INCORPORATING UNCERTAINTY IN THE ANALYSIS OF OPTIMAL BEEF-FORAGE PRODUCTION SYSTEMS

Richard B. Rawlins and Daniel J. Bernardo

ABSTRACT

A risk programming model was developed to evaluate the tradeoffs between risk and expected returns in beef-forage production systems. The specification represents nutrient and intake considerations when allocating forage among cattle enterprises; it also incorporates the various sources of risk facing livestock producers. Efficient ranch organizations were derived for a representative eastern Oklahoma ranch using MOTAD and Target-MOTAD formulations. Diversification of forage enterprises, introduction of cow-calf enterprises, and retained ownership of weaned calves were identified as important responses to reductions in acceptable levels of risk. Results also indicated efficient ranch plans to be sensitive to the risk criteria and producer's willingness to accept risk.

INTRODUCTION

Production of beef cattle is important throughout the southern region of the United States, accounting for approximately 24 percent of the region's total value of agricultural production (U.S. Department of Agriculture, 1989). A principle reason for the dominance of this industry is the wide range of improved and native forages available to livestock producers in the region. The climate and soils of much of the region are conducive to the production of several perennial pastures such as bermudagrass, fescue, and lovegrass. Native range and small grain pasture also comprise a significant component of several states' forage base, particularly in western areas (U.S. Department of Agriculture, 1984). This diversity of forages provides livestock producers considerable flexibility in determining the types of cattle enterprises produced annually. The availability of high quality forage during most of the year provides ranchers numerous backgrounding, retained ownership, and wintering opportunities, as well as choices among calving seasons and weaning dates. Livestock producers must determine how to organize their ranch resources to best take advantage of available production alternatives. The selection of livestock and pasture enterprises has significant ramifications for both the expected value and variability of annual income. Identifying efficient forage-beef production systems for producers characterized by alternative risk preferences is a necessary research priority.

Numerous studies of farm and ranch organization have been conducted. Most earlier studies employed deterministic linear programming models to derive profit maximizing enterprise combinations for representative farms or ranches. Recent analysis have incorporated risk considerations through application of quadratic programming, minimization of the absolute deviations (MOTAD), and Target-MOTAD formulations (Brink and McCarl, Zimmet and Spreen, Teague and Lee). Despite the large number of risk programming applications in this area, few studies have attempted to derive efficient forage and cattle management strategies. Two factors contribute to this apparent void: (1) complexities involved in representing the dynamic relationships between forage quality, digestibility, and animal intake; and (2) difficulties associated with measuring the risk inherent in livestock production systems.

Representing livestock nutrient considerations in mathematical programming analyses centers around the technical relationship between forage quality and animal intake. As forage quality decreases with maturity, its digestibility decreases, thereby resulting in a reduction in the animal's maximum intake of that forage. However, as quality decreases, a greater quantity of consumption is needed to meet an animal's nutrient requirements. Failure to account for these interactions in mathematical programming models can result in optimal livestock-forage production systems that are infeasible; livestock cannot consume the quantity of feed necessary to meet nutrient requirements (Whitson et al.). These complexities are exacerbated by the fact that the relationship between forage quality and intake is dynamic, changing as animals gain weight and mature, environmental conditions change, and animals enter various reproductive states (e.g., lactation, gestation, etc.). Thus, adaptation of basic risk programming specifications is required to accurately represent the...
relationship between forage quality, voluntary intake, and livestock performance.

A second difficulty encountered when applying risk programming techniques to derive efficient livestock-forage systems concerns risk measurement. A primary source of production risk experienced by livestock producers is derived from variability in the quantity of forage produced. However, the timing and quality of this production also influences risk. The livestock producer also faces uncertainties in converting forage produced into final output. Thus, valuation of production risk in livestock applications of risk programming is more complex than simply calculating yield deviations (as might be used to quantify production risk in crop applications). Procedures for representing stochastic influences in risk programming models of livestock-forage systems are needed.

The objective of this study was to evaluate the tradeoffs between risk and returns for various beef-forage production systems available to ranchers in Oklahoma. A model was developed for determining efficient organizations of livestock and forage enterprises that conform to behavioral criteria of Oklahoma livestock producers. Specific attention was focused on developing a decision framework that explicitly represents forage quality and intake considerations, as well as incorporating the various sources of risk facing livestock producers. Efficient ranch organizations were derived for a representative eastern Oklahoma ranch using MOTAD programming procedures. The effect of introducing a safety-first decision criterion on efficient ranch plans was then analyzed using a Target-MOTAD formulation. A comparison of the two derived ranch organizations provides insights into the influence of alternative behavioral criteria on the selection of livestock and pasture management strategies.

METHOD OF ANALYSIS

The MOTAD model was specified in generalized form as:

\[
\text{Max } \sum \bar{c}_j X_j
\]

subject to:

\[
\sum a_{ij} X_j \leq b_i \quad (i = 1, \ldots, n)
\]

\[
\sum (c_{kj} - \bar{c}_j) X_j - Z_k^+ + Z_k^- = 0 \quad (k = 1, \ldots, m)
\]

\[
\sum p_k (Z_k^+ + Z_k^-) \leq \lambda
\]

where:

- \( c_j \) = expected net return per unit of enterprise \( j \)
- \( X_j \) = level of enterprise \( j \)
- \( a_{ij} \) = amount of resource \( i \) required by one unit of enterprise \( j \)
- \( b_i \) = availability of resource \( i \)
- \( n \) = total number of limited resources
- \( c_{kj} \) = net return of enterprise \( j \) for state of nature \( k \)
- \( Z_k^+, Z_k^- \) = positive and negative income deviations for state of nature \( k \)
- \( m \) = total number of states of nature (years)
- \( p_k \) = probability of state of nature \( k \)

Both the objective function and first set of constraints are identical to a standard linear programming formulation; annual net returns are maximized subject to a set of ranch-level resource constraints. The second constraint set estimates annual net return deviations, which are weighted by the probability of their occurrence in determining mean absolute deviations. For this study, the probability of each state of nature is \( 1/m \). By parameterizing \( \lambda \), a set of expected return-absolute deviation (E-A) efficient ranch organizations is derived. Thus, the model maximizes expected net returns subject to parametric restriction on mean absolute deviations in net returns.

The major features of the MOTAD model are illustrated in the abbreviated linear programming tableau presented in Table 1. The abbreviated tableau includes two subperiods, as well as two forages and two livestock activities (cow-calf and stockers). The model is comprised of four general classes of activities: (1) one-acre forage production activities, (2) per-head livestock production activities, (3) livestock sell activities, and (4) income deviation activities used to measure the risk inherent in alternative ranch organizations. Because forage production, forage quality, and animal nutrient requirements differ substantially over time, the year is divided into six two-month subperiods. Livestock nutrient constraints must be met in each of the six subperiods. Forage-livestock interactions are represented in rows 2 through 5. Livestock nutrient requirements

\[1 \text{ The term "stocker enterprise" refers to cattle enterprises in which weaned calves or yearlings are purchased and grazed for a period of time and sold for finishing in feedlots.} \]
Table 1. Abbreviated Tableau of MOTAD Model

<table>
<thead>
<tr>
<th>Livestock Activities</th>
<th>Hay Deviation</th>
<th>Net Return Deviations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forage Activities</td>
<td>Purchase</td>
<td>Year 1</td>
</tr>
<tr>
<td></td>
<td>Sell</td>
<td>Positive</td>
</tr>
<tr>
<td>A B</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-C_a</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-C_g</td>
</tr>
<tr>
<td></td>
<td></td>
<td>+C_m</td>
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<tr>
<td></td>
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<td>-C_q</td>
</tr>
<tr>
<td></td>
<td></td>
<td>+C_w</td>
</tr>
</tbody>
</table>

- Objective Function

- Acres

- Forage A, Period 1

- Forage A, Period 2

- Forage B, Period 1

- Forage B, Period 2

- Energy, Cow-Calf, Period 1

- Energy, Cow-Calf, Period 2

- Protein, Cow-Calf, Period 1

- Protein, Cow-Calf, Period 2

- Intake, Cow-calf, Period 1

- Intake, Cow-calf, Period 2

- Intake, Stockers, Period 1

- Intake, Stockers, Period 2

- Calf Transfer

- Stocker Transfer

- Forge Deviation, Yr. 1, Pd. 1

- Forge Deviation, Yr. 1, Pd. 2

- Forge Deviation, Yr. 2, Pd. 1

- Forge Deviation, Yr. 2, Pd. 2

- Return Deviation, Year 1

- Return Deviation, Year 2

- Risk

<table>
<thead>
<tr>
<th>Livestock Requirements</th>
<th>Cow-Calf Consumption</th>
<th>Stocker Consumption</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forage A</td>
<td>Forage B</td>
<td>Forage A</td>
</tr>
<tr>
<td>A B</td>
<td>Forage A</td>
<td>Forage A</td>
</tr>
<tr>
<td>1 2</td>
<td>1 2</td>
<td>1 2</td>
</tr>
</tbody>
</table>

2a) Forage A, Period 1 -P_a -T_a -T_a

b) Forage A, Period 2 -P_a -T_a -T_a
c) Forage B, Period 1 -P_b -T_b -T_b
d) Forage B, Period 2 -P_b -T_b -T_b

3a) Energy, Cow-Calf, Period 1 -M_a -M_a -E_a

b) Energy, Cow-Calf, Period 2 -M_a -M_a -E_a
c) Protein, Cow-Calf, Period 1 -R_a -R_a -E_a
d) Protein, Cow-Calf, Period 2 -R_a -R_a -E_a

4a) Energy, Stockers, Period 1 -M_a -M_a -E_a

b) Energy, Stockers, Period 2 -M_a -M_a -E_a
c) Protein, Stockers, Period 1 -R_a -R_a -E_a
d) Protein, Stockers, Period 2 -R_a -R_a -E_a

5a) Intake, Cow-calf, Period 1 1 1 -1

b) Intake, Cow-calf, Period 2 1 1 -1
c) Intake, Stockers, Period 1 1 1 -1
d) Intake, Stockers, Period 2 1 1 -1

6) Calf Transfer -1 1

7) Stocker Transfer -1 1

8a) Forge Deviation, Yr. 1, Pd. 1 -d_a -d_a 1960 -1960

b) Forge Deviation, Yr. 1, Pd. 2 -d_a -d_a 1960 -1960
c) Forge Deviation, Yr. 2, Pd. 1 -d_a -d_a 1960 -1960
d) Forge Deviation, Yr. 2, Pd. 2 -d_a -d_a 1960 -1960

9a) Return Deviation, Year 1 -f_a -f_a -f_a -f_a -f_a -f_a -f_a -f_a -f_a -f_a -f_a -f_a -f_a -f_a

b) Return Deviation, Year 2 -f_a -f_a -f_a -f_a -f_a -f_a -f_a -f_a -f_a -f_a -f_a -f_a -f_a -f_a

10) Risk .5 .5 .5 .5 .5 .5 .5 .5 .5 .5 .5 .5 .5 .5 .5

Note: The table includes abbreviated models for livestock activities and hay deviation, along with net return deviations. The table is used to represent objective functions, acres, forage requirements, energy balances, protein balances, intake balances, and deviation balances for different years and periods.
are expressed in terms of digestible energy (megacalories/day) and crude protein (pounds/day). Livestock activities are comprised of production activities that define the nutrient requirements of livestock in each subperiod and consumption activities that specify the intake and nutritional content of each forage. Energy, protein, and intake constraints were developed using National Research Council specifications and are included for each livestock activity. Based upon animal size and forage quality, maximum intake (pounds of dry matter) during each subperiod was estimated for each forage. For example, Tij values in Table 1 represent the maximum intake of forage i by the stocker activity in subperiod j. The megacalories of digestible energy (Mij) and pounds of crude protein (Rij) provided to the stocker by this level of consumption were then estimated.

Rows 2a through 2d are forage balance equations assuring that total consumption of each forage cannot exceed forage availability in that subperiod. Tij values represent per-head intake requirements, while Pij values are quantities of forage available. Rows 3 and 4 define the requirements that each animal’s subperiod energy (Eij and Eij in Table 1) and protein requirements (Eij and Eij) are met from the forage consumed in that subperiod. The intake restrictions (rows 5a through 5d) allow for any combination of feed sources to meet livestock nutrient requirements within each subperiod. These restrictions prevent the livestock unit from exceeding its consumption capacity during the subperiod to meet nutritional needs. Thus, for each livestock enterprise included in the solution, the model derives the most efficient feed ration in each subperiod, given the animal’s intake constraint. An advantage of this approach is that grazing plans and feed rations are derived within the optimization framework, rather than determined exogenously when developing livestock activities.

Clearly, the addition of the livestock-forage interaction and consumption relationships results in significant expansion in the size of the linear programming model. Sets of energy, protein, and intake constraints are required for each livestock enterprise included in the analysis. In addition, livestock consumption activities are required for each possible forage-livestock activity combination. Such detail is particularly necessary in this application since grazing of low-quality native grasses in the fall and winter months is an important production practice in the study region. Failure to represent intake considerations will result in infeasible solutions where livestock are required to consume more forage than they can physically ingest to meet protein and energy requirements.

Rows 6 and 7 are livestock transfer equations. Calves from cow-calf activities may be sold or retained and transferred to a stocker production activity. Stocker transfer rows permit stockers to be sold or transferred to another stocker activity in the subsequent subperiod.

Forage yield variability between years is represented as deviations from expected yields (dij and dij values in rows 8a through 8d). To account for differences in forage quality among the forage deviations, dry matter deviations were expressed in terms of megacalories of energy. These values were then converted into monetary terms by valuing the megacalories in deficit or surplus through purchase or sale of forage in the form of hay (rij values are the cost of hay in year i, subperiod j). Thus, production risk in low forage production years is measured by the quantity of hay required to meet forage deficits. Such an assumption is tenable in cow-calf production where producers must supplement cows to maintain their condition at a level conducive to reproductive performance. Results from a survey of stocker producers indicated that most managers supplement feed to reach some predetermined target weight gain (Walker et al. 1988).

Risk is measured by the mean absolute deviation from expected net returns due to variability in forage yields, livestock prices, and selected input costs. Seasonal deviations in net returns from hay sales and purchases are then transferred to row 9 where they are added to net return deviations resulting from variability in output and input prices. Total net return deviations are then transformed to mean absolute deviation (row 10) by weighting each by the probability of its occurrence.

Features of the actual model not represented in the abbreviated tableau include: (1) forage transfer activities, (2) hay production activities, (3) supplemental feeding activities, and (4) labor and capital accounting activities and constraints. The forage transfer activities allow for the possibility of carrying over ungrazed forage into subsequent subperiods. In addition, pasture acreage that remains ungrazed following selected subperiods may be harvested as hay. Hay and protein supplement feeding activities were included in the model in a manner similar to forage activities. Prairie hay, bermuda hay, and protein supplement (soybean meal) can be fed in any of the six subperiods to meet nutrient requirements not met by forages. Labor and capital activities estimate labor and interest costs and limit the availability of labor and operating capital.

The risk programming model was also specified as a Target-MOTAD formulation. The Target-MOTAD model offers the additional advantage that its solu-
tion set is contained in the set of production plans that are second degree stochastic efficient (Tauer). The mathematical representation of the Target-MO-TAD model is:

$$\text{Max } \sum c_j X_j$$

subject to

$$\sum a_{ij} X_j \leq b_i \quad (i = 1, \ldots, n)$$

$$T_k - \sum c_{kj} X_j - Y_k \leq 0 \quad (k = 1, \ldots, m)$$

$$\sum p_k Y_k = \gamma$$

where $T_k$ represents the target income level, $Y_k$ is the deviation below $T_k$ for state of nature $k$, and $\gamma$ is the expected deviation below the target income.

The objective function and first set of $n$ constraints are identical to the MOTAD model. Thus, rows 0 through 8 in Table 1 also apply to the Target-MO-TAD model. Differences exist in the valuation of annual income deviations (rows 9 through 10 in Table 1). The second set of $m$ constraints above define the deviations below target income ($T_k$) in each period. These deviations are then multiplied by the probability of the state of nature in which they occur to give the expected sum of deviations below the target income. The model is successively solved by varying $\gamma$ over some range of relevant values. When $\gamma$ is sufficiently large, the model is equivalent to deterministic linear programming formulation, and when $\gamma$ equals zero, no negative income deviations are permitted in any of the $m$ states of nature.

**DATA**

The model was used to derive risk-efficient ranch organizations for a representative ranch in the Cherokee Prairie Region of eastern Oklahoma. This area is characterized by a variety of soil types, ranging from deep loamy bottomland along streams and rivers to shallow eroded range sites in the mountainous areas. Average annual precipitation in the region is 42 inches. Tallgrass native range is the dominant pasture in the area, but the region also supports large acreages of improved pasture including fescue, bermudagrass, and lovegrass. Another significant feature of the region’s grazing resources is the large acreage of small grain pasture available for grazing through the fall and winter months. Cattle production in the area is more intensive than in other regions of Oklahoma and includes cow-calf enterprises as well as several alternative stocker enterprises.

The representative ranch was assumed to be comprised of 800 acres of cropland and 1,200 acres of pastureland. Any of the available pasture enterprises can be produced on the cropland, while the pastureland can support only improved or native pastures. The 2,000-acre ranch is representative of a relatively large commercial ranching operation in the study area.

**Livestock Activities**

A variety of cow-calf and stocker enterprises were included in the analysis. Because the selection of forages to meet nutrient requirements was endogenous to the model, livestock enterprises were not forage specific (e.g., cow-calf steers on native pasture). Instead, the livestock production activities dictated when cattle were purchased and sold, the performance assumptions (e.g., weight gain, weaning percent, etc.), and the associated nutrient requirements.

Both spring and fall calving cow-calf enterprises were included in the analysis. Spring calving was centered on April 1, and calves were weaned at an age of 210 days (November 1). Weaned steer calves were assumed to weigh an average of 450 pounds, and heifer calves average 435 pounds. Fall calving was centered on October 1, and calves might be weaned at ages of 210 days (May 1) or 285 days (July 15). Steer calves weaned at 210 days averaged 400 pounds, while heifer calves weighed 385 pounds. Average weaning weights for steers and heifers under the deferred weaning scheme were 565 and 540 pounds, respectively.

Cow-calf activities included in the analysis accounted for feed requirements, costs, and returns applicable to the cow-calf pair, as well as their share of replacement heifers and bulls. Management practices were developed to be representative of well-managed commercial cow herds in the study area. A calving rate of 90 percent and death loss of 2 percent was assumed, yielding a weaning percentage of 88 percent. The ranch was assumed to produce its own replacement heifers; thus 14 heifer calves were retained annually for every 100 cows. Therefore, given a herd of 100 cows, 44 steer calves and 30 heifer calves were available for sale annually. A cow-bull ratio of 25:1 was assumed, and 25 percent of all bulls were culled and replaced annually.

Stocker production activities were incorporated into the model to represent a situation where the producer possesses a high degree of flexibility throughout the year. Each stocker enterprise might have utilized either retained calves, purchased calves, or both. Stocker steers and heifers were included separately to account for differences in nutrient requirements, intake, gainability, and price. For each stocker enterprise, activities were included to
Figure 1. Flow Chart of Selected Production Alternatives for Spring Calves

accommodate different growth rates resulting from alternative forage quality and quantity conditions. These activities differed in terms of their subperiod intake and nutrient requirements, as well as the resulting weight gain, production costs, and gross receipts.

Several of the important stocker enterprises available for spring calves are summarized in Figure 1. Spring calves might have been (1) sold, (2) placed on available winter pasture, (3) "roughed" through the winter on a maintenance diet, or (4) placed on a low-energy winter diet that provided low rates of gain. In the latter two cases, compensatory growth was built up and reflected in improved weight gain during subsequent subperiods. At the conclusion of each two-month subperiod, several retention alternatives were evaluated—stockers might have been sold, retained on available pasture, or moved to an on-farm dry lot and fed a ration of hay and protein supplement. In addition to these options, stocker steers or heifers might have been purchased at the beginning of any of the six subperiods. Similar production and marketing alternatives were available for fall calves. By such a specification, numerous stocker enterprises were represented, differing in terms of the duration of the grazing season, types of forages consumed, and supplemental feeding practices.

Livestock intake and nutrient requirements were estimated based upon formulations reported by the National Research Council. Based upon animal metabolic weight and forage quality (expressed as net energy available for maintenance), an estimate of average daily dry matter intake was derived for each week of the livestock enterprises. Intake estimates were calculated for each feasible combination of feed source and livestock class. Average daily crude protein and digestible energy requirements were also calculated for each livestock enterprise based upon the animals' average weight, frame size, sex, and projected weight gain. Adjustments in nutrient requirements and feed intake were made for pregnant and lactating cows. Daily requirements and forage intake estimates were then aggregated to derive intake estimates and protein and energy requirements for each subperiod in the model.

Livestock prices employed in the analysis were monthly average prices for the 1980-88 period from the Oklahoma City livestock market (Agricultural Marketing Service). Monthly price series (indexed to 1989) were developed for commercial cows, bulls, and several weight classes of heifers and steers. Expected livestock prices used in estimating objective function coefficients represented averages of the monthly prices. Deviations from these expected livestock prices were computed for the same years as forage data to account for any interaction that may have existed between forage availability and beef prices.

Revenues from stocker enterprises were estimated as the product of the cattle price ($/cwt) and projected sale weight, adjusted downward to account for death loss. Gross receipts from the cow-calf enterprises included revenues from the sale of cull cows,
bulls, and calves. Costs of production for the various livestock enterprises were based upon enterprise budgets published by the Oklahoma Cooperative Extension Service and cost estimates reported in Walter et al. (1987) and Bernardo and McCollum. Livestock production costs other than feed and purchased calves were estimated in 1989 dollars and held constant through the analysis. Thus, annual livestock revenues and costs reflect variability in sales price, feed costs, and the price of purchased livestock.

Forage Activities

Forage activities included in the analysis were bermudagrass, weeping lovegrass, fescue, wheat, rye, and tallgrass native range. These pastures are the most popular alternatives available to eastern Oklahoma producers and provide opportunities for forage to be available over the entire year. Two alternative fertilization schemes were available for bermudagrass and fescue. Expected dry matter production estimates were developed for each of the six subperiods from eight years of research clipping data (1980-87) collected at Oklahoma Agricultural Research Stations at Stillwater and Haskell, Oklahoma (Howle et al., McMurphy et al.). Dry matter yields of improved pastures were adjusted downward by 50 percent to account for trampling, refusal, and other sources of disappearance, and grazing inefficiencies. To account for non-consumptive uses and represent good grazing management practices on native range, 25 percent of tallgrass range production was assumed available for allocation to livestock intake. Deviations from expected production values were estimated for each of the eight years of data and used as a measure of production risk derived from annual variation in forage yields (d_i values in Table 1).

Crude protein and metabolizable energy estimates were developed from monthly data collected in the study region (Waller; McMurphy and Hunter; Howle, et al.). Energy values were used to estimate the maximum intake by subperiod for each livestock enterprise included in the analysis (T_i and T_j values in Table 1). Subperiod energy and protein values were then used to determine the quantities of crude protein (M_i and Mo(_i,i)) and metabolizable energy (R_j and R_j) supplied by the ingested feed.

A common grazing management practice in the study area, particularly on native range, is to defer grazing for fall or winter consumption. Producers must weigh the tradeoff between grazing high-quality forage in the spring and early summer and grazing at lower quality levels in the fall and winter. With the exception of small-grain pasture, the possibility of deferred grazing was included for all forages in the model. Quantity as well as quality adjustments were made to represent the deterioration of forages harvested after maturity. Forage decay functions, representing the portion of forage production carried over into subsequent subperiods, were derived from disappearance data in Branson and in Sims and Singh. In the case of small-grain pasture, forage can be deferred for later consumption in the fall and spring growth periods, but cannot be deferred for summer consumption. Activities were also included to represent the possibility of harvesting excess forage as hay for sale or use in future subperiods. Hay production activities were included for bermudagrass, fescue, and native pasture. Wheat may be grazed from November through mid-March and harvested as grain or "grazed out" during the spring (mid-March through mid-May). In the latter option, no grain crop was harvested.

RESULTS

MOTAD Analysis

The MOTAD solutions to the problem are reported in Table 2. Alternative ranch organizations corresponding to five levels of risk (as measured by mean annual deviations) are reported. For each solution, mean annual deviations, expected net returns, the number of head of each livestock enterprise, pasture acreage, and supplemental feed requirements are reported. The feed sources utilized by each livestock enterprise are also identified. Expected income (return above operating costs) ranged from $177,285 when income deviations were ignored, to $111,275 when average annual income deviations were restricted to $30,000. The E-A frontier, traced out by parameterizing γ in Table 1, is presented in Figure 2.

Plan A (Figure 2) represents the profit maximizing organization of the representative ranch. The optimal solution in this case involved the production of two stocker enterprises on a combination of bermudagrass, fescue, rye, and native pasture. The two stocker enterprises in the plan, graze-out stockers and early-season summer stockers, were two of the higher-risk livestock enterprises included in the model. Winter pasture stockers were purchased in November and grazed through mid-May on small-grain pasture and fescue supplemented with bermuda hay. This enterprise differs from traditional wheat grazing systems where stockers are grazed continuously on wheat in that the wheat is limit grazed to more efficiently utilize the high quality forage. Historically, income from such an enterprise has been unstable due to unreliability of winter pasture production and wide fluctuations in cattle.
Table 2. E-A Efficient Ranch Organizations Derived from MOTAD Model, 2000 Acre Representative Eastern Oklahoma Ranch, 1989 dollars

<table>
<thead>
<tr>
<th>Ranch Plan</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
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<tbody>
<tr>
<td>Risk Measure ($)^a</td>
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<td>120,000</td>
<td>90,000</td>
<td>60,000</td>
<td>30,000</td>
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<td>Expected Net Returns ($)</td>
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<td>172,200</td>
<td>159,655</td>
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<td>Livestock Activities (head):</td>
<td></td>
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<tr>
<td>Early-Season Summer Steers</td>
<td>1,411</td>
<td>1,381</td>
<td>1,004</td>
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<tr>
<td>(d,e)</td>
<td>(d,e)</td>
<td>(d,e)</td>
<td></td>
<td></td>
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<tr>
<td>Graze-out Heifers</td>
<td>1,429</td>
<td>1,373</td>
<td>1,329</td>
<td>1,142</td>
<td>302</td>
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<td>(a,c,e)</td>
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<tr>
<td>Retained Late-Summer Steers</td>
<td>48</td>
<td>79</td>
<td>84</td>
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<tr>
<td>(d)</td>
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<td>Retained Late-Summer Heifers</td>
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<td>57</td>
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<td>(d)</td>
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<tr>
<td>Fall Cow-Calf, 285-Day Wean</td>
<td>88</td>
<td>357</td>
<td>661</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(a,c,d,e,g)</td>
<td>(a,c,d,e,g)</td>
<td>(a,c,d,e,g)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Pasture & Feed Activities:

- (a) Rye (Acres) 800 628 368 318 66
- (b) Grain Wheat (Acres) 0 172 432 482 734
- (c) Fescue (Acres) 79 95 276 419 392
- (d) Bermudagrass (Acres) 179 166 146 156 190
- (e) Native Range (Acres) 942 939 777 625 618
- (f) Bermuda Hay (Tons) 271 0 0 0 0
- (g) Protein Supplement (Tons) 0 0 14.1 23.1 35.7

^a Average annual income deviation.

^b Feeds utilized by each livestock enterprise are reported in parentheses and correspond to the letters noted in the pasture and feed section.

^c Description of stocker enterprises:
- Early-Season Summer Steers: stocker steers grazed during spring and early summer (Apr.-July)
- Graze-out Heifers: stocker heifers grazed on small-grain pasture during winter and early spring (Nov.-May)
- Retained Late-Summer Steers & Heifers: July 15 weaned calves retained on late-summer pasture (July-Sept.)

prices from November through May. The production of early-season summer stockers involved purchasing steers in April and grazing them primarily on native pasture until mid-July. This system (often referred to as intensive-early stocking) took advantage of high-quality native pasture produced during the first half of the summer grazing season. The majority of the bermudagrass acreage was harvested as hay to provide supplemental feed for the small grain pasture stocker enterprise.

As the degree of risk aversion increased, two significant changes in the livestock plan were observed. First, livestock numbers were reduced, decreasing pasture utilization and supplemental feed requirements. Second, more income-stable livestock enterprises were substituted for risky production alternatives. In general, optimal stocking rates derived by the model were higher than those normally employed in eastern Oklahoma. This result primarily reflects the model's ability to efficiently allocate forage, often resulting in nutrient requirement being provided by several forages.

Stocking rates derived in the profit maximizing plan were consistent with forage availability in years of average forage production. At average forage production levels, approximately 11,900 AUMs of forage were produced and available for utilization, while 11,860 AUMs were utilized. Supplemental feeding of off-farm produced forage was only necessary to offset large deficits in years of below-average production. As the level of risk aversion increased, total forage production decreased as did the percentage of the available forage utilized. In the most risk averse plan (plan E), approximately 8,400
Figure 2. E-A Efficient Set of Range Organizations for A Representative Eastern Oklahoma Ranch, MOTAD Model, 1989 dollars

AUMs of forage were available, while 7,200 AUMs were utilized. As a result of reallocating available acreage among the alternative forages, forage availability was reduced by 35 percent. Total forage utilization was decreased to about 85 percent of available AUMs to reduce income deviations resulting from supplemental feed purchases in forage-deficit years.

As the level of risk aversion increased, cow-calf production became a more important component of the ranch plan. Fall-calving with deferred weaning was continually substituted for winter and summer stockers in meeting the parametric reductions in deviations of net return. Although fall is not the dominant calving season in eastern Oklahoma, researchers have demonstrated its economic feasibility, particularly when deferred weaning is employed (Walker et al.). Expected net returns from the fall calving enterprise were lower than those expected from spring calving. However, the enterprise was also characterized by lower variability of revenues from the sale of calves. The availability of small-grain pasture to provide high quality forage when the cow herd’s nutritional needs were at their maximum also improved the feasibility of the fall-calving enterprise. All calves weaned from the cow herd were retained on bermudagrass through October 1.

The presence of cow-calf production in the optimal ranch organizations is consistent with livestock inventory data from the study region. However, previous studies evaluating the optimal organization of Oklahoma ranches based solely on a criterion of profit maximization found cow-calf production to be dominated by stocker enterprises (Rockeman and Walker). Studies conducted in other states dominated by cow-calf production have also noted an inconsistency between observed cow numbers and optimal farm plans derived from deterministic linear programming analyses (Wise and Saunders, Musser et al., Wise et al.).

An explanation often proposed for the absence of cow-calf enterprises from optimal farm plans is that producers may derive some form of non-monetary satisfaction from cow-calf production. The results of the present analysis offer an additional explanation—cow-calf enterprises may serve to stabilize annual ranch income and, thus, enter the optimal ranch organization only when risk averse behavior is represented. This finding is limited to the case where the pasture system is capable of producing a year-round forage supply. When pasture is limited exclusively to native range, cow-calf production fails to enter the optimal ranch plan, regardless of producer risk preference.

Regardless of the producer's degree of risk aversion, the optimal ranch plan included a diversified pasture system to assure a year-round forage supply. As the acceptable level of risk exposure was reduced, pasture diversification became more pronounced as more forages with less production variability were substituted into the plan. In moving from plan A to plan E, wheat was continually substituted for rye in allocating tillable acreage. Although
average annual forage production of rye exceeded that of wheat, its high forage-yield variability resulted in significant levels of production risk. In addition, returns earned from the production and sale of grain served to stabilize income. With the exception of native range, the deferment of pasture grazing was not used as a principal means of providing forage throughout the year. Large losses in forage quality were incurred if grazing in improved pastures was deferred for a substantial period of time. Therefore, improved pastures were primarily grazed during their growing seasons.

Trends in the allocation of non-tillable acreage illustrate an additional point. Bermudagrass was characterized by the highest level of variability and was replaced by native range and fescue as risk was initially reduced. However, the acreage of bermudagrass then increased as a larger number of calves were retained through the late-summer period. Higher quality bermudagrass was needed to meet the nutritional requirements of these calves in August and September. Thus, it appears that the variability of livestock production was the dominant source of risk, and forages were altered to meet the changing nutritional needs of the various livestock activities.

The frontier of the E-A efficient set facing the representative eastern Oklahoma livestock producer appears fairly steep, indicating that the model was unable to reduce risk without significantly impacting expected net returns (see Figure 2). This result contrasts with the findings of two earlier studies in which livestock producers were shown to have substantial risk-return tradeoff opportunities (Saez et al., Gebremeskel and Shumway). The fact that these analyses focused exclusively on cow-calf production, while the E-A efficient set derived here involved both cow-calf and stocker production, may explain these contrasting results. Differences between the study regions, particularly in the relationship between cattle prices and forage yields, may also contribute to the opposing findings.

**Target-MOTAD Analysis**

The Target-MOTAD results reported in Table 3 may be used to determine the influence of applying a safety-first risk criterion on efficient ranch organizations. Based upon the average debt load for ranchers in the study area, annual principal and interest payments were determined. This value was added to projected living expenses in estimating a target income level of $40,000. A set of ranch plans was derived which, for any given level of compliance with the target income, provided the maximum expected net return. Alternative solutions were derived from parameterizing \( \gamma \), the expected deviation below target income. For \( \gamma \) greater than $10,000, the Target-MOTAD model provided solutions identical to the deterministic linear programming model. In this case, the optimal solution was identical to solution A in Table 2.

In meeting parametric restrictions on \( \gamma \), the same general strategy described in the MOTAD results was employed. First, livestock numbers were reduced to decrease supplemental feed requirements and pasture utilization, and second, more income-stable enterprises were substituted into the ranch plans. However, several distinctions may be made between the specific strategies used to meet risk reductions under the two behavioral objectives.

Perhaps the most significant difference between the two sets of results was that spring calving replaced fall calving with deferred weaning as the dominant calving scheme. Although the spring-calving enterprise was characterized by higher variability in total receipts than was fall calving, fewer negative deviations occurred. Steer calves retained from the spring calving herd were grazed on small-grain pasture, while heifers were fed a maintenance diet through the winter and placed on summer pasture on May 1. Fall-calving also entered the solutions in the most risk averse plans. Given the diversified forage system available to the producer, it is conceivable that two cow-calf enterprises could be employed due to alternate seasonal demand for high-quality forage, spring- and fall-calving herds can complement each other by utilizing the forage base more efficiently. Fall calves were retained and utilized the same forages as in the MOTAD solutions.

As in the MOTAD solution, both winter and summer stocker enterprises were employed in the optimal ranch plans. Winter pasture stockers were grazed on small-grain pasture and fescue from November through May and supplemented with bermuda hay. A season-long summer stocker enterprise was employed in lieu of the intensive early-season stocker enterprise appearing in the MOTAD solutions. The early-season stocker enterprise was penalized significantly by the presence of two extremely low net return observations. Considerably higher stocking levels were maintained in both stocker enterprises than in the MOTAD solutions, resulting in less sensitivity of expected net returns to reductions in risk.

In contrast to the MOTAD results, small-grain acreage was not harvested for grain as a means of stabilizing income. In each of the solutions, 800 acres of rye were produced and “grazed-out” by stocker cattle and cows. The distribution of rye for-
Table 3. E-A Efficient Ranch Organizations Derived from Target-MOTAD Model, 2000 Acre Representative Eastern Oklahoma Ranch, 1989 dollars

<table>
<thead>
<tr>
<th>Ranch Plan</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk Measure ($)&lt;sup&gt;a&lt;/sup&gt;</td>
<td>23,430</td>
<td>12,000</td>
<td>9,000</td>
<td>6,000</td>
<td>3,000</td>
</tr>
<tr>
<td>Expected Net Returns ($)</td>
<td>177,285</td>
<td>166,725</td>
<td>162,189</td>
<td>157,562</td>
<td>152,722</td>
</tr>
</tbody>
</table>

Livestock Activities (head)<sup>b,c</sup>

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fall-Season Summer Steers</td>
<td>1,411</td>
<td>155</td>
<td>(c,d)</td>
<td>(c,d)</td>
<td>(c,d)</td>
</tr>
<tr>
<td>Graze-out Heifers</td>
<td>1,429</td>
<td>1,931</td>
<td>1,723</td>
<td>1,516</td>
<td>1,312</td>
</tr>
<tr>
<td>Retained Late-Summer Steers</td>
<td>17</td>
<td>67</td>
<td>(c)</td>
<td>(c)</td>
<td>(c)</td>
</tr>
<tr>
<td>Retained Late-Summer Heifers</td>
<td>13</td>
<td>48</td>
<td>(c)</td>
<td>(c)</td>
<td>(c)</td>
</tr>
<tr>
<td>Retained Fall-Pasture Steers</td>
<td>24</td>
<td>71</td>
<td>104</td>
<td>115</td>
<td>115</td>
</tr>
<tr>
<td>Retained Winter-Roughed Heifers</td>
<td>18</td>
<td>51</td>
<td>75</td>
<td>83</td>
<td>83</td>
</tr>
<tr>
<td>Spring Cow-Calf, 210-Day Wean</td>
<td>56</td>
<td>161</td>
<td>237</td>
<td>261</td>
<td>261</td>
</tr>
<tr>
<td>Fall Cow-Calf, 285-Day Wean</td>
<td>40</td>
<td>153</td>
<td>(a,b,c,d,f)</td>
<td>153</td>
<td>(a,b,c,d,f)</td>
</tr>
</tbody>
</table>

Pasture & Feed Activities:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Rye (Acres)</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>800</td>
</tr>
<tr>
<td>(b) Fescue (Acres)</td>
<td>79</td>
<td>199</td>
<td>174</td>
<td>178</td>
<td>229</td>
</tr>
<tr>
<td>(c) Bermudagrass (Acres)</td>
<td>179</td>
<td>186</td>
<td>235</td>
<td>279</td>
<td>312</td>
</tr>
<tr>
<td>(d) Native Range (Acres)</td>
<td>942</td>
<td>815</td>
<td>789</td>
<td>743</td>
<td>658</td>
</tr>
<tr>
<td>(e) Bermuda Hay (Tons)</td>
<td>271</td>
<td>0</td>
<td>12.8</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(f) Protein Supplement (Tons)</td>
<td>0</td>
<td>6.1</td>
<td>16.0</td>
<td>23.1</td>
<td>35.7</td>
</tr>
</tbody>
</table>

<sup>a</sup> Expected annual deviation below target income of $30,000.

<sup>b</sup> Feeds utilized by each livestock enterprise are reported in parentheses and correspond to the letters noted in the pasture and feed section.

<sup>c</sup> Description of stocker enterprises:

- Fall-Season Summer Steers: stocker steers grazed during the spring and summer (Apr. - July)
- Graze-out Heifers: stocker heifers grazed on small-grain pasture during winter and early spring (Nov. - May)
- Retained Late-Summer Steers & Heifers: July 15 weaned calves retained on late-summer pasture (July-Sept.)
- Retained Fall-Pasture Steers: November 1 weaned calves retained on winter pasture
- Retained Winter-Roughed Heifers: November 1 weaned calves "roughed" through the winter and placed on summer pasture.

Production was skewed toward high levels of production and did not contain any extremely low observations. Thus, expected returns could be increased and the safety criterion satisfied by spring grazing rather than harvesting the crop for grain. Trends in acreages of improved and native pasture were similar to those observed in the MOTAD solutions. The acreage of native range was reduced monotonically in meeting reductions in risk levels, while bermudagrass and fescue acreage was increased.

The Target-MOTAD results indicated the presence of considerably more risk-return tradeoff opportunities than were identified in the MOTAD analysis. Significant reductions in expected annual deviations below the target income were obtained with relatively small decreases in expected net returns. One explanation for this contrasting result concerns the measurement of production risk resulting from variability in the quantity of forage produced annually. In the MOTAD formulation, deviations both above and below average production levels were translated into income deviations and included in the risk.
measurement. Only forage production deviations below average levels were included in the risk measure in the Target-MOTAD formulation. Such a specification seems more appropriate since excess forage is typically not harvested, and thus, has little bearing on annual income. Also, supplementation costs resulting from moderate reductions in forage production below average levels could usually be met without falling below the target income. Therefore, significant changes in the ranch plan were not required to meet risk constraints.

SUMMARY AND CONCLUSIONS

The purpose of this analysis was to evaluate the risk-return tradeoffs for various beef-forage systems available to Oklahoma livestock producers. Sets of risk-efficient ranch organizations were derived for a representative eastern Oklahoma ranch using MOTAD and Target-MOTAD programming models. Results of the analysis indicated that efficient ranch plans were relatively sensitive both to the specification of the risk criteria and to the producer’s willingness to accept risk. Thus, ignoring risk when identifying efficient cattle-pasture production systems may lead to erroneous normative prescriptions. Solutions from the MOTAD and Target-MOTAD analyses provided contrasting results concerning the ability of livestock producers to reduce risk without significantly affecting expected net returns.

Although these results were derived for a representative eastern Oklahoma ranch, some general implications for managing risk in Oklahoma cattle operations can be drawn from the study. First, a diversified forage system is requisite to efficient livestock management regardless of the producer’s risk preference. Second, cow-calf production provides a means of stabilizing annual income when forage availability is not limited to specific periods of the year. Third, fall-calving may represent a viable alternative for producers having a diversified forage system, including small-grain winter pasture. Finally, retained ownership of weaned calves provides producers opportunity for additional income without significantly increasing risk.

Results from the study also illustrate the adaptability of the risk programming formulation to representations of the complexities inherent in livestock-forage interactions. An adaptation of the basic risk programming specification was employed to represent both nutrient and intake considerations when allocating available forage among cattle enterprises. Procedures were also employed to incorporate variations in the timing and magnitude of forage production as sources of production risk facing livestock producers. The adaptations of the basic risk programming model provide improved opportunities for the application of MOTAD and Target-MOTAD formulations to the analysis of pasture-livestock production systems.
REFERENCES


