultural economists long have intensively studied decisionmaking by producers facing stochastic technologies. And yet, no commonly accepted body of 'stylized facts' exists for most truly interesting formulations of this problem. Some have even questioned the relevance of the cost minimization hypothesis for risk-averse decisionmakers (Pope and Chavas). More generally, apart from a number of results that have been established for trivially stochastic situations, e.g., price but not production uncertainty, there is no common agreement as to what one can expect from a risk-averse producer facing a stochastic world.

For example, almost nothing of consequence is known about how the input utilization of risk-averse farmers differs from that of risk-neutral farmers or about the closely related question of how input utilization responds to the provision of insurance or income support. Heuristically, one expects risk-neutral farmers to undertake riskier production activities that bring with them the promise of higher return. Similarly, one also expects that insuring farmers or providing them government income support encourages them to undertake riskier production activities. If the worst happens, they always have the government or the insurer to fall back on. Reasoning thus, one expects that inputs that might be perceived as enhancing the riskiness of the production outcome would be more heavily utilized. Conversely, inputs which do little to enhance productivity, but which do act as damage-control agents, would be expected to be used less intensively. Stated in this manner, this would seem almost self evident. However, the existing literature suggests that this is not generally the case even if attention is restricted to single-output, single-input technologies (Quiggin 1992; Ramaswami; Horowitz and Lichtenberg; Hennessy). Because such technologies are highly restrictive, the natural implication seems to be that little, if anything, can be said for more realistic technologies.

Our contention is that much of this indeterminancy arises from the way in which agricultural economists have modelled production uncertainty in the past. Because of the confusion that has arisen over whether risk-averse producers minimize cost or whether duality applies under uncertainty, agricultural economists have overlooked a decomposition of input adjustments under uncertainty that sheds light on these issues. The goal of this paper is to demonstrate the importance of the duality between cost and stochastic technologies by suggesting such a decomposition of input adjustment under uncertainty.

The basic model is a state-contingent formulation, which encompasses both production and price uncertainty, that allows full exploitation of the duality between the technology and the cost structure in comparative-static analyses. We use this duality and a stochastic version of Shephard's lemma to suggest a method for examining input adjustments under uncertainty in a new and informative manner which closely parallels the familiar Hicksian and Slutsky decomposition from consumer theory, but which does not rely on the single-input, single-output stochastic production function model that has dominated many previous studies. Hence, it can intuitively illustrated with familiar graphical techniques. After formulating this decomposition, we illustrate its usefulness by applying it to study input utilization for the simplest possible crop insurance problem, actuarially fair crop insurance.

1 The State-Contingent Technology

Following Chambers and Quiggin (1996, 1997, 2000), the stochastic technology is represented by a multi-product, state-contingent input correspondence. To make this explicit, suppose that the states of nature are given by the set $\Omega = \{1, 2, ..., S\}$, let $\mathbf{x} \in \Re_+^N$ be a vector of inputs committed prior to the resolution of uncertainty, and let $\mathbf{z} \in \Re_+^{M \times S}$ be a vector of state-contingent outputs. So, if state $s \in \Omega$ is realized (picked by 'Nature'), and the producer has chosen the *ex ante* input-output combination $(\mathbf{x}, \mathbf{z})$, then the realized or *ex post* output vector is $\mathbf{z}^s$ corresponding to the sth column of $\mathbf{z}$. In other words, the observed output is an M-dimensional vector $\mathbf{z}^s$ where z_m^s corresponds to the $m-$th output that would be produced in state s.

More formally, the technology is represented by an input correspondence, $X : \Re_+^{MxS} \rightarrow \Re_+^N$, which maps matrices of state-contingent outputs into input sets that are capable of producing that state-contingent output matrix. It is defined

$$X(\mathbf{z}) = \{\mathbf{x} \in \Re_+^N : \mathbf{x} \text{ can produce } \mathbf{z}\}.$$

We impose the following axioms on $X(\mathbf{z})$:

X.1 $X(\mathbf{0}_{MxS}) = \Re_+^N$ (no fixed costs), and $\mathbf{0}_N \notin X(\mathbf{z})$ for $\mathbf{z} \geq \mathbf{0}_{MxS}$ and $\mathbf{z} \neq \mathbf{0}_{MxS}$ (no free lunch).

X.2 $\mathbf{z}' \leq \mathbf{z} \Rightarrow X(\mathbf{z}) \subset X(\mathbf{z}')$.

X.3 $\mathbf{x}' \geq \mathbf{x} \in X(\mathbf{z}) \Rightarrow \mathbf{x}' \in X(\mathbf{z})$.

X.4 $\lambda X(\mathbf{z}) + (1-\lambda)X(\mathbf{z}') \subset X(\lambda\mathbf{z} + (1-\lambda)\mathbf{z}') \qquad 0 \leq \lambda \leq 1.$

X.5 $X(\mathbf{z})$ is closed for all $\mathbf{z} \in \Re_+^{MxS}$.

The first part of X.1 says that doing nothing is always feasible, while the second part of X.1 says that realizing a positive output in any state of nature requires the commitment of some inputs. X.2 says that if an input combination can produce a particular matrix of state-contingent outputs then it can always be used to produce a smaller matrix of state-contingent outputs. X.3 implies that inputs have non-negative marginal productivity. X.4 tells us that the state-contingent technology is convex, and intuitively it leads to diminishing marginal productivity of inputs. X.5 is a technical assumption that ensures the existence of the revenue-cost function that we develop next.

2 The revenue-cost function

Denote by $\mathbf{p} \in \Re_{++}^{M \times S}$ the matrix of state-contingent output prices corresponding to the matrix of state-contingent outputs. The interpretation of $\mathbf{p}$ is basically the same as $\mathbf{z}$. If 'Nature' picks $s \in \Omega$, then the vector of realized spot prices is $\mathbf{p}^s \in \Re_{++}^M$. We assume that producers are competitive, they take these state-contingent output prices and the prices of all inputs, denoted by $\mathbf{w} \in \Re_{++}^N$, as given. The state-contingent revenue vector $\mathbf{r} = \mathbf{pz} \in \Re_+^S$ has typical elements of the form $r_s = \sum_{m=1}^{M} p_m^s z_m^s$.

Producers will be concerned with state-contingent revenue rather than output *per se*, and it is useful to consider the *revenue-cost function* defined as

$$C(\mathrm{w},\mathrm{r},\mathrm{p}) = \min\left\{\mathrm{w}\cdot\mathrm{x} : \mathrm{x} \in X(\mathrm{z}), \sum_m p_{ms}z_{ms} \geq r_s, s \in \Omega\right\},$$

if there exists a feasible state-contingent output array capable of producing r and ∞ otherwise. The properties of $C(\mathrm{w},\mathrm{r},\mathrm{p})$ that follow from X.1-X.5 (Chambers and Quiggin, 2000) are:

Properties of the Revenue-Cost Function (CR):

CR.1$C(\mathrm{w},\mathrm{r},\mathrm{p})$ is positively linearly homogeneous, non-decreasing, concave, and continuous in $\mathrm{w} \in \Re^N_{++}$.

CR.2 Shephard's Lemma.

CR.3 $C(\mathrm{w},\mathrm{r},\mathrm{p}) \geq 0$ with equality if and only if $\mathrm{r} = 0$.

CR.4 $\mathrm{r}' \geq \mathrm{r} \Rightarrow C(\mathrm{w},\mathrm{r}',\mathrm{p}) \geq C(\mathrm{w},\mathrm{r},\mathrm{p})$.

CR.5 $\mathrm{p}' \geq \mathrm{p} \Rightarrow C(\mathrm{w},\mathrm{r},\mathrm{p}') \leq C(\mathrm{w},\mathrm{r},\mathrm{p})$.

CR.6 $C(\mathrm{w},\mathrm{r}_{-s},\theta r_{s,},\mathrm{p}_{-s},\theta \mathrm{p}_s) = C(\mathrm{w},\mathrm{r}_{-s},\theta r_{s,},\mathrm{p}_{-s},\theta \mathrm{p}_s), \quad \theta > 0$.

CR.7$C(\mathrm{w},\mathrm{r},\mathrm{p}) = C(\mathrm{w},\mathrm{r}/k,\mathrm{p}/k), \quad k > 0$.

CR.8 $C(\mathrm{w},\mathrm{r},\mathrm{p})$ is convex in r.

We shall typically assume that $C(\mathrm{w},\mathrm{r},\mathrm{p})$ is smoothly differentiable in all state-contingent revenues and input prices. By assuming a differentiable in revenues cost structure, we, therefore, rule out the stochastic-revenue function approach and the non-stochastic production approach of Sandmo.

3 Preferences

Following Yaari (1969) and Quiggin and Chambers, the producer's preferences are represented by a continuous and increasing function, $W : \Re^s \to \Re$, of his vector of state contingent net returns

$$\mathrm{y} = \mathrm{r} - (\mathrm{w}\cdot\mathrm{x})\,\mathbb{1}_s,$$

where $\mathbb{1}_S$ is the S-dimensional unit vector. The producer's preferences can thus be expressed in terms of the revenue-cost function as

$$\mathrm{y} = \mathrm{r} - \mathbb{C}(\mathrm{w},\mathrm{r},\mathrm{p})\mathbb{1}_s.$$

The producer is *risk-averse with respect to the probability vector* $\boldsymbol{\pi}$ if

$$W(\bar{y}\mathbf{1}^S) \geq W(\mathbf{y}), \forall \mathbf{y},$$

where $\bar{y}\mathbf{1}^S$ is the state-contingent outcome vector with $\bar{y} = \sum_{s\in\Omega} \pi_s y_s$ occurring in every state of nature. Both the usual decision-theoretic approach due to Savage and that employed here may be contrasted with the assumption, common in applied work, that there exist known objective probabilities. Here and in the Savage approach the probabilities, because they depend on preferences, are inherently subjective. In some cases (e.g., climatic uncertainty) where stable relative frequencies can be derived from long runs of historical data, the assumption of known objective probabilities may be appropriate. In such cases, we assume that all individuals would possess the same subjective probabilities.

If preferences are smoothly differentiable, the vector of subjective probabilities is unique and proportional to the marginal rate of substitution between state-contingent incomes along the equal-incomes vector. More concretely, without loss of generality, if preferences are smoothly differentiable (subscripts on functions denote partial derivatives),

$$\pi_s = \frac{W_s\left(c\mathbf{1}^S\right)}{\sum_{t\in\Omega} W_t\left(c\mathbf{1}^S\right)}, \quad s \in \Omega, \quad c \in \Re.$$

Pictorially, therefore, the *fair-odds line*, which gives the locus of points having the same expected value and whose slope is given by minus the relative probabilities is given by the slope of the tangent to the producer's indifference curve at the bisector. Figure 1 illustrates.

In order to impose some structure upon preferences other than simple aversion to risk, consider the partial ordering $\preceq_\pi$ of risky outcomes which possess a common mean for the probability vector $\boldsymbol{\pi}$. This partial ordering is defined by

$$\mathbf{y} \preceq_\pi \mathbf{y}'$$

if and only if $\mathbf{y}$ and $\mathbf{y}'$ have the same mean and $\mathbf{y}$ is less risky than $\mathbf{y}'$ in the sense of Rothschild and Stiglitz. Chambers and Quiggin (1997) define a function $W : \Re^S \to \Re$ to be *generalized Schur-concave* for π if $\mathbf{y} \preceq_\pi \mathbf{y}' \Rightarrow W(\mathbf{y}) \geq W(\mathbf{y}')$.

A comment about generalized Schur concavity is worthwhile. Unlike the assumption of expected-utility maximization, generalized Schur concavity doesn't impose additive separability across states of nature. Consequently, it does not rely upon the independence axiom which has proved vulnerable to a variety of criticisms. Even so, the expected-utility functional with concave u is generalized Schur-concave as can be recognized from the result due to Rothschild and Stiglitz that if $\mathrm{y} \preceq_{\pi} \mathrm{y}'$ then y would be preferred to y′ by all individuals with risk-averse expected-utility preferences. More generally, generalized Schur concavity characterizes a number of preference classes, which are consistent with risk-aversion in our sense, but which are not consistent with expected utility. An example is given by individuals with maximin preferences

$$W(\mathrm{y}) = \min\{y_1, ..., y_S\}.$$

This class of preferences is risk-averse in our sense for all possible probability vectors (note it is not differentiable), and it is also generalized Schur concave. Another class of generalized Schur concave preferences is the mean-variance class. More generally, virtually all preference functions currently in use, including the rank-dependent models (Quiggin 1982, Yaari 1987) and weighted-utility models are consistent with generalized Schur concavity.

When W is smoothly differentiable, a basic result due to Chambers and Quiggin (1997), will prove useful:

Lemma 1 If $W : \Re^S \rightarrow \Re$ is generalized Schur-concave and once continuously differentiable everywhere on its domain, then

$$\left(\frac{W_s(\mathrm{y})}{\pi_s} - \frac{W_r(\mathrm{y})}{\pi_r}\right)(y_s - y_r) \leq 0,$$

for all s and r.

4 Risk-Neutral and Risk-Averse Production Equilibria

We first present some basic results on the production choices of risk-neutral and risk-averse producers.[1] Suppose the risk-neutral producer's subjective probabilities are given by the vector π. Then her first-order conditions on r may be written in the notation of complementary slackness as

$$\pi_s - C_s(\mathbf{w}, \mathbf{r}, \mathbf{p}) \leq 0, \quad r_s \geq 0, \quad s \in \Omega.$$

That is, the marginal cost of increasing revenue in any state is at least equal to the subjective probability of that state. Pictorially, therefore, we represent the producer equilibrium by a hyperplane being tangent to her isocost curve. Figure 2 illustrates. The slope of the hyperplane is determined by the ratio of the producer's subjective probabilities, *the fair-odds line*, and the isocost curve is determined by the equilibrium level of revenue-cost. This is exactly analogous to the representation of production equilibrium in the non-stochastic, multi-product case. Instead of determining an optimal mix of outputs as in the non-stochastic multi-product case, however, the producer equilibrium now determines the optimal mix of state-contingent revenues.

Summing the first-order conditions on $\mathbf{r}$ yields an *arbitrage condition*

$$\sum_{s\in\Omega} C_s(\mathbf{w}, \mathbf{r}, \mathbf{p}) \geq \sum_{s\in\Omega} \pi_s = 1. \tag{1}$$

$\sum_{s\in\Omega} C_s(\mathbf{w}, \mathbf{r}, \mathbf{p})$ is the marginal cost of increasing all state-contingent revenues by the same small amount in each state of nature, i.e., it is the marginal cost of a sure increase in revenue of one unit. Hence, (1) requires this cost be at least as large as the associated sure increase in returns. If it were not, the decisionmaker could increase profit with probability 1 by either expanding or decreasing all revenues equally. For an interior solution, (1) must hold as an equality.

We refer to the set of revenue vectors $\mathbf{r}$ satisfying (1) as the *efficient set*, denoted $\Xi(\mathbf{w}, \mathbf{p})$,

$$\Xi(\mathbf{w}, \mathbf{p}) = \left\{ \mathbf{r} : \sum_{s\in\Omega} C_s(\mathbf{w}, \mathbf{r}, \mathbf{p}) \geq 1 \right\}.$$

The boundary of $\Xi(\mathbf{w}, \mathbf{p})$ is the *efficient frontier*. Its elements are given by:

$$\Xi^o(\mathbf{w}, \mathbf{p}) = \left\{ \mathbf{r} : \sum_{s\in\Omega} C_s(\mathbf{w}, \mathbf{r}, \mathbf{p}) = 1 \right\}.$$

By the homogeneity properties of $C(\mathbf{w}, \mathbf{r}, \mathbf{p})$, $\Xi(\theta\mathbf{w}, \theta\mathbf{p}) = \theta\Xi(\mathbf{w}, \mathbf{p})$ and $\Xi^o(\theta\mathbf{w}, \theta\mathbf{p}) = \theta\Xi^o(\mathbf{w}, \mathbf{p})$, $\theta > 0$ (Chambers and Quiggin, 2000). The efficient set and the efficient frontier are positively linearly homogeneous in input and output prices.