RESEARCH NOTES

DUALISTIC AGRICULTURE AND OPTIMUM LAND REFORM

The core of land reform—the redistribution of a given area of cultivated land—is both an economic and a political problem. The issue involved in the redistribution of land has both equity and efficiency implications. That is, redistribution will have effects both on the level and distribution of income. These effects have both static and dynamic dimensions in the sense that these effects will be immediate as well as over a period of time.

While economists, political scientists, and other social scientists have spent enormous amounts of time and effort on studying various socio-politico-economic-institutional aspects of land reforms in developing countries, surprisingly not enough attention has been paid to the issue and theory of optimum land reform, particularly in the context of dualistic agriculture. For our problem, we will define dualistic agriculture as the existence of substantial differences in the objective circumstances confronting the small farms in contrast to the large farms. These differences could be characterized in several ways: in terms of objectives of production, namely, subsistence versus commercialization; or in terms of methods of production, namely, traditional technology versus modern technology; or in terms of imperfections in the market structure resulting in differential input or output prices for small and large farms, etc. The most ubiquitous form of dualism that exists in many developing nations' agriculture is in terms of labour market dualism; that is, the implicit wage rate of family labour for small farms being less than the explicit market wage rate for hired labour that the large farms confront. Also, since it has been shown more or less, that in many countries the nature of technology and the rate of adoption of technical progress among large and small farms do not seem to differ significantly, it is preferable to use labour market dualism as the most appropriate and ubiquitous dualistic characteristic.

The concept of land reform has two aspects: equity aspect and efficiency aspect. The efficiency aspect is related to the concept of optimum land reform. The concept of optimum land reform has two dimensions—static and dynamic. Static aspect of the optimum land reform refers to the choice of that distribution of land between small and large farms that maximizes some pre-determined objective function at a point in time given the technology and resources. The dynamic aspect of optimum land reform refers to the choice of that distribution of land between small and large farms that maximizes over time the value of the pre-determined objective function taking into account the changing technology, resource constraints, and the saving-investment behaviour or the nature of the capital accumulation process of small and large farms.

Since land reform has both static and dynamic implications for economic efficiency, it is worthwhile to explore the conditions for optimum land reform in the context of a dualistic agricultural economy.
Objectives

1. To explain the existence of such objective realities as the stylized facts of dualistic agriculture with the help of a production function model.
2. In the above context (1), to show that there exists an optimum distribution of land which will maximize total agricultural income or total agricultural employment in the economy.
3. To discuss the static and dynamic aspects of optimum land reform.

Some Stylized Facts of Dualistic Agriculture

Some of the stylized facts of dualistic agriculture are:
1. Output per hectare is higher on smaller farms than on larger farms.
2. Employment per hectare is higher on smaller farms than on larger farms.
3. Savings rate is higher on larger farms than on smaller farms.
4. Productivity per work unit (labour) is higher on larger farms than on small farms.
5. The rate of adoption of technical progress seems to be the same for small and large farms. Nothing much is known about the nature of technical progress itself across farm size-groups.

The above stylized facts have implications for efficiency and equity issues in the context of land reform. In fact many advocates of land redistribution or land reform have used the first two very extensively in their arguments to encourage radical land distribution. The third one, which has lots of implications for growth of output over time, has not yet received the emphasis it deserves in the literature. Since the fourth one does not seem to make any difference between large and small farms, we will leave it out in our analysis.

Under the static concept of optimum land reform the first two empirical realities ensure increase in output and employment from land redistribution from large to small farms. In the dynamic case the third reality implies that land redistribution could affect growth of output over time.

Since our objective is to study the static and dynamic implications of land reform, we will assume that the objective function is one of maximizing total agricultural production in the economy. Though we can define various levels of dualism by incorporating one or more of the several dualistic characteristics listed above, for our purposes we will assume that the only kind of dualistic character is the labour market dualism. Also, though there may be other socio-economic effects of land distribution, we will not consider them for our purposes here.

The Model

Since Cobb-Douglas function has been more or less accepted as the most appropriate aggregate agricultural production function, we will define the
aggregate output \((Q)\) with three inputs: land \((L)\), labour \((N)\), and capital \((K)\) as:

\[ Q = A N^\alpha K^\beta (xL)^{(1-\alpha-\beta)} \]  

We will assume:

(a) \(Q\) is subject to constant returns to scale.
(b) The amount of available land is fixed in the economy and cannot be varied even in the long run.
(c) Currently available land is distributed between the (arbitrarily fixed) two farm size-groups, namely, small and large farms, in the ratio \((x)\) with \((1-x) < x\).
(d) In the short run (say less than or equal to a year) output can be increased only by increasing labour since capital is fixed in the short run.
(e) In the long run (say, beyond one year) capital is variable. Increase in capital for each farm size-group exactly equals the amount of net investment of that size-group in that year.
(f) The rate of savings on large farms \((S_2)\) is greater than on small farms \((S_1)\).
(g) Production function is identical for large and small farms.
(h) Implicit wage rates of small farm labour \((W_1)\) is less than the market wage rate of large farm labour \((W_2)\).
(i) The amount of large farms’ capital is \(\lambda\) time the amount of small farm’s capital (\(\lambda\) to be defined later as a condition).

In the short run, given \(K\) and \(L\), assuming the price of output to be unity, labour being the only variable factor, output will be maximized if

\[ \frac{dQ}{dN} = \alpha A N^{\alpha-1} K^\beta (xL)^{(1-\alpha-\beta)} = W \]

That is

\[ N = (xL) \left( \frac{1-\alpha-\beta}{1-\alpha} \right) K^{\beta} \left( \frac{aA}{W} \right)^{\frac{1}{1-\alpha}} \]

So optimum output \((Q^*)\) in the short run will be:

\[ Q^* = A (xL) \left( \frac{1-\alpha-\beta}{1-\alpha} \right) K^{\beta} \left( \frac{aA}{W} \right)^{\frac{1}{1-\alpha}} \]

Output per hectare \((q)\) will be:

\[ q = A (xL) \left( \frac{1-\alpha-\beta}{1-\alpha} \right) K^{\beta} \left( \frac{aA}{W} \right)^{\frac{1}{1-\alpha}} \]

Employment per hectare \((n)\) will be:

\[ n = (xL) \left( \frac{1-\alpha-\beta}{1-\alpha} \right) K^{\beta} \left( \frac{aA}{W} \right)^{\frac{1}{1-\alpha}} \]
Productivity per work unit \((p)\) will be:

\[
(7) \quad p = \frac{w}{a}\text{ dividing (4) by } N.
\]

Since under the conditions of labour market dualism assumed earlier, \(W_2 > W_1\), \(i.e.,\) wage rate being higher on large farms, it can be easily seen, as observed in reality, productivity per work unit will be higher on large farms. But the relationships between farm size, on the one hand, and employment and output per hectare, on the other hand, are not obvious since we have \(K\) explicitly appearing in equations \((5)\) and \((6)\). Let us see under what conditions we will have higher per hectare productivity and employment on small farms as observed in reality.

**Per Hectare Productivity**

\[
(8) \quad (xL) \frac{\beta}{1-a} K_1 = \left(\frac{a^A}{W}\right)^{\frac{1}{1-a}} \left\{ (1-x) L \right\} \frac{\beta}{1-a} K_2 = \left(\frac{a^A}{W}\right)^{\frac{1}{1-a}},
\]

where subscripts 1 and 2 describe small and large farms respectively. Simplifying \((8)\), we find per hectare productivity will be higher on small farms if

\[
(9) \quad K_1 > \left(\frac{x}{1-x}\right) \left(\frac{W_1}{W_2}\right)^{\frac{1}{\beta}} K_2.
\]

That is, since \(\left(\frac{x}{1-x}\right)\) and \(\left(\frac{W_1}{W_2}\right) < 1\), the small farm share of capital in the economy should be greater than a fraction of the large farm share of capital in the economy which condition will be easily met. Thus our model predicts correctly the per hectare productivity among small and large farms as observed in reality.

Similarly, the per hectare employment relation between small and large farms is given by:

\[
(10) \quad (xL) \frac{\beta}{1-a} K_1 = \left(\frac{a^A}{W_1}\right)^{\frac{1}{1-a}} \left\{ (1-x) L \right\} \frac{\beta}{1-a} K_2 = \left(\frac{a^A}{W_2}\right)^{\frac{1}{1-a}}
\]

which will be so if

\[
(11) \quad K_1 > \left(\frac{x}{1-x}\right) \left(\frac{W_1}{W_2}\right)^{\frac{1}{\beta}} K_2.
\]

Since \(\left(\frac{W_1}{W_2}\right)\) is a fraction and \(\frac{1}{\beta} > \frac{\alpha}{\beta}\) for all values of \(0 < \alpha < 1\), whenever condition \((9)\) is met, \((11)\) is automatically met. Thus \((9)\) is sufficient for ensuring higher per hectare employment on small farms.

Now let us find out whether output in the economy can be increased by redistributing land from large farms to small farms and if so, the optimum level of redistribution. Here, for the sake of simplicity and without loss
of generality we will assume redistribution is done with no cost to the society by legislative action.\(^1\)

Aggregate output \(Q_T\) in the economy equals small farm output \(Q_1\) plus large farm output \(Q_2\):

\[
Q_T = Q_1 + Q_2
= A (xL) \left( \frac{1-a-\beta}{1-a} \right) \frac{\beta}{K_1 \frac{1-a}{1-a}} \left( \frac{aA}{W_1} \right)^{\frac{a}{1-a}}
+ A \left[ (1-x) L \right] \frac{1-a-\beta}{1-a} \frac{\beta}{K_2 \frac{1-a}{1-a}} \left( \frac{aA}{W_1} \right)^{\frac{a}{1-a}}
\]

So

\[
\frac{dQ_T}{dx} = A \left( \frac{1-a-\beta}{1-a} \right) L \left( \frac{1-a-\beta}{1-a} \right) (aA)^{\frac{a}{1-a}}
- \left( \frac{K_1}{x} \right)^{\frac{1-a}{1-a}} \frac{\beta}{W_1^{\frac{1-a}{1-a}}} - \left( \frac{K_2}{x} \right)^{\frac{1-a}{1-a}} \frac{\beta}{W_2^{\frac{1-a}{1-a}}}
\]

Thus \(\frac{dQ_T}{dx} > 0\) if

\[
K_1 > \left( \frac{x}{1-x} \right) \left( \frac{W_1}{W_2} \right)^{\beta} K_2
\]

Which is the same condition as (9) for deriving results consistent with the stylized facts of dualistic agriculture we discussed earlier. Similar results can be drawn for total employment also. So it appears that redistribution is likely to lead to increased production and employment so long as (9) is satisfied.

**The Optimum Level of Redistribution**

The optimum level of redistribution is given by the optimum value of \(x\) (small farm share of land) which will maximize the total output. Optimum value of \(x\) is obtained by setting

\[
\frac{dQ_T}{dx} = A \left( \frac{1-a-\beta}{1-a} \right) L \left( \frac{1-a-\beta}{1-a} \right) (aA)^{\frac{a}{1-a}}
- \left( \frac{K_1}{x} \right)^{\frac{1-a}{1-a}} \frac{\beta}{W_1^{\frac{1-a}{1-a}}} - \left( \frac{K_2}{1-x} \right)^{\frac{1-a}{1-a}} \frac{\beta}{W_2^{\frac{1-a}{1-a}}} = 0
\]

\(^1\) Inclusion of the cost of transfer of land in the model will not change the qualitative nature of the conclusions reached in any significant way so long as the cost function is of the normal variety.
which gives, after simplification,

\[
x = \frac{\left(\frac{K_1}{K_2}\right) \left(\frac{W_0}{W_1}\right)^{\alpha}}{1 + \left(\frac{K_1}{K_2}\right) \left(\frac{W_2}{W_1}\right)^{\beta}} = \frac{1}{\left(\frac{K_2}{K_1}\right) \left(\frac{W_1}{W_2}\right)^{\alpha}}
\]

In fact (16) is the optimum because with

\[
B = A \left(\frac{1 - \alpha - \beta}{1 - \alpha}\right) \left(\frac{1 - \alpha - \beta}{1 - \alpha}\right)^{\alpha} \left(\frac{a}{1 - \alpha}\right)^{\alpha}
\]

the second derivative of (12) is

\[
\frac{d^2Q_T}{dx^2} = -B \frac{\beta}{1 - \alpha} K_1 \frac{\beta}{1 - \alpha} - \frac{\beta}{1 - \alpha} - 1 - \frac{\alpha}{1 - \alpha}
\]

\[
- B \frac{\beta}{1 - \alpha} K_2 \frac{\beta}{1 - \alpha} (1 - x) - \frac{\beta}{1 - \alpha} - 1 - \frac{\alpha}{1 - \alpha} < 0
\]

that is, \( K_1 > \left(\frac{x}{1 - x}\right)^{\frac{1 - \alpha}{\beta}} \left(\frac{W_1}{W_2}\right)^{\frac{\alpha}{\beta}} \) \( K_2 \) which holds, so long as (14) holds.

Thus under the given Cobb-Douglas production function with dual wage rates and condition (14), if the current small farms' share of land \( x \) is greater than the RHS of (16), then redistribution will only lead to adverse effects on output and employment. But what is interesting to see here is that total redistribution of land in favour of small farms is not economically optimal. In other words, total elimination of large farms is not warranted by economic efficiency criteria.

\textit{Dynamic Implications of Land Redistribution}

While static efficiency has received wide attention in the analysis of implications of land redistribution, dynamic efficiency implications received almost no theoretical attention. In dynamic analysis, we are concerned with maximization of output over time subject to given behavioural physical and technological constraints. A particular distribution of land which maximizes static output may not be the one that maximizes the output over a given period of time. This is because in the static case, we are maximizing output subject to a given technology and given level of capital. In contrast
in the dynamic analysis we allow for changes in them and thus differences in rates of adoption of technical progress and capital accumulation between small and large farms might make a difference to the conclusions derived under static conditions.

To analyse the dynamic implications of land redistribution, let us assume the following:

1. The rate of savings in small farms ($S_1$) is smaller than that ($S_2$) on large farms.
2. The supply of labour over the given planning horizon always exceeds the demand for it resulting in institutionally fixed wage rates ($W_1$ for small farms and $W_2$ for large farms) which do not change over time. At each point of time given the amount of capital, the optimum level of output is attained by using labour up to that level at which the marginal productivity of labour equals $W_1$ for small farms and $W_2$ for large farms. In other words, labour market dualism continues into the future.
3. Whatever is saved is invested and results in automatic increase in capital for both farm groups.²
4. For the sake of simplicity, we will assume the rate of discount or the time preference rate to be zero.
5. The planning horizon is given and finite.

Given the above assumptions the objective is to maximize:

\[ Z = \int_0^T Q_{1t} \, dt + \int_0^T Q_{2t} \, dt \]

where

\[ Q_1(t) = A \left\{ N_{1t} \right\}^\alpha \left\{ K_{1t} \right\}^\beta \left(xL\right)^\gamma \]

\[ Q_2(t) = A \left\{ N_{2t} \right\}^\alpha \left\{ K_{2t} \right\}^\beta \left(1-x\right)L^\gamma \]  with $\gamma = 1 - \alpha - \beta$

subject to:

\[ \frac{\delta Q_{1t}}{\delta N_{1t}} = W_1 \]

\[ \frac{\delta Q_{2t}}{\delta N_{2t}} = W_2 \]

with $W_2 > W_1$.

\[ \frac{dK_{1t}}{dt} = I_{1t} = S_1 \, Q_{1t} \]

². This assumption can be easily relaxed by allowing for different proportions of actual savings being invested in agriculture but this will not result in any qualitative changes so long as investment as a proportion of output on large farms is greater than that on small farms.
\[
\begin{align*}
\frac{dK_{2t}}{dt} &= I_{2t} = S_1 \quad Q_{2t} \\
\text{with } S_2 &> S_1.
\end{align*}
\]

Since
\[
\alpha A \left\{ N_{1t} \right\}^{a-1} \left\{ K_{1t} \right\}^\beta \left( xL \right)^\gamma = W_1
\]
and
\[
\alpha A \left\{ N_{2t} \right\}^{a-1} \left\{ K_{2t} \right\}^\beta \left\{ \left( 1-x \right) L \right\}^\alpha = W_2,
\]
we have
\[
\left\{ N_{2t} \right\}^{a-1} \left( K_{2t} \right)^\beta \left\{ \left( 1-x \right) L \right\}^\gamma > \left( N_{1t} \right)^{a-1} \left( K_{1t} \right)^\beta \left( xL \right)^\gamma.
\]
That is,
\[
\left\{ \frac{N_{2t}}{N_{1t}} \right\}^{a-1} \left\{ \frac{K_{2t}}{K_{1t}} \right\} > \left( \frac{x}{1-x} \right)^\gamma.
\]

Now from (25)
\[
N_{1t} = \left[ -\frac{W_1}{\alpha A \left( xL \right)^\gamma} \right]^{1/a-1} \left[ K_{1t}^{-\beta} \right]
\]

Now feeding (29) into (23) gives
\[
\frac{dK_{1t}}{dt} = S_1 \quad W_1 \frac{a}{(a-1)} \quad A \quad \frac{1}{a-1} \quad \frac{\gamma}{(xL)^\gamma} \quad \left( xL \right) \quad \frac{\gamma}{a-1}
\]

Now setting\( b_1 = W_1 \frac{a}{a-1}, \quad h = a \quad A \quad \frac{1}{a-1} \quad \frac{1}{a-1} \quad \frac{1}{a-1} \quad \frac{1}{a-1} \quad \left( xL \right)^{\gamma} \frac{1}{a-1} \)
we can derive
\[
\frac{dK_{1t}}{dt} = K_{1t} \quad \frac{\beta}{a-1} \quad \frac{\gamma}{a-1} \quad \left( xL \right)^{\gamma} \frac{1}{a-1}
\]

After integrating (31) and solving for \( K_{1t} \), we get
\[
K_{1t} = S_1 b_1 h k_{1t} t \left( \frac{\beta}{a-1} + 1 \right) + K_{10} \quad \frac{\gamma}{a-1} \quad \left( xL \right)^{\gamma} \frac{1}{a-1}
\]
Now after substituting $N_{it}$ from equations (29) and (32), we can express $Q_{it}$ (see equation (19)) as:

$$Q_{it} = L_1 \left( L_2 + K_{1t}^{1-a} \right)^\gamma \left( L_2 + K_{10}^{1-a} \right)^\gamma \alpha \left( \frac{\beta (1-a)}{a} \right)$$

where

$$L_1 = A (xL)^\gamma \left( \frac{W_1}{a L} \right)^\gamma = \sqrt[\alpha]{A} (\frac{W_1}{a A} (xL)^{1-a};$$

$$L_2 = S_1 b_i h_i \left( \frac{\beta}{a - 1} + 1 \right)$$

Similarly,

$$Q_{2t} = M_1 \left( M_2 + K_{20}^{1-a} \right)^\gamma \left( M_2 + K_{20}^{1-a} \right)^\gamma \alpha \left( \frac{\beta (1-a)}{a} \right)$$

where

$$M_1 = A \left[ (1-x) L \right]^\gamma \left( \frac{W_2}{a A (1-x) L} \right)^\gamma$$

$$M_2 = S_2 b_2 h_k \left( \frac{\beta}{a - 1} + 1 \right)$$

$$K_{2t} = \left[ \left( M_2 + K_{20}^{1-a} \right)^\gamma \right]$$

Now our problem is to choose that value of $x$ that will maximize total production in the economy over time.

Since

$$Z = \int_0^T Q_{1t} \, dt + \int_0^T Q_{2t} \, dt$$

$$= \frac{1}{S_1} \int_0^T dK_{1t} + \frac{1}{S_2} \int_0^T dK_{2t}$$

Solving the integrals and writing values of $K_{1t}$, $K_{2t}$ derived earlier, it can be shown that, given the planning horizon $(t)$, $\frac{dz}{dx} > 0$ whenever

$$K_{t0} > \left( \frac{x}{1-x} \right) \left[ h \frac{\gamma}{1-a} + \frac{\gamma}{1-a} \left( 1-x \right)^{\frac{\gamma}{1-a}} \right] + \left( \frac{b_2}{b_1} \right)^\gamma K_{20}^{1-a}$$
Thus if the above condition is fulfilled in the distribution of capital across the two farm groups, then it is possible to increase the overall income over a given period of time by redistributing land. What is worth noting about condition (36) is that the more distant the planning time horizon is, the more stringent the initial condition on $K_{10}$ in order for land redistribution to be profitable which is evident from the fact that the first and second derivatives of $K_{10}$ with respect to time are positive. If, in fact, condition (36) is satisfied, the optimal distribution of land between small and large farms can be found by setting $\left(\frac{dz}{dx}\right)$ to zero.

**A Comparison between Static and Dynamic Results**

Under economic statics for $\frac{dQ_T}{dx} > 0$ we needed the condition that

$$K_1 > \left(\frac{x}{1-x}\right) \left(\frac{W_1}{W_2}\right)^\beta K_2$$

In contrast under economic dynamics, for $\frac{dz}{dx} > 0$, we need the condition that

$$K_{10} > \left(\frac{x}{1-x}\right) \left[ h \frac{\gamma}{1-\alpha} t L^{\frac{\gamma}{1-\alpha}} (1-x)^{\frac{\gamma}{1-\alpha}} \left\{ \left(\frac{b_2}{b_1}\right)^\beta (S_2 b_2 - S_1 b_1) \right\} \right]$$

$$+ \left(\frac{b_3}{b_1}\right)^\beta K_{20}$$

Since the initial condition for both static and dynamic models is the same, i.e.,

$$K_1 = K_{10}$$

we can easily prove that

$$\left(\frac{x}{1-x}\right) \left[ h \frac{\gamma}{1-\alpha} t L^{\frac{\gamma}{1-\alpha}} (1-x)^{\frac{\gamma}{1-\alpha}} \left\{ \left(\frac{b_2}{b_1}\right)^\beta (S_2 b_2 - S_1 b_1) \right\} \right]$$

$$+ \left(\frac{b_3}{b_1}\right)^\beta K_{20} > \left(\frac{x}{1-x}\right) \left(\frac{W_1}{W_2}\right)^\beta K_2.$$
Since \( K_2 = K_{2e} \) we can obtain from equation (39) after simplification the equation:

\[
(\frac{W_1}{W_2})^{\frac{\alpha}{1-a}} \frac{\beta}{S_2} - \frac{S_1}{W_2^{\frac{1}{1-a}}} > 0. \quad \text{That is}
\]

\[
(41) \quad S_2 > S_1 \left(\frac{W_2}{W_1}\right)^{\frac{\alpha}{\beta}}.
\]

Thus we might need a more stringent initial condition in the distribution of capital across farm size-groups whenever (41) holds. That is, for redistribution to be profitable, the lower bound on \( K_1 \) needs to be lower under static models than the lower bound on \( K_{10} \) under the dynamic model. The converse holds whenever

\[
S_2 < S_1 \left(\frac{W_2}{W_1}\right)^{\frac{\alpha}{\beta}}.
\]

Since the former one is more plausible, we find under dynamic conditions for redistribution to be profitable we need a more stringent condition. Thus, if one goes by static modeling, over time the output might turn out to be sub-optimal if the dynamic conditions are also not simultaneously met.

**Conclusion**

We have shown the conditions under which it is profitable to redistribute land under static and dynamic modeling. We also showed how under certain conditions static optimization might lead to sub-optimal dynamic output.

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