

Automobile-related environmental policies in a spatial model: lessons from theory

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Abstract. This paper extends the environmental and urban economics literature by linking urban smog to the level of aggregate travel within a city, thereby incorporating the inherently spatial aspect of urban air pollution into environmental policy analysis. The results indicate that increases in marginal transportation costs are not necessarily welfare-reducing in the presence of travel-related pollution. In general, whether a given policy improves welfare depends on the relative magnitudes of the marginal benefits and costs that depend on the specific structure of the policy. The analysis shows that environmental policies aimed at decreasing urban smog differ considerably in the number and complexity of their effects on welfare, providing important information for the choice of instruments. It seems reasonable that, all else equal, preferred policies are those with fewer, simpler, and therefore more tractable effects.

1. Introduction

Wheaton (1974) models a partially closed city (absentee landlords) and provides the first general comparative static analysis of the Alonso model of urban spatial structure. Pines and Sadka (1986) extend Wheaton's analysis to a fully closed city in which urban land rent is redistributed to city residents. An interesting finding in both of these studies is that the equilibrium level of utility in the city is strictly decreasing with respect to increases in marginal transportation cost, implying that the optimal level of a tax on motor fuel is zero. The reason for this result is that these particular models ignore any potential benefits associated with higher transportation costs, such as those that might arise due to lower levels of travel-related pollution.

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This paper extends the environmental and urban economics literature by linking urban smog to the level of aggregate travel within a city, thereby incorporating the inherently spatial aspect of urban air pollution into environmental policy analysis. The results indicate that increases in marginal transportation costs are not necessarily welfare-reducing in the presence of travel-related pollution. In general, whether a given policy improves welfare depends on the relative magnitudes of the marginal benefits and costs that depend on the specific structure of the policy. The analysis shows that environmental policies aimed at decreasing urban smog differ considerably in the number and complexity of their effects on welfare, providing important information for the choice of instruments. It seems reasonable that, all else equal, preferred policies are those with fewer, simpler, and therefore more tractable effects.

Although transportation accounts for a significant portion of urban air pollution in many parts of the world, this issue largely has been ignored in the economics literature dealing with urban spatial structure. Existing studies of pollution in a spatial setting focus either on industrial air pollution (Henderson 1985, chap. 3; Papageorgiou 1990, chap. 17) or on air quality as a location-dependent amenity factoring into the location decisions of households (Diamond and Tolley 1982). In both cases, the amount of pollution is unrelated to aggregate miles of travel.

On the other hand, nonpollution externalities such as traffic congestion have received extensive analysis in the literature. Two issues commonly examined in relation to congestion are optimal road pricing and optimal land use for transportation. The literature on these issues is large, recent summaries include, for example, Kanemoto (1980), Henderson (1985), and Fujita (1989). Other types of externalities that have been analyzed in a spatial framework include racial prejudice, other neighborhood externalities, and the optimal provision and location of public goods (Henderson 1985; Fujita 1989).

The present analysis contributes both to the urban economics literature by linking urban air pollution (and welfare) to aggregate miles of travel and to the environmental economics literature by extending the analysis of certain policy instruments to include the inherently spatial aspect of travel-related air pollution.

2. A spatial model of urban air pollution

The model is based on the following assumptions:

- (a) The city is monocentric—all job opportunities are located in the central business district (CBD);
- (b) All households have identical tastes and incomes;
- (c) The only type of travel is the commuting of workers between residences and the CBD using private automobiles;

- (d) The transportation system is congestion-free, radial, and dense in every direction; and
- (e) The city is located on a featureless plain consisting of identical parcels of land that are ready for residential use without further improvement.

Thus, the only spatial characteristic that concerns each household is its distance from the CBD.

Residents form a government that acquires land from rural landlords at a fixed price, \bar{R} , that reflects the marginal productivity of land in its agricultural use. The government rents the land to city residents at a market-determined price $R(t)$, where t equals distance from the CBD. It is assumed that $R(t) > \bar{R}$ for all t within the city. Thus, the total profit Π from land transactions equals

$$\Pi = \int_0^b L(t)[R(t) - \bar{R}] dt,$$

where:

b = The outer boundary of the city; and

$L(t)$ = The amount of land available for residential use at location t .

Each resident receives an equal share of Π and, therefore, the income Y of a representative household is given by

$$Y = I + \frac{1}{N} \int_0^b L(t)[R(t) - \bar{R}] dt = I + \frac{\Pi}{N}, \quad (1)$$

where:

I = Nonrent (fixed) income; and

N = The population of the city.

Note that income is endogenous in a fully closed city.¹

The level of air pollution, or smog, in the city (S) is a function of the total number of miles of commuting (M):²

$$S = S(M), \quad (2)$$

where:

dS/dM $S_M > 0$.

Aggregate miles of travel are given by

$$M = \int_0^b t n(t) dt, \quad (3)$$

where:

¹ Also in a fully closed city, population is fixed and utility is endogenous. By contrast, in an open city, an endogenous population assures that its utility level is fixed at the level of the rest of the country.

² The specification in equation (2) is simplified. The relationship between urban smog and automotive emissions is less direct—automobiles emit several types of pollutants that combine with each other and with sunlight to produce smog. Furthermore, the amount of smog in an area is also a function of the geography and weather patterns specific to that area.

$n(t)$ = The density profile of the city which gives the number of persons living at location t for all t .

The level of smog that results from automobile emissions is dispersed uniformly across the city and is a pure externality, implying that the amount of pollution does not affect the optimizing behavior of the household. For simplicity, sources of urban smog other than automobiles are ignored.

In general, there are three approaches to modeling household behavior in a spatial framework. Wheaton (1974) relies on the concept of the bid-rent function, while Pines and Sadka (1986) use the more familiar expenditure function. A third approach would be to use the indirect utility function. All three methods yield the same comparative static results because the bid-rent, the expenditure, and the indirect utility functions are inverses of each other.³ The present analysis builds on the approach taken by Pines and Sadka using the expenditure function.

Households gain positive utility through the consumption of land (q) and other goods (z), and experience negative utility from smog. The expenditure function of the household gives the minimum amount that the household can spend on q and z while locating at distance t and enjoying a fixed level of utility (u). Formally, the expenditure function is defined as

$$E(R(t), u, S(M)) = \min_{\{q, z\}} \{z + R(t)q | U(q, z, S(M)) = u\}$$

The first-order conditions of the above minimization problem lead to the following equilibrium conditions of the household:

$$\frac{U_q}{U_z} = R(t),$$

and

$$u = U(q, z, S(M)),$$

where subscripts are used to denote partial derivatives. These conditions can be solved for the equilibrium quantities of land and other goods, \tilde{q} and \tilde{z} , respectively, which then are used to establish the expenditure function:

$$\tilde{q} = q(R(t), u, S(M)),$$

$$\tilde{z} = z(R(t), u, S(M)),$$

$$E(R(t), u, S(M)) = \tilde{z} + R(t)\tilde{q}.$$

The disposable income of the household equals $Y - K(t)$, where $K(t)$ is the transportation cost incurred at distance t and Y is given by equation (1). For simplicity, assume for the time being that $K(t) = kt$, where k is interpreted as the constant marginal cost of transportation. At all locations, the household's disposable income must suffice for its expenditures:

³ A description of each approach and a brief discussion of their relationships are provided in the appendix.

$$E(R(t), u, S(M)) = Y - kt. \tag{4}$$

In addition, equating the demand for and the supply of land at each location requires

$$n(t)\tilde{q}(R(t), u, S(M)) = L(t), \tag{5}$$

and accommodating the city's population within its boundary requires

$$\int_0^b n(t) dt = N. \tag{6}$$

Finally, competing uses for land at the boundary implies

$$R(b) = \bar{R}. \tag{7}$$

The equilibrium system comprises equations (1) through (7).⁴ This system has seven unknowns ($Y, b, u, n(t), R(t), M,$ and S) and five exogenous parameters ($I, k, \bar{R}, L(t),$ and N).

2.1 Changes in marginal transportation cost

The impact of a change in marginal transportation cost on the common level of utility in the city can be determined as follows. Substituting equation (1) into equation (4), differentiating with respect to k , and using equation (7) yields

$$E_R R_k + E_u u_k + E_S S_M M_k = \frac{1}{N} \int_0^b LR_k dt - t, \tag{8}$$

where the arguments of the functions are suppressed when no ambiguity arises. Using the envelope theorem, we have

$$\begin{aligned} E_R &= q, \\ E_u &= \frac{1}{U_z}, \\ E_S &= -\frac{U_S}{U_z}. \end{aligned}$$

Substituting for $E_R, E_u,$ and E_S in equation (8), multiplying by $n(t)$, and integrating over t yields

$$\int_0^b nqR_k dt + u_k \int_0^b n \frac{1}{U_z} dt - \int_0^b n \frac{U_S}{U_z} S_M M_k dt = \int_0^b n \left[\frac{1}{N} \int_0^b LR_k dt \right] dt - \int_0^b tndt. \tag{9}$$

⁴ Demand equals supply in the market for z by Walras' law. Formally, equilibrium in this market is represented by

$$\int_0^b \left\{ n(t) [\tilde{z}(R(t), \hat{u}, S(M)) + kt] + L(t)\bar{R} \right\} dt = NI$$

Using equation (5), the first terms on both sides of equation (9) are identical. Thus,

$$u_k = \frac{\int_0^b n \frac{U_S}{U_z} S_M M_k dt - \int_0^b t n dt}{\int_0^b n \frac{1}{U_z} dt} \quad (10)$$

Assuming that $U_S < 0$, $U_z > 0$, $S_M > 0$, and $M_k < 0$, the sign of u_k is indeterminate.⁵

The expression in equation (10) can be interpreted as follows: When the marginal cost of travel increases, for example, the first term represents the positive impact on welfare of a reduction in pollution due to a fall in aggregate travel. The second term represents an increase in transportation expenditures and affects welfare negatively. Thus, with pollution, the sign of u_k is indeterminate due to the existence of both positive and negative welfare effects. Notice that the expression in equation (10) contains the unambiguous result of Pines and Sadka as a special case. In the absence of pollution, S_M equals zero and the first term is deleted, leaving an expression identical to the one obtained by Pines and Sadka (which is strictly negative as only the welfare cost of increasing k remains). Further, note that u_k will be positive whenever

$$\int_0^b n(t) \frac{U_S}{U_z} S_M M_k dt > \int_0^b t n(t) dt .$$

In other words, increasing the marginal cost of travel will improve welfare when the marginal benefit of reduced pollution exceeds the marginal cost of increased travel expenditures.

3. Automobile-related environmental policies in a spatial context

The model developed above is useful for exploring the welfare effects of several popular policies designed to combat urban smog:

- (a) A tax on fuel;
- (b) Alternative fuel mandates;
- (c) Emissions standards and testing programs; and
- (d) Fuel economy standards.

The model is particularly useful for examining these policies, as they all involve changes in the marginal and/or fixed costs of transportation.⁶

⁵ In their model, Pines and Sadka show M_k to be negative. Their proof is complicated and will not be repeated here. Because pollution in the present model is a pure externality, however, it does not affect the location choice of the household so that M_k can be expected to be negative with pollution as well.

Government policies aimed at reducing urban smog largely target the flow of pollutant emissions from the use of automobiles (rather than focus on smog—the accumulation of emission flows—directly). It will be useful for subsequent policy analyses to consider the various components of total automotive emissions using the following identity:

$$\text{Total Emissions} = (\text{Number of Vehicles}) \times \left(\frac{\text{Miles}}{\text{Vehicle}} \right) \times \left(\frac{\text{Gallons}}{\text{Mile}} \right) \times \left(\frac{\text{Emissions}}{\text{Gallon}} \right) \quad (11)$$

Although any automobile-related environmental policy likely will have some degree of influence on each of the four terms on the right side of equation (11), the potential spatial aspects of policy arise primarily due to induced changes in the miles/vehicle term. And, to the extent that it is difficult to get American commuters out of their cars, changes in miles/vehicle are realized though changes in commuting distances.

3.1 A tax on motor fuel

For a given level of fuel efficiency, a tax on fuel directly increases the marginal cost of travel. In terms of the identity in equation (11), an increase in the marginal cost of travel leads directly to a decrease in the miles/vehicle term. Although the effects of a change in marginal transportation cost were derived above, the impact of an increase in k on welfare as given by equation (10) is only correct for a situation in which all revenues from the tax mysteriously disappear. The result is slightly different when tax revenues are considered within the model. For example, consider a tax on fuel that raises everyone's marginal travel cost by a constant amount τ per unit of distance traveled.⁷ The revenue from this tax is redistributed in equal shares to the city's N inhabitants. Thus, household income in equation (1) becomes

$$Y = I + \frac{1}{N_0} \int_0^b L(t) [R(t) - \bar{R}] dt + \frac{1}{N_0} \int_0^b \tau t n(t) dt = I + \frac{1}{N} (\Pi + \tau M), \quad (1')$$

and the expenditure balance equation in (4) is written as

$$E(R(t), u, S(M)) = Y - kt - \tau t. \quad (4')$$

Substituting equation (1') and equation (4') for equation (1) and equation (4), respectively, and solving as before (with respect to τ rather than k), the welfare effects of changes in the fuel tax are given by

⁶ The present spatial treatment of these environmental policies does not consider consumer choices such as travel mode and trip frequency and timing. It is not that these issues are not important, but that including them in a spatial model is beyond the scope of the present paper.

⁷ An identical increase in marginal costs for all households implies that there are no differences in transportation technologies—i.e., every automobile operates with the same miles-per-gallon efficiency.

$$u_{\tau} = \frac{\int_0^b n \frac{U_S}{U_Z} S_M M_{\tau} dt + \tau M_{\tau}}{\int_0^b n \frac{1}{U_Z} dt} . \quad (12)$$

As in the previous case of u_k in equation (10), the sign of u_{τ} is indeterminate. Assuming $M_{\tau} < 0$, the first term in equation (12) is positive and represents the marginal benefit of reduced pollution, while the second term is negative and equals the marginal reduction in tax revenue; both effects result from reduced travel. In the neighborhood of $\tau = 0$, a small increase in τ improves welfare because the benefits exceed the losses—the first term in equation (12) dominates the second. At a sufficiently high level of τ , however, the losses outweigh the benefits and welfare falls with further increases in τ . This implies that at some positive level of τ , holding other factors constant, utility will be at a maximum. Accordingly, the expression in equation (12) can be solved for the optimal tax rate (τ^*) on motor fuel. Welfare maximization requires u_{τ} to equal zero, implying that the optimal tax rate equals

$$\tau^* = -\int_0^b n \frac{U_S}{U_Z} S_M dt . \quad (13)$$

Assuming that $U_S < 0$ and $U_Z > 0$, τ^* is positive in the presence of pollution ($S_M > 0$). The tax rate formula in equation (13) is the familiar Pigouvian result that the efficient tax on an externality-producing activity is one that equals the marginal external damage (in the optimum) caused by that activity. As would be expected, in the absence of pollution, S_M equals zero and, therefore, the optimal tax on fuel is zero, as implied by Wheaton (1974) and Pines and Sadka (1986).

3.2 Alternative fuel mandates

Consider an increase in marginal transportation cost (k) that arises due to a switch to a cleaner-burning, more expensive fuel. In this case, there is a direct impact of k on S —an increase in k will reduce the amount of smog per mile of travel in addition to reducing aggregate travel.⁸ In terms of the identity in equation (11), an alternative fuel policy that leads to an increase in k will induce a decrease in miles/vehicle as was the case with a fuel tax. But the increase in k in this case is used to finance a decrease in the emission/gallon term. Formally, the smog function in equation (2) is now written as

$$S = S(M, k), \quad (2')$$

⁸ One way to think about the direct effect of k on S in this case is that, as k increases, it causes a downward movement of the plot of S as a function of M . For example, in Figure 1 an increase in k shifts the smog function from S^0 to S^1 .

where $S_k < 0$, reflecting the assumption that an increase in marginal cost results in reduced smog due to the use of a cleaner-burning fuel.

The impact on welfare of mandating a cleaner, more expensive fuel is determined by using equation (2') and proceeding as before to obtain

$$u_k = \frac{\int_0^b n \frac{U_S}{U_Z} (S_M M_k + S_k) dt - \int_0^b t n dt}{\int_0^b n \frac{1}{U_Z} dt}. \quad (14)$$

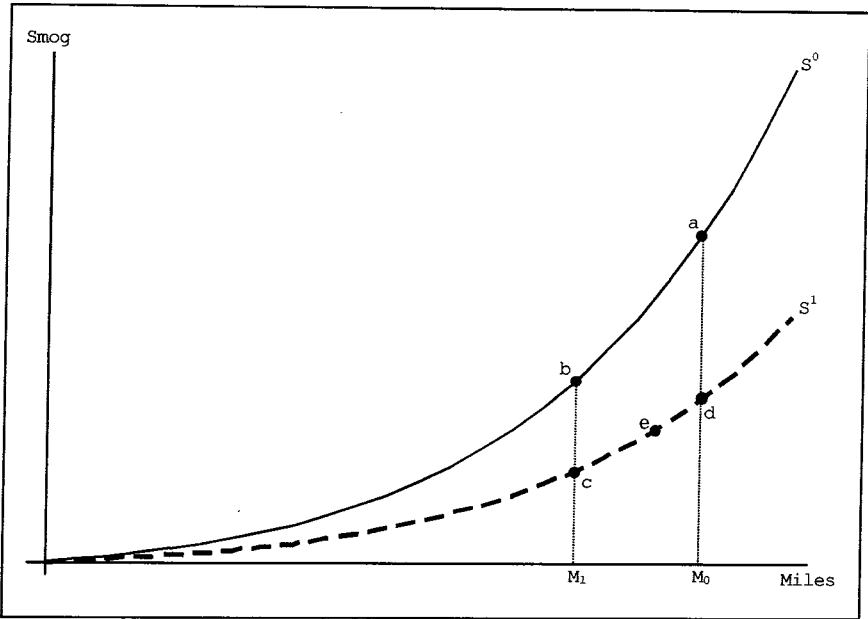
This expression is similar to equation (10) with an additional term involving S_k —the direct effect of k on S . Again, the sign of u_k is ambiguous and depends on the relative sizes of the marginal benefits and costs. Formally, equation (14) implies that utility increases whenever

$$\int_0^b n \frac{U_S}{U_Z} S_M M_k dt + \int_0^b n \frac{U_S}{U_Z} S_k dt > \int_0^b t n dt. \quad (15)$$

In other words, a switch to a cleaner, more expensive fuel improves welfare when the combined marginal benefits of reduced pollution from reduced aggregate travel and lower emissions per mile—the two terms on the left side of equation (15), respectively—exceed the marginal increase in travel expenditures.

The effects of fuel taxes and alternative fuels on the level of smog can be illustrated graphically. In Figure 1, a representative smog function is presented that gives the level of smog as an exponentially increasing function of the level of aggregate travel. A fuel tax that decreases miles of travel from, say, M_0 to M_1 , will reduce the level of smog as shown by the movement from point a to point b (because $S_M M_t < 0$). Now consider a cleaner, more expensive fuel for which the increase in marginal transportation cost equals that of the fuel tax. In this case, the increase in marginal cost is accompanied by a downward shift of the smog function from S^0 to S^1 , reflecting the fact that, for any given level of travel, a cleaner fuel reduces the amount of smog per mile ($S_k < 0$). Now, in addition to the movement from point a to point b (because $S_M M_k < 0$), there is also a movement from point b to point c. Thus, for an equivalent increase in marginal cost, alternative fuels result in a greater reduction in smog than does a fuel tax.

Figure 1. Representative smog function^a



3.3 Emissions standards/testing programs

Unlike fuel taxes and alternative fuels, both of which lead to increases in the marginal costs of transportation, emissions standards and/or testing programs lead to increases in the fixed costs of transportation.⁹ Thus, relative to alternative fuels or a fuel tax, emissions standards/testing will have a minimal impact on the miles/vehicle term in equation (11). Similar to alternative fuels, however, the increased costs of emissions standards/testing are used to purchase lower emissions per mile (which equals emissions/gallon times gallons per mile). To determine the welfare effects of emissions standards/testing in the present model, now assume that $K(t) = F + kt$, where F represents the fixed costs of transportation (which also could include parking fees, licensing and registration, insurance, etc.). In this case, the smog function can be written as

$$S = S(M, F), \tag{2''}$$

^a The smog function in this diagram is given by $S = Mk^\alpha c^{\beta M}$, with $\alpha < 0$ and $0 < \beta < 1$.

⁹ Standards generally require emissions-reducing equipment, which increases the cost of new automobiles, and testing, which imposes time and money costs on drivers irrespective of the total miles driven.

where $S_F < 0$, reflecting the assumption that an increase in fixed cost results in reduced smog. And the expenditure balance equation in equation (4) becomes

$$E(R(t), u, S(M)) = Y - F - kt. \quad (4'')$$

Substituting equation (2'') for equation (2) and equation (4'') for equation (4) and solving as before with respect to changes in F yields

$$u_F = \frac{\int_0^b n \frac{U_S}{U_Z} (S_M M_F + S_F) dt - \int_0^b n dt}{\int_0^b n \frac{1}{U_Z} dt}, \quad (16)$$

where, because an increase in F is a reduction in income I ,

$$M_F = M_I I_F = -M_I,$$

because I_F equals -1 . Given that $M_I > 0$ —that miles of travel increase with increases in income (Pines and Sadka 1986)—it follows that $M_F < 0$. This implies that emissions standards/testing expenditures will improve welfare whenever

$$\int_0^b n \frac{U_S}{U_Z} S_M M_F dt + \int_0^b n \frac{U_S}{U_Z} S_F dt > \int_0^b n dt. \quad (17)$$

The first term on the left side of equation (17) is positive and represents the marginal benefit of reduced smog through an income-induced reduction in aggregate travel. The second term on the left side of equation (17) is also a marginal benefit of reduced smog because $S_F < 0$ as an increase in F is used to purchase lower emissions.¹⁰ The right side of equation (17) is the marginal reduction in aggregate income resulting from an increase in each commuter's fixed costs of transportation.

Returning to Figure 1, the effect of emissions standards on the level of smog is illustrated, for example, as a move from point a to point d due to lower emissions per mile ($S_F < 0$), and also a move somewhat to the left of point d on S^1 due to the fall in income ($S_M M_F < 0$), say, to point e .

3.4 Fuel economy standards

Automotive fuel economy standards are essentially an investment in efficiency—they increase the costs of producing automobiles in return for greater fuel economy.¹¹ Thus, the effects of fuel economy standards can be captured in

¹⁰ This is the case with standards. With emissions testing, however, it is possible that S_F might be close to zero as most new vehicles pass the tests and, in many locations, older vehicles are exempt. Thus, for identical costs, emissions standards reduce smog more than emissions testing.

¹¹ Given the current structure of the fuel economy program in the U.S., it is not certain that the standards result in real technological change that increases the fuel efficiency of individual automobiles. Constrained firms have a number of possible strategies for meeting the standards and

the present model as an increase in the fixed costs of transportation resulting in, among other things, a decrease in the marginal cost of transportation. In terms of the identity in equation (11), fuel economy standards lead to a decrease in gallons/mile, but an increase in miles/vehicle and possibly to a decrease in emissions/gallon. Thus, by increasing miles/vehicle, fuel economy standards might possibly contribute to urban sprawl. The equations of the model are identical to those used in the analysis of emissions standards—equations (1), (2''), (3), (4''), (5), (6), and (7)—except that now it is assumed that, in addition to affecting smog directly, F also has an impact on k . Specifically, $k_F < 0$ whereas in the case of emissions standards, $k_F = 0$. The comparative static result for changes in F in the case of fuel economy standards equals

$$u_F = \frac{\int_0^b n \frac{U_S}{U_Z} [S_M(M_F + M_k k_F) + S_F] dt - k_F \int_0^b n dt - \int_0^b n dt}{\int_0^b n \frac{1}{U_Z} dt} \quad (18)$$

This result implies that increases in F , when used to purchase greater fuel economy, will improve welfare whenever

$$\int_0^b n \frac{U_S}{U_Z} S_M M_F dt + \int_0^b n \frac{U_S}{U_Z} S_F dt - k_F \int_0^b n dt > \int_0^b n dt - \int_0^b n \frac{U_S}{U_Z} S_M M_k k_F dt \quad (19)$$

While this inequality may at first appear somewhat unwieldy, its interpretation is fairly straightforward. The three terms on the left side of the inequality represent the marginal benefits of imposing fuel economy standards. The first term is the marginal value of the reduction in smog resulting from less travel because fixed costs reduce income. The second term equals the marginal value of the reduction in smog due to the fact that an increase in fuel efficiency reduces the amount of fuel burned per mile of travel that, in turn, will most likely reduce the amount of smog produced per mile.¹² The third term is the savings in travel expenditures due to the increase in fuel efficiency.

The two terms on the right side of (19) are the marginal costs associated with imposing fuel economy standards: The first represents the reduction in

it might be the case that the current program mainly increases *average* fuel efficiency by inducing changes in the composition of vehicle fleets. See, for example, Thorpe (1997) for a brief discussion of the strategies facing constrained manufacturers. The present analysis is only concerned with actual technological design changes that reduce emissions and increase the costs of producing an automobile.

¹² There is some uncertainty of whether increased fuel efficiency reduces per-mile automotive emissions. For example, as Khazzoom, Shelby, and Wolcott (1990) point out, the U.S. Environmental Protection Agency (EPA) specifies its maximum allowable emissions standards on a per mile basis, rather than per gallon of fuel burned. Because manufacturers have little incentive to reduce emissions below the required standards, vehicle emissions per mile may be constant. In this case, S_F would equal zero.

aggregate income used to finance improved fuel efficiency. The second term is the marginal value of the increase in smog resulting from increased travel attributable to lower marginal costs of travel. This is the so-called rebound effect (Greene 1992). Thus, fuel economy standards affect welfare in several ways. Separating and quantifying all of the effects of fuel economy standards is likely to be difficult, suggesting perhaps that policies that are more direct and that have fewer side effects might be preferred if simplicity is a desired property of environmental policy instruments.

Furthermore, it is possible that fuel economy standards could increase smog if the rebound effect is large enough. In Figure 1, each of the policies other than fuel economy standards results in a movement to somewhere below point *a*—each policy reduces the level of smog, albeit to varying degrees. In contrast, the effect of fuel economy standards on the level of smog is indeterminate. This is because the reduction in the marginal cost of transportation increases the amount of travel, resulting in a movement up the S^0 curve to the right of point *a* ($S_M M_k k_f > 0$). Two other effects work in the opposite direction: the fall in income will tend to reduce miles of travel ($S_M M_F < 0$), and, to the extent that emissions per mile are reduced, that is if $S_F < 0$ (see footnote 12), there will be a shift in the smog function toward S^1 . If these last two effects are small relative to the rebound effect, then fuel economy standards will increase smog.

4. Summary and conclusions

The urban model developed in Section 2 incorporates travel-related air pollution into previous models and reexamines the comparative static results, particularly the effects of transportation costs on welfare. The analysis in Section 3 of several popular policies designed to curb pollution from automobiles (fuel taxation, alternative fuel mandates, emissions standards/testing programs, and fuel economy standards) reveals that the net welfare effects of each of these policies are ambiguous. Moreover, the spatial model highlights the sources of the ambiguities. Because the benefits and costs associated with these policies vary considerably in number and complexity, one broad policy implication is that when simplicity matters, those policies with fewer and more tractable welfare effects might be preferred.

In order to keep the mathematical model manageable, the preceding analysis abstracts from the fact that in actual cities, pollution levels vary geographically and across time and that households have a choice of different modes of travel. Allowing for location-dependent pollution levels in the monocentric model would not change the qualitative results obtained above. For example, if smog levels were greater near the CBD, the response by households to an increase in marginal transportation cost would be lower but in the same direction as indicated in Section 2 above.

The timing of pollution matters because automotive emissions tend to accumulate (create smog) during rush hours and dissipate at other times. While the policies analyzed above probably do not have an effect on the timing of pollution (i.e., they do not influence a commuter's decision of when to commute), other policies such as staggering daily work hours do have an important time dimension. Thus, one useful extension of the present model would be to include the timing of pollution.

Another useful extension of the model would be to incorporate different modes of travel, such as mass transit, car-pooling, bicycling, and walking. Such a model could be used to reexamine the spatial implications of the above policies as well as other policies that encourage modal switches.

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Appendix

There are three approaches to modeling household behavior in a spatial context:

- (1) The bid-rent function Ψ ;
- (2) The expenditure function E ; and
- (3) The indirect utility function V .

Each of these functions contains all of the relevant information concerning the optimizing behavior of the household. This appendix provides a brief description of each approach and illustrates the relationships between the three methods using specific functional forms. For simplicity, pollution is omitted. Definitions of the variables used are as follows:

- q = Quantity of land;
- z = Quantity of a composite good (serves as numeraire);
- t = Distance from the city center (or CBD);
- $K(t)$ = Total transportation costs when located at distance t ;
- I = Nonrent income;
- Y = Endogenous income;
- $M(t)$ = Disposable income equal to $Y - K(t)$;
- $L(t)$ = Land available for residential use at distance t ;
- $R(t)$ = Market rent function;
- \bar{R} = Agricultural (rural) rent;
- b = City boundary; and
- N = Population (fixed).

1a. The bid-rent approach

The bid-rent function gives the maximum rent per unit of land that the household can pay at a given location t while enjoying a fixed level of utility u . Formally,

$$\Psi(M(t), u) = \max_{\{q, z\}} \left\{ \frac{M(t) - z}{q} \mid U(q, z) = u \right\}.$$

The first-order conditions of this problem lead to the following equilibrium conditions of the household:

$$\frac{U_q}{U_z} = \frac{M(t) - z}{q},$$

$$u = U(q, z).$$

Solving these conditions for q and z yields the following equilibrium values:

$$\begin{aligned}q^* &= q(M(t), u), \\z^* &= z(M(t), u), \\ \Psi(M(t), u) &= \frac{M(t) - z^*}{q^*}.\end{aligned}$$

For example, when $U(q, z) = q^\alpha z^{1-\alpha}$ and $K(t) = kt$, the equilibrium values are:

$$\begin{aligned}q^* &= (1 - \alpha)^{\frac{\alpha-1}{\alpha}} (Y - kt)^{\frac{\alpha-1}{\alpha}} u^{\frac{1}{\alpha}}, \\z^* &= (1 - \alpha)(Y - kt), \\ \Psi &= \left[\frac{\alpha^\alpha (1 - \alpha)^{1-\alpha} (Y - kt)}{u} \right]^{\frac{1}{\alpha}}.\end{aligned}$$

2a. The expenditure approach

The expenditure function gives the minimum expenditure of the household at a given distance t while enjoying a fixed utility level u . Formally,

$$E(R(t), u) = \min_{\{q, z\}} \{z + R(t)q \mid U(q, z) = u\}$$

The equilibrium conditions of the household are given as

$$\begin{aligned}\frac{U_q}{U_z} &= R(t), \\ u &= U(q, z)\end{aligned}$$

Solving these conditions for q and z yields the following equilibrium values:

$$\begin{aligned}\tilde{q} &= q(R(t), u), \\ \tilde{z} &= z(R(t), u), \\ E(R(t), u) &= \tilde{z} + R(t)\tilde{q}.\end{aligned}$$

In this case, when $U(q, z) = q^\alpha z^{1-\alpha}$ and $K(t) = kt$ as before, the equilibrium values are:

$$\begin{aligned}\tilde{q} &= \left(\frac{\alpha}{1 - \alpha} \right)^{1-\alpha} u R(t)^{\alpha-1}, \\ \tilde{z} &= \left(\frac{\alpha}{1 - \alpha} \right)^{-\alpha} u R(t)^\alpha, \\ E &= \frac{u R(t)^\alpha}{\alpha^\alpha (1 - \alpha)^{1-\alpha}}.\end{aligned}$$

3a. The indirect utility approach

The indirect utility function gives the maximum utility attainable at location t given prices and income. Formally,

$$V(R(t), M(t)) = \max_{\{q, z\}} \{U(q, z) | z + R(t)q = M(t)\}.$$

The equilibrium conditions of the household are as follows:

$$\frac{U_q}{U_z} = R(t),$$

$$z + R(t)q = M(t).$$

Solving these conditions for q and z yields the following equilibrium values:

$$\hat{q} = q(R(t), M(t)),$$

$$\hat{z} = z(R(t), M(t)),$$

$$V(R(t), M(t)) = U(\hat{q}, \hat{z}).$$

When $U(q, z) = q^\alpha z^{1-\alpha}$ and $K(t) = kt$, the equilibrium values are:

$$\hat{q} = \frac{\alpha(Y - kt)}{R(t)},$$

$$\hat{z} = (1 - \alpha)(Y - kt),$$

$$V = \frac{\alpha^\alpha (1 - \alpha)^{1-\alpha} (Y - kt)}{R(t)^\alpha}.$$

4a. Relationships between the three approaches

Each of the three functions described above, the bid-rent function, the expenditure function, and the indirect utility function, contains all of the relevant information concerning household behavior. Each of the functions can be inverted to obtain the other two (just as in the more familiar case in nonspatial microeconomic theory where it can be shown that the expenditure and indirect utility functions are inverses). For example, consider the specific case above where $U(q, z) = q^\alpha z^{1-\alpha}$ and $K(t) = kt$. It easily can be shown that:

- (1) Solving $\Psi = R(t)$ for $M(t)$ gives the expenditure function, $E(R(t), u)$, and solving $\Psi = R(t)$ for u gives the indirect utility function, $V(R(t), M(t))$;
- (2) Solving $E = M(t)$ for u gives the indirect utility function, $V(R(t), M(t))$, and solving $E = M(t)$ for $R(t)$ gives the bid-rent function, $\Psi(M(t), u)$; and
- (3) Solving $V = u$ for $M(t)$ gives the expenditure function, $E(R(t), u)$, and solving $V = u$ for $R(t)$ gives the bid-rent function, $\Psi(M(t), u)$.

Thus, each approach embodies the same basic behavioral information.

The comparative static results in the text were derived using the expenditure function. But, as the interested reader can verify, the other two approaches yield expressions identical to those in the text.