

# Feeding and the Equilibrium Feeder Animal Price-Weight Schedule

David A. Hennessy

Feeder animal prices depend on fed animal prices, the biological growth technology, and feed costs. In addition, daily maintenance costs can be avoided through accelerated feeding. These observations allow us to model optimal feeding under equilibrium feeder animal pricing. Our model enables a better understanding of regulation in feedstuff markets. The feeder animal price-weight schedule is likely decreasing and convex in weight. Prices for animals with better growth potential should be less sensitive to feed and fed animal prices. Prices for lighter animals should be more sensitive to these prices. Regression analyses on Southern Great Plains cattle prices provide support for this model.

*Key words:* days on feed, energy use, feed ban, growth hormones, Kleiber's law, ration density, veal market

## Introduction

Animal maintenance energy rates per unit time should be of interest to agricultural economists because they may be viewed as a fixed, but partially avoidable, cost. The cost is fixed because, at least for the animal at hand, there is not much one can do about daily maintenance energy losses except discourage movement and provide shelter to avoid unnecessary heat loss—two major motives for animal confinement. The cost is avoidable to the extent that an accelerated growth regime, through feed management or other means, can bring the animal to maturity earlier. But rate of growth is not entirely a technical decision as feedstuff prices matter and these vary by location. The efficient husbander will look to trade off high growth rates, and so low lifetime expenditure on maintenance costs, against the reduced daily feed costs that lower density feeds can deliver.

Briefly, this article seeks to establish consequences for optimal feeding and live animal pricing of one widely affirmed allometric (i.e., weight-homogeneous) scaling law, Kleiber's law (Kleiber, 1932). This rule for scaling energy maintenance costs, when viewed in isolation, can be seen as a scale economy and affects the rate of biological growth on a given feed ration. We adopt the animal scientist's accounting for energy uses in maintenance and growth to investigate how Kleiber's law should affect feed ration decisions given the variety of ration compositions available to the grower. The law is also used here to derive structure on feeder animal price relations.

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The paper should be of interest because active feeder markets exist for hogs and cattle in most countries with a significant agricultural sector, while local specialty feeder markets exist in other species. Apart from young stock, feed is generally the largest input in meat production. Feeder markets have undergone significant structural changes in the United States and elsewhere over the past 50 years. For example, steers are now being fed more intensively to be slaughtered younger and at a heavier weight (Elam and Preston, 2004). In addition, many of the most significant innovations in animal agriculture, including confinement, genetic manipulation, hormone treatments, and nutrition innovations, have been used to improve feed conversion efficiency (FCE). Among major policy issues concerning meat markets in recent years are bans on the use of some animal-derived materials in animal feed and the use of some growth promotants. To better understand the economic consequences of evolving trends, new technologies, and policy adjustments, it would be very useful to have available a robust microeconomic model. This model should be detailed and explicit in representing the role of feed in meat production, and it should also incorporate equilibrium pricing. Existing models are very limited in these regards.

The most influential paper in the literature is credited to Jarvis (1974). He viewed cattle as assets with biological growth options such that slaughter occurs when growth potential has been exhausted. Jarvis' model specified a single-harvest objective function, assuming also that daily feeding costs were weight- and time-invariant. Feeder animal prices were imputed as Ricardian rent in perfect competition. Slaughter weight was identified as the weight that minimized the unit cost of meat, and the feeder animal price per pound was hypothesized to decline as weight increased toward slaughter weight.

Paarsch's (1985) set of models advocated, more realistically, that the grower be assumed to harvest sequentially (i.e., in rotation) according to Faustmann's paradigm. In addition, weight gain was allowed to depend on weight, and the chosen feed regime was allowed to vary over time. Finally, the price of young stock was held to be exogenous to the meat price. Consequently, by contrast with the Jarvis model, Paarsch's model can relate nothing about the feeder animal price-weight schedule (PWS) in what are typically competitive feeder animal markets. Amer et al. (1994) have utilized Jarvis' economic insight, together with empirical equations from Fox, Sniffen, and O'Connor (1988), to ask when to slaughter and how to compare performance across breeds. Much of the subsequent related work has focused on some peculiar dynamics of feeder animal markets, namely inverted price responses and the role of dynamic biological restrictions in price cycle behavior (Rosen, 1987; Rosen, Murphy, and Scheinkman, 1994; Chavas, 2000; Aadland, 2004; Aadland and Bailey, 2001). However, this research has not sought to model how animal prices should vary across weights and types.

The literature above has not addressed feeding decisions, the role of maintenance costs, or the structure of the feeder animal PWS.<sup>1</sup> A second body of literature has sought to identify the best feeding path in an optimal control framework. Chavas, Kliebenstein, and Crenshaw (1985) modeled swine feeding and marketing under rotational harvest. Talpaz et al. (1988) and Cacho, Kinnucan, and Hatch (1991) modeled other species (broilers and catfish, respectively) without accounting for sequential harvest. These papers did account for maintenance costs and emphasized empirical simulations with appropriate available data rather than develop a conceptual model in the manner of

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<sup>1</sup> An empirical literature, reviewed in Marsh (2001), has estimated determinants of feeder prices.

Jarvis (1974) or Paarsch (1985).<sup>2</sup> Later empirical simulation work, summarized in Hernández et al. (2003), has focused on fish production. The studies most relevant to our research were conducted by Arnason (1992) and Heaps (1993), and will be reviewed here in due course. This optimal feeding literature has taken feeder animal prices as given without reference to equilibrium, and optimum feeding paths for realistic growth dynamics are difficult to interpret.

The intent of this analysis is to revisit some theoretical issues raised by Jarvis (1974) and Paarsch (1985)—ones not addressed in any of the other theoretical or empirical literatures. Specifically, this investigation seeks to provide a better understanding of the price-weight relationship for different animal breeds in a static equilibrium setting. By contrast with the Jarvis and Paarsch models, daily maintenance costs are modeled with specific dynamic technical production relations. This study takes Jarvis' Ricardian perspective on feeder animal prices, and the Paarsch rotation assumption, but departs from both approaches in providing explicit specifications for animal growth and how growth depends on feed through the animal's life. This is done in part because findings in animal science allow for the technical structure which will be imposed here. This approach guides a clearer understanding of the feeder animal PWS and the determinants of optimal feed rations than has been provided in the literature to this point. Daily maintenance costs are crucial in this regard, and have been ignored outside the empirical literature.

We depart from the vast majority of the literature (Jarvis being the exception) by endogenizing prices paid for feeder animals. Here it is assumed competition will force feeder animal buyers to pay the price that exhausts economic profit. Rather than making it more difficult to interpret optimal feeding paths, our equilibrium pricing approach and growth specifications establish a simple and intuitive optimal feeding trajectory. This simplicity allows us to study such policy issues as regulations on hormone implants and the use of animal-derived materials in feedstuffs, and also to impute the equilibrium feeder animal PWS. The insights provided on optimal feeding are also shown to be quite general in that they do not depend upon the technical growth relations assumed in our main model.

The remainder of the paper is laid out as follows. First, a discussion is provided of the origin and grounding of the allometric scaling law of relevance to this analysis, Kleiber's law (Smil, 2000). Next, the baseline production model is presented, as well as an analysis of model implications for pricing immature animals. Conditions under which incentives support the slaughter of young animals are also considered, and the roles of biological parameters in determining how the unit price of animals changes with weight are developed. The model is generalized to apply for less structured assumptions on the growth technology. Some regressions are provided to test model implications on Southern Great Plains feeder animal prices. Strong empirical support is found for four hypotheses. Specifically, the PWS is found to be decreasing and convex in weight; relative to heifer prices, steer prices are found to be less sensitive to feed and fed animal prices; and prices for lighter animals are found to be more sensitive to feed and fed animal prices. The paper concludes with a brief discussion.

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<sup>2</sup> These models, together with Amer et al. (1994) and much of the large production science literature on empirical animal growth models, are similar to ours in two features of relevance to this analysis. Both animal maintenance costs and animal dry matter intake are held to increase in proportion to  $(weight)^\alpha$ , where  $\alpha$  is some number close to 0.75.

### Maintenance Costs

In 1839, scientists F. Sarrus and J. Rameaux suggested a relationship between weight and energy expended by an organism at rest (also known as base metabolic rate, and hereafter labeled as  $r$ ).<sup>3</sup> Animals of all weights,  $w$ , are in approximate equilibrium with the world around them, and lose heat at the rate of body heat production. In addition, skin surface area grows in proportion to  $w^{2/3}$ , and heat loss is in proportion to surface area. Therefore, Sarrus and Rameaux argued that  $r \propto w^{2/3}$  also. Max Kleiber (1932) sought to validate this argument by using observed metabolic rates for different species, but concluded  $r \propto w^{3/4}$  was more consistent with the data. Later works by Brody (1945), Hemmingsen (1960), Bartels (1982), Bennett and Harvey (1987), Heusner (1991), and a large number of others have continued a vigorous debate on whether  $3/4$ , or  $2/3$ , or neither, is the appropriate power relation.

Recent research by Dodds, Rothman, and Weitz (2001) suggests values close to  $3/4$  may be more appropriate for animals above 10Kg (e.g., hogs, sheep, and cattle), while  $2/3$  is more plausible for birds and smaller mammals. For the qualitative results in this paper, the exact number is not relevant except in that it should satisfy  $r \propto w^\alpha$ ,  $\alpha \in (0, 1)$ , so as to provide size economies with respect to maintenance costs.

A formal explanation of Kleiber's law, concerning weight stress, has been provided by McMahan (1973). More recently, West, Brown, and Enquist (1997, 1999) and Banavar, Maritan, and Rinaldo (1999) have emphasized efficiency in nutrient circulation. Whatever the reason,  $3/4$  power scaling has become embedded in practical science. Because many drugs are believed to clear the body according to a  $3/4$  scaling law (Mordenti, 1986), drug prescriptions are often scaled to body mass in this manner. In agriculture, and of direct relevance to this research,  $3/4$  scaling is assumed for both maintenance energy and feed intake relations (National Research Council, 2000).

### Production Model

An animal requires maintenance calories amounting to  $M$  per day. In order to fatten the animal, energy content of live weight gained must amount to  $E$  calories. The animal's intake capacity is  $I$  units of feed per day. Both  $M$  and  $I$  depend upon the animal's live weight, denoted as  $w$  and measured in pounds, in a manner specified a bit later. For the moment, only a small weight gain interval is considered, where these attributes may be assumed to be fixed. Feedlot incoming and outgoing weights are  $w_{in}$  and  $w_{ou}$ , respectively.

A choice available to the grower is the energy density of rations fed, and energy density  $b \in [0, \bar{b}]$  can be attained at cost per unit feed amounting to  $\theta c(b) + \eta$ , where  $c(b)$  is an increasing and convex function.<sup>4</sup> Here,  $\theta > 0$  and  $\eta > 0$  are cost shift parameters that might change due to feed regulations (such as a ban on using animal grease as a feedstuff) or an increase in component prices due to increased demand for feed. Growers are homogeneous in the sense that all are faced with cost  $\theta c(b) + \eta$  and use the same

<sup>3</sup>To be clear, many slightly different technical versions of  $r$  exist, but it is intended to represent energy expenditure on vital biological functions other than feeding and motion. For the mature animal,  $r$  will not include an allocation for non-forced growth. For young animals, it may do so. If it does, then  $r$  should decline with age.

<sup>4</sup> Energy is used as a representative growth-limiting requirement in order to economize on model notation. Inclusion of other constraints (protein balance, for example) would not change the main messages our paper provides.

production technology. Therefore, cost of feed per day is  $I\theta c(b) + I\eta$  to all growers. Let  $T$  be days on feed. Liveweight gain per day is  $(Ib - M)\lambda$ , where  $\lambda$  is the conversion fraction for surplus energy  $Ib - M$  into pounds liveweight. Animals should be fed to capacity because conversion to liveweight is held to be linear in calories surplus to maintenance.<sup>5</sup> Hence, it takes  $E/[(Ib - M)\lambda]$  days to grow an animal from  $w_{in}$  to  $w_{ou}$ , and days on feed must satisfy  $T = E/[(Ib - M)\lambda]$ .

Gross revenue across days on feed is  $\Delta R$ , and its composition will be explained shortly. Gross revenue per day is  $(\Delta R)(Ib - M)\lambda/E$ . Revenue per day net of feed costs is given by:

$$(1) \quad V = \frac{(\Delta R)(Ib - M)\lambda}{E} - I\theta c(b) - I\eta.$$

Reservation utility for the grower will depend on the level of resources applied, which will depend in turn on the animal's intake through demands on labor, machinery, and buildings. In equilibrium, revenue per day net of feed costs should equal the opportunity cost of deployed resources (i.e., reservation utility). If deployed resources are in proportion to feed intake, and reservation utility  $U$  is determined by the opportunity cost of deployed resources, then we may write  $U = \phi I$ , where  $\phi > 0$ . Under perfect competition, feeder animal prices will be such that opportunity costs are just covered, i.e.,  $U = V$ , and so:<sup>6</sup>

$$(2) \quad \phi I = \frac{(\Delta R)(Ib - M)\lambda}{E} - I\theta c(b) - I\eta.$$

If  $U < V$ , then  $\Delta R$  (i.e., feeder animal prices) would adjust downward in perfect competition as growers seek to avoid economic losses. If  $U > V$ , then competition would ensure that feeder animal prices adjust upward.

It remains to characterize  $\Delta R$ , the change in revenue. By making  $I$  and  $M$  functions of weight, attention no longer needs to be confined to the small weight interval  $[w_{in}, w_{ou}]$ . To do this, growth dynamics must be specified. First, the initial weight (i.e., time  $t = 0$ ) of the animal is fixed at  $w(0)$ . The animal's maturity weight is given as  $w_m$ , referred to as the maturity plateau and taken as given, but the maturity date depends on the animal's feeding regime. Weight at time  $t \geq 0$  is written as  $w(t)$ . According to Kleiber's law, the animal's instantaneous maintenance cost is  $M = [w(t)]^{0.75}\gamma_M$ ,  $\gamma_M > 0$ . It is assumed that the animal has intake capacity  $I = [w(t)]^{0.75}\gamma_I$ ,  $\gamma_I > 0$ .<sup>7</sup> For future reference, the couple  $(\gamma_I, \gamma_M)$  is described as an animal's genetic profile.<sup>8</sup> An animal on ration density  $b$  and consuming at capacity will consume at the rate  $[w(t)]^{0.75}\gamma_I b$  calories. This allows (2) to be rewritten as:

$$(2') \quad \Delta R = \frac{[\kappa + \theta c(b)]\gamma_I E}{(\gamma_I b - \gamma_M)\lambda}, \quad \kappa = \phi + \eta,$$

<sup>5</sup> This is a simplification, as the relationship is not quite linear (see Jurgens, 2002, p. 353).

<sup>6</sup> Interest rates have been ignored. Like Jarvis (1974, p. 492), we are of the opinion that interest rates are of minor importance in determining feeder animal prices. See Marsh (2001, table 1) for empirical support on this topic.

<sup>7</sup> See National Research Council (2000, pp. 87–88). Ration composition can affect intake in a variety of ways. National Research Council modeling captures this by adjusting  $\gamma_I$ .

<sup>8</sup> Additional aspects of an animal's genetic profile concern carcass attributes such as cut-out and the distribution of meats across cuts. We will model these through a markup on the fed animal price, and leave the issue aside until the PWS is identified.

where  $[\kappa + \theta c(b)]\gamma_I/[(\gamma_I b - \gamma_M)\lambda]$  is cost per unit gain (and so is free of time units), and  $E$  is the target gain in calories. Since fixed feed cost parameter  $\eta$  behaves in the same manner as opportunity cost parameter  $\varphi$ , only parameter sum  $\kappa$  is referenced from this point on.

Efficiency requires that  $\Delta R$  be minimized over  $b \in [0, \bar{b}]$ . This is the critical assumption differentiating the present work from the optimal feeding literature reviewed earlier. If  $\Delta R$  is not minimized, then an alternative feeding regime will be more efficient, allowing the grower to pay more for a feeder animal in perfect competition, and thus driving  $\Delta R$  downward.<sup>9</sup>

■ **PROPOSITION 1.** *Let an animal have Kleiber's law maintenance costs with three-fourths power intake and genetic profile  $(\gamma_I, \gamma_M)$ . Then the optimal choice of  $b$  satisfies*

$$(3) \quad b^* = \underset{b \in [0, \bar{b}]}{\operatorname{argmin}} H(b; \gamma_I, \gamma_M, \theta),$$

$$H(b; \gamma_I, \gamma_M, \theta) = \frac{[\kappa + \theta c(b)]\gamma_I E}{(\gamma_I b - \gamma_M)\lambda},$$

and it is weight-invariant.

To better understand the relationship in (3), write it as  $c(b)b^* - c(b^*)b \geq (b - b^*)\kappa/\theta + (c(b) - c(b^*))\gamma_M/\gamma_I \forall b \neq b^*$ . The left-hand side is positive if and only if  $c(b)/b \geq c(b^*)/b^*$ . If  $c(b)/b$  is decreasing in  $b$  and there is a  $b^+ > b^*$  such that  $b^+ \in [0, \bar{b}]$ , then  $c(b^+)/b^+ - c(b^*)/b^* < 0$ . But then  $(b^+ - b^*)\kappa/\theta + (c(b^+) - c(b^*))\gamma_M/\gamma_I \geq 0$ , a contradiction. It follows that  $c(b)/b$  decreasing requires the corner solution  $b^* = \bar{b}$ . In that case, cost per unit energy is decreasing, while an increase in ration density also saves on both maintenance costs and the opportunity costs of committed resources. It is reasonable, however, to ignore this case because it is likely that  $\theta c(\hat{b}) + \eta = 0$  for some  $\hat{b} > 0$ , where the ration is comprised of by-products (e.g., from bakeries) with insufficient nutrient density to clear maintenance costs. If  $\theta c(\hat{b}) + \eta = 0$  for some  $\hat{b} > 0$ , and  $c(b)$  is convex on  $(\hat{b}, \bar{b}]$ , then  $c(b)/b$  must be an increasing function on  $(\hat{b}, \bar{b}]$ . Furthermore, interval  $[0, \hat{b}]$  is of no economic interest because nutrient density in this interval does not justify feeding. From this point on, only interior solutions will be considered.

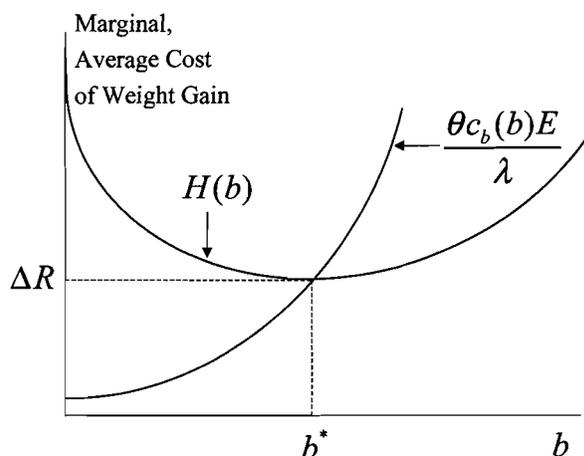
For interior solutions, implications of (3) include:<sup>10</sup>

■ **PROPOSITION 2.** *Under the growth technology assumptions in proposition 1, the incentive to feed a high-energy ration increases in the value of: (a)  $\gamma_M$ , (b)  $-\gamma_I$ , (c)  $-\rho = -\gamma_I/\gamma_M$ , (d)  $-\theta$ , and (e)  $\kappa$ .*

Figure 1 displays the nature of equilibrium implied by (3). Two functions of ration density are described. One,  $\theta c_b(b)E/\lambda$ , is the marginal cost per weight gain. The other,  $H(b)$ , is the average cost per unit weight gain. These are set equal when the unit cost

<sup>9</sup> An optimal control proof of weight invariance in proposition 1 is provided in appendix B. The setup there is more general than the one presently under consideration, so the reader might want to defer inspection. The proof clarifies how  $\min_{b \in [0, \bar{b}]} \Delta R$  simplifies the optimal feeding path problem.

<sup>10</sup> Proofs of propositions 2, 3, and 5 are provided in appendix A.



**Figure 1. Equilibrium ration density under perfect competition in feeder animal markets**

of live meat is minimized [see equation (A1) in appendix A]. Perfect competition in feeder markets will ensure that  $\Delta R = H(\cdot)$ , and the PWS can be imputed from inserting optimal  $b$  into  $\Delta R = H(\cdot)$ .

Part (a) in proposition 2 demonstrates some consequences of higher daily maintenance costs. All else equal, a higher value of  $\gamma_M$  will depress feeder animal prices relative to fed animal prices and may induce early culling. This issue will be revisited later. If the animal is not culled early, then a higher maintenance cost parameter will elicit a more intensive feeding regime in order to more efficiently gain beyond daily maintenance costs. A genetic innovation whose only effect is to push down the value of  $\gamma_M$  for a breed should reduce the incentive to feed intensively. In part (b),  $\gamma_I$  can be considered to be determined by genetic endowments or through feed management practices that enhance palatability, e.g., rolling grain and reducing dust (Ensminger, 1987, chapter 30). Alternatively, non-use of growth-promoting implants (i.e., hormone implants but not ionophores) is held to decrease dry matter intake by about 6% (Fox et al., 1992).<sup>11</sup> An increase in  $\gamma_I$  reduces the need to avoid daily maintenance costs through a costly high-energy regime. Part (c) is a complete characterization of the contents in parts (a) and (b). One may view  $\rho$  as an index of an animal's genetic potential for meat production.

Concerning part (d) of proposition 2, the recent bans on ruminant-to-ruminant feed in the United States and elsewhere have raised the possibility that animal and poultry fats will be removed entirely from the animal and poultry feed markets. This will increase the cost of a high-energy diet. By contrast, an increase in opportunity costs of resources (e.g., high  $\kappa$  due to high labor costs, environmental regulations, or animal welfare laws) will strengthen the incentive to increase the marginal productivity of those resources by feeding to increase throughput. Parts (d) and (e) do provide an interesting contrast whereby an increase in the value of  $\kappa$  can come from an increase in the fixed cost component of rations. If the cost of rations increases such that the marginal cost of density increases, then one should decrease ration density. Both curves shift up

<sup>11</sup> This 6% figure has been adopted by the National Research Council Subcommittee on Beef Cattle Nutrition (2000).

in figure 1, but the effect on marginal cost is the more important because average cost includes the unaffected fixed cost component. If the cost increase is independent of density, then it is optimum to raise the ration density because only the average cost curve shifts up in figure 1. If an increase occurs in red-tape costs of animal feed, such as regulations on origin identification, testing for contamination, or storage conditions, then one should expect more intensive feeding programs.

The reader may find the effects in proposition 2 to be quite intuitive. As will be shown below, intuition is more likely to fail regarding consequences for days on feed. Given weight homogeneity (degree 0.75 in our case) associated with growth, in order to establish the roles of  $\gamma_I$  (through, e.g., hormones, feed preparation methods, or genetics) and  $\gamma_M$  (through, e.g., confinement or genetics) for days on feed, we need only understand how  $\gamma_I b^* - \gamma_M$  changes.

- PROPOSITION 3. *Make the growth technology assumptions in proposition 1. Then there exist increasing, convex ration cost functions such that optimal days on feed either increase or decrease with an increase in either (a)  $\gamma_I$ , or (b)  $\gamma_M$ . But optimal days on feed always increase with (c) an increase in  $\theta$ , or (d) a decrease in  $\kappa$ .*

Intuition for part (a) of proposition 3 is that the direct effect of an increase in  $\gamma_I$  on  $\gamma_I b^* - \gamma_M$  need not be so strong as to dominate the negative effect on the optimal level of rations. Intuition for part (b) is that a decrease in  $\gamma_M$  relaxes the incentive to feed intensively to clear daily maintenance costs and, again, the indirect effect can dominate the direct effect. An interesting feature of part (a) is that  $(db^*/d\gamma_I)|_{\gamma_M=0} = 0$ , and so days on feed always decrease with  $\gamma_I$  whenever  $\gamma_M = 0$ . The existence of daily maintenance costs has a qualitative effect on how optimal rations respond to intake innovations. While in each of parts (a) and (b), we believe the direct effect will likely dominate, the other possibility is introduced in order to better illustrate how technologies and regulations can affect age at slaughter. Parts (c) and (d) follow almost directly from proposition 2, as there is not any direct effect on optimal days on feed. Part (c) is of interest because the U.S. corn-to-live cattle price ratio over 2000–2005 was approximately half its level in the late 1940s. Official data have not been kept, yet it is widely believed that cattle slaughter ages have declined since World War II (Schroeder, Mintert, and Brester, 1995; Elam and Preston, 2004). Although genetic change is likely a primary determinant in this trend, the trend in relative feed prices may have encouraged more intensive ration feeding programs.

### Feeder Price-Weight Schedule

Having considered ration decisions, we now turn to the feeder price-weight schedule. Notice in (2'), homogeneity allowed revenue growth to be written independently of animal weight. Propositions 1 and 2 do not involve weight (as maintenance requirements and intake both scale to the 0.75 power), so the feeder animal's ration density is weight-separated. Thus,  $b$  can be taken as given and time-invariant when solving for physical growth and the feeder PWS. Our model setup identifies the flow of calories converted to liveweight as  $\lambda\gamma_I b^* [w(t)]^{0.75} - \lambda\gamma_M [w(t)]^{0.75}$ . Therefore, the rate of change of weight is governed by:

$$(4) \quad \frac{1}{[w(t)]^{0.75}} \frac{dw(t)}{dt} = \lambda\gamma_I b^* - \lambda\gamma_M.$$

Observe that (4) implies

$$(5) \quad \frac{d^2 w(t)}{dt^2} = \frac{3\lambda^2(\gamma_I b^* - \gamma_M)^2}{4} [w(t)]^{0.5} > 0,$$

$$\frac{d^2 \text{Ln}[w(t)]}{dt^2} = -\frac{\lambda^2(\gamma_I b^* - \gamma_M)^2}{4[w(t)]^{0.5}} < 0,$$

so that weight (with ration path endogenized) is convex in time before the maturity plateau but the log of weight is concave in time before the plateau. Convexity  $d^2 w(t)/dt^2 > 0$  is in contrast with the concavity assumptions made in Jarvis (1974) and Paarsch (1985), but is consistent with the beef production literature (Ensminger, 1987, p. 838; Goodwin, 1977, p. 158; Neumann, 1977, p. 400; Owen, 1991, p. 39).

Since  $b^*$  is weight-independent, (4) may be readily integrated from initial weight  $w(0)$  to obtain the weight path as the formula:

$$(6) \quad w(t) = ([w(0)]^{0.25} + 0.25(\gamma_I b^* - \gamma_M)\lambda t)^4.$$

With maturity weight  $w_m$ , the maturity date is given as  $T_m$ :

$$(7) \quad T_m = \frac{4(w_m^{0.25} - [w(0)]^{0.25})}{(\gamma_I b^* - \gamma_M)\lambda}.$$

In general, the time on feed to weight  $w$  will be  $t_w = 4(w^{0.25} - [w(0)]^{0.25})/[(\gamma_I b^* - \gamma_M)\lambda]$ . This relation is graphed in figure 2, which provides the relationship for two feeding levels,  $b'$  and  $b''$ , with  $b'' > b'$ .

For  $P(w)$  as the feeder animal price when weight is  $w$ , equation (2'), upon taking to the infinitesimal limit as  $dR(w)/dw = P(w) + w dP(w)/dw = d(\text{Cost})/dw$ , allows us to write:

$$(8) \quad \int_{s=w}^{s=w_m} \frac{dR(s)}{ds} ds = \hat{P}_m w_m - P(w)w =$$

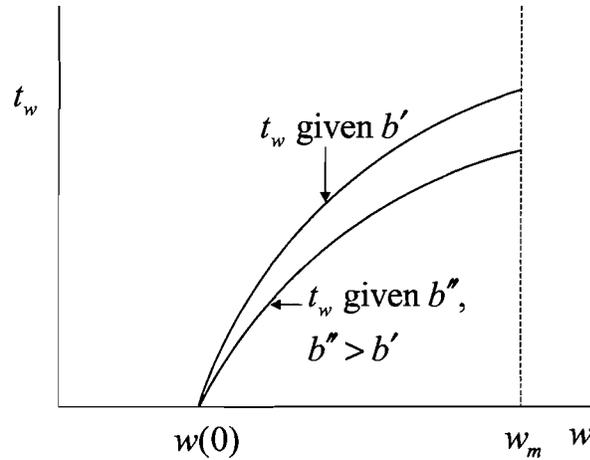
$$\int_{s=w}^{s=w_m} H(b^*; \gamma_I, \gamma_M, \theta) ds = H(b^*; \gamma_I, \gamma_M, \theta)(w_m - w).$$

Here,  $\hat{P}_m = P_m \gamma_Q$  is the mature animal price, where  $\gamma_Q$  accounts for quality-enhancing genetic innovations.<sup>12</sup>

From (8), growth rate homogeneity due to equation (4) ensures that feeder animal value  $P(w)w$  is linear in weight with value  $P(w)w = [\hat{P}_m - H(\cdot)]w_m + H(\cdot)w$ . Feeder animal value is not linear in time. The unit price of feeder animals is expressed as:

$$(9) \quad P(w) = H(b^*; \gamma_I, \gamma_M, \theta) + \frac{[\hat{P}_m - H(b^*; \gamma_I, \gamma_M, \theta)]w_m}{w}.$$

<sup>12</sup> Notice we have assumed that the feed regime does not affect quality. Since ration composition affects quality, this specification is clearly a simplification. Our model could be extended so that ration choice accounts for quality effects, but the consequences of such an extension are straightforward, and we see little point in introducing further notation.



**Figure 2. Time on feed as a function of weight**

Differentiation of (9) supports

$$(10) \quad \frac{dP(w)}{dw} = \frac{[H(b^*; \gamma_I, \gamma_M, \theta) - \hat{P}_m]w_m}{w^2},$$

$$\frac{d^2P(w)}{dw^2} = 2 \frac{[\hat{P}_m - H(b^*; \gamma_I, \gamma_M, \theta)]w_m}{w^3}.$$

If  $\hat{P}_m > H(b^*; \gamma_I, \gamma_M, \theta)$ , as we will demonstrate should be the case, then the first derivative is negative while the second derivative is positive.

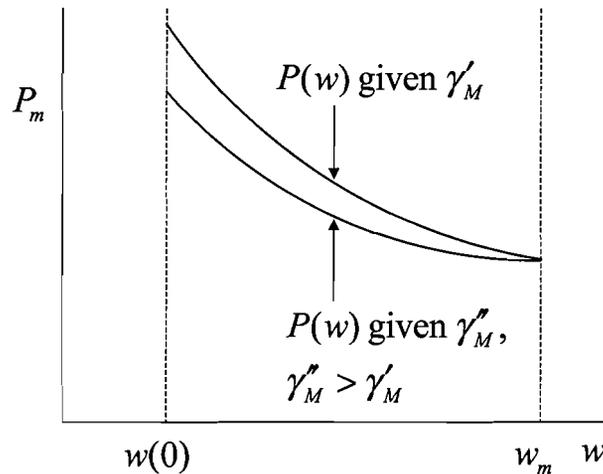
- PROPOSITION 4. *Make the growth technology assumptions in proposition 1. If  $\hat{P}_m > H(b^*; \gamma_I, \gamma_M, \theta)$ , then the feeder animal PWS is decreasing and convex.*

Equation (9) is depicted in figure 3 for two different values of  $\gamma_M$ , i.e.,  $\gamma'_M$  and  $\gamma''_M$ , with  $\gamma''_M > \gamma'_M$ . A comparison of these two parameter values will be addressed shortly. That feeder animal prices are decreasing in weight is consistent with the notion that young animals have growth potential (Jarvis, 1974) to be expended as the animal grows. Convexity only asserts that the growth potential component to value is most rapidly expended at lower weights. Since (10) shows that relative curvature is  $-[d^2P(w)/dw^2]/[dP(w)/dw] = 2/w$ , it is decreasing in weight, and the proportional rate of decline in price with weight is larger at lower weights.

The price-weight schedule should also be affected by the growing environment. In particular:

- PROPOSITION 5. *Make the growth technology assumptions in proposition 1. Then  $\forall w \in [w(0), w_m]$ : (a)  $dP(w)/d\hat{P}_m = w_m/w \geq 1$  and  $d^2P(w)/d\hat{P}_m dw \leq 0$ ; (b)  $dP(w)/d\gamma_M \leq 0$  and  $d^2P(w)/d\gamma_M dw \geq 0$ ; (c)  $dP(w)/d\gamma_I \geq 0$  and  $d^2P(w)/d\gamma_I dw \leq 0$ ; (d)  $dP(w)/d\theta \leq 0$  and  $d^2P(w)/d\theta dw \geq 0$ ; and (e)  $dP(w)/d\kappa \leq 0$  and  $d^2P(w)/d\kappa dw \geq 0$ .*

Part (a) of proposition 5 may be viewed as a sensitivity result, where the Ricardian rent due the owner of young stock as a result of high meat prices becomes less significant



**Figure 3. Price-weight schedule for different maintenance cost parameters**

on a per pound basis at larger weights. An increase in  $\hat{P}_m$  may be viewed either as an increase in the live price of some reference quality carcass or an increase in quality parameter  $\gamma_Q$ . Part (a) suggests that heavily muscled beef breeds, such as Charolais and Blonde d'Aquitaine, should have a steeply declining PWS. A further comment on (a) is that it implies  $d\text{Ln}[P(w)]/d\text{Ln}[\hat{P}_m] = [\hat{P}_m w_m]/[P(w)w] \geq 1$ . Buccola (1980), Shonkwiler and Hinckley (1985), and Marsh (1988, 2001) have obtained econometric estimates of transmission elasticity  $d\text{Ln}[P(w)]/d\text{Ln}[\hat{P}_m]$  in the range of 1.34–1.62, which are consistent with our Ricardian analysis.<sup>13</sup>

In contrast with part (a), parts (b)–(e) require an understanding of ration consequences. Parts (a)–(e) do, however, follow a similar theme. The effects of a parameter innovation which tends to push the feeder price up are spread along the schedule so that the feeder price shift becomes less pronounced at higher weights. Returning to figure 3, compare the schedules under  $\gamma'_M$  and  $\gamma''_M$ . The curves have been drawn to show the vertical gap diminishes as weight increases.

### Veal Market

Meat from young animals is typically more tender and palatable, so the principal motive for growing an animal is to realize growth potential when feed costs are low. The intent of this section is to provide a better understanding of growth potential when the option exists to slaughter early. For the sake of exposition, it is assumed the maturity price is the unique live price for slaughtered animals, i.e., there is not a premium for young meat.<sup>14</sup>

<sup>13</sup> To provide a sense of what  $[\hat{P}_m w_m]/[P(w)w]$  should be for feeder cattle deliverable under the Chicago Mercantile Exchange feeder cattle futures contract, take the mid-point of weights upon which the fed cattle futures is calculated (1,225 lbs.) as  $w_m$ , and take the mid-point of weights upon which the feeder cattle futures is calculated (750 lbs.) as  $w$ . The average of the ratio of the monthly fed cattle futures contract (nearby) over the monthly feeder futures price (nearby) during January 1990 through February 2006 is 0.884. The calculation  $1,225(0.884)/750 = 1.444$  is toward the middle of the range 1.34–1.62 reported above.

<sup>14</sup> Although an age discount could be readily introduced, the analysis would become messy without providing additional insights.

Note, from (9),  $P(w) \geq \hat{P}_m$  if and only if

$$(11) \quad \hat{P}_m \geq H(b^*; \gamma_I, \gamma_M, \theta).$$

In this case, a veal market will be absent for animals of type  $(\gamma_I, \gamma_M)$  because it will always be more profitable to feed or sell to a feedlot than to exercise the slaughter option before maturity. Notice this condition is more likely to hold when  $\hat{P}_m/[\kappa + \theta c(b^*)]$  is relatively high, i.e., when the meat-corn price ratio is high. The condition is also more likely to hold when  $\gamma_M$  is relatively low (see figure 3), and when  $\gamma_I$  is relatively high. Thus, animals with low growth parameters (perhaps surplus calves from dairy herds or chicks from laying hen flocks) are more likely to be slaughtered early. From (11) and observation of  $H(b^*; \gamma_I, \gamma_M, \theta)$  in (3), we have:

- **PROPOSITION 6.** *Make the growth technology assumptions in proposition 1. If a genetic profile  $(\gamma_I, \gamma_M) = (\bar{\gamma}_I, \bar{\gamma}_M)$  is such that the grower does not slaughter the  $w = w(0)$  animal, then the grower does not slaughter the  $w = w(0)$  animal with genetic profile  $(v_1 \bar{\gamma}_I, v_2 \bar{\gamma}_M)$ ,  $v_1 \geq v_2 > 0$ . If a genetic profile  $(\gamma_I, \gamma_M) = (\bar{\gamma}_I, \bar{\gamma}_M)$  is such that the  $w = w(0)$  animal is slaughtered, then the  $w = w(0)$  animal with genetic profile  $(v_1 \bar{\gamma}_I, v_2 \bar{\gamma}_M)$ ,  $0 < v_1 \leq v_2$ , is slaughtered.*

Proposition 6 may be viewed as a scaling result. The decision to slaughter at the outset depends only on  $\rho = \gamma_I/\gamma_M$ . For the growth technology in question, there exists a  $\hat{P}_m$ -dependent  $\rho$  value, call it  $\hat{\rho}(\hat{P}_m)$ , whereby animals should be kept to maturity if and only if  $\rho \geq \hat{\rho}(\hat{P}_m)$ . If meat prices are very low, then early slaughter of low-growth feeder animals will be one way of relieving supply pressure.

### Generalized Feeding Path

Suppose instead that  $I(w) = \gamma_I w^\sigma$ ,  $\sigma \in (0, 1]$ . Then work provided in appendix B shows the ration optimization problem resolves to:

$$(12) \quad b^*(w) = \underset{b \in [0, \bar{b}]}{\operatorname{argmin}} H[b; w, \gamma_I, \gamma_M, \theta],$$

$$H[b; w, \gamma_I, \gamma_M, \theta] = \frac{[\kappa + \theta c(b)] \gamma_I E}{(\gamma_I b - \gamma_M w^{0.75-\sigma}) \lambda},$$

under Ricardian rent extraction by young stock producers. From optimality condition  $(\gamma_I b - \gamma_M w^{0.75-\sigma}) \theta c_b(b) = [\kappa + \theta c(b)] \gamma_I$ , it is clear that  $db^*/dw = (0.75 - \sigma) w^{-0.25-\sigma} c_b(b^*) / [(\rho b^* - w^{0.75-\sigma}) c_{bb}(b^*)] \stackrel{\text{sign}}{=} 0.75 - \sigma$ . Specifically, the optimal plane of nutrition increases (decreases) with animal weight (and time) if  $0.75 > (<) \sigma$ . For  $0.75 > \sigma$ , the intuition is that intake contracts relative to maintenance costs as weight increases. Ration density should increase at higher weights in order to accelerate the animal through the later stages of feeding. Anecdotal evidence suggests to the author that some beef growers at any rate tend to increase feed density close to slaughter, but this will depend on the animal type, season of slaughter whenever animal prices are seasonal, and prevailing feed market prices.

In addition,  $db^*/dw$  will tend to be small when the relative curvature of the cost function,  $c_{bb}(b^*)/c_b(b^*)$ , is large. Also, FCE (growth/day over intake/day) is given as:

$$(13) \quad \frac{1}{I(w)} \frac{dw(t)}{dt} = \lambda b^*[w(t)] - \lambda \rho^{-1}[w(t)]^{0.75-\sigma}.$$

If  $c_{bb}(b^*)/c_b(b^*)$  is sufficiently large that  $b^*(w)$  is quite weight-insensitive, and if  $0.75 > \sigma$ , then the right-hand side of (13) will decline with weight so that FCE declines with weight. There is fairly firm evidence showing FCE declines with animal weight, even at weights well below the mature weight (National Research Council, 2000, p. 24).<sup>15</sup>

The feeding path described in (12) is simpler and differs from those derived by Paarsch (1985), Arnason (1992), and Heaps (1993), among others using optimal control methods, because they do not view the price of feeder stock as a derivative. Feeding then is not just about cost minimization but also about throughput in order to capture economic profits on feeding. When positive economic profits exist per lot, there will be an incentive to accelerate feeding and incur a higher cost of gain in order to re-stock early. While this will be true when positive economic rents exist, we argue economic rents will be dissipated through competition whereby the problem reduces to one of cost-minimized feeding.

The insights in (3) and (12) actually allow a feeding path analysis for much less structured growth technology specifications. Suppose there is a vector  $\bar{\mathbf{b}}$  of feeding choices, energy density, protein density, and so on. Conditional on weight, the time rate of change in costs is written in dot form as  $\dot{C}(\bar{\mathbf{b}}; w)$ . Here,  $\bar{\mathbf{b}} \in B$  where  $B \subset \mathbb{R}^N$ , i.e., non-negative, with  $B$  closed and bounded so that any maximum sought for a continuous function does exist. The growth equation is written in dot form as  $\dot{w}(\bar{\mathbf{b}}; w)$ , and feeder animal value as a function of weight is  $R(w)$ . Thus, revenue growth with time is  $[dR(w)/dw]\dot{w}(\bar{\mathbf{b}}; w)$ , and profit per unit time is  $[dR(w)/dw]\dot{w}(\bar{\mathbf{b}}; w) - \dot{C}(\bar{\mathbf{b}}; w)$ . If competition bids prices to the point of zero economic profits, then  $dR(w)/dw = \dot{C}(\bar{\mathbf{b}}; w)/\dot{w}(\bar{\mathbf{b}}; w)$  and the successful bidders will solve:

$$(14) \quad \bar{\mathbf{b}}^*(w) = \underset{\bar{\mathbf{b}} \in B}{\operatorname{argmin}} \operatorname{Ln}[H(\bar{\mathbf{b}}; w)],$$

$$H(\bar{\mathbf{b}}; w) = \frac{\dot{C}(\bar{\mathbf{b}}; w)}{\dot{w}(\bar{\mathbf{b}}; w)},$$

where the strictly increasing log transformation does not change the values of maximizing arguments.

It is reasonable to hold that the components of  $\bar{\mathbf{b}}$  are technical complements because energy, proteins, and other nutrients are required in approximate proportion for muscle formation. Regarding the unit cost of live weight gain, this may be represented by the assumption that  $d^2H(\bar{\mathbf{b}}; w)/db_i db_j \leq 0 \forall b_i, b_j \in \bar{\mathbf{b}}$ . From a standard result on submodular cost functions (Topkis, 1995), if  $d^2 \operatorname{Ln}[H(\bar{\mathbf{b}}; w)]/db_i dw \leq 0 \forall b_i \in \bar{\mathbf{b}}$ , then  $\bar{\mathbf{b}}^*(w)$  is component-wise increasing in weight; similarly, if  $d^2 \operatorname{Ln}[H(\bar{\mathbf{b}}; w)]/db_i dw \geq 0 \forall b_i \in \bar{\mathbf{b}}$ , then  $\bar{\mathbf{b}}^*(w)$  is component-wise decreasing in weight. Now if  $\dot{C}(\bar{\mathbf{b}}; w) = f(\bar{\mathbf{b}})g(w)$  and  $\dot{w}(\bar{\mathbf{b}}; w) = h(\bar{\mathbf{b}})k(w)$ , then  $\operatorname{Ln}[H(\bar{\mathbf{b}}; w)] = \operatorname{Ln}[f(\bar{\mathbf{b}})/h(\bar{\mathbf{b}})] + \operatorname{Ln}[g(w)/k(w)]$  with null cross-derivatives.

<sup>15</sup> As previously mentioned, the standard assumption is that  $\sigma = 0.75$ , but the data to support this assumption are less firm than the data to support the 0.75 exponent on maintenance costs.

This form of separability ensures invariance of rations composition to weight in the optimal feeding schedule. Invariance failed in (12) because separation of energy density from weight was not possible. It was possible to establish monotonicity on  $b^*(w)$  because  $d^2 \text{Ln}[H(b; w)]/dbdw$  has uniform sign, that of  $\sigma - 0.75$ .<sup>16</sup>

### Empirical Analysis

The analysis suggests a variety of testable hypotheses. Using data from the Livestock Marketing Information Center (LMIC), four hypotheses were tested.<sup>17</sup> The monotonicity and convexity hypotheses arise from proposition 4. The third and fourth hypotheses relate to interaction between growth potential and consumer demand, and to how sensitive the price-weight schedule is to feed costs. Both of these hypotheses arise from further scrutiny of equation (9), as developed below. Feeder animal prices in the main beef-feeding states were chosen to test these inferences. Data considered for each of the four hypotheses are presented in table 1. The data sets were far from complete. Accordingly, when any of the data series needed for a test had a missing relevant data point, the observation was discarded.

#### *Monotonicity and Convexity*

The first hypothesis (referred to as H1) is that price falls monotonically with weight. Observations were considered from the four large live cattle markets of Amarillo (TX), Clovis (NM), St. Joseph (MO), and Oklahoma City, as well as statewide data from Georgia, Kansas, Missouri, Nebraska, and Oklahoma. Reported live prices for feeder steers were chosen because these markets are more liquid than for heifers. Prices for steers in weight band 500–600 lbs. were compared with those in weight band 300–400 lbs., and steers in weight band 700–800 lbs. with those in weight band 500–600 lbs. Reporting frequency, weekly or monthly, and the time interval over which data are available, vary with the market examined.

Data points from the series considered were differenced. If the difference had a positive value, i.e., contrary to the monotonicity hypothesis, then an indicator variable labeled  $I^{mon}$  was assigned the value of zero; otherwise it was assigned the value of one. The indicator function's estimated mean and variance statistics are identified as  $\mu_{I^{mon}}$  and  $\sigma_{I^{mon}}$ , respectively. For the second hypothesis, convexity (denoted as H2), data point differences were differenced again. If, contrary to the convexity hypothesis, the second difference had a negative value, then an indicator variable labeled  $I^{cux}$  was assigned the value of zero; otherwise a value of one was assigned. Indicator function mean and variance estimates are identified as  $\mu_{I^{cux}}$  and  $\sigma_{I^{cux}}$ , respectively.

<sup>16</sup> Two further generalizations of the model are to (a) account for the supply of young stock in equilibrium, and (b) set  $I(w) = \gamma_l l(w)$ . Details of both extensions are available upon request from the author. An interesting implication of the second extension is that circumstances exist under which feeder animal prices could be increasing and/or concave in weight over a weight interval. Such anomalies might occur if there is a weight interval early on in production at which an animal is not good at feed conversion. Empirical evidence will be provided later to support both the "monotone decreasing" and "convex" properties on the PWS.

<sup>17</sup> The LMIC obtains support from State Extension Services, the USDA, and the beef industry. Data are available to members at <http://www.lmic.info/>.

**Table 1. Data Series for Hypotheses H1–H4**

Location	Weight Bands, Pounds Liveweight (S = steer, H = heifer)	Time Intervals (M = monthly, W = weekly)	Number of Observations
<b>Hypotheses H1 and H2:</b>			
Amarillo, TX	S300–350, S500–550, S700–750	M Jan 1992 to Dec 2001	114
Clovis, NM	S300–350, S500–550, S700–750	M Jan 1992 to Dec 2001	120
Georgia State	S300–400, S500–600, S700–800	M Jan 1973 to Feb 2006	392
Kansas State	S300–400, S500–600, S700–800	M Jun 1999 to Feb 2006	81
Missouri State	S300–350, S500–550, S700–750	W 06/25/99 to 03/10/06	333
St. Joseph, MO	S400–450, S600–650, S800–850 <sup>a</sup>	M Jan 1992 to Dec 2001	105
Nebraska State	S300–400, S500–600, S700–800	M Jun 1999 to Feb 2006	80
Oklahoma State	S300–350, S500–550, S700–750	M Jan 1992 to Feb 2006	137
Oklahoma City	S300–400, S500–600, S700–800	W 01/06/73 to 03/18/06	1,547
<b>Hypothesis H3:</b>			
Amarillo, TX	S300–350, H300–350	M Jan 1992 to Dec 2001	114
Clovis, NM	S300–350, H300–350	M Jan 1992 to Dec 2001	120
Georgia State	S300–400, H300–400	M Nov 1989 to Feb 2006	196
Kansas State	S300–400, H300–400	M Jun 1999 to Feb 2006	81
Missouri State	S300–350, H300–350	W 06/25/99 to 02/24/06	317
St. Joseph, MO	S400–450, H400–450 <sup>a</sup>	M Jan 1992 to Dec 2001	110
Nebraska State	S300–350, H300–350	M Jun 1999 to Feb 2006	79
Oklahoma State	S300–350, H300–350	M Jan 1992 to Feb 2006	131
Oklahoma City	S300–400, H300–400	W 11/04/89 to 03/04/06	637
<b>Hypothesis H4:</b>			
Amarillo, TX	S300–350, S700–750	M Jan 1992 to Dec 2001	114
Clovis, NM	S300–350, S700–750	M Jan 1992 to Dec 2001	120
Georgia State	S300–400, S700–800	M Jan 1984 to Feb 2006	267
Kansas State	S300–400, S700–800	M Jun 1999 to Feb 2006	81
Missouri State	S300–350, S700–750	W 06/25/99 to 02/24/06	331
St. Joseph, MO	S350–400, S750–800 <sup>a</sup>	M Jan 1992 to Feb 2006	92
Nebraska State	S300–400, S700–800	M Jun 1999 to Feb 2006	80
Oklahoma State	S300–350, S700–750	M Jan 1992 to Feb 2006	637
Oklahoma City	S300–400, S700–800	W 01/07/84 to 02/18/06	971

<sup>a</sup> Insufficient reported observations were available for animal prices at lighter weights.

Following Freund and Walpole (1987, p. 522), the nonparametric sign test is used. Relative to the null hypothesis that the mean of the differences is 0.5, the test statistic is:

$$(15) \quad z^j = \frac{\mu_{I^j} - 0.5}{\sigma_{I^j} / \sqrt{n}}, \quad j \in \{mon, cvx\},$$

where there are  $n$  observations. Test results are provided in table 2.

**Table 2. Results for Monotonicity and Convexity Hypotheses**

Hypothesis	Location	Difference	Indicator Mean	Test Statistic
<b>H1: Monotonicity</b>	Amarillo, TX	A: 500–550 less 300–350	0.99	55.7*
		B: 700–750 less 500–550	0.96	23.6*
	Clovis, NM	A: 500–550 less 300–350	1.00	∞
		B: 700–750 less 500–550	0.98	32.9*
	Georgia State	A: 500–600 less 300–400	0.97	51.7*
		B: 500–600 less 700–800	0.97	51.7*
	Kansas State	A: 500–600 less 300–400	0.99	40.1*
		B: 700–800 less 500–600	1.00	∞
	Missouri State	A: 500–550 less 300–350	1.00	∞
		B: 700–750 less 500–550	0.99	81.3
	St. Joseph, MO	A: 600–650 less 400–450	0.98	73.1*
		B: 800–850 less 600–650	0.89	12.4*
	Nebraska State	A: 500–600 less 300–400	1.00	∞
		B: 700–800 less 500–600	1.00	∞
	Oklahoma State	A: 500–550 less 300–350	1.00	∞
		B: 700–750 less 500–550	0.99	63.7*
Oklahoma City	A: 500–600 less 300–400	0.98	120.7*	
	B: 700–800 less 500–600	0.95	81.4*	
<b>H2: Convexity</b>	Amarillo, TX	B less A in H1	0.86	11.0*
	Clovis, NM	B less A in H1	0.89	13.8*
	Georgia State	B less A in H1	0.95	42.4*
	Kansas State	B less A in H1	0.86	9.5*
	Missouri State	B less A in H1	0.74	9.9*
	St. Joseph, MO	B less A in H1	0.90	14.3*
	Nebraska State	B less A in H1	0.68	3.4*
	Oklahoma State	B less A in H1	0.86	12.0*
	Oklahoma City	B less A in H1	0.89	63.3*

Note: An asterisk (\*) denotes statistical significance at the 1% level.

The monotonicity hypothesis is almost never violated. Even when it is, the two worst cases involve 11% and 4% of observations.<sup>18</sup> The reason for any violation could be due to the variety of weights and animal qualities sold under any weight band, to the variety of sale locations in the case of statewide data, to price movements during the weekly and monthly time windows, or to sampling and reporting errors.<sup>19</sup> In addition, the failure could occur because markets are not likely to be in static equilibrium.<sup>20</sup> Evidence to be presented later suggests statewide aggregation may be among the more problematic of

<sup>18</sup> The worst case is for St. Joseph on 600–650 lb. and 800–850 lb. steers. This test is furthest down the PWS, where the curve should be flattest and noise will be more important.

<sup>19</sup> For example, most young animal sales may occur early in a month in which most heavy animals are sold later. If fed animal prices rise over the month, then reported prices for feeder animals might incorrectly suggest that feeder prices increase with weight.

<sup>20</sup> The 700–800 lb. animal will sell into a beef market approximately seven months before the 300–350 lb. animal. The medium-range outlook for beef may differ from the long-range outlook. We could use forward or futures market prices to overcome these problems, but only two futures contract markets of relevant maturity are available, and reliable forward market price data series are not available.

these concerns. More violations occur for the convexity hypothesis, and this is particularly true of the Northern Great Plains states. Nevertheless, in all cases, hypothesis H2 can be accepted with 99% confidence.

### *Growth Potential and Consumer Demand*

In the third hypothesis (H3), we test for the role of growth potential in determining how feeder animals respond to demand for fed animals through the following observation. Suppose two animals differ by the maintenance requirement parameter, with  $\gamma_M'' < \gamma_M'$ . Using (9), and with obvious notation, write:

$$(16) \quad \frac{P(w, \gamma_M'')}{P(w, \gamma_M')} = \frac{1 - (w_m - w)X(\gamma_M'')}{1 - (w_m - w)X(\gamma_M')},$$

$$X(\gamma_M) = \frac{H(b^*(\gamma_M); \gamma_I, \gamma_M, \theta)}{P_m}.$$

With  $H(b^*(\gamma_M''); \gamma_I, \gamma_M'', \theta) < H(b^*(\gamma_M'); \gamma_I, \gamma_M', \theta)$ , the fact that the value of  $H(\cdot)$  does not depend upon  $P_m$  implies  $d[P(w, \gamma_M'')/P(w, \gamma_M')]/dP_m < 0$ .<sup>21</sup>

This suggests the following. Heifers are generally less efficient meat producers than steers (Gillespie, 2002, p. 338). An increase in the feed-normalized price of fed cattle should reduce the price of feeder steers relative to feeder heifers. To test for this, steer over heifer price ratios were regressed on the monthly steer and heifer corn feed ratio, and the results are reported in table 3.<sup>22</sup> The regressions are strongly supportive for the four city markets of Amarillo, Clovis, Oklahoma City, and St. Joseph, but support is less strong for state-level data. These findings, together with regression  $R^2$  values, suggest that state-level aggregation generates noise. Aggregation from weekly to monthly data does not appear to generate as much noise.

### *Schedule Slope Sensitivity to Steer and Heifer Corn Feed Ratio*

Equation (9) can also be used to obtain:

$$(17) \quad \frac{P(w')}{P(w)} = \frac{1 - (w_m - w')X}{1 - (w_m - w)X}, \quad X = \frac{H(b^*; \gamma_I, \gamma_M, \theta)}{P_m}.$$

For  $w' < w$ , a differentiation establishes:

$$(18) \quad \frac{d[P(w')/P(w)]}{dX} = \frac{(w' - w)}{[1 - (w_m - w)X]^2} < 0.$$

Thus, an increase in the price of meat on the hoof relative to the price of feed should increase the price of lighter feeder animals by more than that of heavier feeder animals.

<sup>21</sup> The same analysis applies if the animals differ by intake parameter. It only matters that the cost of gain is smaller for one animal type.

<sup>22</sup> The steer and heifer corn feed ratio is the number of bushels of corn that can be bought with receipts from selling a weighted index of 100 lb., liveweight, fed steers and heifers.

**Table 3. Regression Results for Feeder Steer to Heifer Price Ratio on Steer and Heifer Corn Feed Ratio**

Location	Intercept		Animal-to-Corn Feed Price Ratio		$R^2$
	Coefficient	$t$ -Statistic	Coefficient	$t$ -Statistic	
Amarillo, TX	1.26	(61.07)	-0.00310	(4.79)	0.170
Clovis, NM	1.26	(74.81)	-0.00252	(4.77)	0.162
Georgia State	1.30	(130.05)	-0.00410	(13.98)	0.502
Kansas State	1.12	(35.38)	0.00005	(0.07)	0.00005
Missouri State	1.15	(83.95)	-0.00040	(1.16)	0.004
St. Joseph, MO	1.23	(81.11)	-0.00203	(4.24)	0.143
Nebraska State	1.10	(36.90)	0.00012	(0.15)	0.0003
Oklahoma State	1.21	(45.78)	-0.00132	(1.78)	0.024
Oklahoma City	1.37	(66.54)	-0.00267	(9.62)	0.126

**Table 4. Regression Results for Lighter over Heavier Feeder Steer Price Ratio on Steer and Heifer Corn Feed Ratio**

Location	Intercept		Animal-to-Corn Feed Price Ratio		$R^2$
	Coefficient	$t$ -Statistic	Coefficient	$t$ -Statistic	
Amarillo, TX	1.071	(27.78)	0.00776	(6.41)	0.268
Clovis, NM	1.067	(33.19)	0.00996	(9.87)	0.452
Georgia State	1.187	(56.05)	0.00732	(11.57)	0.336
Kansas State	1.283	(24.95)	0.00710	(1.32)	0.022
Missouri State	1.311	(46.48)	0.00046	(0.65)	0.001
St. Joseph, MO	0.983	(23.11)	0.00892	(6.62)	0.328
Nebraska State	1.227	(21.75)	0.00370	(2.58)	0.078
Oklahoma State	1.117	(22.42)	0.00275	(4.81)	0.146
Oklahoma City	1.128	(80.18)	0.00573	(13.69)	0.162

Regression results are provided in table 4. The conjectured response is confirmed in all cases, although the effect is not significant at the 10% level for either Kansas State or Missouri State data. It is significant at the 1% level in the other seven cases. Again, with the exception of Georgia, statewide data appear to be noisier than city market data.

### Discussion

The intent of this article has been to develop a better understanding of the animal feeding operation, with emphasis on rations maintenance costs and equilibrium pricing for feeder animals. Our model is inevitably a simplification, but simplicity has merit if it allows for useful insights to be identified. From a sampling of hypotheses over nine market price series, with eight of these in the most important cattle feeding region of the United States, strong support was found for our model.

Among arguably significant omissions is the assumption that the interest rate is zero. There will of course be a downward shift in feeder animal prices when the opportunity

cost of capital is included, and this shift will be more marked at earlier weights. A positive rate will also cause minor adjustments in feeding incentives because early feeding becomes comparatively more costly. A similar minor concern is that feed costs are held to depend only on the ration density. In the case of grain feed, this is not true because there must be intra-year price appreciation in order to motivate storage. For forage, ration costs vary seasonally given preservation costs and the high levels of spoilage over time. When ration costs are seasonal, then ration levels and feeder animal prices will be seasonal. Accommodating feed seasonality in the model would be challenging because of storage issues and because processing constraints in the face of supply seasonality will also affect feeder animal prices.

We believe the insights our model provides will be robust to these and other modeling concerns, particularly when markets tend to be stable and non-seasonal in nature. An issue which might be more challenging for the data analyst concerns policy distortions in feeder animal markets. It has been policy across European Union countries for a long period prior to decoupling reform in the early years of the 21st century to make annual per head payments on feeder cattle over some of the animal's life. This presents a problem for testing the hypotheses provided in propositions 4 through 6. But it also presents an opportunity to test for rationality regarding price formation consistent with the Ricardian rent assumption.

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### Appendix A: Proofs

*Proof of Proposition 2.* Write the first-order condition arising from (3) as:

$$(A1) \quad \theta c_b(b^*) \frac{E}{\lambda} = \frac{[\kappa + \theta c(b^*)] \gamma_I E}{(\gamma_I b^* - \gamma_M) \lambda} = H(b^*; \gamma_I, \gamma_M, \theta).$$

Second-order conditions will be satisfied locally, which is all that is necessary, if the cost function is strictly convex in  $b$ . Concerning part (d), differentiate (A1) and then use (A1) to obtain  $db^*/d\theta = -\gamma_I \kappa / [(\gamma_I b^* - \gamma_M) \theta^2 c_{bb}(b^*)] < 0$ . Part (e) follows from  $db^*/d\kappa = \gamma_I / [(\gamma_I b^* - \gamma_M) \theta c_{bb}(b^*)] > 0$ . For parts (a) and (b), observe that:

$$db^*/d\gamma_M = c_b(b^*) / [(\gamma_I b^* - \gamma_M) c_{bb}(b^*)] > 0,$$

and

$$db^*/d\gamma_I = -\gamma_M c_b(b^*) / [(\gamma_I b^* - \gamma_M) \gamma_I c_{bb}(b^*)] < 0.$$

For part (c), write (A1) as  $(\rho b^* - 1) \theta c_b(b^*) = [\kappa + \theta c(b^*)] \rho$  and differentiate:

$$db^*/d\rho = -c_b(b^*) / [(\rho b^* - 1) \rho c_{bb}(b^*)] < 0.$$

*Proof of Proposition 3.* In each case, we provide a proof by construction.

■ *Part (a):* From the first-order condition (A1) and proof of proposition 2, write:

$$(A2) \quad \frac{d(\gamma_I b^* - \gamma_M)}{d\gamma_I} \stackrel{\text{sign}}{=} (b^*)^2 c_{bb}(b^*) \rho - b^* c_{bb}(b^*) - c_b(b^*).$$

This expression will certainly be positive if  $(b^*)^2 c_{bb}(b^*) \rho - b^* c_{bb}(b^*) - (1 + \varepsilon) c_b(b^*) = 0$ ,  $\varepsilon > 0$ . An increasing and convex cost function is constructed next that satisfies this relation for all values of  $b$ , and so for the optimal value. Set  $y = c_b(b)$  and  $x = b$ , so that the condition reduces to:

$$(A3) \quad \frac{1}{y} \frac{dy}{dx} = \frac{1 + \varepsilon}{\rho x^2 - x} = \frac{(1 + \varepsilon) \rho}{\rho x - 1} - \frac{(1 + \varepsilon)}{x}, \quad \varepsilon > 0.$$

Integrate and substitute to obtain:

$$(A4) \quad c_b(b) = A_0 \left( \frac{\rho b - 1}{b} \right)^{1+\varepsilon}.$$

Because  $\rho b > 1$  is required for growth, (A4) is positive if  $A_0 > 0$ , while (A4) is increasing in  $b$  if both  $A_0 > 0$  and  $\varepsilon > -1$ . The last condition has already been imposed through  $\varepsilon > 0$ .

In order to establish a case where  $(b^*)^2 c_{bb}(b^*) \rho - b^* c_{bb}(b^*) - c_b(b^*) < 0$ , we need only identify an acceptable cost function such that  $(b^*)^2 c_{bb}(b^*) \rho - c_b(b^*) = 0$ . Rewrite this as the condition  $\rho x^2 (dy/dx) - y = 0$ , or  $(1/y)(dy/dx) = 1/(\rho x^2)$  with solution:

$$(A5) \quad c_b(b) = A_0 e^{-1/(\rho b)}.$$

Integrate over  $b \in [\xi, \bar{b}]$ ,  $\xi > 0$  to obtain

$$c(b) = A_1 + A_0 \int_{\xi}^b e^{-1/(\rho s)} ds$$

on  $b \in [\xi, \bar{b}]$ ,  $\xi > 0$ . Assume  $A_1 > 0$ ,  $A_0 > 0$ , and then splice on any acceptable cost function with requisite smoothness on  $b \in [0, \xi]$  to obtain a function which is increasing and convex while satisfying  $b^2 c_{bb}(b) \rho - b c_{bb}(b) - c_b(b) < 0$  on  $b \in [\xi, \bar{b}]$ ,  $\xi > 0$ .

■ *Part (b)*: We have

$$(A6) \quad \frac{d(\gamma_I b^* - \gamma_M)}{d\gamma_M} \stackrel{\text{sign}}{=} \rho c_b(b^*) - \rho b^* c_{bb}(b^*) + c_{bb}(b^*).$$

The sign is positive if  $c_{bb}(b)/c_b(b) = (\rho - \varepsilon)/(\rho b - 1)$  and  $\rho > \varepsilon > 0$ , i.e., if  $c_b(b) = A_0(\rho b - 1)^{(\rho - \varepsilon)/\rho}$ . For  $\rho > \varepsilon > 0$ , this cost function is positive and increasing on any energy density level that clears maintenance requirements. The sign in (A6) is negative if  $\rho c_b(b) - \rho b^* c_{bb}(b) + c_{bb}(b) = -\rho c_b(b)$ , and thus the differential equation to be solved is  $(1/y)(dy/dx) = 2\rho/(\rho x - 1)$ . The equation solves as

$$(A7) \quad c_b(b) = A_0(\rho b - 1)^2,$$

a positive and increasing function whenever energy density is sufficient to grow the animal.

■ *Parts (c) and (d)*: From proposition 2, we have

$$d(\gamma_I b^* - \gamma_M)/d\theta \stackrel{\text{sign}}{=} db^*/d\theta < 0 \quad \text{and} \quad d(\gamma_I b^* - \gamma_M)/d\kappa \stackrel{\text{sign}}{=} db^*/d\kappa > 0.$$

*Proof of Proposition 5.*

■ *Part (a)*: From (9),  $dP(w)/d\hat{P}_m = w_m/w$  and  $d^2P(w)/d\hat{P}_m dw = -w_m/w^2$ .

■ *Parts (b)–(e)*: From (9),

$$(A8) \quad \begin{aligned} \frac{dP(w)}{d\gamma_M} &= \frac{(w - w_m)}{w} \frac{\partial H(b^*; \gamma_I, \gamma_M, \theta)}{\partial \gamma_M} \leq 0, & \frac{d^2P(w)}{d\gamma_M dw} &= \frac{w_m}{w^2} \frac{\partial H(b^*; \gamma_I, \gamma_M, \theta)}{\partial \gamma_M} \geq 0, \\ \frac{dP(w)}{d\gamma_I} &= \frac{(w - w_m)}{w} \frac{\partial H(b^*; \gamma_I, \gamma_M, \theta)}{\partial \gamma_I} \geq 0, & \frac{d^2P(w)}{d\gamma_I dw} &= \frac{w_m}{w^2} \frac{\partial H(b^*; \gamma_I, \gamma_M, \theta)}{\partial \gamma_I} \leq 0, \\ \frac{dP(w)}{d\theta} &= \frac{(w - w_m)}{w} \frac{\partial H(b^*; \gamma_I, \gamma_M, \theta)}{\partial \theta} \leq 0, & \frac{d^2P(w)}{d\theta dw} &= \frac{w_m}{w^2} \frac{\partial H(b^*; \gamma_I, \gamma_M, \theta)}{\partial \theta} \geq 0, \\ \frac{dP(w)}{d\kappa} &= \frac{(w - w_m)}{w} \frac{\partial H(b^*; \gamma_I, \gamma_M, \theta)}{\partial \kappa} \leq 0, & \frac{d^2P(w)}{d\kappa dw} &= \frac{w_m}{w^2} \frac{\partial H(b^*; \gamma_I, \gamma_M, \theta)}{\partial \kappa} \geq 0, \end{aligned}$$

where the envelope theorem has been employed.

## Appendix B: Ration Optimization as Optimal Control Problem

With the goal of maximizing profit per period, pose the problem as a control path optimization:

$$(B1) \quad \max_{(b(t), T)} J[b(t), T] = \max_{(b(t), T)} \frac{\hat{P}_m w_m - P(w(0))w(0) - \int_{t=0}^T [\kappa + \theta c(b(t))] \gamma_I E[w(t)]^\alpha dt}{T},$$

$$\text{s.t.: } \frac{dw(t)}{dt} = \lambda \gamma_I b(t)[w(t)]^\alpha - \lambda \gamma_M [w(t)]^{0.75};$$

$$b(t) \geq 0 \quad \forall t \in [0, t], \quad w(t) \geq 0 \quad \forall t \in [0, t], \quad T \geq 0, \quad w(T) = w_m.$$

In our case it is sufficient to develop the solution to the constrained optimization over  $b(t)$  with  $T$  fixed at the optimum (transversality conditions will establish optimum  $T$ ). Fixing  $T$ , the Hamiltonian may be written as:

$$(B2) \quad \mathcal{H}(w(t), b(t), \phi(t)) \equiv -[\kappa + \theta c(b(t))] \gamma_I E [w(t)]^\sigma + \phi(t) (\lambda \gamma_I b(t) [w(t)]^\sigma - \lambda \gamma_M [w(t)]^{0.75}).$$

Necessary conditions for interior solutions involve:

$$(B3.1) \quad \frac{dw(t)}{dt} = \lambda \gamma_I b(t) [w(t)]^\sigma - \lambda \gamma_M [w(t)]^{0.75},$$

$$(B3.2) \quad \phi(t) = \frac{\theta E}{\lambda} \frac{dc(b(t))}{db},$$

$$(B3.3) \quad \frac{1}{[w(t)]^{\sigma-1}} \frac{d\phi(t)}{dt} = [\kappa + \theta c(b(t))] \gamma_I E \sigma - \phi(t) \lambda (\gamma_I b(t) \sigma - 0.75 \gamma_M [w(t)]^{0.75-\sigma}).$$

The system simplifies upon making the well-known observation that  $\phi(t)$  is the value of incremental stock (Léonard and Long, 1992, p. 154; Conrad and Clark, 1987, p. 36), where stock in this case is animal weight. In our case,  $\phi(t)$  is  $\Delta R$  at the infinitesimal. But the cost of the increment is cost per incremental time over weight gain per incremental time, i.e.,

$$(B4) \quad \phi(t) = \frac{[\kappa + \theta c(b^*(t))] \gamma_I [w(t)]^\sigma E}{(\lambda \gamma_I b^*(t) [w(t)]^\sigma - \lambda \gamma_M [w(t)]^{0.75})} = \frac{[\kappa + \theta c(b^*(t))] \gamma_I [w(t)]^\sigma E}{(\lambda \gamma_I b^*(t) - \lambda \gamma_M [w(t)]^{0.75-\sigma})}.$$

Under Ricardian rent extraction, incremental value must equal incremental cost because otherwise there would be positive or negative economic rent over the feeding operation, a contradiction. Therefore, (B3.2) resolves to:

$$(B5) \quad \frac{[\kappa + \theta c(b(t))] \gamma_I E}{(\gamma_I b(t) - \gamma_M [w(t)]^{0.75-\sigma}) \lambda} = \frac{\theta E}{\lambda} \frac{dc(b(t))}{db},$$

i.e., the minimizer of  $H[b; w, \gamma_I, \gamma_M, \theta] = [\kappa + \theta c(b)] \gamma_I E / [(\gamma_I b - \gamma_M w^{0.75-\sigma}) \lambda]$ . In this light, (B3.3) becomes (qualitatively):

$$(B6) \quad \frac{d\phi(t)}{dt} \stackrel{\text{sign}}{=} 0.75 - \sigma,$$

i.e., the unit cost of gain increases (decreases) with time, and consequently with an increase in weight, if  $0.75 > (<) \sigma$ . The case of proposition 1 is when  $\sigma = 0.75$  so that the unit cost of gain is weight-invariant. Another way of viewing (B4) under derived feeder animal pricing is that

$$\int_{w(0)}^{w_m} \phi(w(t)) dw = \hat{P}_m w_m - P(w(0)) w(0),$$

since

$$(B7) \quad \int_{w(0)}^{w_m} \frac{[\kappa + \theta c(b^*(t))] \gamma_I [w(t)]^\sigma E}{(\lambda \gamma_I b^*(t) - \lambda \gamma_M [w(t)]^{0.75-\sigma})} dw = \int_0^T \frac{[\kappa + \theta c(b^*(t))] \gamma_I [w(t)]^\sigma E}{(\lambda \gamma_I b^*(t) - \lambda \gamma_M [w(t)]^{0.75-\sigma})} \left[ \frac{dw}{dt} \right]^{-1} dt$$

$$= \int_0^T [\kappa + \theta c(b^*(t))] \gamma_I [w(t)]^\sigma E dt,$$

where  $dw/dt$  is the growth rate under optimal rations. But the right-most term in (B7) equals  $\hat{P}_m w_m - P(w(0)) w(0)$  under Ricardian rent extraction on the part of young stock suppliers.