Information Theoretic Estimators of the First-Order Spatial Autoregressive Model

by

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Abstract

Information theoretic estimators for the first-order autoregressive model are considered. Extensive Monte Carlo experiments are used to compare finite sample performance of traditional and three information theoretic estimators including maximum empirical likelihood, maximum empirical exponential likelihood, and maximum log Euclidean likelihood. It is found that information theoretic estimators are robust to specification of spatial autocorrelation and dominate traditional estimators in finite samples with the only exception of the quasi-maximum likelihood estimator. Finally, the proposed estimators are applied to an illustrative example of hedonic housing pricing.

Key words: information theoretic estimators, the first-order spatial autoregressive model

JEL classification: C3

Variables, related through their location, might directly influence economic behavior and equilibrium outcomes. For instance, spatial patterns are found in regional adoption of agricultural technology (Case 1992) and in a household behavior when a household gains utility in consuming bundles similar to those consumed by their neighbors (Case 1991). Spatial spillover effects arise from technical innovations (Anselin, Vagra, and Acs 1997), and geographic proximity of a firm's competitors might directly determine its marketing strategy (Haining 1984). Recent empirical studies in agriculture focused on the spatial aspects of technology adoption, structure of production, market efficiency, arbitrage, and integration (Wu 2001; Goodwin and Piggott 2001; Roe, Irwin, and Sharp 2002; Thompson, Donggyu, and Bohl 2002; Sephton 2003; Saak 2003; Sarmiento and Wilson 2005; Negassa and Myers 2007).

Information Theoretic (IT) estimators are alternatives to traditional estimators (Owen 1988, 1991; Kitamura and Stutzer 1997) and have been applied in a variety of modeling contexts (Fraser 2000; Golan, Judge, and Zen 2001; Preckel 2001; Miller 2002; Zohrabian et al. 2003). These are asymptotically efficient estimators that might have superior sampling properties in terms of mean squared error relative to traditional estimators (Imbens, Spady, and Johnson 1998). Application of the IT estimators to the spatial regression models is absent from the literature. The only exception is an extensive Monte Carlo study of a generalized maximum entropy estimator by Marsh and Mittelhammer (2004). It is important to explore the IT estimators in the context of the first-order spatial autoregressive model on the basis of their finite sample properties to provide guidance for applied economists working primarily with small and medium size samples.

The objectives of this paper are to propose a generalized information theoretic (GIT) estimator of the first-order spatial autoregressive model, to compare finite sample properties of selected IT estimators with traditional ones, and provide an illustrative example. The IT estimators analyzed include the maximum empirical likelihood, maximum empirical exponential likelihood, and maximum log euclidean likelihood estimators. The finite sample properties of these estimators are investigated in the context of an extensive Monte Carlo analysis conducted over a range of finite sample sizes, a range of spatial autoregressive coefficients, selected forms of heteroscedasticity, distributional assumptions of disturbance term, and different forms of spatial weight matrix typically used in applied econometric work. The estimators are compared among themselves and traditional estimators on the basis of root mean square errors, and response functions are used to summarize findings of all Monte Carlo experiments. Finally, each of these IT estimators are applied to an empirical application using the Harrison and Rubinfeld (1978) example (Gilley and Pace 1996).

The extensive sampling results examined in this paper suggest that the IT estimators are robust to a range of sample sizes, spatial autocorrelation coefficients, distributional assumptions of disturbance term, and spatial weight matrices. It is found that the IT estimators dominate the standard ordinary least squares, two-stage least squares and generalized method of moments estimators on the basis of parameter estimator risk properties with the only exception of quasi-maximum likelihood estimator. These results provide small sample justification of the IT estimators to estimation parameters of the first-order spatial autoregressive model.

The paper proceeds in the following manner. First, the first-order spatial autoregressive model is reviewed and its traditional estimators are presented. Second, the spatial IT estimators are introduced and specified. Finally, Monte Carlo results, an empirical application, and concluding remarks are provided.

First-Order Spatial Autoregressive Model

Consider a first-order spatial autoregressive model[2]

$$(1) \quad \mathbf{Y} = \rho \mathbf{W}\mathbf{Y} + \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}, \quad |\rho| < 1$$

where $\mathbf{Y}$ is a $n \times 1$ dependent variable vector, $\mathbf{X}$ is $n \times k$ matrix of exogenous variables with full column rank, $\mathbf{W}$ is a $n \times n$ spatial proximity matrix of constants,[3] $\boldsymbol{\beta}$ is a $k \times 1$ vector of parameters, ρ is a scalar spatial autoregressive parameter,[4] and $\boldsymbol{\varepsilon}$ is $n \times 1$ vector of unobserved residuals with $E(\boldsymbol{\varepsilon}) = \mathbf{0}$ and $E(\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}') = \sigma^2\boldsymbol{\Omega}$. This specification of disturbance term allows for general patterns of autocorrelation and heteroscedasticity.

A number of alternative estimation procedures for the model with and without spatial disturbances has been proposed in the literature (Anselin and Bera 1998).[5] The ordinary least squares (OLS) estimator is biased and inconsistent due to simultaneity bias (Whittle 1954; Ord 1975; Lee 2002).[6] Maximum likelihood (ML) estimator (with normally distributed disturbance term) is consistent and asymptotically efficient (Anselin 1988; Lee 2004). Quasi maximum likelihood (QML) estimator[7] is consistent and asymptotically normal (Lee 2004). The (quasi) maximum likelihood estimation was not practical for large datasets until a recent study by Smirnov and Anselin (2001). The generalized method of moments (GMM) estimation has been proposed for its computational simplicity, less restrictive distributional assumptions, and asymptotic properties (Kelejian and Prucha 1998, 2002; Lee 2001, 2007a,b). The generalized spatial two-stage least squares (GS2SLS) estimator is consistent and asymptotically normal (Kelejian and Prucha 1998), and the best spatial two-stage least squares (BS2SLS) estimator is also an asymptotically optimal instrumental variable estimator (Lee 2003). These two-stage least squares estimators are inefficient relative to the ML estimator and may be inconsistent (Kelejian and Prucha 2002; Lee 2003). The best generalized method of moments (BGMM) estimator incorporates additional moments conditions and attains the same limiting distribution as the ML estimator (with normal disturbances) (Lee 2007a). A computationally simple sequential GMM estimator based on optimization of a concentrated objective function with respect to a single spatial effect parameter may be as efficient as the BGMM estimator (Lee 2007b). All of the above estimators of the first-order spatial autoregressive model and their asymptotic properties were derived under the assumption that disturbance term is homoscedastic. The asymptotic properties of these estimators might not hold for the heteroscedastic case.[8]

Maximum Likelihood Estimation

Assuming normally distributed disturbance term, $\boldsymbol{\varepsilon} \sim N(\mathbf{0}, \sigma^2\boldsymbol{\Omega})$, the (quasi) ML estimator of a parameter vector in (1) is given by

$$(2) \quad \begin{bmatrix} \hat{\boldsymbol{\beta}}_{ML} \\ \hat{\rho}_{ML} \\ \hat{\sigma}^2_{ML} \end{bmatrix} = \underset{\boldsymbol{\beta},\rho,\sigma^2}{arg\ max} \left[-\frac{n}{2}ln(2\pi) - \frac{n}{2}ln(\sigma^2) - \frac{ln(|\boldsymbol{\Omega}|)}{2} - \frac{\boldsymbol{\nu}'\boldsymbol{\Omega}^{-1}\boldsymbol{\nu}}{2\sigma^2} + ln(|\mathbf{I} - \rho\mathbf{W}|) \right]$$

where $\mathbf{I}$ is a conformable identity matrix, $\boldsymbol{\nu} = (\mathbf{I} - \rho\mathbf{W})\mathbf{Y} - \mathbf{X}\boldsymbol{\beta}$, and $|.|$ is a determinant operator (Anselin 1988; Lee 2003). In practice, there is insufficient information to specify the parametric form of the likelihood function. The structure of covariance matrix of disturbance term, i.e., the functional form of the relationship between the disturbance variance and variables determining that variance are unknown. Estimation might proceed without making distributional assumptions and making less restrictive assumptions about the existence of zero valued moment conditions.

2SLS and GMM Estimation

The model in (1) can be estimated by the computationally simple two-stage least squares (2SLS) method. The instrumental variables are generated from exogenous regressors and the spatial weight matrix (Kelejian and Prucha 1998; Lee 2007a).[9] The 2SLS estimator is

$$(3)\qquad \begin{bmatrix} \hat{\boldsymbol{\beta}}_{2SLS} \\ \hat{\rho}_{2SLS} \end{bmatrix} = \left[\begin{pmatrix} (\mathbf{WY})' \\ \mathbf{X}' \end{pmatrix} \mathbf{Z}_{(p)}(\mathbf{WY},\mathbf{X}) \right]^{-1} \left[\begin{pmatrix} (\mathbf{WY})' \\ \mathbf{X}' \end{pmatrix} \mathbf{Z}_{(p)}\mathbf{Y} \right]$$

where $\mathbf{Z}$ is an $n \times m$ instrumental variable (IV) matrix with full column rank and $m \geq k$. $\mathbf{Z}_{(p)} = \mathbf{Z}(\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}'$ is the orthogonal projector of the column space of $\mathbf{Z}$.

The consistent and asymptotically normal generalized spatial 2SLS estimator (GS2SLS) suggested by Kelejian and Prucha (1998) defines the IV matrix as $\mathbf{Z} = [\mathbf{X}, \mathbf{WX}, \mathbf{W}^2\mathbf{X}]$. The asymptotically optimal best spatial 2SLS (BS2SLS) estimator proposed by Lee (2003) defines the IV matrix as $\mathbf{Z} = [\mathbf{X}, \mathbf{W}(\mathbf{I} - \hat{\rho}\mathbf{W})^{-1}\mathbf{X}^*\hat{\beta}]$, where $\mathbf{X}^*$ has no intercept column, $\hat{\beta}$ and $\hat{\rho}$ are the estimates from the first step.

A consistent generalized method of moments (GMM) estimator of unknown parameters in (1) can be derived from the empirical moments

$$(4)\qquad n^{-1}\mathbf{Z}'[(\mathbf{I} - \rho\mathbf{W})\mathbf{Y} - \mathbf{X}\boldsymbol{\beta}] = \mathbf{0}$$

where $\mathbf{Z}$ is $(n \times m)$ instrumental variables matrix with full column rank and $m \geq k$. The parameters can be estimated as

$$(5)\qquad \begin{bmatrix} \hat{\boldsymbol{\beta}}_{GMM} \\ \hat{\rho}_{GMM} \end{bmatrix} = \underset{\boldsymbol{\beta},\rho}{arg\ min} \left[n^{-1}\mathbf{Z}'[(\mathbf{I} - \rho\mathbf{W})\mathbf{Y} - \mathbf{X}\boldsymbol{\beta}]\right]' \hat{\mathbf{W}}_n \left[n^{-1}\mathbf{Z}'[(\mathbf{I} - \rho\mathbf{W})\mathbf{Y} - \mathbf{X}\boldsymbol{\beta}]\right]$$

where $\hat{\mathbf{W}}_n$ is an estimate of the asymptotically optimal weight matrix (Hansen 1982).

Information Theoretic Estimation

The IT estimators define empirical moment conditions as

$$(6)\qquad (\mathbf{p}\odot\mathbf{Z})'[(\mathbf{I}-\rho\mathbf{W})\mathbf{Y}-\mathbf{X}\boldsymbol{\beta}]=\mathbf{0}$$

where $\mathbf{p}$ is an $(n\times 1)$ vector of the unknown empirical probability weights[10] supported on a sample outcome $(\mathbf{Y},\mathbf{X})$, and $\odot$ denotes the extended Hadamard (elementwise) product operator. The empirical weights p_i are treated as unknown parameters of a multinomial distribution for the n different types of data outcomes presented by the sample such that $\sum_{i=1}^{n} p_i = 1$ and $p_i \geq 0, \forall i = 1,\ldots,n$. In contrast, the empirical moment conditions of the GMM approach in (4) restrict $p_i = 1/n$ for $i = 1,\ldots,n$.

A generalized IT estimator of unknown parameters in (1) can be formulated based on the concept of closeness between estimated and empirical distributions of a random sample outcome which maximizes the objective function $\phi(\mathbf{p})$ subject to empirical moment, normalization, and nonnegativity constraints given by

$$(7)\qquad \underset{\mathbf{p},\boldsymbol{\beta},\rho}{arg\ max}\left\{\phi(\mathbf{p})\ \ s.t.\ \ (\mathbf{p}\odot\mathbf{Z})'[(\mathbf{I}-\rho\mathbf{W})\mathbf{Y}-\mathbf{X}\boldsymbol{\beta}]=\mathbf{0},\ \sum_{i=1}^{n}p_i=1,\ p_i\geq 0,\ \forall i\right\}$$

The general functional specification of the objective function $\phi(\mathbf{p})$ in (7) encompasses estimating criteria nested within the Cressie-Read power divergence statistic (Cressie and Read 1984). This estimation approach circumvents the need for estimating a weight matrix in the GMM procedure. The Lagrange form of the extremum problem in (7) is given by

$$(8)\qquad L(\mathbf{p},\boldsymbol{\beta},\rho,\boldsymbol{\lambda},\eta)=\phi(\mathbf{p})-\boldsymbol{\lambda}'(\mathbf{p}\odot\mathbf{Z})'[(\mathbf{I}-\rho\mathbf{W})\mathbf{Y}-\mathbf{X}\boldsymbol{\beta}]-\eta\left(\sum_{i=1}^{n}p_i-1\right)$$

where $\boldsymbol{\lambda}=[\lambda_1,\ldots,\lambda_m]'$ is $(m\times 1)$ vector and η is scalar Lagrange multipliers. First order conditions are

$$(9)\qquad \frac{\partial L}{\partial p_i}=\frac{\partial\phi(\mathbf{p})}{\partial p_i}-\boldsymbol{\lambda}'Z[i,.]'[(\mathbf{I}-\rho\mathbf{W})\mathbf{Y}-X[i,.]\boldsymbol{\beta}]-\eta=0,\quad \forall i=1,\ldots,n$$

$$(10)\qquad \frac{\partial L}{\partial\boldsymbol{\beta}}=\mathbf{X}'(\mathbf{p}\odot\mathbf{Z})\boldsymbol{\lambda}=\mathbf{0}$$

$$(11)\quad \frac{\partial L}{\partial \rho} = \boldsymbol{\lambda}'(\mathbf{p}\odot\mathbf{Z})'\mathbf{WY} = 0$$

$$(12)\quad \frac{\partial L}{\partial \boldsymbol{\lambda}} = (\mathbf{p}\odot\mathbf{Z})'[(\mathbf{I}-\rho\mathbf{W})\mathbf{Y}-\mathbf{X}\boldsymbol{\beta}] = \mathbf{0}$$

$$(13)\quad \frac{\partial L}{\partial \eta} = \sum_{i=1}^{n} p_i - 1 = 0$$

and $p_i \geq 0, \forall i$. The first set of equations in (9) links the empirical probability weights p_i's to the unknown parameters $\boldsymbol{\beta}$, ρ, and $\boldsymbol{\lambda}$ through the empirical moment conditions. Set of equations in (10) - (12) modifies the traditional orthogonality conditions of the GMM estimation. The equation (13) is the standard normalization condition for the empirically estimated probability weights.

In order to derive the IT estimator, let $\mathbf{f}(\mathbf{p}) = (f_1(\mathbf{p}),\ldots,f_N(\mathbf{p}))'$ and $f_i(\mathbf{p}) = \frac{\partial\phi(\mathbf{p})}{\partial p_i}\ \forall i$, then if an inverse function $\mathbf{f}^{-1}(.)$ exists, then the general solution for $\mathbf{p}$ in terms of parameters and Lagrange multipliers is

$$(14)\quad \mathbf{p}(\boldsymbol{\beta},\rho,\boldsymbol{\lambda},\eta) = \mathbf{f}^{-1}\left(\boldsymbol{\lambda}'(\mathbf{p}\odot\mathbf{Z})'[(\mathbf{I}-\rho\mathbf{W})\mathbf{Y}-\mathbf{X}\boldsymbol{\beta}]+\eta\right)$$

Substitute (14) into (6) and there is a well defined solution $\boldsymbol{\lambda}$ under generality conditions (Qin and Lawless 1994) which is an implicit function of parameters $\boldsymbol{\beta}$ and ρ denoted by

$$(15)\quad \boldsymbol{\lambda}(\boldsymbol{\beta},\rho) = \underset{\boldsymbol{\lambda}}{arg}\left[\sum_{i=1}^{n} p_i(\boldsymbol{\beta},\rho,\boldsymbol{\lambda},\eta^*)Z[i,.]'(Y_i-\rho W[i,.]\mathbf{Y}-X[i,.]\boldsymbol{\beta}) = \mathbf{0}\right]$$

where η^* is the optimal value obtained from the first order conditions.

Substituting $\mathbf{p}(\boldsymbol{\beta},\rho,\boldsymbol{\lambda}(\boldsymbol{\beta},\rho))$, $\boldsymbol{\lambda}(\boldsymbol{\beta},\rho)$ and η^* in (7) produces the maximized objective function with respect to $\mathbf{p}$ given by

$$(16)\quad \phi(\boldsymbol{\beta},\rho,\boldsymbol{\lambda}(\boldsymbol{\beta},\rho)) = \underset{\mathbf{p}}{arg\,max}\left\{\phi(\mathbf{p})-\boldsymbol{\lambda}(\boldsymbol{\beta},\rho)'(\mathbf{p}\odot\mathbf{Z})'[(\mathbf{I}-\rho\mathbf{W})\mathbf{Y}-\mathbf{X}\boldsymbol{\beta}]-\eta^*\left(\sum_{i=1}^{n} p_i - 1\right)\right\}$$

which assigns the most favorable empirical weights to each value of the parameter vector from within the family of multinomial distributions supported on the sample $(\mathbf{Y},\mathbf{X})$ and satisfying the empirical moment equations in (6).

The IT estimators are conventional likelihood estimators obtained by maximizing the objective function in (16) over the domain of parameter space given by $\boldsymbol{\beta} \in \mathbf{R}^k$ and $-1 \leq \rho \leq 1$ as

$$(17) \quad \begin{bmatrix} \hat{\boldsymbol{\beta}}_{IT} \\ \hat{\rho}_{IT} \end{bmatrix} = \underset{\boldsymbol{\beta},\rho}{arg\ max} \{\phi(\boldsymbol{\beta},\rho,\boldsymbol{\lambda}(\boldsymbol{\beta},\rho))\}$$

In this study, the most common objective functions (7) are analyzed including (1) the traditional empirical log-likelihood function $\sum_{i=1}^{n} ln(p_i)$, (2) the empirical exponential likelihood (or negative entropy) function $\sum_{i=1}^{n} p_i ln(p_i)$, and (3) the log Euclidean likelihood function $n^{-1} \sum_{i=1}^{n}(n^2 p_i^2 - 1)$. The corresponding IT estimators are Maximum Empirical Likelihood (MEL), Maximum Exponential Empirical Likelihood (MEEL), and Maximum Log Euclidean Empirical Likelihood (MLEL).

Maximum Empirical Likelihood

The objective function is the traditional empirical log-likelihood function $\phi(\mathbf{p}) = \sum_{i=1}^{n} ln(p_i)$ and the optimal resulting optimal weights p_i implied by (9) are given by

$$(18) \quad p_i(\boldsymbol{\beta},\rho,\boldsymbol{\lambda}(\boldsymbol{\beta},\rho)) = n^{-1} \left[1 + \boldsymbol{\lambda}(\boldsymbol{\beta},\rho)' Z[i,.]'(Y_i - \rho W[i,.]\mathbf{Y} - X[i,.]\boldsymbol{\beta})\right]^{-1}$$

where optimal $\eta^* = 1$.[11]

Maximum Exponential Empirical Likelihood

The objective function is the empirical exponential likelihood function, $\phi(\mathbf{p}) = \sum_{i=1}^{n} p_i ln(p_i)$, and the optimal p_i can be expressed as

$$(19) \quad p_i(\boldsymbol{\beta},\rho,\boldsymbol{\lambda}(\boldsymbol{\beta},\rho)) = \frac{exp\left[\boldsymbol{\lambda}(\boldsymbol{\beta},\rho)' Z[i,.]'(Y_i - \rho W[i,.]\mathbf{Y} - X[i,.]\boldsymbol{\beta})\right]}{\sum_{i=1}^{n} exp\left[\boldsymbol{\lambda}(\boldsymbol{\beta},\rho)' Z[i,.]'(Y_i - \rho W[i,.]\mathbf{Y} - X[i,.]\boldsymbol{\beta})\right]}$$

Maximum Log Euclidean Empirical Likelihood

The objective function is the log Euclidean likelihood function, $\phi(\mathbf{p}) = n^{-1}\sum_{i=1}^{n}(n^2 p_i^2 - 1)$. The optimal p_i can be expressed as

$$(20)\quad p_i(\boldsymbol{\beta},\rho,\boldsymbol{\lambda}(\boldsymbol{\beta},\rho)) = (2n)^{-1}\left[\eta^* + \boldsymbol{\lambda}(\boldsymbol{\beta},\rho)' Z[i,.]'(Y_i - \rho W[i,.]\mathbf{Y} - X[i,.]\boldsymbol{\beta})\right]$$

where $\eta^* = 2 - \frac{1}{n}\sum_{i=1}^{n}\boldsymbol{\lambda}' Z[i,.]'(Y_i - \rho W[i,.]\mathbf{Y} - X[i,.]\boldsymbol{\beta})$.

The IT estimators $\hat{\boldsymbol{\beta}}_{IT}$ and $\hat{\rho}_{IT}$ for $IT \in \{MEL, MEEL, MLEL\}$ are obtained by maximizing the corresponding concentrated objective function in (16) with respect to the $(k+1)$ unknown parameters with the appropriate $p_i(\boldsymbol{\lambda}(\boldsymbol{\beta},\rho),\boldsymbol{\beta},\rho)$ substituted for p_i.[12]

Monte Carlo Experiments

In order to compare repeated sampling properties of the estimators, extensive Monte Carlo sampling experiments are used. For a range of small sample sizes, spatial autocorrelation parameters, and commonly used weight matrices, finite sample performance of the IT estimators is compared to the traditional estimators.

The Monte Carlo experiments are constructed in the following manner. Three values of the sample size n, namely, $25, 49$, and 90 are considered. For each of these values of n, three specification of weight matrices are used. The weight matrices differ in their degree of sparseness, have equal weights, and an average number of neighbors per unit, J, is chosen to be $J = 2, 6$ and 10 (Kelejian and Prucha 1999). Eleven values for the spatial autoregressive coefficient ρ in (1) are chosen, namely, $-0.9, -0.8, -0.6, -0.4, -0.2, 0, 0.2, 0.4, 0.6, 0.8$, and 0.9. Three values of σ^2 are $1, 2.5$, and 5. The remaining elements of the parameter vector are specified as $\beta = (\beta_1, \beta_2) = (1, 1)$. Two regressors, $X = (X_1, X_2)$, are specified to be gamma and uniformly distributed corresponding to income per capita and percent of rental housing, which are commonly used regressors in empirical applications (Kelejian and Robinson 1993). The n observations on independent variables are normalized, so that their sample means and variances are, respectively, zero and one. The same regressors are used in all experiments in which sample size is n with 1000 repetitions for each Monte Carlo experiment. To summarize, there are three values of n, eleven values of ρ, three values of σ^2, and three values of J. All combinations of n, ρ, σ^2, and J result in $3 \times 11 \times 3 \times 3 = 297$ Monte Carlo experiments.

In addition, three distributional assumptions for the disturbance term ε in (1) are selected. First, normally distributed disturbance term, $\varepsilon_i \sim N(0, \sigma^2)$ for $\forall i = 1, \ldots, n$, referred to as $H = 1$ is considered.[13] Three forms of heteroscedasticity are specified for normal distribution such that $\sigma_i^2 = [0.5X_2]\sigma^2$, $\sigma_i^2 = [0.5X_2^2]\sigma^2$, and $\sigma_i^2 = exp(0.5X_2)\sigma^2$ referred to as $H = 2$, $H = 3$, $H = 4$, correspondingly. Second, the log-normal distribution ($H = 5$) is chosen because of its assymetric nature such that $\varepsilon_i = exp(\xi_i) - exp(0.5\sigma^2)$, where $\xi_i \sim N(0, \sigma^2)$. Finally, the mixture of normals ($H = 6$), a normally distributed variable contaminated by another with a larger variance, is considered because of its thicker tails than normal distribution, i.e., $\varepsilon_i = \varsigma_i \xi_i + (1 - \varsigma_i)\zeta$, the ς_i are i.i.d. Bernoulli variables with $Prob(\varsigma_i = 1) = 0.95$, $\xi_i \sim N(0, \sigma^2)$ and $\zeta_i \sim N(0, 100)$.[14] There are 297 experiments for 6 distributional assumptions considered, which results in total $297 \times 6 = 1782$ Monte Carlo experiments performed.

To keep the Monte Carlo study manageable, the bias and the spread of the finite sample distribution of the estimators is measured by root mean square error (RMSE). Since sample moments for calculation of the standard RMSE might not exist, this measure might be meaningless and the RMSE's measure proposed by Kelejian and Prucha (1999) is used which guarantees the existence of the necessary components.[15]

In addition, a response functions are used to summarize the relationship between RMSEs of the estimators and the model parameters over the set of all considered parameter values. Let ρ_i, β_{1i}, β_{2i}, J_i and σ_i^2 be the values, respectively, of ρ, β_1, β_2, J and σ^2 in the ith experiment, $i = 1, \ldots, 297$, for each distributional assumption. Let n_i correspond to the values of a sample size then the functional form of the response function (Das, Kelejian, and Prucha 2003) is

$$(21) \quad RMSE_i = \frac{\sigma_i}{\sqrt{n_i}} exp[a_1 + a_2\left(\frac{1}{J_i}\right) + a_3\left(\frac{\rho_i}{J_i}\right) + a_4\rho_i + a_5\rho_i^2 + a_6\left(\frac{J_i}{n_i}\right) + + a_7\left(\frac{J_i}{n_i}\right)^2 + a_8\left(\frac{1}{n_i}\right) + a_9\sigma_i^2],\ i = 1, \ldots, 297$$

where $RMSE_i$ is the RMSE of an estimator of a given parameter in the ith experiment and the parameters $a_1, \ldots, a_9$ are different for each estimator of each parameter. For each case considered, The parameters of (21) are estimated using the entire set of 297 Monte Carlo experiments for each distributional assumption by least squares after taking logs on both sides.

Results

There are three types of results presented. First, tables of root mean square errors of the estimators for a subset of experiments are reported. Second, the average RMSEs for the entire Monte Carlo study are provided. Finally, the response functions for the RMSEs of the estimators are estimated, and graphs of these response functions are presented.

In order to conserve space, Tables 1, 3, and3 report RMSEs of the estimators for a selected subset of experimental parameter values. These tables report RMSEs of the estimators of the parameters ρ, β_1 and β_2, correspondingly, for 33 Monte Carlo experiments, which is a combination of 11 parameter values for ρ and 3 parameter values for σ^2, with the average number of neighbors $J = 6$, sample size $n = 60$, and normally distributed disturbance term $H = 1$.

According to Table 1, the RMSEs of the QML estimator of ρ are the lowest and OLS estimator are the largest, since the OLS estimator is inconsistent and the QML estimator with normally distributed homoscedastic disturbance term is consistent and efficient. The IT estimators of ρ have the second lowest RMSEs outperforming the OLS, 2SLS, and GMM estimators. The EL estimator outperforms other IT estimators and is, on average, less efficient than the QML estimator. Tables 2 and 3 report the RMSEs of the QML and the

IT estimators of β_1 and β_2 which are on average roughly the same. A loss of efficiency of the IT estimators relative to the QML estimators is small and is mostly arises for estimation of the spatial autocorrelation coefficient ρ. The same pattern holds for other distributional assumptions, namely $H = 2, ..., 5$, and only averages of the RMSEs are reported.

In order to summarize the results of total 1782 Monte Carlo experiments, average RMSEs of the estimators, namely, for ρ, β_1 and β_2, are presented in Tables 4, 5, and 6. A table's entry is the averages of RMSEs of the estimators over 33 Monte Carlo experiments for a given sample size n, average number of neighbors J, and distributional assumptions H.

The results of total Monte Carlo experiments for other distributional assumptions are consistent with the results for a subset of parameter space for the case of $H = 1$ reported in Tables 1, 2, and 3. In fact, Table 4 reports that RMSEs of the QML estimator of ρ are the lowest and OLS as well as GMM estimators are the largest regardless of sample size, average number of neighbors, and distributional assumptions considered. The IT estimators of ρ have the second lowest RMSEs across all Monte Carlo experiments, i.e., less efficient than the QML estimator, and outperform the OLS, 2SLS, and GMM estimators.

Tables 5 and 6 report that the RMSEs of the QML and the IT estimators of β_1 and β_2 are roughly the same with the only exception of the EL estimator which is, on average, less efficient than the QML estimator for β_1 and β_2, correspondingly. A loss of efficiency of the IT estimators relative to the QML estimators is, on average, small and is mostly arises for estimation of the spatial autocorrelation coefficient ρ.

The response functions (21) are used to depict the relationship between the RMSEs of the estimators over the set of all considered parameter values. The least squares response functions estimation results are reported in Table 7. For all distributional assumptions considered, the R^2 is high. Figures 1, 2, and 3 describe 18 response functions of the RMSEs of the QML and the IT estimators of ρ for average number of neighbors equal $J = 2$ and three sample sizes $n = 25, 60, 90$. Each figure describes 6 response functions of the RMSEs for 6 distributional assumptions.

The RMSEs are related to the spatial autocorrelation parameter, ρ, in a concave fashion. The difference in the RMSEs between QML and IT estimators increases as ρ approaches zero, and it decreases as ρ approaches ± 1. The QML estimator outperforms the IT estimators for all distributional assumptions considered which is also the case for $J = 2, 6, 10$. The response functions for the three IT estimators considered are related to the ρ in similar fashion and have comparable magnitude. As the sample size increases, the difference in between the response functions of the QML and IT estimators declines.

Illustrative Application: Hedonic Housing Value Model

Harrison and Rubinfeld (1978) have used housing data for census tracts in the Boston Standard Metropolitan Statistical Area (SMSA) in 1970 to estimate the demand for clean air. There are 506 observations (one observation per census tract) on 14 non-constant independent variables. The dependent variable is the median value of the owner-occupied homes in the census tract (MV). The independent variables in the equation included two structural attribute variables, eight neighborhood variables, two accessibility variables, and one air pollution variable such as levels of nitrogen oxides (NOX), average number of rooms (RM), proportion of structures built before 1940 (AGE), black population proportion (B), lower status population proportion (LSTAT), crime rate (CRIM), proportion of area zoned with large lots (ZN), proportion of nonretail business area (INDUS), property tax rate (TAX), pupil-teacher ratio (PTRATIO), location contiguous to the Charles River (CHAS), weighted distances to the employment centers (DIS), and an index of accessibility (RAD) (Gilley and Pace, 1996).

The estimated equation is

$$\text{(22)} \quad log(MV) = \beta_0 + \rho W log(MV) + \beta_1 CRIM + \beta_2 ZN + \beta_3 INDUS + \beta_4 CHAS + \beta_5 NOX^2 + \beta_6 RM^2 + \beta_7 AGE + \beta_8 log(DIS) + \beta_9 log(RAD)$$

$$+\beta_{10}TAX+\beta_{11}PTRATIO+\beta_{12}B+\beta_{13}LSTAT+\varepsilon$$

where W is a distance matrix constructed from latitude and longitude for each observation as described above. The estimates are reported in Table 8. For a sample size of 506 observation, information theoretic estimators and other estimators considered provide similar estimates of the model parameters with the only exception of the spatial autoregressive parameter ρ.

Conclusions

A general information theoretic estimator of the first order spatial autoregressive model was proposed, and special cases of the Maximum Empirical Likelihood, Maximum Exponential Empirical Likelihood, and Maximum Log Euclidean Likelihood estimators were explored.

In extensive Monte Carlo experiments, the small sample performance of the information theoretic estimators was evaluated over the range of the selected forms of spatial correlation, finite sample sizes, distributional assumption of disturbance term, and spatial weight matrices. The performance was compared to the traditional estimators, and it is found that information theoretic estimators in the context of the first-order spatial autoregressive model have superior sampling properties and outperform the traditional OLS, 2SLS and GMM estimators with the only exception of the quasi-likelihood estimator. Findings suggest to use the information theoretic estimators for small sample size estimation problems in the cases where quasi-maximum likelihood estimators cannot be computed. Estimation of housing value equation illustrates the equivalence among 2SLS, QML, and information theoretic estimators for a large sample size.

Notes

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[2]See also Cliff and Ord (1981), Anselin (1988), and Cressie (1993).

[3]It is row normalized (Anselin 1988) or "row stochastic" (LeSage and Pace 2004) matrix, so that row sums are unity and all diagonal elements are zero, i.e., the spatial weight matrix is nonsingular and the reduced form of (1) is well defined (Horn and Johnson 1985). The weights are fixed in repeated sampling and are constructed based on the spatial configuration of points making up a spatial sample.

[4]It reflects spatial correlation between **Y** and **WY** with domain $\rho \in (\xi_{min}^{-1}, \xi_{max}^{-1})$, where ξ_{min} and ξ_{max} are the minimum and maximum eigenvalues of the spatial weight matrix with $\rho \in (-1, 1)$ for a row normalized spatial matrix (Sun, Tsutakawa, and Speckman 1999).

[5]Bayesian estimators are not considered in this study (see LeSage (1997)).

[6]The OLS estimator can be consistent when the weight matrix is upper or lower triangular (Ord 1975) and when each spatial unit can be influenced aggregately by a significant portion of units in the population (Lee 2002).

[7]The QML estimator is derived from a normal likelihood but the disturbances in the model are not necessary normally distributed.

[8]It is important to validate the asymptotic properties of the proposed estimators for the spatial regression models and develop methodology that allows heteroscedasticity of disturbance term in the future.

[9]The 2SLS method can be implemented only when the spatial regressors are relevant and instrumental variables can be constructed (Kelejian and Prucha 1998).

[10]The concept of empirical probability weights is developed in the framework of empirical likelihood function (Thomas and Grunkemeier 1975). The value of the empirical likelihood function is the maximum empirical probability $\prod_{i=1}^{n} p_i$ that can be assigned to a random sample outcome of $(\mathbf{Y}, \mathbf{X})$ among all distributions of probability $\mathbf{p}$ supported on the (Y_i, X_i)'s that satisfy the empirical moment conditions in (6), adding up restriction $\sum_{i=1}^{n} p_i = 1$, and nonnegativity constraint $p_i \geq 0, \forall i = 1, \ldots, n$ (Owen 1990).

[11] It follows from $\sum_{i=1}^{n} p_i \frac{\partial L(\mathbf{p}, \boldsymbol{\lambda}, \eta)}{\partial p_i} = 0$, (6), and (13).

[12]Of the three specifications considered in this study, the MLEL estimator has received the least attention in the econometrics and statistics literature.

[13]Quasi-maximum likelihood estimator is maximum likelihood estimator for the $H = 1, \ldots, 4$ case, where the disturbance term is normally distributed.

[14]In order to compare small sample performance of IT estimators to quasi-maximum likelihood estimator, these two alternative distributions of the disturbance term are considered.

[15]According to Kelejian and Prucha (1999), $RMSE = \left[bias^2 + (\frac{IQ}{1.35})^2\right]^{0.5}$ where $bias$ is the difference between the median, and the true parameter values and $IQ = c_1 - c_2$ is the inter-quantile range where c_1 and c_2 are 0.75 and 0.25 quantiles, correspondingly.

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Figures

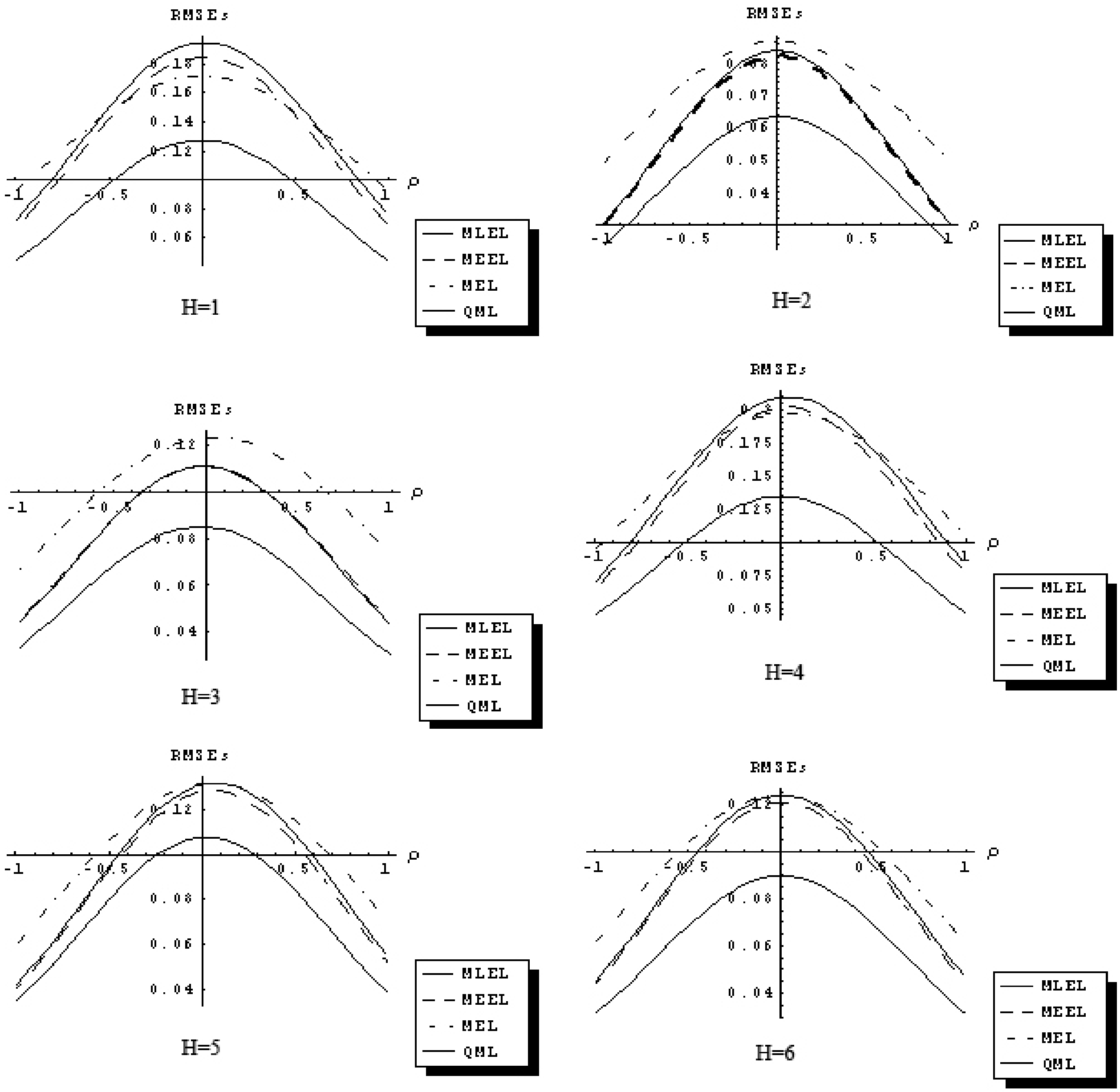

Figure 1. RMSEs of the QML, MEEL, MLEL and MEL estimators of ρ ($n = 25$ and $J = 2$)

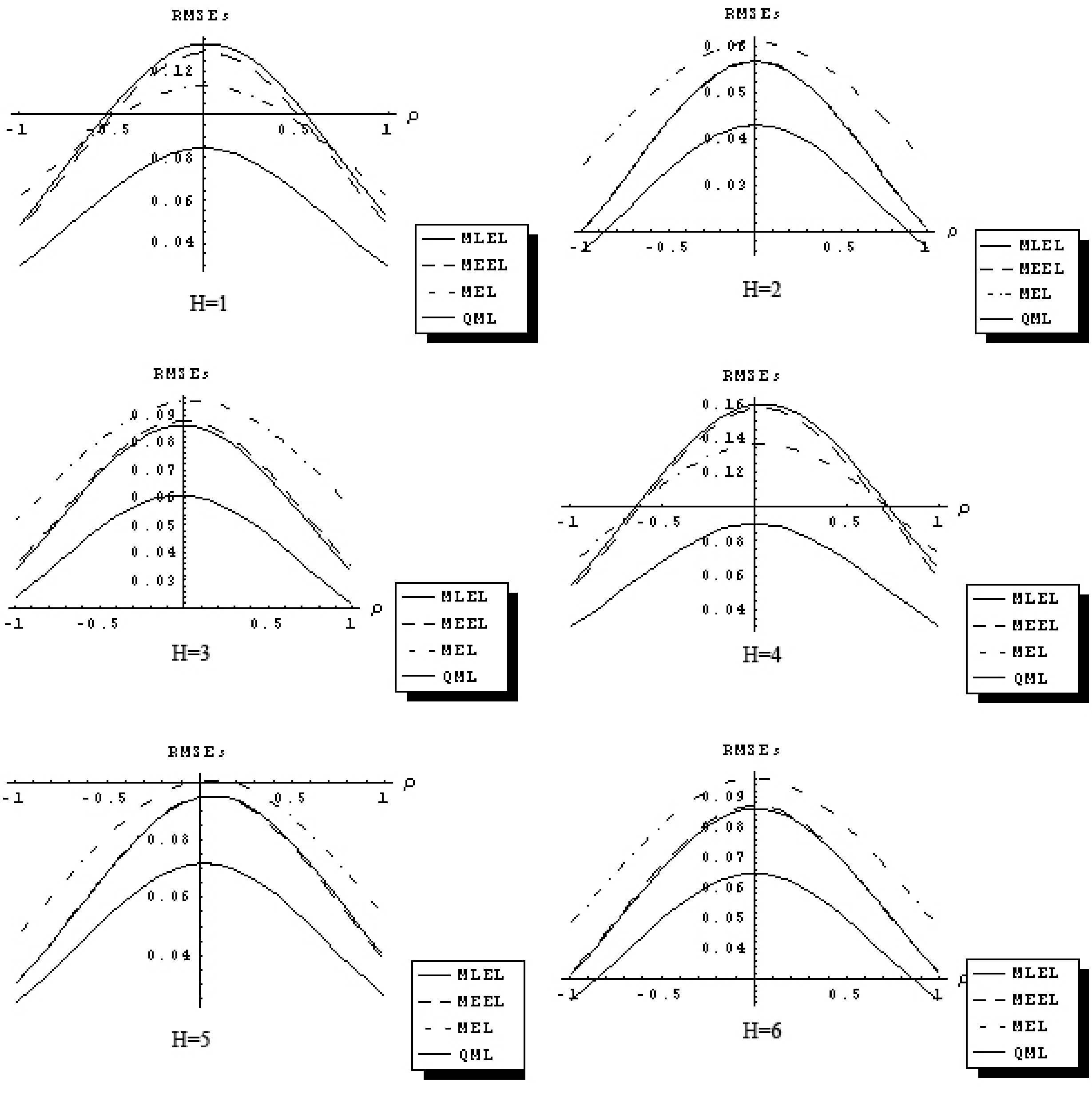

Figure 2. **RMSEs of the QML, MEEL, MLEL and MEL estimators of ρ ($n = 60$ and $J = 2$)**

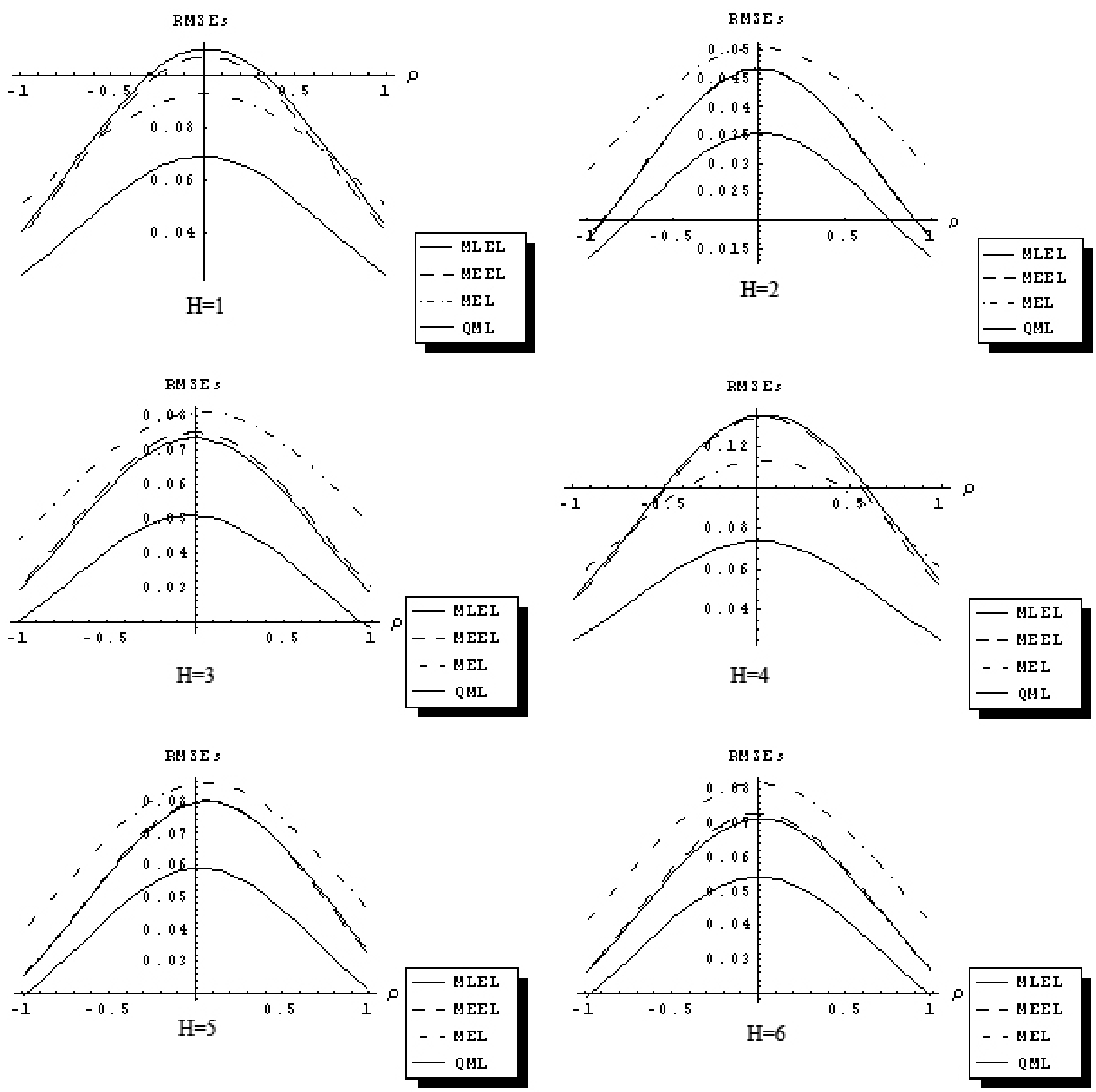

Figure 3. **RMSEs of the QML, MEEL, MLEL and MEL estimators of** ρ ($n = 90$ **and** $J = 2$)

Tables

Table 1. Root mean square errors of the estimators of ρ, $n = 60$, $J = 6$, $H = 1$

ρ	σ^2	OLS	2SLS	GMM	QML	EEL	EL	LEL
0.9	1	0.055	0.048	0.048	0.036	0.051	0.074	0.051
0.9	2.5	0.077	0.061	0.063	0.040	0.065	0.093	0.069
0.9	5	0.110	0.115	0.111	0.046	0.113	0.098	0.127
0.8	1	0.077	0.069	0.070	0.055	0.071	0.120	0.074
0.8	2.5	0.155	0.128	0.138	0.073	0.143	0.121	0.153
0.8	5	0.189	0.178	0.183	0.079	0.165	0.146	0.180
0.6	1	0.173	0.148	0.159	0.097	0.149	0.116	0.155
0.6	2.5	0.237	0.227	0.233	0.113	0.236	0.133	0.243
0.6	5	0.258	0.236	0.260	0.124	0.258	0.146	0.267
0.4	1	0.159	0.166	0.170	0.116	0.160	0.128	0.166
0.4	2.5	0.229	0.241	0.245	0.139	0.247	0.157	0.251
0.4	5	0.271	0.295	0.288	0.150	0.267	0.181	0.274
0.2	1	0.196	0.194	0.204	0.137	0.200	0.190	0.207
0.2	2.5	0.260	0.295	0.294	0.153	0.276	0.222	0.289
0.2	5	0.315	0.400	0.389	0.216	0.354	0.233	0.369
0	1	0.204	0.209	0.214	0.153	0.210	0.164	0.219
0	2.5	0.305	0.315	0.331	0.201	0.302	0.220	0.310
0	5	0.389	0.563	0.593	0.220	0.506	0.255	0.532
-0.2	1	0.231	0.213	0.225	0.166	0.205	0.184	0.207
-0.2	2.5	0.404	0.390	0.414	0.219	0.389	0.238	0.397
-0.2	5	0.424	0.465	0.496	0.216	0.393	0.274	0.410
-0.4	1	0.360	0.282	0.321	0.200	0.284	0.230	0.288
-0.4	2.5	0.476	0.421	0.438	0.214	0.405	0.256	0.415
-0.4	5	0.555	0.556	0.612	0.232	0.533	0.328	0.533
-0.6	1	0.360	0.256	0.282	0.193	0.242	0.221	0.246
-0.6	2.5	0.581	0.434	0.497	0.240	0.397	0.315	0.408
-0.6	5	0.669	0.598	0.627	0.239	0.549	0.350	0.546
-0.8	1	0.532	0.368	0.382	0.208	0.302	0.266	0.307
-0.8	2.5	0.591	0.442	0.463	0.247	0.364	0.307	0.368
-0.8	5	0.789	0.609	0.569	0.240	0.504	0.337	0.498
-0.9	1	0.328	0.224	0.186	0.175	0.178	0.144	0.189
-0.9	2.5	0.656	0.411	0.387	0.217	0.323	0.244	0.329
-0.9	5	0.757	0.576	0.392	0.235	0.396	0.225	0.403
Column Average		0.345	0.307	0.312	0.163	0.280	0.203	0.287

Table 2. Root mean square errors of the estimators of β_1, $n = 60$, $J = 6$, $H = 1$

ρ	σ^2	OLS	2SLS	GMM	QML	EEL	EL	LEL
0.9	1	0.141	0.142	0.141	0.139	0.142	0.154	0.146
0.9	2.5	0.217	0.217	0.219	0.222	0.221	0.247	0.224
0.9	5	0.294	0.309	0.313	0.303	0.315	0.334	0.312
0.8	1	0.132	0.131	0.132	0.128	0.137	0.151	0.138
0.8	2.5	0.225	0.228	0.229	0.222	0.221	0.254	0.228
0.8	5	0.301	0.312	0.315	0.293	0.307	0.321	0.304
0.6	1	0.132	0.133	0.135	0.130	0.136	0.175	0.137
0.6	2.5	0.206	0.209	0.209	0.203	0.202	0.249	0.208
0.6	5	0.305	0.299	0.300	0.293	0.305	0.346	0.310
0.4	1	0.129	0.130	0.129	0.129	0.135	0.192	0.134
0.4	2.5	0.204	0.206	0.206	0.202	0.204	0.272	0.209
0.4	5	0.303	0.305	0.316	0.304	0.310	0.351	0.306
0.2	1	0.130	0.134	0.135	0.128	0.135	0.190	0.139
0.2	2.5	0.217	0.219	0.220	0.219	0.223	0.262	0.221
0.2	5	0.295	0.305	0.314	0.302	0.314	0.356	0.319
0	1	0.133	0.136	0.137	0.129	0.144	0.171	0.146
0	2.5	0.211	0.209	0.213	0.211	0.210	0.252	0.210
0	5	0.296	0.309	0.320	0.294	0.315	0.339	0.315
-0.2	1	0.126	0.129	0.127	0.126	0.132	0.165	0.142
-0.2	2.5	0.199	0.202	0.203	0.201	0.199	0.236	0.204
-0.2	5	0.297	0.305	0.302	0.307	0.320	0.341	0.318
-0.4	1	0.139	0.142	0.145	0.137	0.142	0.197	0.144
-0.4	2.5	0.205	0.212	0.222	0.208	0.207	0.259	0.215
-0.4	5	0.295	0.292	0.303	0.282	0.299	0.334	0.299
-0.6	1	0.131	0.130	0.130	0.129	0.131	0.185	0.137
-0.6	2.5	0.240	0.224	0.234	0.220	0.223	0.281	0.223
-0.6	5	0.303	0.317	0.314	0.296	0.317	0.333	0.323
-0.8	1	0.142	0.134	0.139	0.136	0.140	0.180	0.140
-0.8	2.5	0.216	0.219	0.236	0.210	0.219	0.276	0.219
-0.8	5	0.293	0.298	0.312	0.290	0.301	0.317	0.309
-0.9	1	0.142	0.129	0.132	0.126	0.134	0.175	0.137
-0.9	2.5	0.237	0.214	0.225	0.209	0.211	0.274	0.218
-0.9	5	0.285	0.294	0.307	0.284	0.300	0.337	0.295
Column Average		0.216	0.217	0.222	0.213	0.220	0.258	0.222

Table 3. Root mean square errors of the estimators of β_2, $n = 60$, $J = 6$, $H = 1$

ρ	σ^2	OLS	2SLS	GMM	QML	EEL	EL	LEL
0.9	1	0.131	0.130	0.131	0.126	0.139	0.149	0.141
0.9	2.5	0.233	0.236	0.243	0.230	0.246	0.249	0.243
0.9	5	0.297	0.307	0.308	0.301	0.314	0.330	0.310
0.8	1	0.140	0.137	0.143	0.136	0.141	0.156	0.146
0.8	2.5	0.225	0.225	0.239	0.221	0.237	0.244	0.237
0.8	5	0.274	0.278	0.277	0.275	0.292	0.306	0.293
0.6	1	0.127	0.125	0.124	0.125	0.128	0.173	0.132
0.6	2.5	0.199	0.198	0.199	0.204	0.205	0.249	0.214
0.6	5	0.301	0.311	0.308	0.302	0.304	0.345	0.307
0.4	1	0.137	0.132	0.131	0.133	0.137	0.186	0.140
0.4	2.5	0.211	0.215	0.219	0.210	0.226	0.265	0.226
0.4	5	0.299	0.303	0.312	0.312	0.311	0.340	0.316
0.2	1	0.135	0.136	0.137	0.134	0.140	0.182	0.144
0.2	2.5	0.219	0.224	0.224	0.221	0.220	0.269	0.226
0.2	5	0.308	0.309	0.314	0.306	0.319	0.334	0.321
0	1	0.131	0.132	0.131	0.133	0.140	0.179	0.141
0	2.5	0.228	0.230	0.227	0.226	0.233	0.259	0.236
0	5	0.290	0.302	0.304	0.292	0.304	0.333	0.309
-0.2	1	0.128	0.128	0.131	0.126	0.141	0.182	0.147
-0.2	2.5	0.205	0.210	0.208	0.206	0.221	0.251	0.221
-0.2	5	0.282	0.302	0.303	0.281	0.296	0.310	0.306
-0.4	1	0.134	0.137	0.135	0.133	0.141	0.178	0.143
-0.4	2.5	0.216	0.217	0.221	0.214	0.222	0.250	0.230
-0.4	5	0.302	0.300	0.312	0.294	0.312	0.331	0.319
-0.6	1	0.133	0.131	0.131	0.126	0.140	0.179	0.142
-0.6	2.5	0.236	0.214	0.212	0.202	0.214	0.257	0.213
-0.6	5	0.316	0.321	0.336	0.298	0.314	0.361	0.318
-0.8	1	0.156	0.149	0.155	0.142	0.147	0.190	0.148
-0.8	2.5	0.227	0.212	0.234	0.212	0.222	0.269	0.218
-0.8	5	0.311	0.324	0.320	0.308	0.324	0.343	0.315
-0.9	1	0.137	0.137	0.139	0.136	0.140	0.192	0.145
-0.9	2.5	0.216	0.199	0.205	0.200	0.213	0.265	0.212
-0.9	5	0.305	0.327	0.342	0.296	0.330	0.360	0.344
Column Average		0.218	0.219	0.223	0.214	0.225	0.257	0.227

Table 4. Average root mean square errors of the estimators of ρ

H	J	n	OLS	2SLS	GMM	QML	EEL	EL	LEL
1	2	25	0.212	0.221	0.247	0.115	0.210	0.191	0.223
1	2	60	0.170	0.140	0.148	0.077	0.139	0.112	0.146
1	2	90	0.156	0.113	0.118	0.060	0.113	0.097	0.117
1	6	25	0.493	0.522	0.540	0.272	0.456	0.402	0.465
1	6	60	0.345	0.307	0.312	0.163	0.280	0.203	0.287
1	6	90	0.301	0.251	0.256	0.132	0.230	0.159	0.236
1	10	25	0.693	0.721	0.698	0.372	0.598	0.524	0.610
1	10	60	0.463	0.440	0.466	0.226	0.413	0.284	0.418
1	10	90	0.400	0.351	0.352	0.180	0.319	0.208	0.325
2	2	25	0.114	0.106	0.114	0.077	0.106	0.113	0.109
2	2	60	0.082	0.066	0.069	0.050	0.065	0.070	0.065
2	2	90	0.080	0.057	0.058	0.038	0.055	0.061	0.056
2	6	25	0.260	0.262	0.276	0.189	0.244	0.240	0.246
2	6	60	0.197	0.170	0.177	0.120	0.154	0.145	0.154
2	6	90	0.162	0.125	0.133	0.095	0.117	0.116	0.117
2	10	25	0.395	0.422	0.435	0.273	0.370	0.360	0.370
2	10	60	0.272	0.244	0.257	0.189	0.225	0.214	0.224
2	10	90	0.225	0.184	0.196	0.133	0.169	0.156	0.169
3	2	25	0.148	0.134	0.140	0.081	0.126	0.144	0.127
3	2	60	0.143	0.117	0.117	0.061	0.106	0.111	0.105
3	2	90	0.132	0.099	0.104	0.046	0.084	0.093	0.083
3	6	25	0.365	0.375	0.386	0.217	0.336	0.341	0.334
3	6	60	0.285	0.240	0.257	0.134	0.207	0.217	0.203
3	6	90	0.269	0.236	0.246	0.115	0.203	0.178	0.197
3	10	25	0.444	0.473	0.510	0.277	0.418	0.425	0.420
3	10	60	0.367	0.336	0.360	0.183	0.300	0.288	0.294
3	10	90	0.325	0.325	0.343	0.146	0.260	0.241	0.253
4	2	25	0.219	0.232	0.250	0.120	0.211	0.204	0.222
4	2	60	0.188	0.167	0.177	0.076	0.157	0.131	0.164
4	2	90	0.185	0.144	0.152	0.064	0.138	0.112	0.142
4	6	25	0.506	0.560	0.579	0.266	0.488	0.448	0.496
4	6	60	0.380	0.380	0.387	0.162	0.333	0.239	0.338
4	6	90	0.338	0.307	0.306	0.132	0.271	0.189	0.274
4	10	25	0.701	0.795	0.805	0.365	0.647	0.589	0.657
4	10	60	0.494	0.515	0.510	0.225	0.446	0.303	0.452
4	10	90	0.452	0.450	0.460	0.186	0.410	0.252	0.412
5	2	25	0.175	0.169	0.182	0.100	0.149	0.151	0.155
5	2	60	0.148	0.119	0.125	0.066	0.102	0.105	0.103
5	2	90	0.142	0.101	0.105	0.050	0.087	0.090	0.088
5	6	25	0.439	0.452	0.474	0.244	0.373	0.368	0.374
5	6	60	0.304	0.275	0.282	0.143	0.228	0.210	0.226
5	6	90	0.283	0.235	0.244	0.120	0.196	0.168	0.195
5	10	25	0.659	0.685	0.716	0.349	0.544	0.532	0.549
5	10	60	0.420	0.408	0.442	0.206	0.362	0.325	0.360
5	10	90	0.357	0.307	0.336	0.163	0.270	0.240	0.266
6	2	25	0.160	0.160	0.169	0.083	0.143	0.148	0.148
6	2	60	0.147	0.120	0.126	0.057	0.096	0.103	0.096
6	2	90	0.142	0.100	0.105	0.046	0.080	0.086	0.080
6	6	25	0.343	0.375	0.386	0.193	0.312	0.311	0.315
6	6	60	0.286	0.269	0.278	0.131	0.213	0.206	0.209
6	6	90	0.261	0.222	0.227	0.108	0.171	0.159	0.168
6	10	25	0.486	0.597	0.580	0.271	0.473	0.451	0.478
6	10	60	0.383	0.401	0.431	0.183	0.330	0.332	0.322
6	10	90	0.343	0.324	0.344	0.152	0.257	0.251	0.250
Column Average			0.304	0.295	0.305	0.153	0.255	0.230	0.257

Table 5. Average root mean square errors of the estimators of β_1

H	J	n	OLS	2SLS	GMM	QML	EEL	EL	LEL
1	2	25	0.349	0.356	0.374	0.337	0.365	0.384	0.372
1	2	60	0.226	0.223	0.231	0.215	0.227	0.254	0.231
1	2	90	0.186	0.179	0.183	0.172	0.181	0.215	0.182
1	6	25	0.345	0.352	0.364	0.338	0.359	0.383	0.366
1	6	60	0.216	0.217	0.222	0.212	0.220	0.258	0.222
1	6	90	0.177	0.177	0.179	0.173	0.178	0.227	0.180
1	10	25	0.343	0.355	0.368	0.334	0.360	0.384	0.368
1	10	60	0.219	0.221	0.227	0.212	0.224	0.266	0.227
1	10	90	0.172	0.172	0.173	0.169	0.174	0.226	0.175
2	2	25	0.327	0.329	0.331	0.329	0.339	0.356	0.353
2	2	60	0.229	0.230	0.231	0.227	0.235	0.249	0.241
2	2	90	0.186	0.184	0.185	0.184	0.181	0.201	0.185
2	6	25	0.335	0.334	0.335	0.331	0.338	0.354	0.349
2	6	60	0.224	0.224	0.225	0.223	0.221	0.244	0.225
2	6	90	0.184	0.183	0.184	0.183	0.181	0.200	0.182
2	10	25	0.317	0.316	0.320	0.314	0.321	0.342	0.330
2	10	60	0.222	0.223	0.225	0.222	0.221	0.243	0.225
2	10	90	0.187	0.186	0.187	0.186	0.184	0.205	0.186
3	2	25	0.758	0.773	0.763	0.781	0.802	0.820	0.833
3	2	60	0.589	0.600	0.599	0.599	0.602	0.616	0.600
3	2	90	0.546	0.561	0.564	0.559	0.554	0.578	0.540
3	6	25	0.883	0.884	0.881	0.888	0.901	0.929	0.916
3	6	60	0.768	0.778	0.781	0.778	0.784	0.794	0.781
3	6	90	0.631	0.635	0.635	0.640	0.623	0.649	0.614
3	10	25	0.782	0.784	0.786	0.781	0.795	0.820	0.801
3	10	60	0.732	0.733	0.731	0.732	0.735	0.742	0.734
3	10	90	0.655	0.664	0.663	0.660	0.649	0.671	0.637
4	2	25	0.799	0.828	0.828	0.839	0.844	0.871	0.854
4	2	60	0.751	0.795	0.800	0.804	0.799	0.835	0.796
4	2	90	0.505	0.523	0.521	0.524	0.518	0.543	0.513
4	6	25	0.844	0.857	0.866	0.851	0.870	0.906	0.878
4	6	60	0.616	0.628	0.634	0.627	0.626	0.657	0.624
4	6	90	0.655	0.666	0.665	0.669	0.656	0.696	0.648
4	10	25	0.828	0.850	0.873	0.840	0.870	0.899	0.882
4	10	60	0.706	0.715	0.711	0.716	0.707	0.751	0.703
4	10	90	0.579	0.585	0.586	0.580	0.579	0.611	0.574
5	2	25	0.270	0.271	0.282	0.257	0.258	0.294	0.257
5	2	60	0.190	0.185	0.190	0.178	0.170	0.221	0.165
5	2	90	0.168	0.158	0.162	0.152	0.145	0.194	0.140
5	6	25	0.276	0.274	0.285	0.265	0.262	0.301	0.259
5	6	60	0.182	0.180	0.183	0.177	0.165	0.219	0.161
5	6	90	0.155	0.154	0.155	0.149	0.139	0.194	0.134
5	10	25	0.270	0.268	0.284	0.257	0.259	0.298	0.256
5	10	60	0.187	0.185	0.190	0.181	0.171	0.229	0.167
5	10	90	0.153	0.152	0.153	0.148	0.138	0.196	0.133
6	2	25	0.234	0.236	0.244	0.229	0.231	0.263	0.230
6	2	60	0.181	0.178	0.183	0.170	0.155	0.212	0.147
6	2	90	0.161	0.152	0.156	0.146	0.129	0.193	0.122
6	6	25	0.241	0.240	0.249	0.234	0.232	0.267	0.230
6	6	60	0.178	0.176	0.179	0.172	0.152	0.217	0.145
6	6	90	0.148	0.148	0.150	0.144	0.125	0.196	0.117
6	10	25	0.238	0.243	0.254	0.228	0.233	0.269	0.232
6	10	60	0.173	0.173	0.177	0.168	0.151	0.215	0.143
6	10	90	0.149	0.148	0.151	0.145	0.125	0.194	0.118
Column Average			0.382	0.386	0.390	0.382	0.383	0.418	0.383

Table 6. Average root mean square errors of the estimators of β_2

H	J	n	OLS	2SLS	GMM	QML	EEL	EL	LEL
1	2	25	0.351	0.361	0.376	0.342	0.371	0.386	0.379
1	2	60	0.223	0.221	0.230	0.213	0.228	0.250	0.231
1	2	90	0.185	0.178	0.183	0.172	0.182	0.213	0.184
1	6	25	0.343	0.351	0.367	0.332	0.360	0.385	0.369
1	6	60	0.218	0.219	0.223	0.214	0.225	0.257	0.227
1	6	90	0.178	0.175	0.178	0.172	0.177	0.221	0.181
1	10	25	0.343	0.353	0.364	0.336	0.362	0.387	0.370
1	10	60	0.216	0.217	0.221	0.212	0.220	0.258	0.223
1	10	90	0.175	0.175	0.177	0.172	0.177	0.225	0.179
2	2	25	0.152	0.151	0.155	0.147	0.151	0.183	0.152
2	2	60	0.108	0.102	0.106	0.100	0.099	0.151	0.098
2	2	90	0.092	0.085	0.087	0.083	0.082	0.144	0.081
2	6	25	0.160	0.160	0.161	0.156	0.155	0.191	0.154
2	6	60	0.109	0.107	0.108	0.106	0.103	0.158	0.101
2	6	90	0.086	0.085	0.085	0.084	0.081	0.150	0.080
2	10	25	0.162	0.162	0.165	0.158	0.157	0.193	0.157
2	10	60	0.100	0.098	0.100	0.097	0.096	0.153	0.095
2	10	90	0.085	0.084	0.085	0.084	0.082	0.154	0.081
3	2	25	0.254	0.254	0.255	0.250	0.239	0.282	0.231
3	2	60	0.188	0.187	0.190	0.181	0.169	0.227	0.161
3	2	90	0.182	0.171	0.176	0.166	0.151	0.221	0.140
3	6	25	0.238	0.235	0.243	0.227	0.220	0.263	0.217
3	6	60	0.205	0.204	0.203	0.200	0.183	0.241	0.172
3	6	90	0.174	0.179	0.179	0.179	0.148	0.238	0.134
3	10	25	0.239	0.239	0.250	0.231	0.230	0.272	0.227
3	10	60	0.209	0.208	0.208	0.204	0.185	0.257	0.176
3	10	90	0.185	0.186	0.185	0.185	0.164	0.242	0.153
4	2	25	0.436	0.452	0.473	0.437	0.461	0.486	0.464
4	2	60	0.316	0.321	0.329	0.305	0.318	0.354	0.312
4	2	90	0.257	0.255	0.262	0.247	0.248	0.297	0.243
4	6	25	0.426	0.440	0.459	0.421	0.441	0.474	0.441
4	6	60	0.302	0.306	0.313	0.293	0.297	0.350	0.292
4	6	90	0.255	0.257	0.262	0.254	0.248	0.315	0.242
4	10	25	0.445	0.458	0.480	0.444	0.458	0.495	0.461
4	10	60	0.303	0.310	0.314	0.299	0.304	0.358	0.299
4	10	90	0.243	0.243	0.246	0.240	0.236	0.302	0.233
5	2	25	0.278	0.280	0.289	0.271	0.267	0.304	0.261
5	2	60	0.198	0.192	0.199	0.186	0.173	0.238	0.167
5	2	90	0.172	0.163	0.168	0.158	0.149	0.213	0.143
5	6	25	0.279	0.285	0.291	0.273	0.267	0.310	0.261
5	6	60	0.189	0.189	0.192	0.184	0.171	0.243	0.166
5	6	90	0.159	0.157	0.158	0.153	0.141	0.217	0.136
5	10	25	0.275	0.276	0.287	0.266	0.259	0.304	0.253
5	10	60	0.192	0.192	0.193	0.189	0.172	0.249	0.166
5	10	90	0.159	0.158	0.161	0.156	0.144	0.224	0.137
6	2	25	0.252	0.253	0.262	0.246	0.245	0.276	0.242
6	2	60	0.194	0.191	0.198	0.186	0.165	0.230	0.154
6	2	90	0.172	0.164	0.168	0.159	0.138	0.208	0.128
6	6	25	0.249	0.250	0.256	0.243	0.240	0.275	0.237
6	6	60	0.188	0.187	0.189	0.185	0.160	0.231	0.148
6	6	90	0.160	0.159	0.159	0.156	0.132	0.209	0.124
6	10	25	0.248	0.253	0.260	0.241	0.241	0.276	0.237
6	10	60	0.185	0.184	0.188	0.181	0.157	0.230	0.147
6	10	90	0.160	0.158	0.161	0.156	0.132	0.210	0.123
Column Average			0.220	0.220	0.225	0.214	0.210	0.263	0.207

Table 7. The estimated response functions

Response function parameters	ρ_{OLS}	ρ_{2SLS}	ρ_{GMM}	ρ_{QML}	ρ_{EEL}	ρ_{EL}	ρ_{LEL}
			$H=1$				
a_1	1.173	0.960	1.029	0.731	0.925	0.521	0.935
a_2	-1.482	-1.986	-1.812	-2.014	-1.658	-1.070	-1.655
a_3	2.791	2.037	2.020	2.022	1.855	1.523	1.815
a_4	-1.369	-1.001	-0.993	-1.010	-0.903	-0.758	-0.873
a_5	-0.618	-0.948	-1.078	-1.072	-1.013	-0.619	-0.981
a_6	2.127	2.762	3.054	2.635	3.266	3.330	2.969
a_7	-1.343	-2.595	-3.644	-2.970	-3.967	-3.015	-3.692
a_8	-15.976	-6.504	-6.417	-5.579	-9.049	-6.907	-7.521
a_9	-0.071	-0.031	-0.047	-0.143	-0.049	-0.109	-0.046
R^2	0.904	0.884	0.876	0.939	0.895	0.884	0.891
			$H=2$				
a_1	0.282	0.189	0.262	0.152	0.153	0.193	0.138
a_2	-1.558	-1.918	-1.973	-2.341	-1.856	-1.681	-1.850
a_3	2.217	1.645	1.699	1.943	1.467	1.163	1.434
a_4	-1.084	-0.819	-0.830	-0.956	-0.728	-0.577	-0.712
a_5	-0.666	-0.959	-0.985	-0.976	-1.016	-0.569	-1.025
a_6	3.681	4.047	3.917	3.232	3.837	2.333	3.794
a_7	-3.295	-3.557	-3.797	-3.747	-3.898	-1.411	-3.956
a_8	-18.251	-11.342	-10.142	-7.629	-9.464	-7.720	-8.648
a_9	0.001	-0.012	-0.013	-0.060	-0.017	-0.053	-0.014
R^2	0.830	0.822	0.828	0.831	0.832	0.825	0.825
			$H=3$				
a_1	1.068	1.021	1.164	0.586	0.804	0.702	0.759
a_2	-1.676	-2.166	-2.251	-2.253	-1.993	-1.613	-1.977
a_3	2.494	2.101	2.146	2.111	1.958	1.761	1.865
a_4	-1.236	-1.062	-1.055	-1.110	-0.987	-0.835	-0.947
a_5	-0.391	-0.844	-1.001	-0.995	-0.923	-0.570	-0.948
a_6	1.396	2.263	1.822	2.377	2.381	2.082	2.349
a_7	-0.857	-1.951	-1.477	-2.742	-2.409	-1.903	-2.217
a_8	-20.025	-15.211	-14.357	-8.478	-12.888	-11.310	-12.109
a_9	-0.071	-0.060	-0.062	-0.147	-0.056	-0.081	-0.050
R^2	0.805	0.770	0.765	0.822	0.756	0.719	0.746
			$H=4$				
a_1	1.445	1.386	1.504	0.809	1.299	0.837	1.250
a_2	-1.434	-2.055	-1.975	-1.974	-1.795	-1.222	-1.706
a_3	2.852	2.229	2.475	2.062	2.185	1.874	2.103
a_4	-1.375	-1.041	-1.141	-1.019	-1.014	-0.886	-0.964
a_5	-0.568	-0.999	-1.187	-1.097	-1.067	-0.688	-1.014
a_6	1.996	2.472	2.026	2.607	2.717	2.486	2.802
a_7	-1.188	-1.876	-1.769	-2.943	-2.979	-1.661	-3.142
a_8	-19.228	-11.218	-10.296	-6.160	-12.958	-7.603	-12.176
a_9	-0.114	-0.065	-0.076	-0.164	-0.075	-0.125	-0.070
R^2	0.905	0.875	0.872	0.925	0.884	0.875	0.881
			$H=5$				
a_1	1.034	0.881	0.968	0.644	0.743	0.543	0.714
a_2	-1.488	-1.993	-1.916	-2.156	-1.843	-1.201	-1.832
a_3	2.855	2.208	2.435	2.178	2.202	1.919	2.176
a_4	-1.365	-0.987	-1.084	-1.036	-0.975	-0.881	-0.945
a_5	-0.541	-0.951	-1.067	-1.085	-1.035	-0.706	-1.014
a_6	2.307	3.264	3.626	2.620	3.729	4.728	3.540
a_7	-0.789	-2.279	-3.469	-2.695	-3.763	-4.794	-3.507
a_8	-18.970	-11.996	-12.391	-6.047	-12.602	-15.674	-11.182
a_9	-0.062	-0.028	-0.039	-0.139	-0.041	-0.082	-0.036
R^2	0.894	0.888	0.891	0.942	0.896	0.900	0.895
			$H=6$				
a_1	1.057	0.965	1.008	0.685	0.684	0.519	0.647
a_2	-1.475	-2.094	-1.926	-2.305	-1.947	-1.256	-1.964
a_3	2.603	2.017	2.192	2.116	1.960	1.638	1.898
a_4	-1.287	-1.004	-1.087	-1.054	-0.973	-0.824	-0.932
a_5	-0.353	-0.923	-1.032	-1.057	-1.014	-0.711	-1.005
a_6	1.508	2.341	3.012	1.983	2.735	4.717	2.402
a_7	-0.485	-1.355	-3.206	-2.200	-2.837	-5.855	-2.294
a_8	-20.427	-12.785	-14.060	-8.229	-9.522	-15.959	-7.343
a_9	-0.078	-0.030	-0.039	-0.151	-0.035	-0.070	-0.029
R^2	0.889	0.878	0.891	0.941	0.889	0.889	0.883

Table 8. The estimated parameters of hedonic housing model

Parameters	OLS	2SLS	GMM	QML	MEEL	MEL	MLEL
ρ	0.441	0.276	0.033	0.393	0.276	0.276	0.432
β_0	1.034	1.237	1.850	1.137	1.237	1.237	1.137
β_2	-0.004	-0.004	-0.004	-0.004	-0.004	-0.004	-0.004
β_3	0.000	0.000	0.000	0.000	0.000	0.000	0.000
β_4	0.001	0.001	0.000	0.001	0.001	0.001	0.001
β_5	0.009	0.022	0.002	0.012	0.022	0.022	0.012
β_6	-0.131	-0.141	0.396	-0.146	-0.143	-0.141	-0.146
β_7	0.003	0.004	0.000	0.003	0.004	0.004	0.003
β_8	0.000	0.000	0.001	0.000	0.000	0.000	0.000
β_9	-0.157	-0.142	0.108	-0.161	-0.141	-0.142	-0.161
β_{10}	0.083	0.083	0.150	0.083	0.084	0.083	0.083
β_{11}	0.000	0.000	0.000	0.000	0.000	0.000	0.000
β_{12}	-0.003	-0.004	-0.008	-0.004	-0.004	-0.004	-0.004
β_{13}	0.000	0.000	0.000	0.000	0.000	0.000	0.000
β_{14}	-0.286	-0.296	-0.623	-0.295	-0.294	-0.296	-0.295