

**Subsidization of the Biofuel Industry:
Security vs. Clean Air?**

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ABSTRACT

Agriculture plays an important role both in reducing Greenhouse Gas Emissions (GHG) and dependence on imported oil from economically and politically volatile areas. Certain crops can be used as inputs for the production of alternative fuels. In addition to these two benefits, the development of biofuel industry has other secondary advantages including rural development. As the current price of biofuel is considerably greater than the price of fossil fuel alternatives, the governments around the world are heavily subsidizing the development of this industry. This paper focuses on the growth of biofuel industry in Canada and US. We develop a theoretical model to examine whether or not the same governmental policy (subsidization) yields different results i.e. a different level of optimal subsidies under different current objectives. We consider that subsidizing the development of the biofuel industry in the present is equivalent to buying an option on its use for future goals – energy security or reduced GHG emissions- so our research uses option value theory to assess these alternatives. The theoretical model yields an optimal subsidy option function for each of the two countries. Furthermore, under the scenario of obtaining different optimal levels of subsidies in the two countries, trade disputes can arise. A numerical simulation method is proposed to quantify the optimal level of subsidy option for each country.

Introduction

Today, countries face critical decisions regarding both climate change mitigation and energy security. Agriculture will play a role both in reducing Greenhouse Gas Emissions (GHG) and dependence on imported oil from economically and politically volatile areas because certain crops can be used as inputs for the production of biofuels. In addition to these two main benefits, a vibrant biofuel industry would contribute to rural development by creating new markets for agricultural commodities, creating new jobs in the rural sector and by increasing farm income.

In light of these developments, governments around the world are supporting the establishment of the biofuels industry. Biofuel production is expanding rapidly and it is at varying levels of development in different countries. In addition, governments heavily subsidize the industry as at the current level of industry development and historic fossil fuel price levels, the cost of biofuels is considerably greater than fossil fuel alternatives. Interestingly, in North America the US and Canada possess very different motivations with respect to the development of the biofuels industry. For the US, the key factor appears to be energy security, whereas within Canada increased use of biofuels is directly related to Kyoto commitments. Given these very different motivations, the optimal level of subsidies might be different in the two countries. These differing circumstances have the potential to create trade problems in the near future.

If we assume that subsidizing the development of the biofuel industry in the present is equivalent to buying an option on its use for future objectives – energy security or reduced GHG emissions – a theoretical model can be developed to examine whether or not the same government policy (subsidization) will yield different results (different level

of subsidies) if the option is based on different current objectives. Our theoretical model will be developed using financial option value theory, and optimal levels of energy subsidy will be generated using numerical simulations. The possibility of trade problems and trade disputes over this issue will also be analyzed within the framework of current international trade law.

The paper is organized as follows: in Section II, the characteristics of biofuel industry are presented with respect to the industries in Canada and US. In Section III, we conduct a brief literature review of option theory. The next section illustrates the results and some potential empirical analysis. The paper will end with some concluding remarks.

Biofuels Market Characteristics

There are two primary types of biofuels: biodiesel and ethanol. Since production of biodiesel is limited in the US and Canada, this paper will concentrate on ethanol.

Ethanol Market

Fuel ethanol is a high octane, water-free alcohol produced from any biological feedstock that contains sugar, or any materials that can be converted into sugar (starch, cellulose). Most of the world's ethanol is produced from sugar cane or sugar beets. Fuel ethanol can be used by itself as a fuel, but, normally, it is blended with gasoline in concentrations of 5, 10 up to 85 percent (commonly known as gasohol). The most common blend contains 10 percent ethanol (E10). Ethanol is obtained through two production processes: wet

milling and dry milling. The difference between the two methods of production lies in the initial treatment of the grain.

The two main advantages of using ethanol are, first, that it reduces the dependence on imports of foreign oil and, second, it has environmental benefits, including reduction of greenhouse gases and ground level ozone. Other secondary advantages of ethanol are that it is completely biodegradable. As well, being renewable, it helps to conserve resources, and finally it may potentially create new markets for agricultural commodities helping rural development by creating new jobs in rural sector and by increasing farm income.

Table 1. Emission reductions from ethanol blends

Emission	Low-level blends (E10)	High-level blends (E85)
Carbon Monoxide (CO)	25-30% decrease	25-30% decrease
Carbon Dioxide (CO ₂)	10% decrease	up to 100% decrease
Nitrogen Oxides (NO _x)	5% increase/decrease	up to 20% decrease
Volatile Organic Carbons (VOC):		
Exhaust	7% decrease	30% or more decrease
Evaporative	-	decrease
Sulphur Dioxide (SO ₂) and particulate matter	decrease	significant decrease
Aldehydes	30-50% increase (but negligible due to catalytic converter)	-
Aromatics (Benzene and Butadiene)	decrease	more than 50% decrease

Source: Canadian Renewable Fuels Association

The major disadvantage of ethanol at present is its high cost. Without being highly subsidized, an ethanol market would not exist. Even with a high level of support from federal and state/provincial governments, the cost of ethanol is higher than the price of gasoline. Ethanol produced from grain feedstock using conventional conversion

processes is not likely to compete with gasoline unless world oil price rises considerably. Other secondary disadvantages would be its high volatility that limits its use in hot weather, that is has a lower energy content per litre than gasoline and its potential to impair engine operation and possible corrosion in fuel system components in the case of phase separation¹.

In the US, the primary feedstock for ethanol is corn. Production of ethanol increased from 175 millions gallons in 1980 to 3.4 billion gallons in 2004 and 3.9 billion gallons in 2005. The increase in production is explained by the high level of support offered by the US federal and state governments for the development of this industry and also by banning the use of MTBE (methyl tertiary-butyl ether) as a gasoline additive in some states (California, New York and Connecticut) starting as of January 2004 (the federal ban of MTBE was postponed for 10 years). Through the Energy Policy Act of 2005, programs and policies were created that have the role of increasing and diversifying domestic energy production. The 2005 Act includes a renewable fuels standard (RFS) provision, which requires a minimum amount of renewable fuel each year. This starts at 4 billion gallons in 2006, reaching at 7.5 billion gallons in 2012, while afterwards the production of renewable fuel should grow at least at the same rate as gasoline production (Duffield and Collins, 2006).

Currently, there is a federal tax exemption of 5.4 US cents per US gallon for ethanol/gasoline blends that are 10 percent ethanol. For lower ethanol blends, tax exemption is reduced proportionally. Thus, the tax exemption for 1 US gallon of ethanol

¹ The phase separation can occur if excessive water is absorbed by ethanol. The result would be a mixture of alcohol and water in the bottom of a fuel tank.

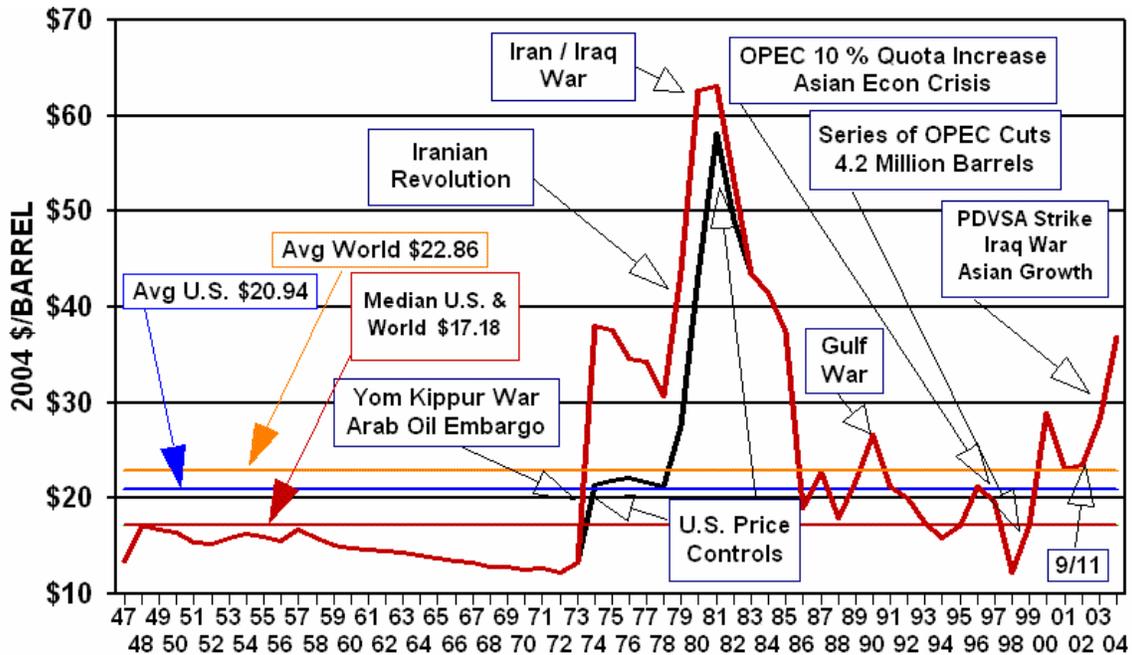
is 54 US cents (5.4 US cents per blended gallon). In 1998, the Federal tax exemption was extended until 2008, but was reduced to 5.3 US cents per blended gallon in 2001, to 5.2 US cents in 2004 and 5.1 US cents in 2005. The tax exemption of 5.1 US cents per blended gallon was subsequently extended until December 2010.

In addition to the federal tax exemption, at least 30 states have decided to subsidize the ethanol industry in different ways. One way is to offer an exemption from gasoline taxes when it is blended with ethanol. Another way is to give direct subsidies to the producers of ethanol. In addition, some states provide low-interest loans and require government vehicles to use ethanol. Different reasons for providing subsidies to the ethanol industry are provided by state governments (i.e. rural development, supporting prices etc), but mostly these differ from the principal motivation of the US government with respect to subsidizing the biofuels industry – energy security.

In different periods of time, different arguments have been offered to justify federal subsidy to alternative fuels. In the early 1970s, oil price and supply shocks were held responsible (Klass, 1995); in the 1980s, the “farm crisis” gave another argument for the development and use of corn-based ethanol (Duncan, 2004); the 1990 Clean Air Act Amendments (CAAA90) gave ethanol another boost because it can be used as a clean-air additive and in this century, oil price shocks are again being used as an argument for the development of a steady supply of alternative fuels.

It is well known that oil prices have been characterized by high volatility over the last 25 years (figure 1). This behavior is the result of numerous unforeseen natural, economic and political events. Furthermore, increases in the price of oil are not typically a long-term phenomenon, because high real prices deter consumption of oil, increase the

consumption of other energy sources and spur investments in finding new sources of energy resources.



Source: WTRG Economics

Figure 1. Crude Oil Prices – 2004 dollars (1947-2004)

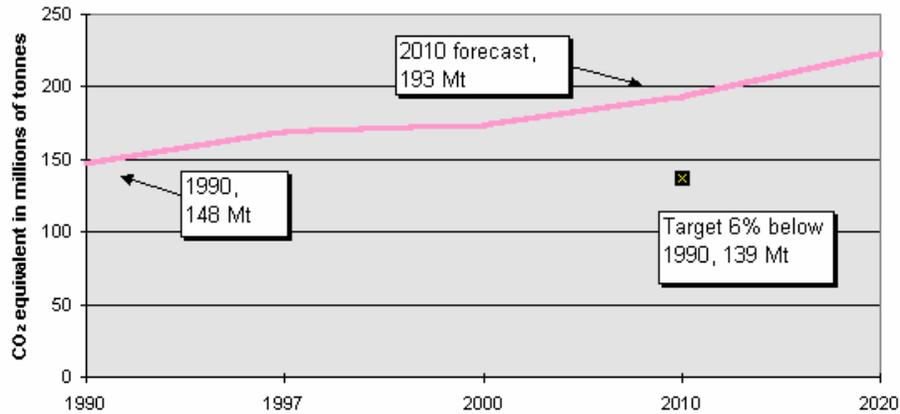
According to the US EIA, US dependency on foreign oil is estimated to grow from 62% in 2002 to more than 77% by 2025. Two thirds of the US petroleum demand was used in the transportation sector in 2002, and projections show that this percentage will increase. While the domestic supply of oil in US currently follows a decreasing trend, imports are increasing. US oil reserves declined from 39 billion barrels in 1970 to 22.7 billion barrels at the end of 2002. Domestic oil production is estimated to decrease further from 9.2 million barrels per day in 2002 to 8.6 million barrels per day in 2025, while oil consumption is expected to rise from 19.6 million barrels per day in 2002 to 28.3 million barrels per day in 2025. This decline of US oil reserves and the projected

increases in US demand for oil make it impossible for the US to increase energy security by using domestic oil supply (US Department of Energy). Reducing the transportation sector's reliance on oil, in particular, and improving vehicle fuel efficiency using biofuels, would reduce the demand for the imported oil (by an estimated 1.6 million barrels by 2012) and also energy dependency on a politically volatile part of the world (Renewable Fuel Association - RFA). Thus, while biofuels cannot eliminate US oil dependence anytime soon, the increasing production of biofuels would reduce this dependence and improve national response to oil supply disruptions.

In Canada, the development of the fuel ethanol industry has been far slower than in Brazil or the US. In Canada, ethanol is obtained from corn (73%), wheat (17%), barley (3%) and agricultural and forestry waste (7%).

Canada is a net exporter of petroleum based fuels and, as a result, does not have an energy security motivation for promoting biofuels. The domestic production of oil increased from 1.8 million barrels per day in 1980 to 3.1 million barrels per day in 2004, while the consumption of oil increased from 1.9 million barrels per day in 1980 to 2.3 million barrels per day in 2004. Thus, Canada is a net exporter of oil on the world market. In 2004, Canada was the seventh-largest world oil producer and consumer (EIA).

However in December 2002, the Government of Canada ratified the Kyoto Protocol. Under Kyoto, Canada has agreed to a GHG emissions reduction target of 6 percent below 1990 levels during the period 2008 to 2012. This means that Canada is committed to reduce 240 megatons of GHG emissions (figure 2).



Source: Transport Canada

Figure 2. Kyoto Protocol implications transport sector GHG emissions projections: 1990-2020

To support the ratification decision, the Government of Canada released the report entitled “*Climate Change Plan for Canada*” as a framework for action. During the period 1997 to 2002, the Government of Canada initiated a series of activities targeting the reduction of GHG emissions, including *Action Plan 2000*. *Action Plan 2000* proposes to achieve 65 megatons GHG emissions reduction per year during the commitment period 2008-2012. The transportation sector is expected to account for 10% of the total reduction (Government of Canada). The 2003 federal budget committed \$2 billion to climate change, of which \$1.3 billion was allocated to a series of concrete environmental policy measures by the summer 2003.

In order to reduce the GHG emissions to the specified level, the government planned to increase the production and consumption of ethanol. The federal government supports the development of the ethanol industry through different types of measures (Climate Change Saskatchewan). These include R&D programs for market development of technologies; \$0.28 US/gallon tax exemption for the ethanol portion of blended

gasoline; the use of ethanol by the federal government vehicles and the Future Fuels initiatives, with an increase of 750 million litres in Canada's annual capacity to produce ethanol, yielding a 25% increase of Canada's total gasoline supply containing 10% ethanol.

Provincial support depends on the goals of each province. For instance, Saskatchewan and Manitoba are interested in developing their rural economies, while British Columbia wants to stimulate the production of cellulose-based ethanol using forest wastes. Alberta, on the other hand, has shown little interest in the ethanol industry due to the size of the provincial conventional reserves (Klein et al, 2004). Thus, not all provinces have reductions of GHG emissions as their main objective for providing support to the biofuel industry.

In sum, the arguments used to explain the high level of governmental support for the development of the ethanol market in Canada are, first of all, environmental targets that need to be reached under Kyoto commitments and, second, rural development and new markets for the agricultural products.

Investment strategies and real options

Under certainty, there is no option value. Thus, the decision to invest can be made based on a simple Net Present Value (NPV) rule - invest when the present discounted value of the investment is greater than or equal to the investment cost. Traditional valuation methods in capital budgeting, as NPV and other discounted cash flow (DCF) techniques, are developed based on value maximization in a world without uncertainty and flexibility.

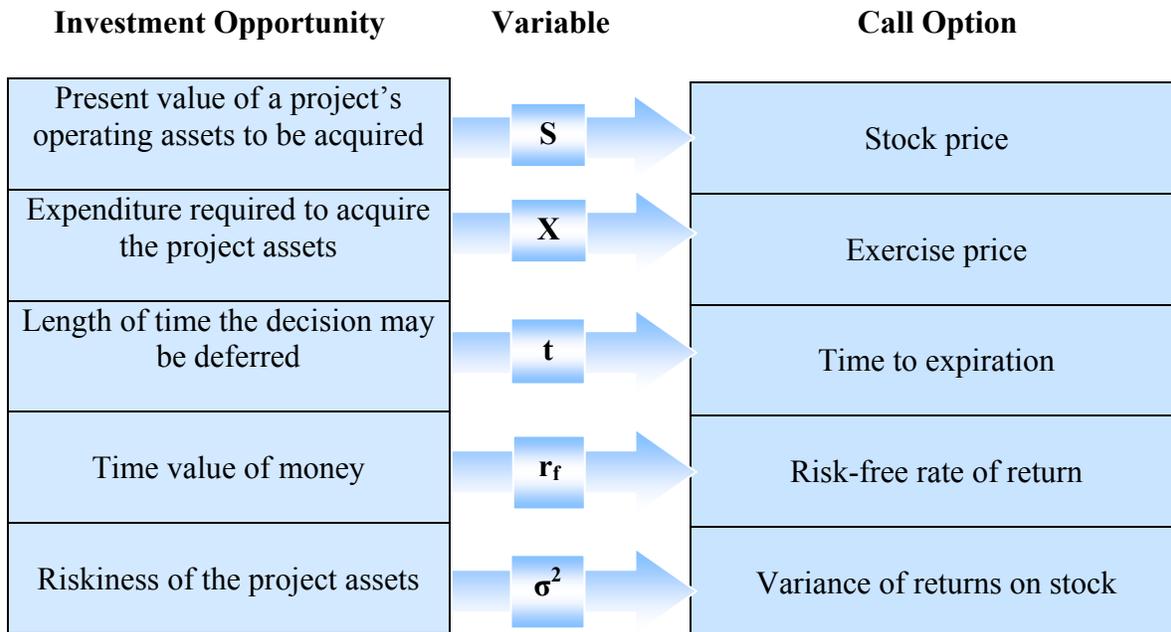
In reality, however, investment decisions have three important characteristics that fall outside the applicability of DCF (Dixit and Pindyck, 1994). Investments are often partially or completely irreversible - the cost of investment is partially sunk; investments are often undertaken under uncertainty over the future rewards; and investments can typically be postponed to get more information. The latter means that even a project with a negative NPV can be valuable as long as the investment can be postponed and new favorable information can arrive.

Real options and financial options

By definition, a financial option gives the owner the right, but not the obligation to sell (put option) or to purchase (call option) a security at a specified price (strike price) during a specified period of time. The seminal work of Black and Scholes (1973) first analyzed the valuation of financial options. The so-called Black-Scholes formula prices a European put or call option (meaning the option can be exercised only on the expiration date) on a stock that does not pay a dividend. Their model assumes that the stock price follows a geometric Brownian motion with constant volatility. One other important assumption of the model is that the underlying asset is tradable, allowing for the use of risk-neutral valuation.

Subsequent research has shown that the same basic definition of an option can be applied to other situations that do not involve the use of a financial asset. Thus, a firm that has the opportunity to invest holds an option, which is similar with the financial option. It has the right, but not the obligation to buy or sell an asset at some future time. When firms make an irreversible investment, they give up the possibility of waiting for

new information, which might affect the desirability and timing of the expenditure. This lost option is an opportunity cost that should be included in the cost of investment. Such non-financial options are called “real options”, stressing the strong link with the financial options (figure 3):



Source: Luehrman, T.A. (1998)

Figure 3. Mapping an Investment Opportunity onto a Call Option

The value of a real option increases as the stock price (S), time to expiration (t), risk-free rate of return (r_f) and variance of returns (σ^2) increase, and the exercise price (X) decreases.

As the discounted-cash-flow (DCF) approaches (e.g. NPV) applied in investment projects cannot capture the management's flexibility to revise decisions according to the changes and uncertainty that characterize the marketplace, returns from the projects will most probably differ from what management expected. As new information arrives,

management should be able to adapt to the conditions and to react accordingly to minimize the losses. Thus managerial flexibility in this manner is very important in an investment. Trigeorgis (1993) defines the managerial flexibility as a collection of real options - the option to defer, to abandon, to contract, to expand or to switch the investment.

Literature review

Option value theory has led to a rich literature pertaining to empirical applications that analyze investment opportunities. The method has also been used in many studies in environmental policy analysis. In agriculture, Purvis et al (1995) studied the adoption of free-stall dairy housing for Texas producers considering stochastic milk production and feed costs. Khanna et al (2000) analyzed the adoption of site-specific crop management under stochastic output prices and expectations of declining fixed costs of the equipment. Isik (2002) studied the impact of the uncertainty in cost-share subsidy policies on the adoption of site-specific technologies. In energy policy, Hasset and Metcalf (1995) analyzed residential energy investments considering that the energy price follow a Brownian motion. Millock and Nauges (2004) estimate an option value model on firms' actual abatement choice, the major uncertainty facing the firm being the future energy price.

Our investment problem can be included in a category of real options referred to by Trigeorgis (2001) as a time to build option (staged investment). This is important for R&D intensive industries and long-development capital-intensive projects. The development of a new market (e.g. biofuel market) has the following three

characteristics: the decisions and the cash outlays take place sequentially over time; there is a maximum rate of investment; and there are no returns until the project is completed (Majd and Pindyck, 1987).

This research is founded on several related studies in the options literature. Roberts and Weitzman (1981) built a model of a “sequential development project” (SDP) which has the same features outlined above. The project can be stopped in any stage and as the investment takes place, the cost of completing the project and its uncertainty (variance) are reduced. They derive an optimal sequential decision rule for R&D or exploration projects and they show that even the NPV is negative, the investor can go ahead with the first stages of the project. Weitzman, Newey and Rabin (1981) apply the sequential methodology to see whether the development of liquid synthetic fuels from coal market should be subsidized by the US government. McDonald and Siegel (1986) considered a basic model of irreversible investment with two stochastic variables, each of which evolves as geometric Brownian motion: the sunk cost and the value of the project. Their results show that the optimal investment is reached by waiting until the benefits are twice the investment cost.

Grossman and Shapiro (1986) studied the optimal dynamic R&D investments considering that the total effort to reach a payoff is unknown. Their work is more concentrated on the rate of investment rather than the decision of investing or not, considering that the rate of progress is a concave function of effort. They model the uncertainty in the returns as a Poisson process with the arrival rate as a function of the cumulative effort expended. They find that when uncertainty is introduced in the model,

firms prefer risky projects rather than the safe ones, when both have the same level of expected cost of effort.

Majd and Pindyck (1987) determine an optimal investment rule for a sequential investment, when firm can invest at a maximum rate, and the value of the project follows a geometric Brownian motion. An important characteristic of their model is that expenditure flow can be adjusted as new information arrives. They show that the biggest effects of time to build appear when the uncertainty is very high, the opportunity cost of delay is high and when the maximum rate of investment is low. Pindyck (1993) exploited the same idea as in Majd and Pindyck (1987) with the exception that the cost of completing the project is uncertain rather than the value of the project. He considered that the cost of completing the project includes two different kinds of uncertainty: technical uncertainty, which is related with the physical difficulty of completing the project and input cost uncertainty, which is external to what the firm does. Finally, Schwartz and Moon (2000) analyzed the investment in R&D (the development of a new drug) considering three sources of uncertainty: uncertainty about the investment cost, about the future payoffs and the possibility that a catastrophic event can stop the project. Their results include not only the value of the project, but also the optimal values for the state variables at which the investment should proceed.

Methodology

To our knowledge, there are no theoretical or empirical applications of a real option valuation model that have been developed to assess a governmental policy in the fashion

we suggest here. Further, existing environmental policy analyses study the behavior of firms for cases of investments that have environmental benefits.

Subsidy for the biofuel industry is considered as a normal investment that is undertaken by the government, and not by a firm. The goal of the government is to maximize social welfare. We will not consider the benefits that agricultural support programs bring to producers and consumers through increased crop prices or employment. The model will capture only the primary motives that the respective governments are seeking - environmental benefits (Canada) and decreased energy dependence (the US).

At a given period of time, the value of the government investment is measured by the increase in the social welfare due to the new investment. Knowing that without subsidization there will be no market for biofuels, the social welfare from the biofuel industry in case of no subsidy will be 0. Thus, the investment value would look as in equation 1.

$$(1) \quad V(I) = P * Q - wx - \tilde{S} + O * y + D(y)$$

where,

$V(I)$ = investment value in a certain time period;

P = the price of biofuels;

Q = the quantity consumed of biofuels;

w = per unit cost of inputs;

x = the inputs quantity used in producing the biofuels;

\tilde{S} = the total subsidy used by the governments to help the industry;

y = the quantity of oil that is replaced by the biofuels;

O = the price of oil;

D = damage function.

We assume that P and Q are uncertain. In the future, the production cost of biofuels might decrease and, as a result, the price would decrease and the quantity consumed would increase. Since prices cannot be negative, we let P follow a process of geometric Brownian motion with drift to reflect the stochastic innovations as well as any long-term trend in price evolution:

$$(2) \quad dP(t) = \alpha_P P(t)dt + \sigma_P P(t)dz_P(t)$$

where dz_P is the increment of a Wiener process: $dz_P = \varepsilon\sqrt{dt}$, $\varepsilon \sim N(0,1)$; α_P is the drift parameter, which represents the rate of growth and σ_P is the volatility in the drift parameter.

Also Q follows a process of geometric Brownian motion to reflect the stochastic innovations:

$$(3) \quad dQ(t) = \alpha_Q Q(t)dt + \sigma_Q Q(t)dz_Q(t)$$

The parameter α_Q is the expected rate of demand growth, while σ_Q is the standard deviation of the expected percentage change in demand. The variable dz_Q is a standard Wiener process with zero mean and standard deviation of dt . The relationship between Q and P is reflected in $E[dz_x dz_y] = \rho_{PQ}dt$ where $-1 \leq \rho_{PQ} \leq 0$ is the instantaneous correlation between Q and P. Negative correlation implies a downward sloping demand curve.

The cost of investment, which is represented by the subsidy \tilde{S} is considered uncertain as well. The uncertainty of \tilde{S} can be explained by the fact that developing an

energy market is a large project that takes considerable time. In top of being an uncertain cost, the investment is also irreversible. For different reasons, such as insufficient demand for the product or excessive costs, the government cannot recover the money spent on trying to develop the ethanol market.

In our case, the uncertainty in project costs is called technical uncertainty (Pindick, 1993). As mentioned earlier, technical uncertainty is related to the physical difficulty of completing the project, including both time and effort. In fact, the total cost of the project can be known only when it is completed. There is no value of waiting in this case, as all the information about cost arrives when the investment is taking place.

Following the investment model of Pindick (1993), we consider the expected cost of completing the project $S = E(\tilde{S})$, while the maximum rate of investment is I_m .

The changes in the expected cost of investment S is modeled using the following controlled diffusion process:

$$(4) \quad dS = -I dt + \sigma_S (IS)^{1/2} dz_S$$

where I represents the rate of investment, while dz_S is a Wiener process. Note in equation 4, the expected cost to completion declines with the rate of investment and also changes stochastically. The chosen functional form for the expected cost is very easy to manipulate, and yields just two solutions: no investment or investment at the maximum rate I_m . Notice that if $I=0$, the $dS=0$ and there is no technical uncertainty over the level of S required to develop the market. The stochastic term in equation 4 has a mean of 0, meaning that the expected level of S is unbiased. The variance of S is:

$$(5) \quad \sigma_S^2(S) = \left(\frac{\sigma_S^2}{2 - \sigma_S^2} \right) S^2$$

The mathematical solution for the mean and the variance of S are presented in Appendix A.

Equation 5 shows that as S decreases, the uncertainty in S decreases, which reflects learning with investment.

Further, the revenue that is obtained by replacing the imported oil with the biofuels: $y \cdot O$, is considered to be a benefit. The future price of oil, O, is a stochastic variable and it can be represented either by a simple random walk with a mean reversion or by a random walk with mean reversion and a jump process. We offer that the mean-reversion process is the natural choice for modeling the oil price because, even though oil price suffers short-term shocks, historically it has tended to revert back to a normal long-term equilibrium.

The simplest mean-reverting process known also as Geometric Ornstein-Uhlenbeck or Dixit and Pindyck model is:

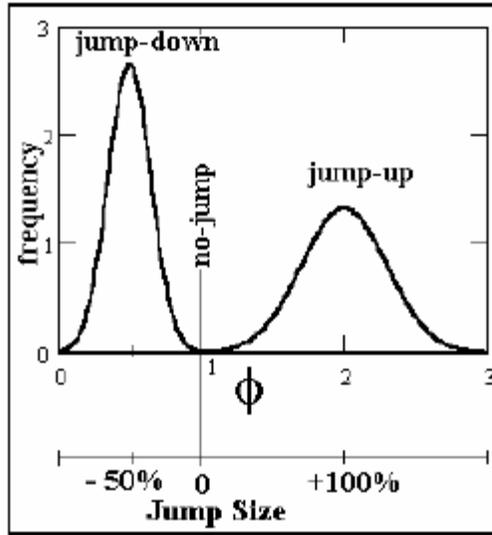
$$(6) \quad \frac{dO}{O} = \eta \left(\bar{O} - O \right) dt + \sigma_O dz_O$$

where the first term of equation 6 is the mean-reverting drift, \bar{O} is the long-run equilibrium mean and η is the speed of reversion. The second term represents continuous time uncertainty, where σ_O is the volatility and dz_O is a Wiener increment.

The jump-diffusion model links the changes in price and the arrival of information. This type of model combines two types of information: the smooth variation in the oil prices as an effect of normal news and the jump in prices caused by abnormal news. Smooth variation is modeled using a mean-reversion process (continuous process), while the jumps are modeled with a Poisson process (discrete time process). The Poisson-

jump for petroleum prices can be either positive or negative, depending on the specific economic/politic abnormal situations. This means that price can suffer a sudden increase or decrease. The inter-arrival times of successive jumps are independently and identically distributed (i.i.d.) random variables.

Figure 4 shows that the jump sizes for both cases, jump-up and jump-down, are random.



Source: Dias, M.A.G. and Rocha, K.M.C. (1999)

Figure 4. Random Jumps Distribution

Thus, the rate of change in oil price can be written as follows:

$$\frac{dO}{O} = \eta \left(\bar{O} - O \right) dt + \sigma_O dz_O + dq$$

$$(7) \quad dq = \begin{cases} 0, & \text{with probability } 1 - \lambda dt \\ \phi - 1, & \text{with probability } \lambda dt \end{cases}$$

$$k = E(\phi - 1)$$

Thus, the first term of equation 7 is the mean-reverting drift, where \bar{O} is the long-run equilibrium mean and η is the speed of reversion. The second term represents continuous time uncertainty, where σ is the volatility and dz_O is a Wiener increment. The last term is the jump term, which appears with a probability λdt (λ is a Poisson arrival parameter). ϕ is the jump size probability distribution, while the expected jump size is represented by k .

As the biofuels are used as a substitute for oil, there should be a relationship between the prices of the two products. As the price of oil increases, the price of biofuels should decrease. Thus, the relationship between P and O is reflected by ρ_{PO} , with $-1 \leq \rho_{PO} \leq 0$, where ρ_{PO} is the correlation coefficient between the two prices.

The last term of equation 1 above represents a damage function. Each unit of oil consumed produces an amount of GHG emissions. Assume e is the total emissions released by the oil consumption and each unit of oil consumed releases γ_i units of GHG emissions (CO_2 , NO_x , SO_2 etc.). It follows that: $e = y * \gamma_i$. Development of the biofuels industry does not increase the emission coefficient, so $\gamma_1 \leq \gamma_0$ (γ_1 and γ_0 are the emission coefficients after and before the introduction of biofuels) (Millock and Nauges, 2004). Thus, a reduction in GHG emissions represents an environmental benefit for society. In this case, the damage function equals the product between the total reduction in GHG emissions, e , and the price per unit of GHG emissions, τ : $D(y) = e * \tau$.

As the total emission, e , is a function of the quantity of oil consumed, y , is considered to be a stochastic variable. The quantity of biofuel consumed is unknown, rendering the quantity of oil that it is replaced a random variable. Assuming that y

follows a Brownian motion process, this leads to the following stochastic damage function:

$$(8) \quad dD = \gamma_i \tau (\alpha_y y dt + \sigma_y y dz_y)$$

where α_y is the drift parameter and σ_y is the volatility in the drift parameter, and dz_y is a Wiener process with the characteristic that $dz_y = \sqrt{dt}$.

We can derive decision rules for irreversible investment knowing that total cost is technically uncertain and also that the value of the investment represented by returns is stochastic. Our decision rule also includes the possibility that the project could be abandoned. The rule we will follow is that government will invest in developing the market as long as the expected cost of the investment is less than a critical value (Pindick, 1993). Furthermore, the investment value function does not include the damage function in case of the US, while for Canada, it does not include the benefits from the imported oil replacement. In this way, we differentiate between the two goals that motivate each country to subsidize their respective biofuel industries.

Ultimately, the problem is characterized by a compound option and is a sequential investment problem with technical uncertainty. It is a compound option because each annual investment, I , creates a new investment option on the present value of cost savings from the investment already done, with a diminished exercise price $S-I$. In fact, this kind of investment program can be temporarily or permanently suspended costlessly.

The optimal investment rules

The case of Canada

Using the assumptions outlined above, for Canada, the value of the biofuel investment that the government will maximize represented by the social welfare, is:

$$(9) \quad V(t) = P(t) * Q(t) + D(y)(t) - w * x - \tilde{S}(t) = P(t) * Q(t) + y(t) * \gamma_i * \tau - w * x - \tilde{S}(t)$$

An optimal investment rule can be found using contingent claim analysis. The model has four stochastic variables: P, Q, D(y) and S. Changes in each variable are modeled as a geometric Brownian motion using the processes in equations 2, 3, 8 and 4. The variables w and x are considered deterministic as they do not impact the value of the subsidy. We assume that the risks in P, Q, D(y) and S are spanned by existing assets and that is crucial to this method. We are able to make this assumption since the new product that will be developed is closely related to commodities (oil) that are usually traded on spot and future markets. Following Dixit and Pindyck (1993), we consider a portfolio for which we hold the option to invest. The opportunity to invest is worth F(P, Q, D(y), S). Appendix B shows that the investment opportunity F(P, Q, D, S) must satisfy the following stochastic differential equation:

$$(10) \quad \max_{I(t)} \frac{1}{2} F_{PP} \sigma_P^2 P^2 + \frac{1}{2} F_{QQ} \sigma_Q^2 Q^2 + \frac{1}{2} F_{DD} \sigma_y^2 y^2 \gamma_i^2 \tau^2 + \frac{1}{2} F_{SS} \sigma_S^2 S^2 + \\ + F_{PQ} \sigma_P \sigma_Q P Q \rho_{PQ} + F_P P (r - \delta_p) + F_Q Q (r - \delta_q) + F_D y \gamma_i \tau (r - \delta_d) + F_S I - I - rF = 0$$

Equation 10 is similar to a Bellman equation, obtainable using stochastic dynamic programming. The only differences between equation 10 and a standard Bellman equation would be that the riskless interest rate r, which is specified exogenously in our

model, is used in place of an exogenously specified discount rate and the growth rate α of the geometric Brownian motion is replaced by $r-\delta$. Thus, in case of contingent claim analysis, $\mu = \alpha + \delta$ represents the total expected rate of growth. The total expected rate of return μ represents the compensation that investors get for taking risk. The risk that is important here is nondiversifiable risk.

Equation 10 is linear in I . Thus the maximization problem gives us just two solutions:

$$(11) \quad I(t) = \begin{cases} I_m, & \frac{1}{2} F_{SS} \sigma_S^2 S - F_S - 1 \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

To interpret the solutions, government should invest as long as the expected cost to complete the project falls below a critical value. The general solution indicates that the market should be developed as long as the total subsidy (the cost of the project) is less or equal to the critical value $S^*(P, Q, D)$. In this case, government should invest at the maximum rate I_m . In the case where the total subsidy is greater than $S^*(P, Q, D)$, the investment should not be undertaken. Next, the value of S^* can be found as part of the solution of $F(P, Q, D, S)$. In fact, equation 10 is an elliptic partial differential equation with a free boundary along the space $S^*(P, Q, D)$.

To determine $S^*(P, Q, D)$ and $F(P, Q, D, S)$, we need to solve (10) subject to the following boundary conditions:

$$(12) \quad F(P, Q, D, 0) = V(P, Q, D)$$

$$(13) \quad \lim_{P \rightarrow 0} F(P, Q, D, S) = 0$$

$$(14) \quad \lim_{Q \rightarrow 0} F(P, Q, D, S) = 0$$

$$(15) \quad \lim_{D \rightarrow 0} F(P, Q, D, S) = 0$$

$$(16) \quad \lim_{S \rightarrow \infty} F(P, Q, D, S) = 0$$

$$(17) \quad \frac{1}{2} F_{SS}(S^*, P, Q, D) \sigma_S^2 S^* - F_S(S^*, P, Q, D) - 1 = 0$$

$$(18) \quad F(P, Q, D, S) \text{ continuous at } S^*(P, Q, D)$$

To summarize, equation 12 implies that at the end of the project, when the value of subsidy would be 0, the payoff would be $V(P, Q, D)$ - the value of the project. Equations 13, 14 and 15 all show that 0 is the absorbing barrier for P, Q and D, while equation 16 reflects that when S is very large, the probability of beginning the project in some finite time approaches 0. Equation 17 is derived from equation 10 and is equivalent to the “smooth pasting” condition (Pindyck, 1993) that $F_S(S^*, P, Q, D)$ is continuous at $S^*(P, Q, D)$. Finally, equation 18 is the “value matching” condition, meaning that $F(P, Q, D, S)$ is continuous at $S^*(P, Q, D)$.

Equation 10, together with the boundary conditions specified above, can be solved numerically using simulation, finding $S^*(P, Q, D)$ and $F(P, Q, D, S)$ at the same time. In order to perform a simulation, we need to obtain relevant data. Within the numerical simulation, a finite difference procedure is used to discretize Equation 10 considering either a maximum rate of investment or a 0 rate of investment.

Data for the model variables are available, except for the damage cost of GHG emissions for Canada. Prior studies specify that it is difficult to estimate a damage cost for CO₂ and other greenhouse gases because the damage due to global warming is just beginning to occur, so data are not available to estimate damage costs. Still, the damage

cost of GHG is estimated for US, OECD countries, countries of International Energy Agency and for the world.

To this end, Murphy and Delucchi (1998) review the literature on the social cost of motor vehicle use in the US. Among all the studies that are reviewed, only three estimated the damage cost of GHG emissions. Mackenzie et al (1992) estimated the annual cost for GHG not borne by drivers for 1989 at US \$27 billion; Ketcham and Komanoff (1992) estimated the climate change costs of roadway transportation borne by non-users for 1990 at US \$25 billion and the California Energy Commission (1994) used carbon emissions control costs of US \$28 per-ton of carbon to represent carbon value.

In a report published by the International Center for Energy Assessment (1995), the estimated annual cost of climate change for US falls between US \$3 to US \$27.5 billion in 1997 dollars.

Tol (1999) estimated the marginal cost of GHG emissions for the world using the FUND model. The estimates are between US\$9 and US\$23/tC, depending on the discount rate.

The Federal Office for Scientific, Technical and Cultural Affairs (2001) in Belgium estimated the external costs of transport use in Belgium. Part of the environmental costs is represented by the impact from greenhouse gases, which were estimated using the Open Framework and the FUND model. Their estimates (table 2) are for the period 2000-2009, with the costs discounted to the year 2000.

Table 2. Recommended marginal costs of GHG emissions

	Minimum	Low	Central estimates	High	Maximum
CO ₂ (€/tCO ₂)	0.1	1.4	2.4	4.1	16.4
N ₂ O(€/tN ₂ O)	24.3	440.2	748.3	1,272.1	5,242.1
CH ₄ (€/tCH ₄)	1.9	28.2	44.9	71.5	257
N(€/kgN)	-5.5	198.2	337	527.9	1,270.2
S(€/kgS)	-35.8	-16.6	-9.8	-5.8	0.0

Source: Federal Office for Scientific, Technical and Cultural Affairs, Belgium (2001)

In a subsequent study done by Victoria Transport Policy Institute (2004) in British Columbia, vehicle air pollution costs in British Columbia urban areas were assessed (table 3).

Table 3. Recommended shadow prices (1996 Canadian Cents per km)

	PM_{2.5}	Ozone	CFCs (Vehicles with AC)	Total (With AC)	Total (Without AC)
Light Gasoline Vehicle	0.6-1.0c	0.1c	2.7c	3.4-3.8c	0.7-1.1c
Light Diesel Vehicle	2.5-6.3c	0.1c	2.7c	5.3-9.1c	2.6-6.4c
Heavy Gasoline Vehicle	2-4c	0.1c	2.7c	4.8-6.8c	2.1-4.1c
Heavy Diesel Vehicle	9-27c	0.1c	2.7c	11.8-29.8c	9.1-27.1c

Source: Victoria Transport Policy Institute, 2004

This study also lists the European Commission estimates of greenhouse gas cost for 14 countries of US\$0.18 to US\$0.56 per gallon of gasoline or US\$0.009 to US\$0.028 per mile. In sum, without the benefit of a specific estimate of damage cost for GHG emissions in Canada, a proxy (damage cost of GHG emissions for US and for the world) will be used in this study.

The case of US

In case of the US, the investment value or the social welfare, that the government will maximize is expressed in the following equation:

$$(19) \quad V(I) = P(t) * Q(t) + O(t) * y(t) - w * x - \tilde{S}(t)$$

Once again, an optimal investment rule can be found using contingent claim analysis. This equation has five stochastic variables: P, Q, O, y and S. Changes in each of them follow a geometric Brownian motion, save for the price of oil, for which in the first section we use a mean-reverting process, while in the second section we use a jump-diffusion process. Thus changes are expressed by the equations 2, 3, 6 or 7, 8 and 4, while the risks in P, Q, O, y and S are spanned by existing assets in the same manner as for Canada. Finally, the opportunity to invest is worth $F(P, Q, O, y, S)$.

The partial differential equation objective function(s) that the US investment opportunity should satisfy are obtained in the same manner as Canada:

a) in case of using a mean-reverting process for the change in oil price:

$$(20) \quad \begin{aligned} & \max_{I(t)} \frac{1}{2} F_{PP} \sigma_P^2 P^2 + \frac{1}{2} F_{QQ} \sigma_Q^2 Q^2 + \frac{1}{2} F_{OO} \sigma_O^2 O^2 + \frac{1}{2} F_{yy} \sigma_y^2 y^2 + \\ & + \frac{1}{2} F_{SS} \sigma_S^2 IS + F_{PQ} \sigma_P \sigma_Q PQ \rho_{PQ} + F_{PO} \sigma_P \sigma_O PO \rho_{PO} + \\ & + F_P P(r - \delta_p) + F_Q Q(r - \delta_q) + F_O O(r - \delta_o) + F_y y(r - \delta_y) - IF_S - I = rF \end{aligned}$$

b) in case of using a jump-diffusion model for the change in oil price, we obtain:

$$(21) \quad \begin{aligned} & \max_{I(t)} \frac{1}{2} F_{PP} \sigma_P^2 P^2 + \frac{1}{2} F_{QQ} \sigma_Q^2 Q^2 + \frac{1}{2} F_{OO} \sigma_O^2 O^2 + \frac{1}{2} F_{yy} \sigma_y^2 y^2 + \\ & + \frac{1}{2} F_{SS} \sigma_S^2 IS + F_{PQ} \sigma_P \sigma_Q PQ \rho_{PQ} + F_{PO} \sigma_P \sigma_O PO \rho_{PO} + \\ & + F_P P(r - \delta_p) + F_Q Q(r - \delta_q) + F_O O(r - \delta_o) - \lambda KOF_O + \lambda F(O\Phi) - \\ & - (r + \lambda)F(O) + F_y y(r - \delta_y) + F_S S(r - \delta_s) - I = 0 \end{aligned}$$

In these cases, the total expected returns equals $\mu = \eta(\bar{O} - O) + \delta_o$.

Equations 20 and 21 are linear in I, meaning this maximization problem again gives us a corner solution:

$$(22) \quad I(t) = \begin{cases} I_m, \frac{1}{2} F_{SS} \sigma_S^2 S - F_S - 1 \geq 0 \\ 0, \text{otherwise} \end{cases}$$

Not surprisingly, we obtain the same basic investment rule as that found for Canada: the government should invest at the maximum rate I_m as long as $S \leq S^*(P, Q, O, y)$, where S^* represents a critical value. In addition, the investment should not be continued if $S > S^*(P, Q, O, y)$. The critical value S^* can be found as part of the solution of $F(P, Q, O, y, S)$. Once again, equations 20 and 21 are elliptic with a free boundary along the space $S^*(P, Q, O, y)$. Again, in this case we have boundary conditions that together with equations 20 or 21 will help us finding $S^*(P, Q, O, y)$ and $F(P, Q, O, y, S)$:

$$(23) \quad F(P, Q, O, y, 0) = V(P, Q, O, y)$$

$$(24) \quad \lim_{P \rightarrow 0} F(P, Q, O, y, S) = 0$$

$$(25) \quad \lim_{Q \rightarrow 0} F(P, Q, O, y, S) = 0$$

$$(26) \quad \lim_{O \rightarrow 0} F(P, Q, O, y, S) = 0$$

$$(27) \quad \lim_{y \rightarrow 0} F(P, Q, O, y, S) = 0$$

$$(28) \quad \lim_{S \rightarrow \infty} F(P, Q, O, y, S) = 0$$

$$(29) \quad \frac{1}{2} F_{SS}(S^*, P, Q, O, y) \sigma_S^2 S^{*2} - F_S(S^*, P, Q, O, y) - 1 = 0$$

$$(30) \quad F(P, Q, O, y, S) \text{ continuous at } S^*(P, Q, O, y)$$

Equations 20 or 21 together with the boundary conditions can be solved numerically, by finding $S^*(P, Q, O, y)$ and $F(P, Q, O, y, S)$ at the same time. Future work will use a finite difference method to discretize Equations 20 and 21 considering a maximum rate of investment or a 0 rate of investment.

Summary and conclusions

Issues regarding climate change and energy security have led to the continued development of alternative fuels. In addition, the agricultural industry views biofuels as a future growth area that could help save a declining industry. Therefore, development of the biofuel market is perceived to have two main benefits for North America. These are a reduction of Greenhouse Gas Emissions (GHG) and reduced dependence on imported oil from economically and politically volatile areas. Secondary advantages associated with using biofuels are represented through rural development and the creation of new markets for agricultural commodities, leading to new jobs in the rural sector. One main disadvantage of biofuel at the moment is its high cost relative to petroleum. Thus, without being highly subsidized by governments, a biofuel market will not grow.

This paper first concentrates on the problem faced by Canada. Increased use of biofuels will help Canada meet Kyoto commitments. Next, we look at the US perspective on the issue, where the key factor is energy security. By investing today in the biofuel

market, governments effectively purchase an option for the future either through Kyoto commitments or energy security.

A theoretical real options model is developed to examine whether the same governmental policy (subsidization) will yield different levels of optimal subsidies with different objectives. The model indicates that optimal biofuel subsidy in Canada is a function of price and quantity of biofuel consumed and the quantity of oil that is replaced by biofuel. In contrast, in the US optimal biofuel subsidy is a function of these parameters plus the price of oil. Solving these respective stochastic differential equations will be done using numerical methods (finite difference methods). The two levels of optimal subsidies per capita will be precisely quantified in future research.

Note that if the level of subsidies for the development of biofuels industry in the two countries is different, trade disputes can arise. Disputes will be a function of whether the subsidies are included in the so-called “green box” or not. If they are included in the green box being used for environmental reasons, they will not be actionable or limited, so there will be no trade problems. In this case, there should exist scientific evidence that environmental benefits are provided and, second, the industry is subsidized because the biofuels are not priced competitively. However, the lack of clarity in current trade law in these areas means that future trade disputes are likely to arise (Kerr and Loppacher, 2004).

REFERENCES

Black, F., and M. Scholes. 1973. “The pricing of options and corporate liabilities.”
Journal of Political Economy May-June: 637-654.

- Canadian Renewable Fuels Association. <http://www.greenfuels.org/>.
- Climate Change Saskatchewan. “Fueling a new economy.” Downloaded from:
<http://www.climatechangesask.ca/downloads/Ethanol.may9.pdf>.
- Dias, M.A.G., and K.M.C. Rocha. 1999. “Petroleum concessions with extendible options using mean reversion with jumps to model the oil prices.” Working paper no. 620, Institute for Applied Economic Research. Downloaded from:
http://papers.ssrn.com/sol3/papers.cfm?abstract_id=159692.
- Dixit, A.K., and R.S. Pindyck. 1994. *Investment under uncertainty*, New Jersey: Princeton University Press, Princeton.
- Duffield, J.A., and K. Collins. 2006. “Evolution of Renewable Energy Policy.” *Choices Magazine* 21(1). Downloaded from:
<http://www.choicesmagazine.org/2006-1/biofuels/2006-1-02.htm>.
- Duncan, M. 2004. “U.S. Federal initiatives to support biomass research and development.” *Journal of Industrial Ecology*. 7 (3-4): 193-201.
- Energy Information Administration. <http://www.eia.doe.gov/>.
- Federal Office for Scientific, Technical and Cultural Affairs, Belgium. 2001. “The external costs of transportation.” Final report, downloaded from:
http://www.belspo.be/belspo/home/publ/pub_ostc/mobil/rapp04_en.pdf.
- Government of Canada. “Action Plan 2000 on Climate Change.” Downloaded from:
http://www.climatechange.gc.ca/english/publications/ap2000/Action_Plan_2000_en.pdf.
- Grossman, G.M., and C. Shapiro. 1986. “Optimal Dynamic R&D Programs.” *Rand Journal of Economics* 17(4): 581-593.

- Hasset, K. A., and G.E. Metcalf. 1995. "Energy tax credits and residential conservation investment: Evidence from panel data." *Journal of Public Economics*. 57: 201-217.
- Isik, M. 2004. "Incentives for technology adoption under environmental policy uncertainty: Implications for green payment programs." *Environmental and resource economics* 27(3): 247-263.
- Karlin, S., and H.M. Taylor. 1981. *A second course in stochastic processes*, New York: Academic Press.
- Kerr, W.A., and L.J. Loppacher. 2004. "Trading biofuels – Will international trade law be a constraint?." Paper prepared for *Current Agriculture, Food and Resource Issues*.
- Khanna, M., M. Isik, and A. Winter-Nelson. 2000. "Investment in site-specific crop management under uncertainty: implications for nitrogen pollution control and environmental policy." *Agricultural Economics*. 24: 9-21.
- Klass, D. 1995. "Biomass energy in North American policies." *Energy Policy* 23 (12): 1035-1048.
- Klein, K., R. Romain, M. Olar, and N. Bergeron. June 2004. "Ethanol policies, programs and production in Canada." Paper presented at Agriculture as a Producer and Consumer of Energy conference, June.
- Luehrman, T.A. 1998. "Investment Opportunities as Real Options: Getting Started on the Numbers." *Harvard Business Review*, pp. 54-67.
- Majd, S., and R.S. Pindyck. 1987. "Time to build, option value, and investment decisions." *Journal of Financial Economics* 18: 7-27.

- McDonald, R., and D. Siegel. 1986. "The value of waiting to invest." *Quarterly Journal of Economics* 101(4): 707-727.
- Millock, K., and C. Nauges. 2004. "Energy price uncertainty and firm abatement investment choice." Paper presented at EAERE thirteen annual conference, Budapest, Hungary.
- Murphy, J.J., and M.A. Delucchi. 1998. "A review of the literature on the social cost of motor vehicle use in the United States." *Journal of Transportation and Statistics* 1 (1): 15-42.
- Pindyck, R.S. 1993. "Investment of uncertain cost." *Journal of Financial Economics* 34: 53-76.
- Purvis, A., G. Boggess, C.B. Moss, and J. Holt. 1995. "Technology adoption decisions under irreversibility and uncertainty: an ex ante approach." *American Journal of Agricultural Economics* 77: 541-551.
- Renewable Fuels Association. 2004. "Synergy in Energy: Ethanol industry outlook in 2004." Downloaded from: <http://www.ethanolrfa.org/2004outlook.pdf>.
- Renewable Fuels Association. 2001. "Small producer credit amendment sought in economic stimulus bill." Bob Dinneen – RFA President. Downloaded from: <http://www.ethanolrfa.org/ereports/er102401.html>.
- Roberts, K., and M.L. Weitzman. 1981. "Funding criteria for research, development, and exploration projects." *Econometrica* 49: 1261-1288.
- Schwartz, E.S., and M. Moon. 2000. "Evaluating Research and Development Investments." In Michael J. Brennan and Lenos Trigeorgis, ed. *Project Flexibility, Agency, and Competition*. New York: Oxford University Press Inc., pp. 85-106.

Transport Canada. 1999. "Transportation and climate change: options for action."

Downloaded from:

<http://www.tc.gc.ca/programs/environment/climatechangesubgroups1/OptionsPaper/English/default.htm>.

Tol, R.S.J. 1999. "The marginal cost of greenhouse gas emissions." *The Energy Journal* 20 (1): 61-81.

Trigeorgis, L. 1993. "The Nature of Option Interactions and the Valuation of Investments with Multiple Real Options." *The Journal of Financial and Quantitative Analysis* 28 (1):1-20.

Trigeorgis, L. 2001. "Real Options: An Overview." In Eduardo S. Schwartz and Lenos Trigeorgis, ed. *Real Options and Investment under Uncertainty: Classical Readings and Recent Contributions*. Cambridge, Massachusetts: MIT Press, pp. 103-134.

US Department of Energy. <http://www.energy.gov>.

Victoria Transport Policy Institute. 2004. "Transportation Costs and Benefits: Resources for measuring transportation costs and benefits." TDM Encyclopedia. http://www.vtpi.org/tdm/tdm66.htm#_Toc18284958.

Weitzman, M.L., W. Newey, and M. Rabin. 1981. "Sequential R&D strategy for synfuels." *Bell Journal of Economics* 12(2): 574-590.

WTRG Economics. "Oil Price History and Analysis." Downloaded from:

<http://www.wtrg.com/prices.htm>.

APPENDIX A

Mean and variance of \tilde{S}

We know that $S(t)$ follows the following controlled diffusion process:

$$(A1) \quad dS = -I dt + \sigma_S (IS)^{1/2} dz$$

Following Pindyck (1993), we define the function $M(S)$:

$$(A2) \quad M(S) = E_t \left[\int_t^T I d\tau | S(t) \right]$$

where T is the first time interval for $S=0$. We show that the expected value of S , which is expressed by $M(S)$ would equal S .

In order to derive the mean of S , we solve the Kolmogorov backward equation:

$$(A3) \quad \frac{1}{2} \sigma_S^2 IS M_{SS} - IM_S + I = 0$$

subject to the following boundary conditions:

$$(A4) \quad \begin{aligned} M(0) &= 0 \\ M(\infty) &= \infty \end{aligned}$$

One clear solution of equation A33 subject to boundary conditions A34 is $M(S)=S$.

The variance of S would equal:

$$(A5) \quad Var(S) = E_t \left[\int_t^T I d\tau | S \right]^2 - S^2(t)$$

We consider $G(S) = E_t \left[\int_t^T I d\tau | S \right]^2$ and we solve Kolmogorov backward equation (Karlin

and Taylor, 1981):

$$(A6) \quad \frac{1}{2} \sigma_S^2 ISG_{SS} - IG_S + 2IS = 0$$

subject to the following boundary conditions:

$$(A7) \quad \begin{aligned} G(0) &= 0 \\ G(\infty) &= \infty \end{aligned}$$

The solution of this equation is:

$$(A8) \quad G(S) = \frac{2S^2}{2 - \sigma_S^2}$$

From equations A35 and A38, we obtain the variance of S:

$$(A9) \quad Var(S) = \left(\frac{\sigma_S^2}{2 - \sigma_S^2} \right) S^2$$

APPENDIX B

Canada

Contingent claim analysis

Having that the opportunity to invest is worth $F(P, Q, D(y), S)$, n units of the asset with price P , m units of the asset with price Q , a units of the asset with price $D(y)$ and b units of the asset with price S are sold short.

The value of the portfolio is:

$$(B1) \quad \Phi = F - n * P - m * Q - a * D(y) - b * S$$

As P , Q , $D(y)$ and S change, also the n , m , a and b will change from one short time interval to the next. Thus, the composition of the portfolio will continuously be changed.

For each short time interval dt , we keep n , m , a and b fixed.

As no rational investor would hold a long-run position in the project without a dividend payment, the short position in this portfolio requires payments of $\delta_p nP$, $\delta_q mQ$, $\delta_d a y \gamma_i \tau$ and $\delta_s bS$. δ represents in fact an opportunity cost of delaying the project and keeping the option to invest alive (Dixit and Pindick, 1994). In addition, as the investment is taking place a payment stream of $I(t)$ should be paid to hold the investment opportunity. Thus, the total value of the portfolio is:

$$(B2) \quad \Phi = F - n * P - m * Q - a * y \gamma_i \tau - b * S - \delta_p nP - \delta_q mQ - \delta_d a y \gamma_i \tau - \delta_s bS - I(t)$$

As a result, the instantaneous change in the value of the portfolio is:

$$(B3) \quad \begin{aligned} d\Phi = & dF - ndP - n\delta_p Pdt - mdQ - m\delta_q Qdt - adD - a\delta_d y \gamma_i \tau dt - \\ & - bdS - b\delta_s Sdt - I(t)dt \end{aligned}$$

Using Ito's Lemma, we obtain:

$$\begin{aligned}
 (B4) \quad dF &= F_P dP + F_Q dQ + F_D dD + F_S dS + \frac{1}{2} F_{PP} dP^2 + \frac{1}{2} F_{QQ} dQ^2 + \\
 &+ \frac{1}{2} F_{DD} dD^2 + \frac{1}{2} F_{SS} dS^2 + F_{PQ} dP dQ
 \end{aligned}$$

where:

$$\begin{aligned}
 (B5) \quad dP &= \alpha_P P dt + \sigma_P P dz_P \Rightarrow dP^2 = \sigma_P^2 P^2 dt \\
 dQ &= \alpha_Q Q dt + \sigma_Q Q dz_Q \Rightarrow dQ^2 = \sigma_Q^2 Q^2 dt \\
 dP dQ &= \sigma_P \sigma_Q \rho_{PQ} dt \\
 dD &= \gamma_i \tau (\alpha_y y dt + \sigma_y y dz_y) \Rightarrow dD^2 = \sigma_y^2 y^2 \gamma_i^2 \tau^2 dt \\
 dS &= -I dt + \sigma_S (IS)^{1/2} dz_S \Rightarrow dS^2 = \sigma_S^2 IS dt
 \end{aligned}$$

To make the portfolio riskless, we choose $n = F_P, m = F_Q, a = F_D, b = F_S$.

Thus, the change in the value of the portfolio becomes:

$$\begin{aligned}
 (B6) \quad d\Phi &= F_P dP + F_Q dQ + F_D dD + F_S dS + \frac{1}{2} F_{PP} \sigma_P^2 P^2 dt + \frac{1}{2} F_{QQ} \sigma_Q^2 Q^2 dt + \\
 &+ \frac{1}{2} F_{DD} \sigma_y^2 y^2 \gamma_i^2 \tau^2 dt + \frac{1}{2} F_{SS} \sigma_S^2 IS dt + F_{PQ} \sigma_P \sigma_Q \rho_{PQ} dt - F_P dP - F_P \delta_P P dt - \\
 &- F_Q dQ - F_Q \delta_Q Q dt - F_D dD - F_D \delta_D y \gamma_i \tau dt - F_S dS - F_S \delta_S S dt - I(t) dt
 \end{aligned}$$

$$\begin{aligned}
 (B7) \quad d\Phi &= \frac{1}{2} F_{PP} \sigma_P^2 P^2 dt + \frac{1}{2} F_{QQ} \sigma_Q^2 Q^2 dt + \frac{1}{2} F_{DD} \sigma_y^2 y^2 \gamma_i^2 \tau^2 dt + \frac{1}{2} F_{SS} \sigma_S^2 IS dt + \\
 &+ F_{PQ} \sigma_P \sigma_Q \rho_{PQ} dt - F_P \delta_P P dt - F_Q \delta_Q Q dt - F_D \delta_D y \gamma_i \tau dt - F_S \delta_S S dt - I(t) dt
 \end{aligned}$$

Over the time interval $(t, t+dt)$, the holder of the portfolio will have the capital gain:

$$\begin{aligned}
 (B8) \quad &\frac{1}{2} F_{PP} \sigma_P^2 P^2 dt + \frac{1}{2} F_{QQ} \sigma_Q^2 Q^2 dt + \frac{1}{2} F_{DD} \sigma_y^2 y^2 \gamma_i^2 \tau^2 dt + \frac{1}{2} F_{SS} \sigma_S^2 IS dt + \\
 &+ F_{PQ} \sigma_P \sigma_Q \rho_{PQ} dt
 \end{aligned}$$

In the same time interval, the cost of portfolio is:

$$(B9) \quad F_P \delta_P P dt + F_Q \delta_Q Q dt + F_D \delta_D y \gamma_i \tau dt + F_S \delta_S S dt + I(t) dt$$

The next step is to equate the value of the portfolio over the time interval (t, t+dt) to the riskless return:

$$(B10) \quad r(F - nP - mQ - aD - bS)$$

After collecting the terms, we get the basic equation:

$$(B11) \quad \frac{1}{2} F_{PP} \sigma_P^2 P^2 + \frac{1}{2} F_{QQ} \sigma_Q^2 Q^2 + \frac{1}{2} F_{dd} \sigma_y^2 y^2 \gamma_i^2 \tau^2 + \frac{1}{2} F_{SS} \sigma_S^2 IS + \\ + F_{PQ} \sigma_P \sigma_Q PQ \rho_{PQ} + F_P P(r - \delta_p) + F_Q Q(r - \delta_q) + F_d y \gamma_i \tau (r - \delta_d) + \\ + F_S S(r - \delta_S) - I = rF$$

where,

$$(B12) \quad S(r - \delta_S) = -I$$