ENTRY, EXIT, AND COMPETITIVE PROFITS IN THE LONG RUN

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Entry, Exit, and Competitive Profits in the Long Run

Val Eugene Lambson
Department of Economics
University of Wisconsin
Madison, WI 53706

This paper constructs an explicitly dynamic model of an industry where competitive firms make entry, exit, and output decisions with the objective of maximizing the present expected value of their profits. The model predicts that average profit rates over time differ even across competitive industries, i.e., higher rates of profit are not sufficient evidence to conclude that there is market power. Furthermore, higher profit rates are not the result of "quasi-rents" attributable to entry costs. Indeed, the assumption of dynamic maximizing behavior seems to have few implications for average profit rates over time. The model also predicts that industries with high variability (over time) in the number of firms will have low variability (over time) in the value of existing firms. Other results explain how the stochastic processes governing endogenous variables can be derived from the stochastic processes governing endogenous variables, how sunk costs and existence costs bound current profits, how the existence of entry costs results in "hysteresis", and how average profits and average firm values (over time) are related.

Keywords: Industrial Organization, Profits, Firm Value, Entry, Exit, Dynamic Competition.
1. Introduction

It has long been asserted in the industrial organization literature that profit rates differ systematically across industries. These differences do not appear to "average out" over time. This paper develops a theoretical structure to analyze persistent differences in profit rates, as well as differences in firm value, entry, and exit across industries characterized by price-taking behavior. The model is also well-suited for analyzing the evolution over time of these variables within a given industry. Such analysis requires an explicitly dynamic model where firms react to changes in market conditions and where entry and exit are taken into account.

In the framework developed below all firms, including potential firms, are price-takers and have access to the same technology. This technology is described by a cost function that gives total cost as a function of the market condition and the firm's output. The market condition describes the relevant exogenous variables, such as input prices and demand. Market conditions are governed by a stochastic process that is known to all firms. At the beginning of each period firms observe the current market condition and then make entry and exit decisions. An entering firm must pay an entry cost and an exiting firm receives its scrap value. All active firms, i.e., newly entered firms and previously active firms that have not exited, then pay a maintenance fee, or fixed cost, and make their output decisions. Entry costs, fixed costs, and scrap values may all depend on the current market condition. Equilibrium conditions require that firms enter if the expected present value of doing so exceeds the entry cost, that firms exit if the expected present value of not doing so is less than the firm's scrap value, and that active firms produce in each period so that current profits are maximized given current prices.
In this framework, it is natural to define sunk costs in the current period as the difference between the current entry cost and the scrap value expected for the next period. The analysis makes clear that current one-period profits can be strictly less than current sunk costs. Indeed, they will be strictly less than sunk costs if there is any possibility that the next period will not be unfavorable enough to (weakly) provoke exit. The difference between current sunk costs and current profits can be thought of as the "option value" of having a firm in place. Specifically, existing firms have the option of producing without incurring the entry cost.

Similarly, define "existence costs" in the current period as the difference between the current scrap value and the entry cost expected for next period. Hence existence costs are the expected present value of a firm that chooses to exit in the current period and reenter in the subsequent period. The analysis makes clear that current profits can be strictly greater than current existence costs. Indeed, they will be strictly greater than existence costs if there is any possibility that the next period will not be sufficiently favorable to (weakly) provoke entry. The difference between current profit and existence costs can be thought of as the "option value" of being out of the market. A firm that is not in the market has the option of entering when conditions improve but need not pay the fixed costs if conditions are unprofitable. 4

Given the stochastic process governing the exogenous variables, the model has implications for the stochastic processes governing endogenous variables. Refer to a market condition and an (endogenous) number of firms as a state. Then the probability distribution over the next period's states depends only on the current number of firms and on the history of market conditions. Furthermore, the degree of dependence on the history of market conditions is the same as for the stochastic process governing market
conditions. In particular, if market conditions follow a first-order Markov process then states follow a first-order Markov process. Hence any variable that depends only on the current state, such as current profits, current firm value, or price, follows a first-order Markov process.

Note that this model generates what has come to be called "hysteresis," i.e., effects that persist after the causes that brought them about have been reversed. For example, if there is a change from one market condition to another there can be a change in the number of firms. If the change in market conditions is reversed, the change in the number of firms generally will not be. This result is due to the existence of entry costs: once the firms have entered they are no longer subject to entry costs so their optimal decisions can differ.

When market conditions, and hence states, follow a first-order Markov process, assumptions can be made on the exogenous stochastic process that imply that the endogenous stochastic process has a limiting distribution. More formally, if \( \rho_{1j}^t \) is the probability of moving from state 1 to state \( j \) in \( t \) periods, then \( \lim_{t \to \infty} \rho_{1j}^t = \eta_j \) independently of \( i \) where \( \sum_j \eta_j = 1 \). This is important because if \( R \) is any variable that depends only on the current state then \( \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \sum_j \rho_{1j}^t R_j = \sum_j \eta_j R_j \) independently of \( i \). Hence, if long-run averages of such variables are calculated using data from industries for which states follow a first-order Markov process, the results will, for a large number of observations, be close to the mean value of the variable with respect to the limiting distribution. This motivates the analysis of the properties of means with respect to the limiting distribution.

The analysis provides some insights into the effects of entry costs and scrap values on average profit rates over time. It seems natural to believe that higher entry barriers might lead to higher profit rates by reducing the
number of firms that enter the industry. This belief is theoretically wanting in at least two respects. The first is apparent after a moment's thought: if entry costs are part of the investment, higher entry costs must generate more than proportionally higher current profits to increase the rate of profit. The second is more subtle and is made clear in this paper: although higher entry costs reduce the number of entrants when times are good they also increase the number of firms that remain when times are bad, because existing firms understand that high entry costs will protect them from excessive entry when times change for the better. Hence the effect on the average number of firms over time is ambiguous. The net effect of all of this is to render ambiguous the effects of entry costs and scrap values on long-run average profit rates. However, in the special case where every change in the market condition either provokes entry or exit (because, for example, changes in factor prices are always large relative to entry costs and scrap values), long-run average profits are decreasing in entry costs and increasing in scrap values, contrary to what one might initially guess.

Note that underlying the discussion in the previous paragraph is the implicit assertion that long-run average profit rates differ even across competitive industries when firms are assumed to maximize their expected present values. This calls into question some inferences that are commonly made on the basis of profit data. Specifically, higher long-run rates of profit need not imply a greater degree of monopoly power.

Another implication of the model suggests that industries characterized by high variability in firm value should be characterized by low variability in the number of firms. Define the range of a variable as the difference between the extreme values the variable takes on. Since a firm's value fluctuates between the cost of entry and the scrap value of a firm, it is trivial that the range of firm value is increasing in entry costs and
decreasing in scrap value. What is less trivial but no less true under Markovian assumptions is that the range of the number of firms is nondecreasing in entry costs and nonincreasing in scrap value. The intuition is that changes in market conditions affect the profitability of firms. If entry costs and scrap values are negligible then firms are more likely to react to changes by entering (when changes are good) or exiting (when changes are bad). Otherwise, firms are more likely to find it more profitable to weather the changes without entering or exiting the industry.

Section 2 illustrates the basic results in the context of a simple example in which formality is reduced to a minimum. Section 3 formally describes the general model. Section 4 contains statements and discussion of the general theorems. Section 5 contains a summary and some concluding remarks.

2. An Illustrative Example

Suppose there are three market conditions. In each market condition, fixed costs are denoted $\phi_i$, with $\phi_1 < \phi_2 < \phi_3$. In each market condition active firms, that is, the firms that have entered and have not exited, can produce up to one unit of output at zero marginal cost. In every market condition the cost of entry is $\xi$ and the scrap value is $\chi$, where $\xi > \chi$. Output demand is the same in all market conditions and is given by $P = 1 - y$ where $P$ is the output price and $y$ can be interpreted both as total output and as the number of firms. (More formally, each firm is of Lebesgue measure zero and $y$ is the mass of active firms. The mass of active firms will be referred to as "the number of firms.") The stochastic process governing market conditions is i.i.d. with $\rho_i$ denoting the probability of market condition $i$. The discount rate for all firms is denoted by $\delta$. The remainder of this section analyzes the properties of equilibrium in this example. Rigorous definitions and proofs are ommitted in this section since
the results here are special cases of the general results in section 4. The numbering of the theorems in this section corresponds to the numbering of their generalizations in section 4.

Informally, equilibrium requires that in each period active firms behave as price-takers and choose output to maximize current profit, in each period the market clears, and, given the equilibrium stochastic price sequence, this behavior coupled with optimal exit decisions causes the expected present value of firms to equal if there is entry, $x$ when there is exit, and some value in between $x$ and $\xi$ when there is neither entry nor exit.

It turns out that one can associate an interval, $[N_1, X_1]$, with each market condition $i$ and characterize equilibrium in terms of these three intervals. Specifically, in market condition $i$ there will be entry if there are fewer than $N_1$ firms, there will be exit if there are more than $X_1$ firms, and there will be neither if there are between $N_1$ and $X_1$ firms. More formally, if there were $y_{t-1}$ firms in period $t-1$ and the market condition in period $t$ is market condition $i$ then the number of firms in period $t$ will be given by $y_t = \min\{X_1, \max\{N_1, y_{t-1}\}\}$. These intervals completely characterize the equilibrium since output per firm will always be one in this example and, hence, the price in period $t$ is $1-y_t$. The structure of equilibrium is stated in Theorem 2.1. To reduce the number of cases without discarding anything of interest, it is assumed that $\varphi_2 = (\varphi_1 + \varphi_3)/2$. To avoid corner solutions it is assumed that $(1-\delta)\xi + \varphi_3 \leq 1$. There are three cases: (1) $\xi - \chi < \varphi_3 - \varphi_2 = \varphi_2 - \varphi_1$, (2) $\xi - \chi \leq \varphi_3 - \varphi_2 = \varphi_2 - \varphi_1$, and (3) $\xi - \chi > \varphi_3 - \varphi_1$. 
Theorem 2.1: In case 1 equilibrium is characterized by the intervals, 
\[ [N_1, X_1] \] where
\[
N_1 = 1 - (1-\delta_1)\xi + \delta(1-\rho_1)\chi - \varphi_1 \\
N_2 = 1 - [1-\delta(1-\rho_3)]\xi + \delta_3\chi - \varphi_2 \\
N_3 = 1 - (1-\delta)\xi - \varphi_3 \\
X_1 = 1 - (1-\delta)\chi - \varphi_1 \\
X_2 = 1 + \delta_1\xi - [1-\delta(1-\rho_1)]\chi - \varphi_2 \\
X_3 = 1 + \delta(1-\rho_3)\xi - (1-\delta_3)\chi - \varphi_3.
\]
In case 2 equilibrium is characterized by the intervals \([N_1, X_1]\) where
\[
N_1 = 1 - [1-\delta(1-\rho_3)]\xi + \delta_3\chi - (1-\delta_2)\varphi_1 - \delta_2\varphi_2 \\
N_2 = 1 - (1-\delta)\xi - (1-\delta_3)\varphi_2 - \delta_3\varphi_3 \\
N_3 = 1 - (1-\delta)\xi - \varphi_3 \\
X_1 = 1 - (1-\delta)\chi - \varphi_1 \\
X_2 = 1 - (1-\delta)\chi - \delta_1\varphi_1 - (1-\delta_1)\varphi_2 \\
X_3 = 1 + \delta_1\xi - [1-\delta(1-\rho_1)]\chi - \delta_2\varphi_2 - (1-\delta_2)\varphi_3.
\]
In case 3 equilibrium is characterized by the intervals \([N_1, X_1]\) where
\[
N_1 = 1 - (1-\delta)\xi - (1-\delta)\varphi_1 - \delta \sum_{i=1}^3 \rho_1 \varphi_i \\
N_2 = - (1-\delta)\xi + (1-\delta_3)(1-\varphi_2) + \delta_3(1-\varphi_3) \\
N_3 = 1 - (1-\delta)\xi - \varphi_3 \\
X_1 = 1 - (1-\delta)\chi - \varphi_1 \\
X_2 = - (1-\delta)\chi + \delta_1(1-\varphi_1) + (1-\delta_1)(1-\varphi_2) \\
X_3 = 1 - (1-\delta)\chi - (1-\delta)\varphi_3 - \delta \sum_{i=1}^3 \rho_1 \varphi_i.
\]
This theorem is proved by direct calculation. To solve case 1, note that in that case there will be entry or exit each time the market condition changes. To see this let \(\phi(y)\) be the expected present value of a firm from the next period on when there are \(y\) firms in the market in the current period. (Since the process is i.i.d. \(\phi(y)\) depends only on \(y\) and not on the current market condition.) Suppose there were an equilibrium where the
number of firms did not change when the market condition changed. In market condition \( k \) the expected present value of a firm when there are \( y \) firms is thus \( V_k(y) = 1 - \gamma_k + \delta \phi(y) \). Hence \( |V_i(y) - V_j(y)| = |\phi_i - \phi_j| > \xi - \chi \). But equilibrium requires that \( |V_i(y) - V_j(y)| = \xi - \chi \). This contradiction establishes that each time the market condition changes the number of firms must change. Then \( N_i \) is easily calculated for each \( i \) by assuming that the expected present value in the next period will be \( \xi \) if the market condition is \( j \neq i \) and \( \chi \) if the market condition is \( j > i \) and by choosing \( N_i \) so that the expected present value in the current period if the market condition is \( i \) equals \( \xi \). In other words, choose \( N_i \) to solve \( \xi = 1 - N_i - \phi_i + \delta [\sum_{j \geq i} \rho_j \xi + \sum_{j > i} \rho_j \chi] \). Similarly, choose \( \chi \) to solve \( \chi = 1 - \phi_i - \delta [\sum_{j < i} \rho_j \xi + \sum_{j > i} \rho_j \chi] \).

To solve case 2, it is sufficient to note (using reasoning similar to the case 1 reasoning) that changes between market conditions 1 and 3 provoke entry or exit but subsequent changes to market condition 2 do not. With this in mind, setting up the appropriate equations is not a difficult exercise. Similarly, to solve case 3 it is sufficient to show that once an extreme market condition (1 or 3) has occurred there will be no further entry or exit. Once again, setting up the appropriate equations is straightforward. The details are omitted.

Let \( \pi(y,i) = 1 - \gamma_i - \phi_i \) be the current profit in market condition \( i \) when there are \( y \) firms. The next theorem follows from the definition of equilibrium. Specifically, if market condition \( i \) is realized in period \( t \) and the equilibrium number of firms is \( y \), then it must be true that \( \chi = \pi(y,i) + \delta \phi(y) \leq \xi \). Equilibrium also requires that \( \chi \leq \phi(y) \leq \xi \).

Theorem 2.2: For any of the cases and for any initial number of firms, if \( y_t \) is the equilibrium number of firms in period \( t \) (given the history) and the current market condition is \( i \) then

\[
(2.1) \quad \chi - \delta \xi \leq \pi(y_t,i) \leq \xi - \delta \chi.
\]
The right hand side of (2.1) is readily interpretable as the sunk cost that establishing a firm entails. It is the difference between the cost of establishing a firm now and the value of scrapping the firm one period hence. Equilibrium thus has the property that current profits are typically strictly less than sunk costs.

The left hand side of (2.1) is readily interpretable as an "existence cost." It is the difference between the gain from scrapping a firm today and the discounted cost of reestablishing the firm tomorrow. Equilibrium thus has the property that current profits typically strictly exceed the existence cost.

The difference between sunk costs and profits on the one hand and profits and existence costs on the other reflect the "option values" of being active or inactive, respectively. Specifically, firms are willing to enter even though current profits do not immediately cover sunk costs because having a firm in place gives them the option of producing in the future if times are good without paying the entry cost. If times are bad, however, the firm need not exercise the option; it may exit and receive its scrap value. Similarly, firms are willing to exit even though current profits strictly exceed existence cost. This is because being out of the market gives firms the option of avoiding the fixed cost in the future if times are bad. If times are good, however, the firm need not exercise the option; it may pay the entry cost and enter.

A pair \((i,y)\), where \(i\) is a market condition and \(y\) is a number of firms, will be called a state. The stochastic process governing market conditions, which in this example is i.i.d., generates a stochastic process governing states. This process is described in Theorem 2.3.
Theorem 2.3: In each case, the stochastic process governing states is derived from the stochastic process governing market conditions as follows:

$$\tilde{p}(j,y')(i,y) = \begin{cases} p_j & \text{if } y \leq N_j \text{ and } y' = N_j \\ p_j & \text{if } y \geq X_j \text{ and } y' = X_j \\ p_j & \text{if } N_j \leq y \leq X_j \text{ and } y = y' \\ 0 & \text{otherwise.} \end{cases}$$

Intuitively, if there are $y$ firms in the current period, the only possible values that the number of firms can attain are $y$, the $N_i$ that are greater than $y$, and the $X_i$ that are less than $y$. The number $y$ will be repeated if the market condition does not change or if the change is insufficient to provoke entry or exit. By contrast, if the market condition changes to $i$, and if $i$ is sufficiently more (less) favourable to provoke entry (exit), then the number of firms will change to $N_i$ ($X_i$).

The theorem is generalized in section 4 to show that the stochastic process governing states depends only on the past history of market conditions (to the same degree that the stochastic process governing market conditions depends on the past history) and on the current number of firms. In particular, first-order Markov processes on market conditions induce first-order Markov processes on states. Note, however, that i.i.d. processes on market conditions, though they must induce first-order Markov processes on states, do not generally induce i.i.d. processes. This is clear from the example, as will now be discussed.

Consider case 1. Note that $N_3 < X_3 < N_2 < X_2 < N_1 < X_1$. Refer to Theorem 2.3 and note that once market condition 1 or 2 has been realized, the number of firms cannot take on values greater than $N_1$. Similarly, once either market condition 2 or 3 has been realized, the number of firms cannot take on values less than $X_3$. Finally, regardless of the initial number of firms, once there is a change in market condition, the only numbers of firms that
can occur are $X_3$, $N_2$, $X_2$ and $N_1$. In particular, regardless of the initial number of firms, there are exactly four recurrent states (i.e., states that are eventually repeated with probability one). These are $(1,N_1)$, $(2,X_2)$, $(2,N_2)$, and $(3,X_3)$. In words, it is eventually the case that there is entry until there are $N_1$ firms each time the market condition changes to market condition 1, exit until there are $X_3$ firms each time the market condition changes to market condition 3, entry until there are $N_2$ firms each time the market condition changes from 3 to 2, and exit until there are $X_2$ firms each time the market condition changes from 1 to 2.

Now consider case 2 and note that $N_3 < N_2 < X_3 = N_1 < X_2 < X_1$. Refer to Theorem 2.3 and note that once there is a change to market condition 1 there are never fewer than $X_3$ firms. Similarly, once there is a change to market condition 3 there are never more than $N_1$ firms. Hence, after either such change there are only two numbers of firms possible: $X_3$ and $N_1$. Hence there are four recurrent states: $(1,N_1)$, $(2,N_1)$, $(2,X_3)$, and $(3,X_3)$. Note that with the higher difference between entry costs and scrap values, there is no change in the number of firms when the market condition changes to state 2. Only extreme changes from market condition 1 to market condition 3 or the reverse (perhaps via market condition 2) provoke entry or exit.

Finally, consider case 3. Note that $N_3 < N_2 < N_1 < X_3 < X_2 < X_1$. In this case the recurrent states depend on initial condition. Let $y_0$ be the initial number of firms. If $y_0 < N_1$ then there will be be entry each time the market condition changes to a market condition that is more favorable than any previous market condition. Once market condition 1 is realized there will be $N_1$ firms forever after. If $y_0 > X_3$ then there will be exit each time the market condition changes to a market condition that is less favorable than any previous market condition. Once market condition 3 is realized there will be $X_3$ firms forever after. If $y_0 \in [N_1,X_3]$ then there will always be $y_0$
firms. Hence, depending on the initial condition there are three recurrent
states of the form (1,y), (2,y), and (3,y).

Note that, in each case, equilibrium exhibits what has been called
"hysteresis." Specifically, if a change in the exogenous variables is
reversed, the associated change in the endogenous variables need not be. In
the first two cases, the number of firms that will be active in market
condition 2 depends on whether market condition 2 is entered from market
condition 1 or from market condition 3. In case 3, if y_o does not lie in
the interval [N_1, X_3] then the number of firms must eventually undergo a
change that will never be reversed.

Note that in the first two cases the set of recurrent states is
independent of the initial number of firms. This property is not shared by
the third case. This is because in the first cases, in contrast to the
third, \( \max_i N_i \geq \min_i X_i \). When this condition is not satisfied the number of
firms is eventually constant at a level that depends on initial conditions.

Theorem 4.4 establishes general conditions for the existence of a
limiting distribution for the stochastic sequence governing states when
market conditions follow a first-order Markov process. In this example that
distribution is easily calculated for the first two cases using the
following relationships: \( \sum_j \rho_{ij} \eta_j = \eta_i \) and \( \sum_i \eta_i = 1 \).\(^6\) For case 1 number the
states (1,N_1), (2,X_2), (2,N_2), and (3,X_3) by 1, 2, 2', and 3, respectively.
Similarly, for case (2) number the states (1,N_1), (2,N_1), (2,X_3), and (3,X_3)
by 1, 2, 2', and 3, respectively. The limiting distribution in case 3
depends on the initial number of firms. Specifically, the recurrent states
are (1,y), (2,y), and (3,y) where \( y = N_1 \) if \( y_o \leq N_1 \), \( y = X_3 \) if \( y_o \geq X_3 \), and \( y = y_o \)
otherwise. Number these states by 1, 2, and 3, respectively.
Theorem 2.4: For case 1 and case 2, the limiting distribution is as follows: \( \eta_1 = p_1, \eta_2 = p_1 p_2/(1-p_2), \eta_2' = p_2 p_3/(1-p_2), \) and \( \eta_3 = p_3. \) For case 3 the limiting distribution is \( \eta_1 = p_1, \eta_2 = p_2, \) and \( \eta_3 = p_3. \)

This limiting distribution is important because the long-run average of any variable that depends only on the current state approaches the variable's mean with respect to this limiting distribution as the number of observations grows large. (This is a variant of the ergodic theorem for Markov processes. See section 4 for a formal statement.) This motivates the study of mean values with respect to the limiting distribution.

The following table lists the current profits and firm values for each state and each case.

### Case 1

<table>
<thead>
<tr>
<th>State</th>
<th>Current Profits</th>
<th>Firm Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (1, N_1) )</td>
<td>((1-\delta p_1)\xi - \delta(1-p_1)\chi )</td>
<td>( \xi )</td>
</tr>
<tr>
<td>( (2, N_2) )</td>
<td>([1-\delta(1-p_1)]\chi - \delta p_1 \xi )</td>
<td>( \chi )</td>
</tr>
<tr>
<td>( (3, X_3) )</td>
<td>((1-\delta p_3)\chi - \delta(1-p_3)\xi )</td>
<td>( \xi )</td>
</tr>
</tbody>
</table>

### Case 2

<table>
<thead>
<tr>
<th>State</th>
<th>Current Profits</th>
<th>Firm Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (1, N_1) )</td>
<td>([1-\delta(1-p_3)]\xi - \delta p_3 \chi + \delta p_2 (\varphi_2 - \varphi_1) )</td>
<td>( \xi )</td>
</tr>
<tr>
<td>( (2, X_2) )</td>
<td>([1-\delta(1-p_1)]\chi - (1-\delta p_2) (\varphi_2 - \varphi_1) )</td>
<td>( \xi - (\varphi_2 - \varphi_1) )</td>
</tr>
<tr>
<td>( (2, X_3) )</td>
<td>(-\delta p_1 \xi + (1-\delta(1-p_2)) \chi + (1-\delta p_2) (\varphi_3 - \varphi_2) )</td>
<td>( \chi + (\varphi_3 - \varphi_2) )</td>
</tr>
<tr>
<td>( (3, X_3) )</td>
<td>(-\delta p_1 \xi + (1-\delta(1-p_2)) \chi - \delta p_2 (\varphi_3 - \varphi_2) )</td>
<td>( \chi )</td>
</tr>
</tbody>
</table>

### Case 3

<table>
<thead>
<tr>
<th>State</th>
<th>Current Profits</th>
<th>Firm Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (1, y) )</td>
<td>(1 - y - \varphi_1)</td>
<td>((1-\gamma - \varphi_1) + (\delta/1-\delta)\sum_k^1 \rho_k (1-y-\varphi_k))</td>
</tr>
<tr>
<td>( (2, y) )</td>
<td>(1 - y - \varphi_2)</td>
<td>((1-\gamma - \varphi_2) + (\delta/1-\delta)\sum_k^1 \rho_k (1-y-\varphi_k))</td>
</tr>
<tr>
<td>( (3, y) )</td>
<td>(1 - y - \varphi_3)</td>
<td>((1-\gamma - \varphi_3) + (\delta/1-\delta)\sum_k^1 \rho_k (1-y-\varphi_k))</td>
</tr>
</tbody>
</table>
The effects of entry costs and scrap values on long-run average profit rates can be explicitly determined in this example. Interpreting the entry cost as investment, define the rate of profit in state $i$ by $\pi_i/\xi$. Note that the direct effect of higher entry costs is to increase the value of the denominator, thus reducing the rate of profit. The indirect affect on the numerator is more subtle. First note that Theorem 2.1 makes clear that, although higher entry costs result in fewer entrants when times are good, they also cause more firms to remain active when times are bad. (This is seen by noting that all of the $N_1$ are nonincreasing in $\xi$ while all of the $X_1$ are nondecreasing in $\xi$.) This is reflected in the current profits listed in the table. (Note that periods of entry exhibit higher profits when $\xi$ is higher and periods of exit exhibit lower profits when $\xi$ is higher. Just the opposite is true for the effects of $\chi$.) The net effect of these considerations is that the effects of entry costs on profit rates are ambiguous. This is illustrated in the example by Theorem 2.5, which explicitly solves for the long-run average profit rates, (a task made easier by the application of Theorem 2.6.)

**Theorem 2.5:** Long-run average profit rates for case 1 and case 2 are given, respectively, by

$$E_\eta(\pi/\xi) = (1-\delta)\{ \rho_1 + (\rho_2\rho_3/(1-\rho_2)) + [\rho_3 + (\rho_1\rho_2/(1-\rho_2))]\}(\chi/\xi)$$

and

$$E_\eta(\pi/\xi) = (1-\delta)\{ \rho_1 + (\rho_1\rho_2/(1-\rho_2)) + [\rho_3 + (\rho_2\rho_3/(1-\rho_2))]\}(\chi/\xi)$$

$$+ [(\rho_2\rho_3-\rho_1\rho_2)/(1-\rho_2)][(\phi_2-\phi_1)/\xi].$$

The long-run average profit rate for case 3 is

$$E_\eta(\pi/\xi) = [(1-y) - \Sigma_i \rho_i \phi_i]/\xi$$

where $y=N_1$ if $y_0=N_1$, $y=X_3$ if $y_0=X_3$, and $y=y_0$ otherwise.

Note that in case 1 the long-run average rate of profit declines in $\xi$ and increases in $\chi$. This is a general result: if changes in the market condition always induce entry or exit then the long-run average rate of
profit declines in $\xi$ and increases in $\chi$. By contrast, note that the effect of entry in case 2 depends on the sign of

$$[p_3+(p_2p_3/(1-p_2))]\chi+[(p_2p_3-p_1p_2)/(1-p_2)][f_2-f_1],$$

which is ambiguous. In case 3 it is straightforward to show (by plugging in the various possible values for $y$, that long-run average profit rates are increasing in $\xi$ if $y<N_1$ and declining in $\xi$ otherwise. Interpretation in the latter case is somewhat difficult, however, since, strictly speaking, the entry cost is never observed to be paid.

It is important to note that the rate of profit discussed here is an accounting figure, not an economic one. Specifically, firms do not attempt to maximize their average rates of profit, but rather their expected present values. Hence factors do not flow in such a way as to equalize the long-run average rate of profit across industries. Rather, investment occurs in those industries where the expected present value of profits per marginal unit of investment is greatest. This illustrates a potential pitfall in empirical work using profit data. If firms are assumed to maximize the rate of profit, so that the rate of profit should be equalized across competitive industries, observed differences in the rate of profit across industries may lead researchers to infer the existence of market power in some industries. This model makes clear that such an inference may be unfounded: such differences are to be expected even if all industries are competitive. Note that higher profit rates are not due to quasi-rents attributable to entry costs either: the effects of entry costs on profit rates are ambiguous.

The next result makes the proof of Theorem 2.5 less tedious, and it is interesting in its own right. Some algebra verifies that, in each case, the long-run average current profits are proportional to long-run average firm values.

**Theorem 2.6:** In all cases $\sum_j \eta_j \pi_j = (1-\delta) \sum_j \eta_j V_j$. 
This result generalizes to all first-order Markov processes on states that have limiting distributions. It suggests that one need only study long-run average profits or long-run average firm values; the study of one subsumes the study of the other. This could be useful in situations where data are only available for one of the categories when the researcher is interested in the other. (If profit rates are defined by $\pi/\xi$ and Tobin's $q$ is defined by $V/\xi$, similar statements can be made concerning these two variables if $\xi$ is independent of the state.)

The final result suggests that, other things than entry costs and scrap values equal, industries with high variability in firm value over time should exhibit low variability in the number of firms. Specifically, define the range of a variable as the difference between its extreme values taken over the recurrent states.

**Theorem 2.7:** In cases 1 and 2 the range of an active firm's value is $\xi-\chi$. In case 1 the range of the number of firms is $N_1-X_3=-(\phi_3-\phi_1)(1+\delta p_2)(\xi-\chi)$ and in case 2 it is $N_1-X_3=-(1-\delta p_2)[(\xi-\chi)+\phi_3-\phi_1]$. Hence in cases 1 and 2 the range of firm value is increasing in $\xi$ and decreasing in $\chi$ while the range of the number of firms is decreasing in $\xi$ and increasing in $\chi$. In case 3 the range of an active firm's value is $\phi_3-\phi_1$ and the range of the number of firms is 0. Hence in case 3 the range of firm value and the range of the number of firms are independent of $\xi$ and $\chi$.

Note that as $\xi$ increases and $\chi$ decreases we move from case 1 to case 2 and finally to case 3, and the range of firm value monotonically increases while the range of the number of firms monotonically decreases until the former achieves a value of $\phi_3-\phi_1$ and the latter achieves a value of zero. This result is generalized in Theorem 4.7. If $\xi$ and $\chi$ are both close to zero, implying tight bounds on firm value, then the industry reacts more efficiently to changes in profitability by exhibiting high levels of entry
and exit. As $\xi$ increases and $\chi$ decreases, more firms find it profitable to weather changes without entering and exiting. The high entry costs discourage entry directly and discourage exit indirectly by promising protection in future good times. The low scrap values discourage exit directly and discourage entry indirectly by making future bad times more costly to avoid.

This concludes the discussion of the illustrative example. Section 3 exposits the general model and section 4 contains the general results.

3. The General Framework

Consider a market that is to exist for infinitely many periods indexed by $t$. Let $M$ be the index set for market conditions. It is assumed throughout that there are at most countably many possible market conditions. Let $H_t$ be the set of all $t$-period histories, i.e., an element of $H_t$ is a $t$-dimensional vector of market conditions. Let $H=\bigcup_{t=1}^{\infty} H_t$ be the set of all histories (of all lengths) and, for $h \in H_t$ and $s \geq t$, let $H|h$ and $H_s|h$ be the set of histories and of $s$-period histories, respectively, whose first $t$ market conditions are $h$. (If $g \in H|h$ write $g \succ h$. If $g \succ h$ or $g=h$ write $g \approx h$.) For $s \geq t$, $h \in H_t$, and $g \in H_s$, let $\rho(g|h)$ be the probability that $g$ is realized if $h$ is realized. A subscript on a history refers to the market condition in the corresponding period, e.g., $h_t$ is the market condition in period $t$ of the history $h$.

Let firms be indexed by the positive real line; hence each firm is of (Lebesgue) measure zero. Firms can either be active, i.e., capable of producing without paying an entry cost, or inactive, i.e., incapable of producing without paying an entry cost. At the beginning of each period firms observe the current market condition, some $m \in M$. At that time inactive firms decide whether to pay the entry cost, $\xi(m)$, and become active while active firms decide whether to accept the scrap value, $\chi(m)$, and become
inactive. After these decisions are made, all active firms pay the fixed cost, \( \varphi(m) \), and make their production decisions. Note that a newly active firm can produce immediately upon paying the entry cost and that it must also pay the fixed cost. Both entry costs and fixed costs are strictly positive in all market conditions; the scrap value may be positive or, if there are disposal costs or severance pay, it may be negative. In any case, \( \xi(m) \equiv \chi(m) \) for all \( m \in M \).

For each \( m \in M \) let \( C(\cdot, m) : [0, q] \to \mathbb{R}_+ \) be the cost function that active firms face in market condition \( m \). Note that all active firms have the same cost function. \( \bar{q} \) is an upper bound on output that can be interpreted as a capacity constraint. Alternatively, one can assume that in all market conditions marginal cost rises sufficiently high that production of more than \( \bar{q} \) units of output is never optimal for any single firm. For all \( m \in M \), \( C(\cdot, m) \) is convex, hence continuous, and nondecreasing in output with \( C(0, m) = 0 \).

Let \( d(\cdot, m) \) be the demand curve in market condition \( m \). For all \( m \in M \), assume \( d(\cdot, m) \) is nonincreasing and that \( \lim_{x \to \infty} d(x, m) = 0 \). For \( h \in H_t \), let \( y(h) \) and \( q(h) \) be, respectively, the mass of active firms and the individual firm's output in period \( t \) if the history \( h \) is realized. Then the price in period \( t \) if \( h \in H_t \) is realized is \( p(h) = d(y(h)q(h), h_t) \). It will be said that \( p \) and \( q \) are derived from \( y \) given \( m \) if \( p = d(yq, m) \) and \( q = \arg \max_x (px - C(x, m)) \), i.e., if firms are maximizing current profits given \( p \) and if their aggregative behavior results in the price \( p \). It will be said that the stochastic sequences \( P = \{p(h)\} \) and \( Q = \{q(h)\} \) are derived from the stochastic sequence \( Y = \{y(h)\} \) if, for all \( t \) and all \( h \in H_t \), \( p(h) \) and \( q(h) \) are derived from \( y(h) \) given \( h_t \). Define \( \pi(y, m) = pq - C(q, m) - \varphi(m) \) where \( p \) and \( q \) are derived from \( y \) given \( m \). Define \( \pi(0, m) = \lim_{y \to 0} \pi(y, m) \); \( \pi(0, m) \) may be infinite. \( \pi(y, m) \) is interpretable as the current profit of an individual firm if there are \( y \)
firms in market condition m. Standard arguments verify that \( \pi(y, m) \) is continuous and nonincreasing with \( \lim_{y \to \infty} \pi(y, m) = -\varphi(m) \).

Given \( h \in H \), let \( A \) be any subset of \( H \) such that, for all \( f \in h \), \( f \geq h \) and \( g \in h \) for any \( g > f \). Let \( A \) be the collection of such subsets. Note that each \( h \in A \) can be thought of as an exit rule starting in the last period of the history \( h \): if \( f \in h \) then the firm exits if the history \( f \) is realized. Let \( \theta(h, s) \equiv \{ g \in H_s | f > g \text{ for some } f \in h \} \), that is, \( \theta(h, s) \) is the set of all \( s \)-period histories such that a firm using the exit rule \( h \) does not exit by or during period \( s \). Finally, define \( A_s \equiv A_n H_s \), that is, \( A_s \) is the set of all \( s \)-period histories such that a firm using the exit rule \( h \) exits in period \( s \). The value of an existing firm in period \( t \) given the history \( h \in H_t \) and the stochastic sequence \( Y \) is

\[
\tilde{V}(Y, h) = \pi(y(h), h_t) + \sup_A \left( \sum_{s=1}^{t} \delta^{s-t} \sum_{g \in \theta(h, s)} \rho(g|h) \pi(y(g), g_s) + \sum_{s=t}^{\infty} \delta^{s-t} \sum_{g \in A_s} \rho(g|h) \chi(g_s) \right).
\]

Given \( h \in H_t \), let \( h^{-T} \) be the history comprised of the first \( t-T \) market conditions of \( h \). Equilibrium can now be formally defined.

Given an initial mass of firms, \( y_0 \), an equilibrium is a stochastic sequence \( \{Y, Q, P\} \) such that \( Q \) and \( P \) are derived from \( Y \) and such that, for all \( t \) and for all \( h \in H_t \),

\[
\begin{align*}
\tilde{V}(Y, h) &\leq \xi(h_t) & \text{if } y(h) > y(h^{-1}) \\
\tilde{V}(Y, h) &\geq \xi(h_t) & \text{if } y(h) = 0 \\
\tilde{V}(Y, h) &\leq \chi(h_t) & \text{if } y(h^{-1}) > y(h) > 0 \\
\tilde{V}(Y, h) &\geq \chi(h_t) & \text{if } y(h^{-1}) > y(h) = 0
\end{align*}
\]

Informally, an equilibrium is a stochastic sequence of masses of firms, levels of production, and prices such that (1) the current price always clears the current market for output, (2) each extant firm always maximizes its current profits given the current price, (3) more firms enter if and
only if the expected discounted profits from doing so exceed the entry cost, and (4) firms exit if and only if their expected discounted value is less than their scrap value.

Three additional assumptions are used:

Assumption 1: For all $t$, all $h \in H_t$, and all $s > t$, $E(\xi_s | h) = \sum_{g \in H_s} \rho(g|h) \xi(g_s)$ and $E(\chi_s | h) = \sum_{g \in H_s} \rho(g|h) \chi(g_s)$ are finite.

Assumption 2: For all $t$ and all $h \in H$, \[ \lim_{s \to \infty} \delta^{s-t} E(\xi_s | h) = \lim_{s \to \infty} \delta^{s-t} E(\chi_s | h) = 0. \]

Assumption 3: For all $t$ and all $h \in H_t$, $\bar{V}(h) < \xi(h_t)$, where \[ \bar{V}(h) = -\varphi(h_t) + \sup_{\mu} \left\{ \sum_{s=t+1}^{\infty} \delta^{s-t} \sum_{g \in H_s} \rho(g|h)(-\varphi(g_s)) + \sum_{s=t+1}^{\infty} \delta^{s-t} \sum_{g \in H_s} \rho(g|h) \chi(g_s) \right\}. \]

Assumption 1 guarantees that expected entry costs and scrap values are always well-defined. Assumption 2 guarantees that the sufficiently distant future is unimportant to the firms, because the expected present value of profits from time $s$ on, evaluated in the $t^{th}$ period of $h$, must lie between $E(\chi_s | h)$ and $E(\xi_s | h)$ in equilibrium. Assumption 3 is used to prove that the mass of firms entering at any given time is finite, although the mass of active firms may increase without bound over time. It states that if there is never any hope of ever making positive variable profit, the present expected value of entering and optimally scrapping the firm thereafter does not cover the cost of entry. In other words, Assumption 3 rules out expected capital gains that are sufficient to cover the expected maintenance costs.

4. Dynamic Competitive Behavior

This section contains the general results alluded to in section 2. Since the intuition behind the results was discussed in section 2, it will not be repeated here. The first result is a general existence and
uniqueness result. Note that no assumptions on the nature of the stochastic
process governing market conditions are required. Furthermore, since the
proof does not follow the strategy of establishing equilibrium as the
solution to a social planner's problem, that problem need not be
well-defined. For example, the rate of growth in demand can exceed the rate
of impatience.

Theorem 4.1: Given an initial mass of firms, \( y_0 \geq 0 \), an equilibrium exists.
It can be characterized by a stochastic sequence of intervals,
\((\{N(h),X(h)\})\), as follows: \( Y \) satisfies \( y(h) = \min\{X(h),\max[N(h),y(h-1)]\} \) and \( Q \)
and \( P \) are derived from \( Y \). Given \( y_0 \), all equilibria exhibit the same
stochastic price sequence. If, for all \( t \) and all \( h \in H_t \), \( d(x, h_t) \) is strictly
decreasing for all \( x \) satisfying \( d(x, h_t) > 0 \) and if \( C(\cdot, h_t) \) is twice
derdifferentiable with \( C''(0, h_t) > 0 \), then equilibrium is unique.

Proof: See Appendix.

The existence proof employs a limiting argument. More specifically, it
establishes the existence of an equilibrium in each \( \tau \)-period truncation of
the model that can be characterized by intervals of the form \([N_\tau(h),X_\tau(h)]\)
in a way analogous to that stated in the theorem. (The intervals are
defined by backward induction.) It is then shown that \( \lim_{\tau \to \infty} N_\tau(h) = N(h) \) and
\( \lim_{\tau \to \infty} X_\tau(h) = X(h) \) characterize an equilibrium for the model in the way stated
in the theorem.

Note that even without additional assumptions, all equilibria exhibit
the same stochastic price sequence. Of course, there can be the usual kinds
of nonuniqueness of the output sequence associated with "flats" in the
demand and marginal cost curves. There is also another kind of
nonuniqueness. Suppose, for example, that marginal cost is constant and
that, for some \( h \in H_t \), \( y(h) > 0 \) and \( \pi(y(h), h_t) = -\phi(h_t) \), i.e., \( p(h) = C'(\cdot, h_t) \).
Suppose, furthermore, that equilibrium exhibits strictly positive entry in
the next period for every realization of the stochastic process. Finally, suppose that \(-\varphi(h_t)+E(\xi_{t+1}|h)=\chi(h_t)\), so firms are indifferent between exiting and remaining. Then a new equilibrium can be constructed from the old by having a few more firms exit in the last period of \(h\) and having the same mass enter in the subsequent period. A simple way to disrupt this kind of nonuniqueness is to assume \(C'(0,m)>0\) so that \(\pi(y,h_t)=-\varphi(h_t)\) for all finite \(y\). This guarantees that each price is derived from a unique mass of firms; hence the uniqueness of the equilibrium price sequence implies the uniqueness of equilibrium. Another, more cumbersome, method is to assume away the relevant ties, of which \(-\varphi(h_t)+E(\xi_{t+1}|h)=\chi(h_t)\) is one example. In what follows it is assumed that equilibrium is unique.

As is true of Theorem 4.1, Theorem 4.2 requires no assumptions on the stochastic process governing market conditions.

**Theorem 4.2:** For all \(t\) and all \(h \in H_t\), if \(y(h)>0\) then

\[
\chi(h_t)-\delta E(\xi_{t+1}|h) \leq \pi(y(h),h_t) \leq \chi(h_t)-\delta E(\xi_{t+1}|h).
\]

**Proof:** The result follows directly from the definitions of equilibrium.

The right hand side is interpretable as the expected sunk cost that establishing a firm entails and the left hand side is interpretable as the opportunity cost of existing in the current period, i.e., it is the expected value of scrapping the firm today and rebuilding it tomorrow. See section 2 for an intuitive discussion.

The characterization of the equilibrium by means of the intervals \([N(h),X(h)]\) provides insights into how the stochastic process governing the endogenous variables (such as current profits, or current output price) can be derived from the exogenous stochastic process governing the market conditions. A state is a market condition and a mass of firms. Let \(S\) be the set of states. Let \(\bar{H}_t\) be the set of all \(t\)-period histories of states. Then \(\bar{h} \in \bar{H}_t\) can be written in the form \(\bar{h}=(h;y_1,\ldots,y_t)\) where \(h \in H_t\) and \(y_s \in \mathbb{R}_+\).
Theorem 4.3: The stochastic process, \( \tilde{\rho} \), governing states is derived from the stochastic process, \( \rho \), governing market conditions as follows: For \( \tilde{h} = (h; y_1, \ldots, y_t) \in \tilde{H}_t \) and \( \tilde{g} = (g; y_1, \ldots, y_t, y_{t+1}) \in \tilde{H}_{t+1} \) such that \( h = g^{-1} \),

\[
\tilde{\rho}(\tilde{g} | \tilde{h}) = \rho(g | h)
\]

if \( y_t \leq N(g) \) and \( y_{t+1} = N(g) \),

if \( y_t \geq X(g) \) and \( y_{t+1} = X(g) \),

or if \( N(g) \leq y_t \leq X(g) \) and \( y_t = y_{t+1} \).

\( \tilde{\rho}(\tilde{g} | \tilde{h}) = 0 \)

otherwise.

This theorem is a corollary of Theorem 4.1, which establishes that \( y(h) = \min\{X(h), \max\{N(h), y(h^{-1})\}\} \). Note that, whatever the exogenous stochastic process on market conditions, the probability distribution of the next period’s state depends only on the history of market conditions and the current mass of firms. Furthermore, the dependence on the history of market conditions is to the same degree that the probability distribution of the next period’s market condition depends on that history. In particular, if market conditions follow a first-order Markov process, then states follow a first-order Markov process as well. (Note, however, that the mass of firms does not: more information than the current mass of firms is required to construct the probability distribution for the mass of firms in the next period.) Thus anything that depends only on the current state, i.e., on the exogenous current variables and the endogenous mass of firms, must also follow a first-order Markov process (unless the variable happens to take on the same value in more than one state).

Assume henceforth that the process governing the market conditions is a first-order Markov process. The analysis to follow requires the existence of a limit distribution for the associated Markov process that governs the states. That is, let \( \tilde{\rho}^t_{ij} \) be the probability that state \( j \) will occur \( t \) periods after state \( i \) and let \( \tilde{\rho}^t_{ij} = \rho^{t-1}_{ij} \). Then a limit distribution is said to
exist independently of initial conditions if there exists a nonnegative vector, $\eta$, such that $\sum_j \eta_j = 1$ and $\lim_{t \to \infty} \bar{p}_{ij}^t = \eta_j$ independently of $i$. Necessary and sufficient conditions for a limit distribution to exist are known. Let $\bar{p}_{ij}$ be the probability that state $j$ occurs some time after state $i$.

**Proposition:** A necessary and sufficient condition for the existence of a limit distribution independently of initial conditions is that there is, in the set $S$ of states of the chain, exactly one aperiodic positive recurrent class $C$ such that $\bar{p}_{ij} = 1$ for all $j \in C$ and for all $i \in S$.

**Proof:** See Shiryayev (1984), Theorem 3, page 545.

Informally, a state is recurrent and positive if it occurs infinitely often and if the average time required for the state to be repeated is finite. A sufficient (but not necessary) condition for a state to be aperiodic is $\bar{p}_{ii} > 0$. The reader is referred to Shiryayev for a complete and formal discussion of the proposition's conditions.

Note that if $h \in H_t$, $g \in H_u$, and $h_t = g_u$, then the Markovian assumption implies that $N(h) = N(g)$ and $X(h) = X(g)$. Hence, to simplify notation, $N_i$ and $X_i$ will be written in the place of $N(h)$ and $X(h)$ for all $h$ whose last market condition is $i$. Similarly, all the variables that depend only on the current state will be denoted using subscripts: $p_i$, $q_i$, etc., will denote the output price in state $i$, individual output in state $i$, etc. Finally, let $m_i$ be the market condition in state $i$.

Since the stochastic process governing the states is endogenous it is inappropriate to place restrictions directly on it. The following theorem establishes sufficient conditions for the exogenous process governing market conditions to generate an endogenous process governing states with the desired properties.
Theorem 4.4: Assume $\sup_i N_i > \inf_i X_i$. If the set of market conditions, $M$, is recurrent and positive, with $\rho_{ij} > 0$ for all market conditions $i$ and $j$, then the set of states, $S$, contains exactly one aperiodic positive recurrent class, $C$. Furthermore, $\tilde{p}_{ij} = 1$ for all states $j \in C$ and all states $i \in S$.

Proof: See Appendix.

These conditions together are sufficient but not necessary. The importance of the condition that $\sup_i N_i > \inf_i X_i$, however, was noted in case 3 of section 2.

The limit distribution of the states is important because, under the assumptions, a variant of the ergodic theorem implies that, asymptotically, long-run averages of any variable that depends only on the current state are means with respect to the limit distribution. (A good discussion of ergodic theorems can be found in Karlin and Taylor (1975, pp. 474-501).)

Specifically, consider any variable, $R_j$, that depends only on the current state. (For example, $R_j$ could be current profit or a firm's value in state $j$.) Then, independently of the initial state, $i$, $\lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \rho_{ij} R_j = \sum_{j} \eta_j R_j$.

Now suppose that $\xi(m) = \xi$ and $x(m) = x$ for all market conditions, $m$.

Interpret $\xi$ as the required investment and $\pi_j / \xi$ as the rate of profit in state $j$, where $\pi_j = \rho_{ij} q_j - C(q_j, m_j) - \phi_j$. Theorem 4.5 says that if every change in market condition provokes either construction or scrapping of firms, then long-run average profit rates are declining in $\xi$ and increasing in $x$.

(Recall from case 2 in section 2 that these signs can be reversed if some changes in market conditions do not provoke construction or scrapping of firms.)

Theorem 4.5: Assume $\xi(m) = \xi$ and $x(m) = x$ for all $m \in M$. For all $y \geq 0$ and all $i, j \in S$, assume that $\tilde{p}_{(i,y)}(j,y) = 0$ if $i \neq j$. Then $\sum_j \eta_j (\pi_j / \xi)$ is declining in $\xi$ and increasing in $x$.

Proof: See Appendix.
Theorem 4.5 is proved using a result that is interesting in its own right. It says that the expected value of current profits equals \((1-\delta)\) times the expected value of firm value, where expectation is with respect to the limit distribution.

**Theorem 4.6:** \(\sum_j \eta_j \pi_j = (1-\delta) \sum_j \eta_j V_j\).

**Proof:** See Appendix.

It is mentioned in passing, without proof, that in the special case where states follow an i.i.d. process, the variance of current profits is the same as the variance of firm value. There is no apparent, elegant relationship between the two variances when states follow a general first-order Markov process, however.\(^7\)

The model can also be used to analyze the behavior of endogenous variables over time as well as how this behavior might be expected to differ across industries. For example, one can analyze the effects of differing entry costs across industries. Define the range of a variable as the difference between its extreme values, i.e., range \(R = \sup_i, j [R_i - R_j]\). The final result suggests that, other things than entry costs and scrap values equal, industries with higher variability in firm value might also be expected to exhibit lower variability in the number of firms. This result generalizes Theorem 2.7.

**Theorem 4.7:** Assume \(\xi(m) = \xi\) and \(\chi(m) = \chi\) for all \(m \in M\). Further assume \(\sup_i N_i > \inf_i X_i\). The range of firm value is increasing in \(\xi\) and decreasing in \(\chi\). The range of the mass of firms is nondecreasing in \(\xi\) and nonincreasing in \(\chi\).

The first part of Theorem 4.7, that the range of firm value is increasing in \(\xi\) and decreasing in \(\chi\) under the assumptions, is trivial because \(\xi\) and \(\chi\) are the extreme values of firm value. The second part of the proof, that the range of the mass of firms is nondecreasing in \(\xi\) and
nonincreasing in $\chi$, is proved by showing that, for all $h \in H$, $N(h)$ is nonincreasing in $\xi$ and nondecreasing in $\chi$ while $X(h)$ is nondecreasing in $\chi$ and nonincreasing in $\xi$. In words, higher entry costs discourage entry and exit while higher scrap values encourage entry and exit. This intuitively appealing result is not quite as obvious as it may seem. For example, although $N(h)$ is decreasing in $\xi(h_t)$ for $h \in H_t$, it can be strictly increasing in $\xi(g_s)$ for $g \in H_s | h$ and $s > t$: higher entry barriers in the future encourage more entry in the present. Nevertheless, as long as future entry costs do not increase by too much more than current entry cost (as is the case when $\xi(m) = \xi$ for all $m \in M$), the first must outweigh the second. A similar argument applies to changes in scrap values.

5. Summary and Concluding Remarks

This paper has analyzed a dynamic model of competitive behavior in the face of uncertainty when there are (positive) entry costs and (positive or negative) scrap values. The paper gives some insights into the effects of sunk costs and existence costs, relating them to the equilibrium option values of being in or out of the market. It further demonstrates how the endogenous stochastic processes driving such things as profits, price, and firm value, can be derived from underlying exogenous stochastic process. One implication of the results in the context of this model is that if exogenous variables (jointly) follow a first-order Markov process then endogenous variables that depend only on the exogenous variables and the current number of firms will also follow a first-order Markov process. Under the assumption that exogenous variables follow a first-order Markov process, it was shown that, in general, nothing can be said about the effects of entry costs and scrap values on long-run average profit rates. One of the reasons for this result is that, although higher entry costs (lower scrap values) imply that fewer firms will enter when times are good,
they also imply that more firms will remain in the industry when times are bad. Nevertheless, with the additional assumption that changes in market conditions are always large enough to provoke either entry or exit, it is unambiguously true that higher entry costs (and lower scrap values) decrease the long-run average rate of profit. In passing, it was also shown that the long-run average rate of profit is proportional to the long-run average of firm value.

The implication that different competitive industries composed of firms that maximize expected discounted profits will not exhibit the same long-run average rate of profit raises questions about the appropriate use of profit data. The ambiguous effects of, for example, entry costs on the long-run average profit rates suggests that economic theory has little to tell us about these variables. This should not be surprising since, by assumption, behavior is motivated by a desire to maximize present expected values rather than a desire to maximize long-run average profits.

The final result suggests that industries exhibiting high variation in firm value over time should also exhibit low variation in the number of firms.

Appendix

Proof of Theorem 4.1

For $y \geq 0$, $\pi(y, m)$ is defined in the text. For $y < 0$ define $\pi(y, m)$ in any way such that $\pi(\cdot, m)$ is continuous and nonincreasing with $\sup_y \pi(y, m) = \infty$. Consider a truncated version of the model where all active firms are forced to exit in period $\tau$ and receive their scrap value. For all $\tau$, all $t \leq \tau$, and all $h \in H_t$, define $V_{\tau}(\cdot, h) : \mathbb{R} \to \mathbb{R}$ by backward induction as follows:

\[
(A.1) \quad V_{\tau}(y, h) = \begin{cases} 
\chi(h_{\tau}) & \text{if } h \in H_{\tau}, \\
\pi(y, h_t) + \delta \sum_{g \in H_{t+1}} \rho(g|h) V_{\tau}[\hat{y}_{\tau}(g), g] & \text{if } h \in H_{\tau}, \ t < \tau,
\end{cases}
\]

where $\hat{y}_{\tau}(g) = \min(\hat{X}_{\tau}(g), \max(\hat{N}_{\tau}(g), y))$ for each $g \in H_{t+1}, \ t+1 < \tau$. The intervals
[\hat{N}_t(g), \hat{X}_t(g)] are defined for each \( g \in H_t \), \( t < \tau \), by backward induction. For \( g \in H_t \), \( \hat{N}_t(g) \) is the largest value such that \( V_{\hat{N}_t(g), g} = \xi(h_t) \). If such a value exists, \( \hat{X}_t(g) \) is the smallest value such that \( V_{\hat{X}_t(g), g} = \chi(h_t) \); otherwise \( \hat{X}_t(g) = \infty \). These choices are possible because \( V_{\tau}(y, g) \) is nonincreasing in \( y \) for all \( t \) and all \( g \in H_t \) and because an application of Assumption 3 guarantees that \( \lim_{y \to \infty} V_{\tau}(y, h) < \xi(h_t) \). Now note that

\[ V_{\tau}(y, h) \leq V_{\tau+1}(y, h) \]  

for all \( y \) and all \( h \in H_{\tau} \). Note also that if \( V_{\tau}(y, g) \leq V_{\tau+1}(y, g) \) for all \( y \) and all \( g \in H_{\tau+1} \), then \( V_{\tau}(y, h) \leq V_{\tau+1}(y, h) \) for all \( y \) and all \( h \in H_{\tau} \). (This is because if \( y_{\tau}(g) < y_{\tau+1}(g) \) for any \( g \in H_{\tau+1} \), then either \( y_{\tau+1}(g) = \hat{N}_{\tau+1}(g) \) or \( y_{\tau}(g) = \hat{X}_{\tau}(g) \). In either case, \( V_{\tau}(y_{\tau}(g), g) \leq V_{\tau+1}(y_{\tau+1}(g), g) \). If, on the other hand, \( y_{\tau}(g) = y_{\tau+1}(g) \), then, since \( V_{\tau}(y, g) \leq V_{\tau+1}(y, g) \) for all \( y \) and since \( V_{\tau} \) and \( V_{\tau+1} \) are nonincreasing, \( V_{\tau}(y_{\tau}(g), g) \leq V_{\tau+1}(y_{\tau+1}(g), g) \).) It follows that, for fixed \( y \) and all \( h \in H_{\tau} \), \( V_{\tau}(y, h) \) is monotonically decreasing in \( \tau \) (once \( \tau \) is sufficiently large for it to be defined). So since \( V_{\tau}(y, h) \) is nonincreasing in \( y \) for all \( \tau \), \( \hat{N}_t(h) \) and \( \hat{X}_t(h) \) are nondecreasing in \( \tau \) for all \( h \in H \) (once \( \tau \) is large enough for them to be defined). Hence the limits of \( \hat{N}_t(h) \) and \( \hat{X}_t(h) \), denoted respectively by \( \hat{N}(h) \) and \( \hat{X}(h) \), exist. Furthermore, \( \hat{N}(h) \) is finite. To see this note that for \( t < \tau - 1 \) and \( h \in H_t \), \( V_{\tau}(y, h) \) can be written in the form

\[ V_{\tau}(y, h) = \pi(y, h_t) + \sum_{s=t+1}^{\tau} \delta^{s-t} \sum_{g \in H_s} \rho(g|h) \pi(\hat{y}_s(g), g_s) \]

\[ + \delta^{\tau+1-t} \sum_{g \in H_{\tau+1-t}} \rho(g|h) V_{\tau}(y_{\tau}(g), g_{\tau+1-t}) \]

By Assumption 2, for any \( \varepsilon > 0 \) there exists \( T(\varepsilon) \) such that if \( T > T(\varepsilon) \)
for all \( t > T+1 \). Now fix \( T^* > T(\epsilon)+1 \), let \( \tilde{V}_{T^*}(h) = \lim_{y \to \infty} V_{T^*}(y, h) \) and note that for all \( h \in H_{T^*-1} \), \( \tilde{V}_{T^*}(h) \leq \bar{V}(h) \). A backward induction argument then establishes that \( \tilde{V}_{T^*}(h) \leq \bar{V}(h) \) for all \( h \in H_t \), \( t < T^* \). Hence there exists \( y(\epsilon, T^*) \) such that if \( y > y(\epsilon, T^*) \) then

(A.5) \[ V_{T^*}(y, h) < \tilde{V}(h) + \epsilon. \]

Now (A.2) (with \( T = T^* \)), (A.4) and (A.5) imply

(A.6) \[ \pi(y, h_t) + \sum_{s=t+1}^{T} \delta^{s-t} \sum_{g \in H_s} \rho(g|h)\pi(\hat{y}_{T^*}(g), g_s) < \tilde{V}(h) + 2\epsilon \]

if \( T > T(\epsilon) \) and \( y > y(\epsilon, T^*) \). Since the \( \hat{y}_{T}(g) \) are nondecreasing in \( \tau \),

(A.7) \[ \pi(y, h_t) + \sum_{s=t+1}^{T} \sum_{g \in H_s} \rho(g|h)\pi(\hat{y}_{T}(g), g_s) \]

\[ \leq \pi(y, h_t) + \sum_{s=t+1}^{T} \sum_{g \in H_s} \rho(g|h)\pi(\hat{y}_{T^*}(g), g_s) \]

for all \( t > T^* \). Then (A.2), (A.3), (A.6) and (A.7) imply \( V_{T}(y, h) < \tilde{V}(h) + 3\epsilon \) if \( T > T^* \) and \( y > y(\epsilon, T^*) \). Assumption 3 implies \( \tilde{V}(h_t) = \xi(h_t) - \Delta \) for some \( \Delta > 0 \).

Choose \( \epsilon < \Delta/3 \). Then \( V_{T}(y, h) < \xi(h_t) \) for all \( T > T^* \) if \( y > y(\epsilon, T^*) \), implying \( \hat{N}(h) \) is less than \( y(\epsilon, T^*) \) for all \( T > T^* \).

For all \( t \) and each \( h \in H_t \) define

\[ V(y, h) = \pi(y, h_t) + \sum_{s=t+1}^{T} \sum_{g \in H_s} \rho(g|h)\pi(\hat{y}(g), g_s), \]

where the \( \hat{y}(g) \) are inductively by \( \hat{y}(g) = \min\{X(g), \max(\hat{N}(g), \hat{X}(g^{-1}))\} \). It can be shown that, given a sequence \( \{y_{T}\} \) with \( \lim_{T \to \infty} y_{T} = y \), \( \lim_{T \to \infty} V_{T}(y_{T}, h) = V(y, h) \) for each \( h \in H \). So the definitions of \( \hat{N}_{T}(h) \) and \( \hat{X}_{T}(h) \), the monotonicity of \( V_{T}(y, h) \) in \( y \) for all \( h \in H \), and the definitions of \( \hat{N}(h) \) and \( \hat{X}(h) \), imply that, for all \( t \) and all \( h \in H_t \), \( V(\hat{N}(h), h) = \xi(h_t) \), \( V(\hat{X}(h), h) = \chi(h_t) \), and

\[ \xi(h_t) \geq V(y, h) \geq \chi(h_t) \] if \( \hat{N}(h) \leq y \leq \hat{X}(h) \). Hence, given the initial mass of firms,
If \( \hat{Y} \) is defined inductively by \( \hat{y}(h) = \min\{X(h), \max\{N(h), \hat{y}(h-1)\}\} \) then, for all \( t \) and \( h \in H_t \),
\[
(A.8a) \quad V[\hat{y}(h), h] \leq \xi(h_t) \\
(A.8b) \quad V[\hat{y}(h), h] = \xi(h_t) \quad \text{if } \hat{y}(h) > \hat{y}(h-1) \\
(A.9a) \quad V[\hat{y}(h), h] = \chi(h_t) \\
(A.9b) \quad V[\hat{y}(h), h] = \chi(h_t) \quad \text{if } \hat{y}(h) < \hat{y}(h-1).
\]

Note that
\[
(A.10) \quad \tilde{V}(\hat{Y}, h) = V[\hat{y}(h), h] \quad \text{for all } h \in H.
\]
Now define \( N(h) = \max\{0, \hat{N}(h)\} \) and \( X(h) = \max\{0, \hat{X}(h)\} \). Given the initial number of firms, \( y_0 \), define \( Y \) inductively by \( y(h) = \min\{X(h), \max\{N(h), y(h)\}\} \). Then \( Y \geq \hat{Y} \), so
\[
(A.11) \quad \tilde{V}(Y, h) \leq \tilde{V}(\hat{Y}, h) \quad \text{for all } h \in H.
\]
If \( y(h) = \hat{y}(h) \), however, the exit rule \( \mathcal{A} = \{f > h : \hat{y}(f) < \hat{y}(f-1) \text{ and } \hat{y}(g) \geq \hat{y}(g-1) \text{ for } g \text{ such that } f > g > h\} \) secures expected profits of \( \tilde{V}(\hat{Y}, h) \), implying that 
\( \tilde{V}(Y, h) = \tilde{V}(\hat{Y}, h) \). Hence
\[
(A.12) \quad \tilde{V}(Y, h) = \tilde{V}(\hat{Y}, h) \quad \text{if } y(h) = \hat{y}(h).
\]
Finally, note that
\[
(A.13) \quad y(h) = \hat{y}(h) \quad \text{if } y(h) > 0.
\]
Now \( A.8a \), \( A.10 \), and \( A.11 \) imply \( 3.2a \). Note that \( y(h) > y(h-1) \geq 0 \) implies \( \hat{y}(h) > \hat{y}(h-1) \) and, by \( A.13 \), \( y(h) = \hat{y}(h) \). So \( A.8b \), \( A.10 \), and \( A.12 \) imply \( 3.2b \). For \( y(h) > 0 \), \( A.13 \), \( A.9a \), and \( A.12 \) imply \( 3.3a \).
For \( y(h-1) > y(h) > 0 \), \( A.9b \), \( A.10 \), \( A.12 \), and \( A.13 \) imply \( 3.3b \). For \( y(h-1) > y(h) = 0 \), \( y(h) = \hat{y}(h) \) and \( y(h-1) = \hat{y}(h-1) = 0 = \hat{y}(h) \), so \( A.9b \), \( A.10 \) and \( A.11 \) imply \( 3.3c \).

To see that there is unique equilibrium price sequence, consider two
equilibrium sequences, \( Y \) and \( Y' \). Note that for any \( t \) and \( h \in H_t \), if
\( y(h-1) \leq y'(h-1) \) but \( y(h) > y'(h) \) then \( \tilde{V}(Y, h) = \tilde{V}(Y', h) \). This is because either
\( y(h) > y(h-1) \) or \( y'(h) < y'(h-1) \). If the former, \( \xi(h_t) = \tilde{V}(Y, h) \leq \tilde{V}(Y', h) \) and if
the latter, \( \bar{V}(Y, h) \leq \bar{V}(Y', h) = x(h_t) \). Since \( V(Y', h) \leq x(h_t) \) and \( V(Y, h) \geq x(h_t) \), \\
\( \bar{V}(Y, h) = \bar{V}(Y', h) \).

Now suppose \( p(h) < p'(h) \) for some \( h \in H_t \). Then \( \bar{V}(Y, h) < \bar{V}(Y', h) \). If \\
y(h^{-1}) \neq y'(h^{-1}) \) then, since \( y(h) > y'(h) \), \( \bar{V}(Y, h) = \bar{V}(Y', h) \) and a contradiction has been reached. Hence \( y(h^{-1}) > y'(h^{-1}) \). Now this implies \( \bar{V}(Y, g) \leq \bar{V}(Y', g) \) for all \( g \in H_t \). Since \( \bar{V}(Y, h) < \bar{V}(Y', h) \), this implies \( \bar{V}(Y, h^{-1}) < \bar{V}(Y', h^{-1}) \).

If \( y(h^{-2}) \neq y'(h^{-2}) \) then, since \( y(h^{-1}) > y'(h^{-1}) \), \( \bar{V}(Y, h^{-1}) = \bar{V}(Y', h^{-1}) \) and a contradiction has been reached. Hence \( y(h^{-2}) > y'(h^{-2}) \). Continuation of the argument inductively implies that \( y(h^{-k}) > y'(h^{-k}) \) for all \( k \). In particular, \( y_0 > y'_0 \), i.e., the initial conditions differ. This contradiction proves the uniqueness of the equilibrium price sequence.

The rest of the theorem is seen to follow by noticing that under the additional conditions each price can be derived only from a unique mass of firms. This concludes the proof of Theorem 4.1.

Proof of Theorem 4.4: Let \( \Psi_N = \{ y | y = N_i \} \) for some \( i \) such that \( N_i \geq \inf_i X_i \). Let \( \Psi_X = \{ y | y = X_i \} \) for some \( i \) such that \( X_i \leq \sup_i N_i \). Let \( I_i \) denote the interval \( [N_i, X_i] \). Then the set of states \( C = \{ (i, y) | y \in (\Psi_N \cup \Psi_X) \cap I_i \} \) is indecomposable (because \( \rho_{ij} > 0 \) for all market conditions, \( i \) and \( j \)) and it is the only indecomposable subset of states (because all states communicate with it).

It is obvious that \( C \) is aperiodic because \( \rho_{ii} > 0 \) and hence \( \bar{\rho}(i, y)(i, y) > 0 \) for all \( (i, y) \in C \). It remains to be seen that \( C \) is recurrent and positive. Since the set of market conditions is aperiodic, indecomposable, recurrent, and positive, \( \lim_{t \to \infty} \rho_{ik} = g_k > 0 \) for all \( i \) and \( k \), where \( g \) is the limiting distribution of the stochastic process, \( \rho \). (See Shiryaev, Lemma 3, p. 535.) Now the state \( (i, y) \in C \) has the property that \( y = N_j \) or \( y = X_j \) for some market condition, \( j \). In the former case choose \( k \) so that \( X_k \leq N_j \) and in the latter case choose \( k \) so that \( N_k \geq X_j \). In either case, \( \bar{\rho}(i, y)(i, y) \geq \rho_{ik} \rho_{kj} > 0 \). Hence, \( \lim_{t \to \infty} \bar{\rho}(i, y)(i, y) \geq \rho_{ik} \rho_{kj} > 0 \). It follows that \( \lim_{t \to \infty} \rho(i, y)(i, y) = \infty \).
which is a necessary and sufficient condition for recurrence. (See Shirayev, Lemma 3, p. 534). Given recurrence, \( \lim_{t \to \infty} \rho(1,y)(1,y) > 0 \) establishes positivity. (Shirayev, Lemma 3, p. 535.) Finally, since the state \( j \in C \) is recurrent and all states, \( i \in S \), communicate with each \( j \in C \), \( \hat{f}_{1j} = 1 \). This concludes the proof.

Proof of Theorem 4.5: Under the assumptions of the theorem, either \( V_J = \xi \) or \( V_J = \chi \). Let \( J \) be the subset of states such that \( V_J = \xi \) and let \( K \) be the subset of states such that \( V_J = \chi \). Then Theorem 4.6 implies that \( \sum_j \eta_j \pi_j / \xi = \sum_j \eta_j + \sum_k \eta_k \chi / \xi \), which is declining in \( \xi \) and increasing in \( \chi \).

Proof of Theorem 4.6: By definition, \( V_i = \pi_i + \delta \sum_j \rho_{ij} V_j \), so

\[
(A.14) \quad \sum \eta_i \pi_i = \sum \eta_i [V_i - \delta \sum_j \rho_{ij} V_j],
\]

or, equivalently,

\[
(A.15) \quad \sum \eta_i \pi_i = \sum \eta_i V_i - \delta \sum_j V_j \sum \eta_j \rho_{ij}.
\]

Now the limiting distribution has the property that \( \eta_j = \sum \eta_i \rho_{ij} \). This and (A.15) imply the result.

Proof of Theorem 4.7: Under the assumptions of the theorem, \( \xi \) and \( \chi \) are the extreme values of firm value. Hence the range is \( \xi - \chi \), which is clearly increasing in \( \xi \) and decreasing in \( \chi \). Now for each \( h \in H \) compare the values of \( \hat{N}_\tau(h) \) and \( \hat{X}_\tau(h) \) when \( \xi(m) = \xi \) for all market conditions to the analogous values, say \( N'_\tau(h) \) and \( X'_\tau(h) \) when \( \xi(m) = \xi' > \xi \) for all market conditions. Note that, for \( h \in H_\tau \), \( N'_\tau(h) < N_\tau(h) \) and \( X'_\tau(h) = X_\tau(h) \). Make the induction hypothesis that for all \( g \in H \), \( u < s \leq \tau \), \( N'_\tau(g) \leq N_\tau(g) \) and \( X'_\tau(g) \geq X_\tau(g) \). Let \( G(\tau, h) = \{ g \in H | g > h \} \) and \( G_\tau(h) \) for some \( t \leq \tau \) and let \( \{ y_\tau(g) \} \) and \( \{ y'_\tau(g) \} \) be inductively defined by \( y_\tau(g) = \min \{ X_\tau(g), \max [N_\tau(g), y_\tau(g^{-1})] \} \) and \( y'_\tau(g) = \min \{ X_\tau(g), \max [N_\tau(g), y'_\tau(g^{-1})] \} \), respectively with \( y_\tau(h) = y'_\tau(h) = N_\tau(h) \).

Let \( (g \in H) | y'_\tau(g) < y_\tau(g) \) and \( y'_\tau(g^{-k}) \geq y_\tau(g^{-k}) \) for all \( k \) such that \( g^{-k} \in G(\tau, h) \). Informally, \( \Gamma \) is the set of "first times" that \( y'_\tau(g) < y_\tau(g) \). Note that it must be that \( y_\tau(g) = N_\tau(g) \) for all \( g \in \Gamma \). Hence, for all \( g \in \Gamma \),
(A.16) $V'_\tau(y'_\tau(g),g) - V'_\tau(y'_\tau(g),g) \leq \xi' - \xi$.

This implies, since $y'_\tau(g^{-k}) \geq y'_\tau(g^{-k})$ for all $g \in \Gamma$ and all relevant $k$, that

$$V'_\tau(N'_\tau(h),h) - V'_\tau(N'_\tau(h),h) \leq \sum_{s=t}^{T} \delta^{S_{t}} \sum_{g \in \Gamma_{s}} \rho(g|h)(\xi - \xi') \leq \xi' - \xi$$

where $\Gamma_{s} = \Gamma \cap H_{s}$. Hence $N'_\tau(h) \leq N'_\tau(h)$ for all $h \in H_{t}$, $t \leq \tau$. Similar reasoning establishes that $X'_\tau(h) \geq X'_\tau(h)$ for all $h \in H_{t}$, $t \leq \tau$. The last two sentences imply (taking limits) that $N'(h) \leq N(h)$ and $X'(h) \geq X(h)$ for all $h \in H$. Since the range of the mass of firms is $\sup_{h} N(h) - \inf_{h} X(h)$, the theorem is proved.
REFERENCES


Footnotes

1 For a recent example see Schmalansee (1985).


3 Scrap values may be positive or, if there are disposal costs, negative.

4 See Dixit (1988a) for a discussion of option values in a similar context.

5 See Baldwin (1986), Baldwin and Krugman (1986), and Dixit (1988b) for a discussion of such phenomena in an international trade context and see Dixit (1988a) for a discussion in an industrial organization context.

6 See Shiryayev (1984, p. 541, Theorem 1) for a discussion of why these are the relevant relationships.

7 Indeed, variances are not very well-behaved in this model, leading the researcher to consider ranges as measures of variability in what follows.