Market competition and abatement technology diffusion under environmental liability law

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One challenge in environmental liability law is to apportion the liability appropriately among multiple parties, when these multiple parties generate a single and non-separable damage. This paper provides a method of designing efficient apportionment rule in the presence of product market competition. In particular, I examine the role of apportionment rule in inducing abatement technology diffusion between competing firms. I show that such diffusion can be induced by an efficient apportionment rule. And this apportionment rule allocates a relatively large (more than 1/2) portion of the liability to the firm which originally owns the abatement technology. Furthermore, allocating the liability equally between the firms cannot induce diffusion.

Keywords: Environmental liability law; Apportionment rule; Diffusion; Competition

One challenge in environmental liability law is to apportion the liability among multiple parties, when these multiple parties generate a single and non-separable environmental damage. Non-separable damage is often likely to occur when industry is agglomerated. In the presence of spatial clustering, it is possible that pollutants emitted from different firms interact frequently in the environmental medium, which could cause serious environmental damage. Obviously, individual firm's contributions to the total damage cannot be easily disentangled, and in particular when the pollution plumes are persistent.

A report issued by Environmental Defense Fund finds that auto industry is one of the nation's largest sources of mercury pollution through both manufacturing process and recycling process (Environmental Defense Fund, 2001). And the majority of the auto
companies’ U.S. plants are located in the Great Lakes Region, such as Ford, Chrysler, and General Motors. The mercury emissions have already caused certain damages to the aquatic systems in that area.

An interesting research question arising from such a phenomenon is: what is the efficient apportionment rule to divide the liability or the compensation among these potentially responsible parties in the presence of product market competition. I establish that apportionment rule may influence the competitive advantages of firms, the eventual market structure, and firm incentives to diffuse abatement technology. The main contribution of this paper is to investigate a design for efficient apportionment rule to divide the joint environmental liability between competing firms, and in particular, the role of the efficient apportionment rule in inducing abatement technology diffusion between rivals is studied.

I consider a simple model of homogeneous good duopoly under environmental liability law, where the two competing firms cause a single and non-separable damage. Environmental liability law is analyzed in the regime of strict liability. Under strict liability, the polluting firm is required to compensate the damage regardless of how much care it has undertaken. Thus, each firm compensates a portion of the damage, and the sum of the compensation paid by the two firms is equal to the total damage. Initially, the two firms are asymmetric in abatement technology levels with only one firm having an
abatement technology. Before engaging in Cournot competition, the firm owning the technology decides whether to transfer the technology to the rival, and this rival firm also decides whether to accept it.

I show that in this model diffusion is induced by an efficient apportionment rule. And this apportionment rule allocates a relatively large (more than 1/2) portion of the compensation payment to the technology owner. The intuition is as follows. The technology owner's incentive to transfer the abatement technology depends upon the relative sizes of "burden sharing effect" and "strategic cost effect". Specifically, under strict liability, firm is liable for the environmental damage irrespective of its precautionary behavior. Therefore, the abatement technology owner will aim to reduce the total damage so as to reduce its individual compensation payment. Diffusing the abatement technology to the rival can increase abatement of both firms, and consequently, the total damage is lowered. Sharing the technology could be a benefit to the technology owner--what I call the burden sharing effect. On the other hand, in duopoly settings, the technology owner has another countervailing incentive. If the technology owner does not transfer the technology, then it can have a cost advantage over the rival (the abatement technology lowers cost in an implicit way)--what I call the strategic cost effect. The burden sharing effect is strengthened and appears to dominate when the technology owner is assigned to compensate a major portion of the total damage. As a result, in order
to induce diffusion, the apportionment rule requires the technology owner to be allocated with a relatively large apportionment.

Endres and Friehe (2011) are the first to analyze polluting firm's incentives to diffuse abatement technology to other polluting firms under environmental liability law. And they assume that firms are not competing on the market. In the absence of competition, the incentive to diffuse the abatement technology under strict liability depends entirely on the burden sharing effect. And this explains why in their model the diffusion outcome is independent of the apportionment rule and full diffusion may occur even if the liability is apportioned on an arbitrary basis. This paper argues that this is not the case when product market competition is taken into account.

This paper also contributes to three different strands of literature. First, it contributes to the literature analyzing the effects of environmental policy instruments on abatement technology diffusion. Earlier works (e.g., Milliman and Prince, 1989, Jaffe and Stavins, 1995 and Coria, 2009) have studied diffusion of abatement technology under various instruments, such as direct controls, emission subsidies, emission taxes, free marketable permits, and auctioned marketable permits. This paper shows that environmental liability law can be a specifically tailored instrument to encourage abatement technology diffusion.

Second, there is a large literature examining firm incentives for technology sharing with rivals. In existing literature, technology sharing between rivals has been broadly viewed
as a device to facilitate collusion, deter entry, or force exit. This paper suggests a new explanation for such behavior.

Finally, there is a rich literature in law concerned with apportionment rule. One section of this literature focuses on deriving apportionment rule which is on the grounds of fairness (see, e.g., Rizzo and Arnold, 1980). While another section (closer to this paper) concentrates on efficiency grounds. Kornhauser and Revesz (1990) consider the inefficiency that may arise when firm is unable to pay its apportioned share of the liability and exits the market entirely. The main difference between my paper and the existing literature is that I find in addition to its influence on market structure, apportionment rule may have important welfare consequence in promoting abatement technology diffusion.

One interesting observation is that the efficient apportionment rule suggested in this paper may seem "unfair" in the sense that it requires the originally "cleaner" firm to be assigned with a larger apportionment. In fact, in order to enhance ex-post efficiency, some ex-ante bias against the technology owner is necessary. And it is this ex-ante bias that creates the incentive for the technology diffusion.

The paper is organized as follows. Section 2 presents the model formally. Section 3 derives the main results. Section 4 contains the welfare analysis. Section 5 concludes.
The model

Consider a Cournot duopoly where the inverse market demand function $P(Q)$ is:

$$P(Q) = a - Q, 0 \leq Q \leq a.$$  

(1)

Firms are indexed by $i \in \{1, 2\}$. Each firm $i$ produces output with a common unit cost normalized to 0 and generates pollution as a by-product. Firm $i$’s emission level $e_i$ is determined by its emission per unit output $x_i$ and output quantity $q_i$:

$$e_i = x_i q_i.$$  

(2)

Firm 1 has an abatement technology which allows it to reduce its emission per unit output $x_i$ to $x^*$, where $0 < x^* < 1$. A smaller $x^*$ indicates a cleaner technology. Firm 2 does not have access to such abatement technology initially and $x_2 = 1$.

The total expected environmental damage depends on the emission levels of both firms:

$$D(e_1, e_2) = (e_1 + e_2)^2.$$  

(3)

Firms' emission levels interact via this total environmental damage function. Clearly, if firm $i$ is the only active firm on the market, then $D = e_i^2$.

Under strict liability, one firm is held liable as long as it is causing the damage. In this present setting, the two firms jointly bring forth a compensation payment equal to the total damage when both of them are responsible for the damage. In this model, apportionment rule is denoted by a pair of apportionments $\{\beta_1, \beta_2\}$, and $\beta_i$ is defined as the apportionment of liability assigned to firm $i$, where $\beta_i \geq 0$ for all $i$ and $\sum_{i=1}^2 \beta_i = 1$. 
Hence, firm $i$ compensates a share $\beta_i$ for the total damage and firm $i$'s individual compensation payment is equal to $\beta_i D$.

I model a four-stage game and the four stages are: (1) firm 1 decides whether to transfer the abatement technology to firm 2; (2) firm 2 decides whether to accept the technology; (3) firms simultaneously determine whether to stay or exit the market; and (4) if both firms choose to stay on the market, firms engage in Cournot competition and compensate the environmental damage according to the apportionment rule described earlier; otherwise, one firm becomes a monopolist and compensates all the damage it generates.

The technology is successfully diffused to firm 2 if and only if: (1) both firms agree to transfer and accept the technology and (2) both firms stay on the market. Firm 2's emission per unit output $x_2$ will reduce to $x^*$ if diffusion occurs and remains unchanged otherwise. Therefore, the two firms' emissions per unit output are given by:

$$x_1 = x^*, x_2 = \begin{cases} x^*, & \text{if diffusion occurs;} \\ 1, & \text{otherwise.} \end{cases}$$

In order to focus on the diffusion incentives created by the apportionment rule, I assume that firm 1 does not receive any payment from the technology transfer\(^1\).

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\(^1\) Example of such technology transfer without direct payment includes the General Motors and Toyota case (Creane and Konishi, 2009).


**Equilibrium**

In this section, I determine the equilibrium of the game described in the previous section.

First, the payoffs are derived for different cases. Then I present different exit regimes and analyze the effects of apportionment rule on market structure. Finally, I discuss conditions under which diffusion takes place, and consider the role of apportionment rule in determining the diffusion outcome.

**Payoffs**

In the last stage, given the possible diffusion and exit outcomes yielded in the previous stages, four cases may arise. The first case is DS, in which duopoly persists and technology diffusion successfully takes place. In the second case, DU, duopoly persists but diffusion does not occur. The third case is MT, in which the technology owner (firm 1) becomes a monopolist. In the last case, MN, the firm with no abatement technology (firm 2) becomes a monopolist. The four cases are summarized in Table 1.

<table>
<thead>
<tr>
<th>Case</th>
<th>Diffusion</th>
<th>Production</th>
</tr>
</thead>
<tbody>
<tr>
<td>DS</td>
<td>Successful diffusion; $x_1 = x_2 = x^*$.</td>
<td>A duopoly persists.</td>
</tr>
<tr>
<td>DU</td>
<td>Unsuccessful diffusion; $x_1 = x^*, x_2 = 1$.</td>
<td>A duopoly persists.</td>
</tr>
<tr>
<td>MT</td>
<td>Diffusion does not occur; $x_1 = x^*$.</td>
<td>Firm 1: monopolist.</td>
</tr>
<tr>
<td>MN</td>
<td>Diffusion does not occur; $x_2 = 1$.</td>
<td>Firm 2: monopolist.</td>
</tr>
</tbody>
</table>

Table 1-The Four Cases
Let $R_i^C$ denote firm i's payoff, where the superscript C indicates different cases, and C=DS, DU, MT, and MN. Under each case, the active firm i maximizes its payoff with respect to output, and the payoff function is given by product market profit minus its individual compensation payment:

$$\max_{q_i} R_i^C = q_i^C (a - q_1^C - q_2^C) - \beta_i (x_i q_i^C + x_2 q_2^C)^2, i = 1, 2.$$  \hspace{1cm} (4)

Note that if firm i becomes the only active firm, it becomes responsible for all the damage it generates, and in other words, $\beta_i = 1$. In addition, $x_i = x_2 = x^*$ in case DS; otherwise, $x_i = x^*$ and $x_2$ remains 1. The equilibrium outputs and payoffs are reported in Table 2.

<table>
<thead>
<tr>
<th>Case DS</th>
<th>Case MT</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_1$</td>
<td>$Q_2$</td>
</tr>
<tr>
<td>$R_1$</td>
<td>$R_2$</td>
</tr>
</tbody>
</table>

Table 2-Equilibrium outputs and payoffs

\[
\begin{array}{ccc}
\hline
 & Case DS & Case MT \\
\hline
Q_1 & \frac{1+(4\beta_2-2)(x^*)^2}{3+2(x^*)^2}a & \frac{1}{2+2(x^*)^2}a \\
Q_2 & \frac{1+(2-4\beta_2)(x^*)^2}{3+2(x^*)^2}a & 0 \\
R_1 & \frac{1+(8\beta_2-4)((x^*)^2+(x^*)^4)}{[3+2(x^*)^2]^2}a^2 & \frac{1+(x^*)^2}{[2+2(x^*)^2]^2}a^2 \\
R_2 & \frac{1+(4-8\beta_2)((x^*)^2+(x^*)^4)}{[3+2(x^*)^2]^2}a^2 & 0 \\
\hline
\end{array}
\]
Exit regimes

In stage three, the two firms simultaneously choose whether to stay or exit, given the diffusion outcome realized in the first two stages. Firm $i$ exits the market if it expects a non-positive payoff (for ease, I assume if a firm is indifferent between staying and exiting, it exits), i.e., $R_i^C \leq 0$. Equivalently, firm $i$ exits the market if its product market profit $q_i^C (a - q_i^C - q_j^C)$ cannot cover its individual compensation payment $\beta_i (x_i q_i^C + x_j q_j^C)^2$.

Obviously, there exist some critical thresholds of the apportionment which will determine when it is optimal for firm $i$ to exit the market.

I first define two critical apportionments for firm 2. Let $\beta_2^*$ denote the critical apportionment such that if the apportionment assigned to firm 2 is larger than (or equal to)
this critical apportionment, firm 2 exits the market provided diffusion does not occur in the previous stages. In addition, \( \beta_2 \) denotes the critical apportionment such that if the apportionment assigned to firm 2 is larger than (or equal to) this critical apportionment, firm 2 exits the market even when diffusion successfully occurs. \( \beta_2 \) and \( \beta_2 \) are defined as the solution to \( R_2^{DU} = 0 \) and \( R_2^{DS} = 0 \), respectively, i.e.,

\[
q_2^{DU} (a - q_1^{DU} - q_2^{DU}) = \beta_2 (x^* q_1^{DU} + q_2^{DU})^2 ;
\]

(5)

\[
q_2^{DS} (a - q_1^{DS} - q_2^{DS}) = \beta_2 (x^* q_1^{DS} + x^* q_2^{DS})^2 .
\]

(6)

In a similar manner, I can define another two critical apportionments for firm 1. Let \( \beta_2 \) denote the critical apportionment such that if the apportionment assigned to firm 2 is smaller than (or equal to) this critical apportionment (so that \( \beta_1 = 1 - \beta_2 \) is quite large), firm 1 exits the market provided diffusion does not occur in the previous stages. In addition, \( \beta_2 \) denotes the critical apportionment such that if the apportionment assigned to firm 2 is smaller than (or equal to) this critical apportionment, firm 1 exits the market even when diffusion successfully occurs. \( \beta_2 \) and \( \beta_2 \) are defined as the solution to \( R_1^{DU} = 0 \) and \( R_1^{DS} = 0 \), respectively, i.e.,

\[
q_1^{DU} (a - q_1^{DU} - q_2^{DU}) = (1 - \beta_2) (x^* q_1^{DU} + q_2^{DU})^2 ;
\]

(7)

\[
q_1^{DS} (a - q_1^{DS} - q_2^{DS}) = (1 - \beta_2) (x^* q_1^{DS} + x^* q_2^{DS})^2 .
\]

(8)

The expressions of \( \beta_2 \), \( \beta_2 \), \( \beta_2 \) and \( \beta_2 \) can be found in the Appendix A.

Intuitively, there are two different driving forces behind exit. The first one is the
apportionment rule, and the other is related to firm's strategic incentive. Specifically, the individual compensation payment \( \beta_i (x_i q_i^C + x_2 q_2^C)^2 \) could be considered as a cost incurred by firm i. Given the output quantities and the emission per unit output, an increase in \( \beta_i \) leads to an increase in individual compensation payment. When \( \beta_i \) is sufficiently large, it is possible that no matter diffusion occurs or not, firm i's compensation payment exceeds its product market profit and firm i chooses to exit. Therefore, an extremely large apportionment assigned to a firm can induce that firm's exit, and I call this kind of exit as apportionment induced exit.

Furthermore, one firm can strategically drive out the other firm by preventing technology diffusion--what I call predation. To explain this, let us take a closer look at the term \( \beta_i (x_i q_i^C + x_2 q_2^C)^2 \). As mentioned earlier, \( \beta_i (x_i q_i^C + x_2 q_2^C)^2 \) can be thought as firm i's cost, and I can derive each firm's implicit marginal cost from it. For example, given \( q_1 \) and \( q_2 \), firm 2's marginal cost drops to \( 2\beta_2 (x^* q_1 + x^* q_2) x^* \) if diffusion occurs, but remains \( 2\beta_2 (x^* q_1 + q_2) \) otherwise. Hence, firm 1 can raise rival's cost and gain a competitive advantage by not transferring the technology--what I call the raise rival cost effect.

Under certain conditions, it is possible that firm 2 can survive on the market only if diffusion occurs. Therefore, if keeping the technology a secret, firm 1 could prey to a monopoly but an automatic implication is that firm 1's apportionment will become one; on the other hand, if sharing the technology, firm 1 has to split the market with the rival
but will not have to bear the burden alone. As a result, the predation is profitable to firm 1 if the monopoly payoff $R_{1MT}$ is higher than $R_{1DS}$, the duopoly payoff under diffusion.

Similarly, firm 2 can drive out firm 1 by not accepting the technology in the second stage. And the predation is profitable to firm 2 if the monopoly payoff $R_{2MN}$ is higher than $R_{2DS}$, the duopoly payoff under diffusion:

$$R_{2MN} > R_{2DS};$$

or equivalently

$$\beta_2 > \beta_2^A = \frac{28(x^*)^4 + 20(x^*)^2 - 1}{64(x^*)^4 + 64(x^*)^2}.$$ \hfill (10)

And the following propositions respectively summarize the conditions under which apportionment induced exit and predation occur:

**Proposition 1.** Firm 1 exits the market irrespective of the diffusion outcome if $0 \leq \beta_2 \leq \min\{\bar{\beta}_2, \hat{\beta}_2\}$; firm 2 exits the market irrespective of the diffusion outcome if $\bar{\beta}_2 \leq \beta_2 \leq 1$.

**Proposition 2.** If $\beta_2^* < \beta_2 \leq \min\{\bar{\beta}_2, 1\}$, it is profitable for firm 1 to prey on firm 2 by not transferring the abatement technology, and as a result, firm 2 chooses to exit; if $\max\{0, \hat{\beta}_2, \beta_2^A\} \leq \beta_2 < \beta_2^*$, it is profitable for firm 2 to prey on firm 1 by not accepting the abatement technology, and as a result, firm 1 chooses to exit.

**Proof.** See Appendix B.
Abatement technology diffusion

In this subsection, I obtain conditions that ensure abatement technology diffusion. Diffusion takes place when both firms stay on the market and find diffusion profitable. Proposition 1 and Proposition 2 imply that when the apportionment is sufficiently large to either firm 1 or firm 2, exit could be induced. Hence, one anticipates that diffusion occurs as apportionments become fairly moderate to both firms. And several questions might arise. For example, it is natural to ask how moderate should apportionment be to encourage diffusion, and can apportioning the liability equally \((\beta_1 = \beta_2 = \frac{1}{2})\) induce such diffusion. A second question is: in addition to apportionment rule, is there any other factor that can affect diffusion?

To determine the conditions for diffusion, I will focus on the case in which a duopoly persists. First, consider firm 1's incentive to transfer the abatement technology. Firm 1, the technology owner, chooses to transfer the technology if the duopoly payoff under diffusion \(R_i^{DS}\) is higher than \(R_i^{DU}\), the duopoly payoff under non-diffusion. It turns out that \(R_i^{DS}\) is lower than \(R_i^{DU}\) if the technology is dirty, i.e., \(x \leq x^* < 1\). Therefore, firm 1 does not choose to transfer the technology in the first place, and diffusion fails as a consequence.

\[^2\] Simple computations reveal that \(x^* = 0.8293\).
**Proposition 3.** No apportionment rule induces abatement technology diffusion if the technology is dirty, i.e., \( x \leq x^* < 1 \).

**Proof.** See Appendix B.

The intuition behind this result is as follows. The dirty technology can at most reduce the emission per unit output to \( x \). And this is quite close to the emission per unit output when no technology is applied. Hence, the total environmental damage as well as the individual compensation payment will only reduce slightly even when both firms apply the technology. As a result, the gain from diffusion is negligible. On the other hand, as mentioned earlier, firm 1 can gain a competitive advantage over the rival by suppressing diffusion. Taking these into account, firm 1 is not willing to share the technology with the rival.

Then, the main question is whether some apportionment rule can induce diffusion as the technology becomes clean, i.e., \( 0 < x^* < x \). Under condition of clean technology \((0 < x^* < x)\), let \( \beta_2^{\tau} \) denote the critical apportionment such that if the apportionment assigned to firm 2 is equal to \( \beta_2^{\tau} \), firm 1 is indifferent between transferring the technology and not transferring. And \( \beta_2^{\tau} \) is implicitly defined by:
\[
R_1^{DS} = q_1^{DS} (a - q_1^{DS} - q_2^{DS}) - (1 - \beta_2^T) (x^* q_1^{DS} + x^* q_2^{DS})^2 \\
= q_1^{DU} (a - q_1^{DU} - q_2^{DU}) - (1 - \beta_2^T) (x^* q_1^{DU} + q_2^{DU})^2 = R_1^{DU}.
\]

And it can be shown that \( \beta_2 < \beta_2^T < \frac{1}{2} \) (see Appendix B for details).

Next, I define a diffusion set \( S \) such that:

\[
S = (\beta_2, \beta_2^T] \cup [\max\{0, \beta_2^T\}, \min\{\beta_2^A, \beta_2\}]
\]  

(12)

where \( S \) consists of two different groups of apportionments: \((\beta_2, \beta_2^T]\) and 
\([\max\{0, \beta_2^T\}, \min\{\beta_2^A, \beta_2\}]\). I refer to them as high-apportionment range and 
low-apportionment range respectively.

And I claim that: both firms find diffusion profitable and diffusion occurs if and only if 
the technology is clean and \( \beta_2 \) belongs to the diffusion set \( S \).

Proposition 4. Abatement technology diffusion occurs if and only if \( 0 < x^* < x \) and \( \beta_2 \) 
begins to the diffusion set \( S \), where \( S = (\beta_2, \beta_2^T] \cup [\max\{0, \beta_2^T\}, \min\{\beta_2^A, \beta_2\}] \). 

Proof. See Appendix B.

To understand why these two conditions matter, let us first look at the condition 
\( 0 < x^* < x \). When the technology is clean i.e., \( 0 < x^* < x \), the total environmental damage 
as well as the individual compensation payment will reduce considerably if both firms 
apply the technology. And the reduction could be more substantial as \( x^* \) decreases. 
Thus, the gain from diffusion can be significant, and both firms have strong incentives to
support diffusion.

The other important condition is that $\beta_2$ belongs to the diffusion set $S$: 
$\left(\beta_2^T, \beta_2^a\right) \cup \left[\max\{0, \hat{\beta}_2\}, \min\{\beta_2^a, \beta_2\}\right]$. When $\beta_2$ lies in the high-apportionment range, a duopoly persists irrespective of the diffusion outcome. But both firms achieve higher duopoly payoffs under diffusion, and consequently diffusion takes place. As $\beta_2$ lies below the high-apportionment range (but larger than $\max\{0, \hat{\beta}_2\}$), the compensation burden on firm 1 is so great that it can stay on the market if and only if diffusion occurs.

In other words, firm 2 can further raise firm 1’s cost (compensation burden) and prey to a monopoly by not accepting the technology. And as explained earlier, the predation is not profitable and firm 2 chooses to accept the technology if $\beta_2 \leq \beta_2^a$. Therefore, the low-apportionment range is defined as $\left[\max\{0, \hat{\beta}_2\}, \min\{\beta_2^a, \beta_2\}\right]$.

Observe that every individual $\beta_2$ in the diffusion set $S$ is smaller than $1/2$. An immediate implication is that to promote diffusion, the apportionment assigned to firm 1 should be larger than $1/2$ ($\beta_1 = 1 - \beta_2$). In other words, technology owner (firm 1) should be allocated with a relatively large apportionment. Moreover, diffusion may take place even when the firms have very asymmetric apportionments. At the extreme, diffusion could occur when firm 1 pays the entire liability (so that $\beta_2 = 0$). Figure 1 illustrates the

---

3 It is straightforward to see that $\beta_2^T$ is the largest apportionment in the diffusion set $S$. And $\beta_2^T$ is always smaller than $1/2$. Therefore, every $\beta_2$ in the diffusion set $S$ is smaller than $1/2$. 

diffusion set $S$ with $x^* = 0.3$.

![Diagram: Diffusion set with $x^* = 0.3$.]

This result may seem surprising and one may ask why firm 1 is willing to share its technological advantage with the rival when the apportionment rule has put it at a disadvantage. In fact, the incentive to share the technology is created by the burden sharing effect: diffusing the technology to the other firm can increase abatement of both firms, and therefore, the total damage as well as firm 1's individual compensation payments are lowered. Firm 1 will be more eager to share the technology when it is assigned to compensate a major portion of the total damage. Equivalently, the burden sharing effect is strengthened and dominates the raise rival cost effect when firm 1's apportionment is relatively large.

Another key finding is that apportioning the liability by an equal share ($\beta_1 = \beta_2 = \frac{1}{2}$) which is commonly applied in practice, cannot induce diffusion. The intuition behind this result is straightforward. Suppose that $\beta_1 = \beta_2 = \frac{1}{2}$. If firm 1 does not transfer the technology, its implicit marginal cost would be $(x^* q_1 + q_2)x^*$ compared with $(x^* q_1 + q_2)$, firm 2's implicit marginal cost. Obviously, firm 1 has a cost advantage over the rival and
the cost gap increases as $x^*$ decreases. However, if diffusion occurs, the two firms will have a common marginal cost $(x^*q_1 + x^*q_2)x^*$. To firm 1, although its marginal cost decreases in absolute term after diffusion, the cost gap is closed and the cost advantage is eliminated. Therefore, firm 1 will not let the abatement technology diffuse to the other firm.

**Welfare analysis**

In the previous section, I have presented that apportionment rule can influence both market structure and abatement technology diffusion. And the change in apportionment rule can lead to different equilibrium outcomes. In this section, I will make a welfare analysis to find the efficient apportionment rule.

First, the welfare function $W$ is defined as follows:

$$W = CS + \sum_{i=1}^{2} R^C_i + \sum_{i=1}^{2} \beta_i D(e_1, e_2).$$

(13)

The welfare function is composed of three terms, which respectively denote consumer surplus, producer surplus, and the total compensation payment collected. And clearly, the welfare function can be rewritten as:

$$W = CS + \sum_{i=1}^{2} \pi^C_i,$$

(14)
where the latter term is the sum of firms' product market profit.

For every outcome that may arise eventually, the corresponding expression of the welfare function is displayed in Table 3.

<table>
<thead>
<tr>
<th>Case</th>
<th>Welfare function</th>
</tr>
</thead>
</table>
| DS   | \[
+\frac{4 + 4(x^*)^2}{3 + 2(x^*)^2} a^2
\] |
| DU   | \[
+\frac{6\beta^2 + 10\beta + 4 - (8\beta + 6)x^* + (-12\beta^2 + 2\beta + 12)(x^*)^2 - (8\beta^2 + 8)(x^*)^3 + (6\beta^2 - 12\beta + 6)(x^*)^4}{H^2} a^2
\] |
| MT   | \[
+\frac{3}{2} + (x^*)^2
\] |
| MN   | \[
+\frac{5}{32} a^2
\] |

Observe that the expression of \( W^{DS} \), \( W^{MT} \), and \( W^{MN} \) is independent of \( \beta_2 \), and the expression of \( W^{DU} \) suggests that \( \beta_2 \) directly affects the welfare level in case DU. A straightforward calculation shows that \( \frac{dW^{DU}}{d\beta_2} > 0 \)\(^4\). This implies that a larger \( \beta_2 \) improves social welfare if a duopoly persists but diffusion fails. Intuitively, when both firms stay on the market and are asymmetric in abatement technology levels, the firm

\(^4\) See the Proof of Proposition 5.
with the technology (firm 1) is considered to be more efficient in the sense that it has a lower marginal cost. Furthermore, firm 1's marginal cost would be even lower, as its apportionment $\beta_1$ decreases (so that $\beta_2$ increases). A larger $\beta_2$ makes the efficient firm 1 more efficient, firm 1 expands its output, and social welfare is improved as a consequence.

Depending on the apportionment scheme, four different equilibrium outcomes are possible. To determine the efficient apportionment rule, it is necessary to discuss which equilibrium outcome is the most desirable. I make a comparison of welfare levels under different outcomes. And the comparison yields the following ranking$^5$:

$$W^{DS} > W^{DU} > W^{MT} > W^{MN}. \quad (15)$$

The following proposition defines the efficient apportionment rule.

**Proposition 5.** The efficient apportionment rule is:

1. Assign $\beta_2$ to firm 2 such that $\beta_2$ belongs to the diffusion set $S$:

   $$\left[\beta^2_2, \beta^T_2\right] \cup \left[\max\{0, \hat{\beta}_2\}, \min\{\beta^d_2, \beta^c_2\}\right] \quad \text{if} \quad 0 < x^* < x;$$

2. Assign $\beta_2$ to firm 2 such that $\beta_2 = \beta^*_2$, if $x \leq x^* < 1$.

**Proof.** See Appendix B.

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$^5$ See the Proof of Proposition 5.
This proposition implies that the efficient apportionment rule could be quite different as $x^*$ varies. Specifically, if the technology is clean (say, $0 < x^* < 1$), every $\beta_2$ in the diffusion set $S$ can lead to the most desirable equilibrium outcome $DS$ and generate equivalent welfare level. One basic feature these $\beta_2$ share is that they are all smaller than 1/2. And the efficient apportionment rule requires the technology owner to bear a relatively large share of the liability. To enhance efficiency, the two firms should be treated asymmetrically (even though they become identical ex post).

On the other hand, if the technology is dirty (say, $x \leq x^* < 1$), diffusion never occurs and the most desirable outcome is that a duopoly persists and diffusion fails. In that case, a larger $\beta_2$ improves welfare ($\frac{dW^{DU}}{d\beta_2} > 0$). The largest $\beta_2$ that ensures this equilibrium outcome is $\beta_2^*$. As a result, $\beta_2^*$ is the unique efficient apportionment under dirty technology. And $\beta_2^*$ is always larger than 1/2, which indicates that the efficient apportionment rule requires the technology owner to bear a relatively small share of the liability. This is in contrast to the requirement of the efficient apportionment rule when the technology is clean. To enhance efficiency, the apportionment rule should favor the more efficient firm (firm 1).

**Conclusion**

Apportioning the liability in cases involving multiple parties is an important problem in both theory and practice. In this paper, an efficient apportionment rule is determined in
the presence of market competition. The apportionment rule suggested in this paper is efficient in a sense that it promotes abatement technology diffusion between competing firms and it sustains market competition. I find that this efficient apportionment rule requires the firm originally owning the abatement technology to bear a relatively large (more than 1/2) share of the liability. In addition, apportioning the liability equally between firms cannot lead to diffusion.

To focus on how abatement technology diffusion is affected by apportionment rule, I have assumed away the innovation stage and considered the abatement technology owned by a single firm. Therefore, a useful extension of this paper is to introduce innovation into the model. Another interesting extension of this paper is to consider an oligopoly setting or a model with free entry. These will be difficult steps and I leave these issues for future research.

**The Appendix A**

Computation of $\beta_2, \hat{\beta}_2, \hat{\beta}_3$, and $\bar{\beta}_2$. I will only provide a detailed analysis on how $\hat{\beta}_2$ is derived, and the other three critical apportionments are determined in a similar way. Given that diffusion fails, firm 1 exits the market if $R_{DU}^1 \leq 0$, and I obtain:

$$R_{DU}^1 = \frac{4\beta_2^2 + 5\beta_2 + (4\beta_2^2 - 4)x^* + (4\beta_2^2 + 3\beta_2 + 1)(x^*)^2 + (-4\beta_2^2 + 8\beta_2 - 4)(x^*)^3}{H^2} a^2 \leq 0. \quad (16)$$

Solving for the inequality, I have when:
Similarly, I can derive each firm's exit conditions corresponding to different diffusion outcomes, as well as the expressions of \( \hat{\beta}_2, \beta_2, \) and \( \overline{\beta}_2 \). Specifically, one can show that:

\[
\hat{\beta}_2 = \frac{4(x^*)^4 + 4(x^*)^2 - 1}{8(x^*)^4 + 8(x^*)^2}, \quad \beta_2 = \frac{4(x^*)^4 + 4(x^*)^2 + 1}{8(x^*)^4 + 8(x^*)^2}; \\
\overline{\beta}_2 = \frac{-[8(x^*)^4 + 4(x^*)^3 + (x^*)^2 + 4x^* - 1] + \sqrt{32(x^*)^8 + 72(x^*)^6 + 65(x^*)^4 + 48(x^*)^3 + 30(x^*)^2 + 8x^* + 1}}{2[-4(x^*)^4 - 4(x^*)^3 + 4(x^*)^2 + 4x^*]}
\]

Therefore, firm 1 exits the market irrespective of the diffusion outcome if 
\( 0 \leq \beta_2 \leq \min\{\beta_2, \hat{\beta}_2\} \).

Likewise, firm 2 exits the market irrespective of the diffusion outcome if 
\( \max\{\beta_2, \overline{\beta}_2\} \leq \beta_2 \leq 1 \).

Computation of \( \beta_2^4 \). Firm 2 finds predation profitable if \( R_2^{MN} > R_2^{DS} \), and this leads to:

\[
R_2^{MN} = \frac{1}{8} a^2 \frac{[1 + (4 - 8 \beta_2)(x^*)^2 + (4 - 8 \beta_2)(x^*)^4]}{[3 + (2(x^*)^2)]^2} a^2 = R_2^{DS}.
\]  \hspace{1cm} (17)

One can easily show that: \( R_2^{MN} > R_2^{DS} \) when \( \beta_2 > \beta_2^4 = \frac{28(x^*)^4 + 20(x^*)^2 - 1}{64(x^*)^4 + 64(x^*)^2} \).

---

6 \( \min\{\beta_2, \hat{\beta}_2\} = \beta_2 \) if \( 0.8293 \leq x^* < 1 \), and \( \min\{\beta_2, \hat{\beta}_2\} = \beta_2 \) if \( 0 < x^* < 0.8293 \). When \( x^* \) is small enough, i.e., \( 0 < x^* < \sqrt{\frac{-1 + \sqrt{2}}{2}} \), \( \min\{\beta_2, \hat{\beta}_2\} = \beta_2 \) becomes negative, which implies that firm 1 will never exit the market if diffusion occurs. On the other hand, \( \beta_2 \) is strictly positive for every \( x^* \in (0, 1) \).

7 And \( \max\{\beta_2, \overline{\beta}_2\} = \overline{\beta}_2 \) as long as \( x^* \in (0, 1) \). When \( x^* \) is small enough, i.e., \( 0 < x^* < \sqrt{\frac{-1 + \sqrt{2}}{2}} \), \( \overline{\beta}_2 \) exceeds 1, which implies that firm 2 will never exit the market if diffusion occurs. And when \( x^* \) becomes even smaller, i.e., \( 0 < x^* < 0.2426 \), \( \beta_2 \) exceeds 1 as well. In that case, firm 2 stays on the market no matter diffusion takes place or not.
The Appendix B

Proof of Proposition 2. When \( \beta_2 < \beta_2 \leq \min \{ \bar{\beta}_2, 1 \} \), firm 2 exits the market if diffusion fails, and stays on the market otherwise. And in the first stage, firm 1 chooses to transfer the technology if and only if the duopoly payoff under diffusion \( R_i^{DS} \) is higher than the monopoly payoff \( R_i^{MT} \). Otherwise, firm 1 preys on firm 2 by not transferring the technology. And I proceed by making a comparison between \( R_i^{DS} \) and \( R_i^{MT} \).

Suppose first \( \min \{ \bar{\beta}_2, 1 \} = \bar{\beta}_2 \) (this holds when \( \sqrt{\frac{-1+\sqrt{2}}{2}} \leq x^* < 1 \)). Obviously,

\[
R_i^{DS} = \frac{[1+(8\beta_2-4)(x^*)^2+(8\beta_2-4)(x^*)^4]}{[3+2(x^*)^2]^2} a^2
\]

is an increasing function in \( \beta_2 \), and \( R_i^{DS} \) is maximized at \( \beta_2 = \bar{\beta}_2 \). Define \( R_i^{DS} (\bar{\beta}_2) \) such that:

\[
R_i^{DS} (\bar{\beta}_2) = R_i^{DS} (\beta_2 = \bar{\beta}_2)
\]

\[
= \left[1+ \frac{8(x^*)^4 + 4(x^*)^2 + 1}{8(x^*)^4 + 8(x^*)^2} - 4(x^*)^2 + \frac{8(x^*)^4 + 4(x^*)^2 + 1}{8(x^*)^4 + 8(x^*)^2} - 4(x^*)^4 \right] \frac{a^2}{[3+2(x^*)^2]^2}
\]

(18)

And \( R_i^{MT} = \frac{1+(x^*)^2}{[2+2(x^*)^2]^2} a^2 > R_i^{DS} (\bar{\beta}_2) \) for any fixed \( x^* \in \left[ \sqrt{\frac{-1+\sqrt{2}}{2}}, 1 \right) \). See Figure 2.
Therefore, $R^{MT}_1$ is higher than $R^{DS}_1$ when $\beta'_2 < \beta_2 \leq \min \{ \bar{\beta}_2, 1 \} = \bar{\beta}_2$. By the same logic, one can check that $R^{MT}_1$ is also higher than $R^{DS}_1$ when $\beta'_2 < \beta_2 \leq \min \{ \bar{\beta}_2, 1 \} = 1$. And firm 1 preys on firm 2 by not transferring the technology when $\beta'_2 < \beta_2 \leq \min \{ \bar{\beta}_2, 1 \}$.

Next consider the condition under which predation is profitable to firm 2. Suppose $\max \{ 0, \hat{\beta}_2, \beta'_2 \} \leq \beta_2 < \beta_2$. From the definitions of $\hat{\beta}_2$ and $\underline{\beta}_2$, I can state that when $\max \{ 0, \hat{\beta}_2 \} \leq \beta_2 < \beta_2$, firm 1 exits the market if diffusion fails, and stays on the market otherwise. Thus, there is an opportunity for firm 2 to prey on firm 1 by not accepting the technology as $\beta_2$ is in this range. The predation is profitable if the monopoly payoff $R^{MN}_2$ is higher than the duopoly payoff under diffusion $R^{DS}_2$. And I have shown that $R^{MN}_2 > R^{DS}_2$ when $\beta_2 > \beta'_2$. As a result, firm 2 preys on firm 1 by not accepting the technology when $\max \{ 0, \hat{\beta}_2, \beta'_2 \} \leq \beta_2 < \beta_2$.

**Proof of Proposition 3.** If the technology is dirty, i.e., $x \leq x^* < 1$, $\beta_2, \hat{\beta}_2, \beta'_2$, and $\bar{\beta}_2$ are ranked as: See Figure 3.
From Proposition 1 and Proposition 2, it is easy to see that the only possible range in which diffusion may occur is \( \hat{\beta}_2 < \beta_2 \leq \beta'_2 \). If \( \hat{\beta}_2 < \beta_2 \leq \beta'_2 \), both firms stay on the market regardless of the diffusion outcome. Therefore, firm 1 chooses to transfer the technology if the duopoly payoff under diffusion \( R^{DS}_1 \) is higher than \( R^{DU}_1 \), I define:

\[
R^{DS}_1 - R^{DU}_1 = \frac{a^2 G(\beta_2)}{(3 + 2(\beta^*)^2) H^2}. \tag{19}
\]

Clearly, \( R^{DS}_1 - R^{DU}_1 > 0 \) if and only if \( G(\beta_2) > 0 \) and \( G(\beta'_2) \) is a continuously differentiable function. For simplicity, I rewrite:

\[
G(\beta_2) = G_1(\beta^*)\beta^2_2 + G_2(\beta^*)\beta_2^2 + G_3(\beta^*)\beta_2 + G_4(\beta^*). \tag{20}
\]

At \( \beta_2 = \hat{\beta}_2 \), \( R^{DS}_1 = 0 \), \( R^{DU}_1 > 0 \), and obviously, \( G(\hat{\beta}_2) < 0 \). To determine if \( G(\beta_2) < 0 \) for every \( \beta_2 \in (\hat{\beta}_2, \beta'_2) \), I next examine whether \( G(\beta_2) \) is a decreasing function in \( \beta_2 \). \( G(\beta_2) \) is a decreasing function in \( \beta_2 \) if:

\[
G'(\beta_2) = 3G_1(\beta^*)\beta^2_2 + 2G_2(\beta^*)\beta_2 + G_3(\beta^*) < 0. \tag{21}
\]

Solving the inequality, I have \( G'(\beta_2) < 0 \) as long as:

\[
G(\beta_2) = 128[(\beta^*)^6 - (\beta^*)^6 - (\beta^*)^4 + (\beta^*)^2]\beta^2_2 +
\sum [-320(\beta^*)^8 + 144(\beta^*)^7 - 112(\beta^*)^6 + 32(\beta^*)^5 + 368(\beta^*)^4 - 140(\beta^*)^3 + 84(\beta^*)^2 - 36\beta^* - 20]\beta^2_2 +
\sum [256(\beta^*)^8 - 224(\beta^*)^7 + 436(\beta^*)^6 - 320(\beta^*)^5 + 80(\beta^*)^4 - 88(\beta^*)^3 - 103(\beta^*)^2 - 16\beta^* - 21]\beta_2 +
\sum [-64(\beta^*)^8 + 80(\beta^*)^7 - 180(\beta^*)^6 + 176(\beta^*)^5 - 144(\beta^*)^4 + 116(\beta^*)^3 - 17(\beta^*)^2 + 24\beta^* + 9].
\]
\[ 0 \leq \beta_2 < \frac{-2G_2(x^*) + \sqrt{4(G_2(x^*))^2 - 12G_1(x^*)G_3(x^*)}}{6G_1(x^*)} \]

And \[ -2G_2(x^*) + \frac{\sqrt{4(G_2(x^*))^2 - 12G_1(x^*)G_3(x^*)}}{6G_1(x^*)} > \beta_2^t \] for every \( x^* \in (0,1] \). See Figure 4.

![Graph showing the gap between \(-2G_2(x^*) + \frac{\sqrt{4(G_2(x^*))^2 - 12G_1(x^*)G_3(x^*)}}{6G_1(x^*)}\) and \( \beta_2^t \). Figure 4.

Thus, \( G(\beta_2) \) is a decreasing function in \( \beta_2 \) when \( \hat{\beta}_2 < \beta_2 \leq \beta_2^t \). \( G(\hat{\beta}_2) < 0 \), and hence \( G(\beta_2) < 0 \) for every \( \beta_2 \in (\hat{\beta}_2, \beta_2^t] \). This indicates that \( R_1^{DS} < R_1^{DU} \) and firm 1 does not choose to transfer the technology in the first stage. Consequently, diffusion does not take place. I have completed the proof.

**Proof of Proposition 4.** First, I show that there exists a unique \( \beta_2^T \in (\frac{1}{2}, \frac{1}{2}) \) such that at \( \beta_2 = \beta_2^T \), \( R_1^{DS} = R_1^{DU} \). Furthermore, \( R_1^{DS} \geq R_1^{DU} \) when \( \beta_2^t < \beta_2 \leq \beta_2^T \), and the reverse happens when \( \beta_2 > \beta_2^T \).
To see this, when $0 < x^* < \bar{x}$, $\beta_2 > \max\{0, \hat{\beta}_2\}$. It is easy to see that at $\beta_2 = \beta_2^T$, $R_{1}^{DS} > 0$, $R_{1}^{DU} = 0$ and hence $G(\beta_2) > 0$. In addition, at $\beta_2 = \frac{1}{2}$, $G(\beta_2) < 0$ for every $x^* \in (0,1)$. See Figure 5.

![Graph showing the value of $G(\frac{1}{2})$](image)

Fig.5. The value of $G(\frac{1}{2})$.

I have established that $G(\beta_2) < 0$ and $G(\beta_2)$ is a decreasing function in $\beta_2$ when $0 \leq \beta_2 < \frac{1}{2}$. Hence, by continuity, there exists a unique $\beta_2^T \in (\beta_2, \frac{1}{2})$ such that at $\beta_2 = \beta_2^T$, $G(\beta_2) = 0$. Furthermore, $G(\beta_2) \geq 0$ and $R_{1}^{DS} \geq R_{1}^{DU}$ when $\beta_2 < \beta_2 \leq \beta_2^T$; and the reverse happens when $\beta_2 > \beta_2^T$.

Therefore, when $0 < x^* < \bar{x}$ and $\beta_2 < \beta_2 \leq \beta_2^T$, it is optimal for firm 1 to choose transfer in the first stage. Whether diffusion could take place also depends on firm 2’s choice in

$$\beta_2 > \frac{1}{2} \quad \text{when} \quad -2G_1(x^*) + \frac{\sqrt{4(G_2(x^*))^2 - 12G_1(x^*)G_3(x^*)}}{6G_1(x^*)} > \frac{1}{2}$$
the second stage, and I next check if firm 2 chooses to accept the technology when \(0 < x^* < x\) and \(\hat{\beta}_2 < \beta_2 \leq \beta_2^T\). As \(\beta_2\) is in this range, a duopoly always persists, and firm 2 chooses to accept the technology if the duopoly payoff under diffusion \(R_2^{DS}\) is higher than \(R_2^{DU}\), the duopoly payoff under non-diffusion. And I make a comparison between \(R_2^{DS}\) and \(R_2^{DU}\).

Similarly, I define \(R_2^{DS} - R_2^{DU} = \frac{a^2 g(\beta_2)}{[3 + 2(x^*)^2]^2 H^2}\). Using very similar arguments as the proof of Proposition 3, one can show that \(g(\beta_2) > 0\) and \(g(\beta_2)\) is increasing in \(\beta_2\) when \(\hat{\beta}_2 < \beta_2 \leq \beta_2^T\). Thus, \(g(\beta_2) > 0\) for every \(\beta_2 \in (\beta_2, \beta_2^T]\), which indicates that \(R_2^{DS} > R_2^{DU}\) and firm 2 chooses to accept the technology in the second stage.

As a result, when \(0 < x^* < x\) and \(\hat{\beta}_2 < \beta_2 \leq \beta_2^T\), both firm 1 and firm 2 agree to transfers the technology and accept it. And diffusion occurs consequently.

Notice that the diffusion set \(S\) consists of another subset \([\max\{0, \hat{\beta}_2\}, \min\{\beta_2^A, \beta_2\}]\) in addition to \((\beta_2, \beta_2^T]\). To understand why \(\beta_2 \in [\max\{0, \hat{\beta}_2\}, \min\{\beta_2^A, \beta_2\}]\) can lead to diffusion, consider first if \(\max\{0, \hat{\beta}_2\} \leq \beta_2 < \hat{\beta}_2\). Then it is obvious to see that firm 1 always chooses to transfer the technology. As shown above, when \(\beta_2\) is in this range, firm 2 has an opportunity to prey on firm 1. And firm 2 chooses to accept the technology.

\[g(\beta_2) = 128[-(x^*)^8 + (x^*)^6 + (x^*)^4 - (x^*)^2]\beta_2^3 + \]
\[304(x^*)^8 - 144(x^*)^7 + 96(x^*)^6 - 32(x^*)^5 - 292(x^*)^4 + 140(x^*)^3 - 124(x^*)^2 + 36x^* + 16]\beta_2^2 + \]
\[-224(x^*)^8 + 208(x^*)^7 - 348(x^*)^6 + 288(x^*)^5 - 120(x^*)^4 + 132(x^*)^3 + 29(x^*)^2 + 20x^* + 15]\beta_2 + \]
\[48(x^*)^8 - 64(x^*)^7 + 112(x^*)^6 - 112(x^*)^5 + 76(x^*)^4 - 64(x^*)^3 + 16(x^*)^2 - 12x^*].\]
instead of preying on firm 1 if \( \beta_2 \leq \beta_2^A \). Therefore, to ensure that diffusion is profitable to both firms, the apportionment is specified as \([\max\{0, \hat{\beta}_2\}, \min\{\beta_2^A, \beta_2\}]\).

**Proof of Proposition 5.** First, I determine the ranking of \( W^{DS}, W^{DU}, W^{MT} \) and \( W^{MN} \).

I will only provide a detailed description on the comparison of \( W^{DU} \) and \( W^{MT} \).

\[
W^{DU} = \frac{6\beta_2^2 + 10\beta_2 + 4 - (8\beta_2 + 6)x' + (-12\beta_2^2 + 2\beta_2 + 12)(x')^2 - (-8\beta_2 + 8)(x')^3 + (6\beta_2^2 - 12\beta_2 + 6)(x')^4}{H^4} a^3 > \frac{3}{2} + (x')^2 \frac{a^3}{[2 + 2(x')^3]}. \]

And it is easy to check that the above inequality holds provided that \( \beta_2 \geq 0 \), and hence \( W^{DU} > W^{MT} \) for every \( \beta_2 \in [0,1] \). Using very similar arguments, one can complete the comparison among \( W^{DS}, W^{DU}, W^{MT} \) and \( W^{MN} \). Hence, I have the following ranking:

\( W^{DS} > W^{DU} > W^{MT} > W^{MN} \).

Clearly, the most desirable equilibrium outcome is DS. This outcome only arises when \( 0 < x' < x \) and \( \beta_2 \) belongs to the diffusion set S. As a result, if \( 0 < x' < x \), every \( \beta_2 \) in the diffusion set S is efficient.
On the other hand, if $\chi \leq x^* < 1$, diffusion never takes place. And the most desirable equilibrium outcome is DU. Taking the derivative of $W^{DU}$ with respect to $\beta_2$, one can calculate that:

$$\frac{dW^{DU}}{d\beta_2} = \frac{f_1(x^*)\beta_2^2 + f_2(x^*)\beta_2 + f_3(x^*)}{H^4} a^{11}.$$

When $\chi \leq x^* < 1$, one can check that $f_1(x^*)$, $f_2(x^*)$, and $f_3(x^*)$ are all strictly positive (See Figure 6), and hence

$$\frac{dW^{DU}}{d\beta_2} = \frac{f_1(x^*)\beta_2^2 + f_2(x^*)\beta_2 + f_3(x^*)}{H^4} a^{11} > 0.$$ A larger $\beta_2$ improves welfare and the largest $\beta_2$ that can ensure this equilibrium outcome is $\beta_2^*$ (see the proof of Proposition 3). As a result, if $\chi \leq x^* < 1$, $\beta_2^*$ is the only efficient apportionment. I have completed the proof.

Fig.6. The value of $f_1(x^*)$, $f_2(x^*)$, and $f_3(x^*)$.

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$^{11}$ $f_1(x^*) = -32(x^*)^7 + 16(x^*)^6 + 96(x^*)^5 - 48(x^*)^4 - 96(x^*)^3 + 48(x^*)^2 + 32x^* - 16$;
$f_2(x^*) = 64(x^*)^7 - 48(x^*)^6 - 80(x^*)^5 + 76(x^*)^4 - 32(x^*)^3 - 8(x^*)^2 + 48x^* - 20$;
$f_3(x^*) = -32(x^*)^7 + 32(x^*)^6 - 16(x^*)^5 - 4(x^*)^4 + 32(x^*)^3 - 22(x^*)^2 + 16x^* - 6$. 
References


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