ON EVALUATING CROP RESPONSE TO LIME IN THE TENNESSEE VALLEY REGION

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Excessive soil acidity has long been recognized as one reason soils become unproductive, and liming to neutralize excess acidity has been practiced at least since the second century B.C. [5, p. 125]. Although liming has become a common practice, some researchers contend that farmers use too little lime [e.g., 5, p. 125].

Although lime is sometimes used to supply calcium or magnesium, it is principally used to neutralize soil acidity [3, p. 178], commonly measured by soil pH. Many agronomists agree that most crops require some lime if pH falls below 5.0 and that most require no lime if pH is 7.0 or higher [17, pp. 221-222]. When legumes supplied nitrogen for other crops in rotation, most lime recommendations were designed to raise soil pH to 6.5 or higher to accommodate the legumes. Now, however, nitrogen requirements for most crops are supplied from fertilizer, and some lime recommendations aim for pH levels somewhat below 6.5, at least for non-legumes such as corn and cotton.

TABLE 1. CROP ACREAGES IN THE SEVEN TENNESSEE-VALLEY STATES

CROP	1955[a]	1965[b]	1975[c]
	-----THOUSAND ACRES-----		
Corn	13,325	8,170	7,490
Cotton	5,242	3,788	3,606
Soybeans	1,651	4,426	11,021
Alfalfa	663	748	382
Improved Pasture	17,963	18,564[d]	17,227[d]
Other Hay	5,508	5,462	5,749
TOTAL	44,352	41,158	45,475

[a]Harvested area [16,1959].

[b]Planted area [15, Jan. 1966].

[c]Planted area [15, Jan. 1976].

[d]Source: [16, 1969, 1974].

Ground limestone is the most common liming material. In the United States in 1975, roughly 34.3 million tons of agricultural liming materials were sold; 33.8 million tons (98.6 percent) of that was ground limestone [14, Table 651].

CROPS AND LIME USE IN THE TENNESSEE VALLEY

Table 1 reports crop acreages in the seven-state Tennessee Valley region during the period 1955-1975 and Table 2 reports lime use for the same period. Except for the soil-bank years around 1965, total crop acreages were stable during this time. The acreage of soybeans increased substantially, however, largely at the expense of corn and cotton.

Although total crop acreage changed little in the 1955-75 period, use of agricultural limestone more than doubled. If all the lime reported in Table 2 had been used on the crops reported in Table 1, the average rate of use would have been 0.07 ton per acre per year in 1955 (1 ton per acre every 14.3 years) and 0.18 ton per acre per year in 1975 (1 ton per acre every 5.7 years). This evidence indicates that

TABLE 2. USE OF AGRICULTURAL LIME IN THE TENNESSEE VALLEY REGION[a]

STATE	1955	1965	1975
	-----TONS-----		
Alabama	171,308	496,291	942,000
Georgia	301,800	776,651	1,361,000
Kentucky	1,316,484	1,990,470	1,757,000
Mississippi	207,915	483,500	625,000
North Carolina	356,136	851,345	1,268,000
Tennessee	306,055	1,453,313	1,249,000
Virginia	520,500	804,379	847,000
TOTAL	3,180,198	6,855,949	8,049,000

[a]Source: National Limestone Institute [11].

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both total use and rate of use have increased. Nevertheless, farmers still may not be using enough lime.

CROP RESPONSE TO LIME

In an economic sense, lime is a crop-production input that provides benefits at a cost. A farmer who wants to maximize net returns should increase the lime rate as long as returns from increased production (or savings in the cost of other inputs) exceed the cost of the lime. One difficulty with applying this criterion is estimating the benefits, because a single lime application may affect soil pH for five years or longer.

Response surfaces should be useful in estimating the benefits of lime and in identifying optimal lime rates.

Response-Surface Formulation

The pH change induced by a lime application, not the amount of lime *per se*, determines crop yield, and yield is the variable farmers hope to affect. Physiologically, then, the pH-yield response function is pivotal. But because farmers apply lime, not pH, economic evaluation requires the lime-yield response function. These two functions are related through a third function, the pH-lime response function. Relations among these three functions are discussed elsewhere [9]; discussion hereafter is limited to the lime-yield function.

At a given location, with all controllable factors except lime fixed, a general yield-response function might be written

$$(1) \qquad Y_i = f_1(IpH_i, LIME_i) + e_i$$

where $i = 1, 2, \ldots, n$ represents the n observations in the experiment, and for the i^{th} observation, Y_i represents crop yield, IpH_i represents initial pH (before lime was applied), and $LIME_i$ represents lime rate. The residual e_i measures the experimental error of the i^{th} observation.

If IpH_i is low, yield response to lime may be dramatic; if it is high enough, there may be no response. If IpH_i is uniform over all observations, it may affect the level of yield but it will not contribute to the variation in yield. In that case, IpH_i can be eliminated from equation 1 yielding the function

$$(2) \qquad Y_i = f_2(LIME_i) + e_i.$$

If IpH_i varies among observations, however, parameter estimates from equation 2 will be biased [12, pp. 394-395].

The precise mathematical forms of functions 1 and 2 are not known, but some characteristics of crop response to lime that these functions should display are known: (1) yield response is greatest for the first increment of lime [17, p. 220]; in economic parlance, there are decreasing returns [e.g., 7, p. 2]; (2) yield response often reaches a plateau somewhere below pH 7.0 and may even decrease near 7.0 (cf. 1, p. 16].

Application

Lime-yield response functions of the general form of equation 2 were fitted for alfalfa and corn. Alfalfa is generally regarded one of the most responsive crops to lime and corn one of the least responsive. Because none of the data included initial-pH measurements, the fitted surfaces have the form of equation 2 rather than equation 1. In experiments with relatively homogeneous plots, the biases from equation 2 should be small.

When the mathematical form of a response function is not known, the function can sometimes be approximated satisfactorily, within the experimental region, by a polynomial or other function of the variables [6, p. 335]. Here, the response functions are approximated by grafted polynomials [8], also called spline functions [13], and by piecewise linear regression functions [13]. For comparison, the well known square-root function is also fitted to the data.

Grafted Polynomials. Suppose a response curve in a single variable, X, is approximated by two quadratic functions:

$$(3a) \qquad Y = a_0 + a_1X + a_2X^2, \qquad X \leqslant C$$

$$(3b) \qquad Y = b_0 + b_1X + b_2X^2, \qquad X \geqslant C$$

where C is some specified value of X and the parameters (the a's and b's) are restricted so that the curve and its first derivative are continuous at C. Fuller [8] shows that the required restrictions are:

$$(4a) \qquad a_0 + a_1C + a_2C^2 = b_0 + b_1C + b_2C^2$$

$$(4b) \qquad a_1 + 2a_2C = b_1 + 2b_2C$$

Without restrictions, equation 3 would have six independent parameters; with restrictions, it has only four. Hence, four parameters can be estimated from the data and the remaining parameters can be estimated as linear combinations of those four.

Fuller estimates a_0, a_1, a_2, and b_2-a_2 from the regression equation

$$(5) \qquad Y = a_0 + a_1X + a_2X^2 + (b_2-a_2)Z,$$

where

$$Z = 0, \text{ if } X \leqslant C$$
$$= (X-C)^2, \text{ if } X > C.$$

He then writes the b_i in terms of the a_i from equation 5:

$$b_0 = a_0 + C^2(b_2 - a_2)$$
$$b_1 = a_1 - 2C(b_2 - a_2)$$
$$b_2 = a_2 + (b_2 - a_2).$$

Model 5 is a "grafted polynomial" in the sense that it incorporates the essential features of equation 3a and b in a single equation.

Piecewise Linear Regression Functions. These functions merely join (graft) two or more linear segments into a single function [13, p. 132]. If there is a single independent variable, X, and if there are only two segments, one formulation of the function is:

(6) $$Y = b_0 + b_1X_1 + b_2X_2$$

where

$$X_1 = X \text{ if } X \leqslant C$$
$$= C \text{ otherwise}$$
$$X_2 = X - C \text{ if } X > C$$
$$= 0, \text{ otherwise}$$

and where C is some specified value of X. In effect, equation 6 joins two straight lines at X=C. The sizes of the coefficients are unrestricted, but in response surfaces $b_1 > b_2$ usually holds. Such functions are continuous over the domain of X, but the first derivative is discontinuous at X=C. Suits et al. [13] and Anderson and Nelson [4] discuss the use of three or more segments.

ALFALFA RESULTS

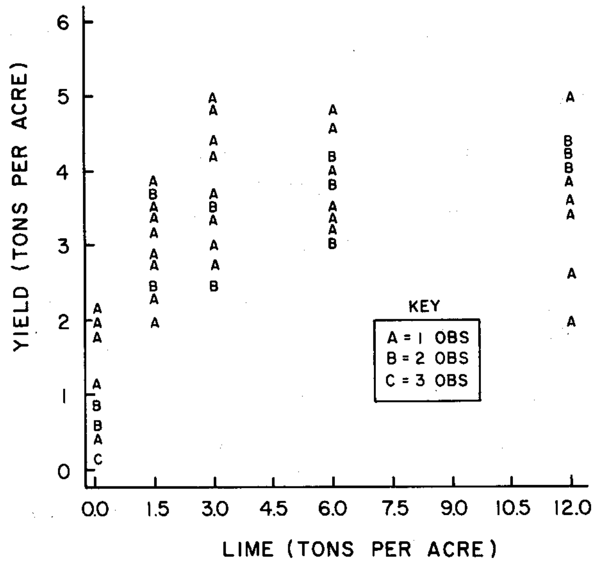
In a lime-alfalfa experiment conducted at Mayfield, Kentucky, on Grenada silt loam, lime was applied in May 1958, alfalfa was seeded in August 1961, and pH readings were made in April 1962. Lime was applied in three forms: dolomite disked into plowed surface; dolomite, one-half plowed down, one-half disked into plowed surface; calcite disked into plowed surface. Each form was applied at the five rates shown in Table 3, making a total of 15 treatments. Each treatment was replicated four times in randomized complete blocks. Differences in response for the three forms of application were so small that form is ignored in subsequent analyses, and the data are handled as though five treatments were replicated 12 times. Figure 1 is a plot of the data.

TABLE 3. MEAN pH AND YIELD FOR ALTERNATIVE LIME RATES, KENTUCKY LIME-ALFALFA EXPERIMENT

Lime Rate (Tons/Acre)	Mean pH[a] (April 1962)	Mean Yield[a] (Tons/Acre)
0	5.28	0.89
1.5	5.90	3.01
3.0	6.28	3.60
6.0	6.78	3.82
12.0	7.13	3.84

[a]Each mean represents 12 observations — 4 replications of 3 forms at each lime rate.

FIGURE 1. ALFALFA YIELD VS. LIME RATE



Yield Response to Lime

A grafted polynomial, a piecewise linear function, and a square-root function were fitted to the data. In fitting these functions, a given replication is assumed to affect every treatment uniformly, and the replication effects are estimated by dummy variables which, in effect, fit a separate intercept for each replication. In the prediction equations, the reported intercept is the average of the replication intercepts; the numbers in parentheses are standard errors.

Grafted polynomial. The join point for the grafted polynomial is C = 3 tons of lime per acre.

(7) $$Y^* = \underset{}{0.902} + \underset{(0.232)}{1.794}\,\text{LIME} - \underset{(0.052)}{0.290}\,\text{LIME**2} + \underset{(0.061)}{0.286}\,Z3 \qquad R^2 = 0.75$$

where

Y* = predicted yield in tons per acre
LIME = tons of lime per acre
LIME**2 = LIME x LIME
Z3 = (LIME - 3)**2 if LIME > 3.0
= 0 otherwise.

C = 3, a design point of the experiment, was chosen as the join point after Figure 1 was examined. Because a join point at C = 6 produces a similar function with only slightly poorer fit, the choice of join point appears to be somewhat flexible.

Piecewise Linear Function. The two linear portions of this function are also joined at C = 3 tons of lime per acre.

$$(8) \quad Y^* = \underset{}{1.142} + \underset{(0.095)}{0.901}\ LIME1 - \underset{(0.033)}{0.001}\ LIME2 \qquad R^2 = 0.70$$

where

$$LIME1 = LIME \text{ if } LIME < 3$$
$$= 3 \text{ if } LIME \geqslant 3$$
$$LIME2 = LIME - 3 \text{ if } LIME > 3$$
$$= 0 \text{ otherwise}$$

Function 7 fits the data better than function 8, but it also requires one more degree of freedom.

Square-Root Function.

$$(9) \quad Y^* = 0.900 - \underset{(0.067)}{0.390}\ LIME + \underset{(0.244)}{2.190}\ LIME^{**}0.5 \qquad R^2 = 0.74$$

where

$$LIME^{**}0.5 = \text{square root (LIME)}.$$

Judged by R^2 alone, function 9 fits the data almost as well as function 7 and requires one less degree of freedom; function 9 fits better than function 8 and requires the same degrees of freedom. If the cube root of lime is added to function 9, all coefficients become nonsignificant, apparently because of multicollinearity among the three lime variables.[1]

Optimal Lime Rates

A single lime application may affect soil pH and crop yield for several years, and estimating such effects requires data for several consecutive years. The alfalfa data include observations for only year, however. A first approximation to the optimal lime rate can be obtained by assuming that lime affects yield uniformly for five years, i.e., that the marginal product of lime is uniform over five years.[2] If lime costs $10 per ton and a farmer's opportunity cost for capital is 8 percent, the average annual cost per ton of lime is $2.94 = $[(10x1.08^5)/5]$. (A tenant who has a one-year nonrenewable lease and whose landlord is unwilling to pay for residual lime benefits remaining at the end of the year has an annual cost of $10 per ton.) An approximate optimal lime rate is obtained by equating the average annual cost of lime (which, because it is uniform, is also the marginal cost) with its marginal value product.

TABLE 4. OPTIMAL AND MAXIMUM-YIELD LIME RATES, TONS PER ACRE, KENTUCKY ALFALFA EXPERIMENT

Response Function	Max-Yield Rate	Optimal Rate	
		5-Year Payoff	1-Year Payoff
Polynomial (7)	6.0	2.9	2.5
Linear (8)	3.0	3.0	3.0
Square Root (9)	7.9	5.0	2.3

Table 4 reports one-year optimal and five-year optimal lime rates, determined in the manner described, for each estimated response function; it also reports maximum yield rates. The optimal rates are based on the assumption that the net price for alfalfa hay (net of harvesting costs) is $30 per ton. The square-root function yields the highest maximum-yield and five-year optimal rates but the lowest one-year optimal rate.

CORN RESULTS

At Brewton, Alabama, 17 lime-fertilizer treatments were replicated four times in a randomized complete block design. Lime was applied in 1957 and the plots were planted to cotton 1957-1961. In 1962, corn was grown with a change in fertilizer treatment. Mean corn yields and mean soil pH readings for the various treatments are given in Table 5. Figure 2 is a plot of the data.

Yield Response to Lime

A grafted polynomial, a piecewise linear function, and a square-root function were fitted to the data. Replication effects were treated exactly as described for the alfalfa data, and in the prediction equations the reported intercept is the average of the replication intercepts. The numbers in parentheses are again standard errors.

Grafted Polynomial. The join point is C = 2 tons of lime per acre.

[1]For the alfalfa data, the correlations are 0.948 between LIME and LIME**0.5 and 0.983 between LIME and LIME**0.3333; 0.999 is the multiple correlation among all three variables. For the corn data, the same correlations are 0.949, 0.977, and 0.998, respectively.

[2]Moschler et al. [10, p. 11] present some evidence that soil pH increased for approximately three years after lime was applied, then decreased; they did not examine the corresponding pattern in yield response.

TABLE 5. MEAN pH AND YIELD FOR VARIOUS TREATMENTS, BREWTON, ALABAMA CORN EXPERIMENT

Treatment Number	Lime Rate[a] (tons/acre)	N-Rate 1957-61[b] (lbs./acre)	Mean pH (Feb. 62)	Mean Yield (bu./acre)
1	0	60	5.6	68.4
2	0	240	5.3	52.6
3	0.5	60	5.7	71.6
4	1.0	0	6.0	22.9
5	1.0	60	5.9	68.7
6	1.0	240	5.5	63.8
7	1.0	240[c]	5.4	56.1
8	1.0	240[c]	6.0	61.9
9	1.0	60	5.8	71.9
10	1.0	240	5.5	69.0
11	8.0[d]	60	6.6	74.5
12	8.0[d]	240	6.5	70.6
13	2.0	60	5.9	75.4
14	2.0	240	5.6	71.3
15	4.0	60	6.3	72.5
16	4.0	240	6.1	75.5
17	4.0	240	6.0	75.1

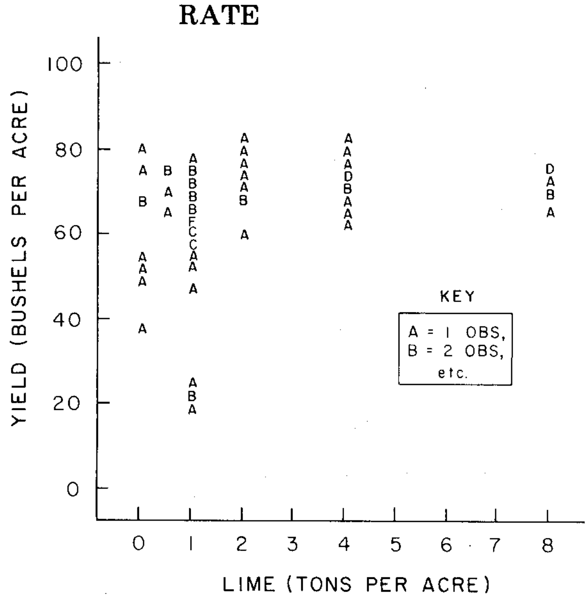
[a]Calcite on treat. nos. 9, 10, 17; dolomite on all others.

[b]In 1962, treatment 4 received no N; all others received 150 lbs. N/acre.

[c]N an ammonium sulfate on treatment no. 7, sodium nitrate on treatment 8; all others ammonium nitrate.

[d]Includes 7 tons added in 1960.

FIGURE 2. CORN YIELD VS. LIME RATE



(10) $$Y^* = 17.181 + \underset{(3.002)}{7.064}\,\text{LIME} - \underset{(0.955)}{1.371}\,\text{LIME**2} + \underset{(1.099)}{0.929}\,\text{Z2} - \underset{(0.656)}{3.377}\,\text{N1} + \underset{(3.303)}{51.849}\,\text{N2} + \underset{(0.189)}{0.483}\,\text{LIME*N1}$$

$R^2 = 0.86$

where

Y* = predicted yield in bushels per acre
LIME = tons of lime per acre
LIME**2 = LIME x LIME
Z2 = (LIME − 2)**2 if LIME >2
= 0 otherwise
N1 = N/60 for annual rate of N 1957-61 (N1 = 0, 1, or 4)
N2 = N/150 for N applied in 1962 (N2 = 0 or 1).

C = 2, a design point of the experiment, was chosen as the join point after Figure 2 was examined.

Piecewise Linear Function. The two linear portions are joined at C = 2 tons of lime per acre.

(11) $$Y^* = 22.534 + \underset{(1.890)}{0.341}\,\text{LIME1} - \underset{(\ 395)}{0.136}\,\text{LIME2} - \underset{(0.890)}{5.399}\,\text{N1} + \underset{(2.946)}{51.373}\,\text{N2} + \underset{(0.610)}{2.551}\,\text{LIME1*N1}$$

where $R^2 = 0.88$

LIME1 = LIME if LIME < 2
= 2 if LIME ⩾ 2
LIME2 = LIME − 2 if LIME > 2
= 0 otherwise.

Function 11 fits the data better than function 10 and it requires one less degree of freedom. The estimated main effect of lime is positive up to LIME = 2 (LIME1), negative for LIME > 2 (LIME2), but neither estimate is significantly different from zero. The principal results of interest are for nitrogen and lime-by-nitrogen interaction, and a separate section is devoted to those results.

Square-Root Function.

(12) $$Y^* = 14.940 - \underset{(1.044)}{3.462}\,\text{LIME} + \underset{(2.811)}{11.397}\,\text{LIME**0.5} - \underset{(0.668)}{3.334}\,\text{N1} + \underset{(3.335)}{52.663}\,\text{N2} + \underset{(0.193)}{0.502}\,\text{LIME*N1}$$

where $R^2 = 0.85$

LIME**0.5 = square root (LIME).

Judged by R^2 alone, function 12 fits the data almost as well as function 10 and requires one less degree of freedom; function 12 requires the same degrees of freedom as function 11 but does not fit the data as well.

Nitrogen and Lime-By-Nitrogen Interaction

Nitrogen usually increases the yield of non-leguminous crops such as corn, and that effect is clear in all three prediction equations. Increasing the 1962 nitrogen rate from none (N2 = 0) to 150 pounds per acre (N2 = 1) increased corn yield by more than 50 bushels per acre (1/3 bushel per pound of N). This is an estimate of the linear effect of nitrogen, and only the linear effect can be estimated because there are only two nitrogen rates.

Heavy nitrogen rates also deplete lime reserves [3, pp. 161-162; 17, p. 208]. For high enough lime rates, heavy previous nitrogen applications may not reduce yield at all; but for low lime rates, heavy previous nitrogen rates may reduce yield. These expectations are confirmed by the data in Table 6. Some corn plots

TABLE 6. EFFECTS OF N1 AND LIME-BY-N1 INTERACTION ON CORN YIELD

Lime Rate (tons/acre)	N1 = 1 (28 Plots) pH	Yield	N1 = 4 (36 Plots) pH	Yield
0	5.59	68.4	5.28	52.6
0.5	5.71	71.5	-----	-----
1.0	5.86	70.3	5.60	62.7
2.0	5.94	75.5	5.62	71.3
4.0	6.32	72.6	6.01	75.3
8.0	6.59	74.5	6.49	70.6

received 240 pounds of N per acre, others only 60 pounds, for the five years preceding the corn experiment. Plots that received no lime and 60 pounds of N (N1 = 1) have an average pH of 5.59 and yields slightly lower than those of the remaining N1 = 1 plots. Plots that received one ton of lime or less and 240 pounds of N have mean pH readings of 5.60 or lower and substantially lower yields.

Of the three response functions fitted to the corn data, the piecewise linear function (11) apparently best reflects the lime-by-nitrogen interaction. Variable LIME1 includes lime rates of 2 tons per acre and less, the crucial ones according to Table 6. In function 11 the estimated main effects of lime (LIME1 and LIME2) are small and nonsignificant, but the interaction between LIME1 and N1 is large and highly significant. The first derivative of Y* with respect to LIME1 is 0.341 + 2.551 N1.

In practical terms, this derivative shows that the increase in corn yield per unit increase in LIME1 depends on N1: if N1 = 0, yield increases by 0.34 bushel per ton of lime; if N1 = 1, yield increases by [0.34 + (2.55 x 1)] = 2.89 bushels per ton of lime; if N1 = 4, yield increases by [0.34 + (2.55 x 4)] = 10.54 bushels per ton of lime. The LIME*N1 interaction terms in the other two response functions are statistically significant but much smaller in absolute value than the one in function 11.

Optimal Lime Rates

The corn data, like the alfalfa data, include observations for only one year; consequently, crop response over time cannot be estimated. If the assumptions discussed for alfalfa are made, however, a first approximation to the optimal lime rate can be obtained. According to those assumptions, the marginal product of lime is uniform over five years and the average annual cost of lime is $2.94 per ton for a farmer-owner and $10 per ton for a tenant with a one-year nonrenewable lease. On the basis of these assumptions and the further assumption that the net price for corn (net of harvesting and drying costs) is $2.25 per bushel, approximate optimal lime rates are those reported in Table 7. Table 7 reports optimal lime rates and maximum-yield rates separately for each N1 rate.

TABLE 7. OPTIMAL AND MAXIMUM-YIELD LIME RATES, TONS PER ACRE, ALABAMA CORN EXPERIMENT

Response Function and N1 Rate	Max-Yield Rate	Optimal Rate 5-Year Payoff	1-Year Payoff
Polynomial (10)			
N1 = 0	3.8	2.3	0.0
N1 = 1	4.3	2.9	0.0
N1 = 4	6.0	4.5	0.9
Linear (11)			
N1 = 0	2.0	0.0	0.0
N1 = 1	2.0	2.0	0.0
N1 = 4	2.0	2.0	2.0
Square Root (12)			
N1 = 0	2.7	1.4	0.5
N1 = 1	3.7	1.8	0.6
N1 = 4	15.4	4.3	0.9

All three functions show strong yield responses to lime even though the yields were measured in the sixth year after lime was applied. If lime lasts only five years there should have been no response; hence, these results cast doubt on the assumption that lime lasts only five years. When N1 = 4, the square-root function has a much larger maximum-yield rate than either of the other two functions. Anderson and Nelson [4, p. 306] found a related result: for two of four corn experiments in Tennessee, the square-root function yielded optimum nitrogen rates of nearly 600 pounds per acre; quadratic functions fitted to the same data had optima of less than 300 pounds of N.

CONCLUSIONS

The optimal lime rates for alfalfa exceed the average rates (for all crops), even for a one-year payoff. The optimal lime rates for corn for a five-year payoff exceed the actual rates; for a one-year payoff, unless there have been repeated previous heavy nitrogen applications, actual rates exceed optimal rates. But optimal rates will be so location specific—depending on initial pH, previous fertilizer applications (especially nitrogen), and the farmer's tenancy status—that such aggregate assessments are probably meaningless. Response-surface results offer more promise for providing lime recommendations to individual farmers.

Several unresolved questions must be answered before response surfaces will provide a basis for sound recommendations, however. What is the best mathematical form of the response surface? Do different crops require different forms? How can the multiyear response characteristic of lime best be described, and how should the cost of lime be allocated among those years? How does year-to-year variation in yields due, for example, to variations in weather but independent of the lime rate affect the choice of lime rate? Answering some of these questions will require new data.

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