AJAE appendix for '*The Gains from Differentiated Policies to Control Stock Pollution when Producers Are Heterogeneous*'

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Note: The material contained herein is supplementary to the article named in the title and published in the American Journal of Agricultural Economics (AJAE).

This document includes the proofs of Propositions 2 and 3, a corollary, the analysis of the steady state, and a sensitivity analysis of the numerical results. The organization generally follows that of the article.

Proof of Proposition 2

For a particular allocation of the land and taking into account that there is only one technology available, we can focus on equation (11) of the paper given by $\left(pf_{u_i} - c - \tau_i\right) x_i + \upsilon_i = 0,\ i = 1,2,$ to determine the change in input as a response to a change in the tax. By the implicit function theorem, we obtain that $\frac{\partial u}{\partial \tau} = \frac{1}{pf_{uu}} < 0$. Thus, at each land quality, the employed input is decreasing in the input tax. Moreover, the marginal land quality ε_m under the tax is determined such that $pf - cu - I - \tau u = 0$. Totally differentiating this expression with respect to τ, we obtain the change in the extensive margin as a result of a change in the tax. Consequently, an increase in the input tax decreases not only the employed input but also the amount of cultivated land. Therefore, there exists a unique relationship between the tax and the aggregated emissions,

$$\frac{\partial z}{\partial \tau} = \int_{\varepsilon_m}^{\varepsilon_1} \gamma(\varepsilon) \frac{\partial u(\varepsilon)}{\partial \tau} x(\varepsilon) l(\varepsilon) d\varepsilon - \gamma(\varepsilon_m) u(\varepsilon_m) x(\varepsilon_m) l(\varepsilon_m) \frac{d\varepsilon_m}{d\tau} < 0,$$

that is, an increase in the input tax decreases the aggregated emissions.

Moreover, let us assume that $\tau^{DU}(t) > \tau^*(t, \varepsilon_m^*)$. The optimal differentiated input tax, $\tau^*(t, \varepsilon^*)$, is decreasing in land quality, since it is proportional to the pollution coefficient $\gamma_i(\varepsilon)$, which in turn decreases with ε. Thus, for every land quality in production, i.e., $\varepsilon > \varepsilon_m$, the spatially uniform input tax will be higher than the optimal spatially differentiated tax. Knowing that $\partial u / \partial \tau < 0$, at each land quality the implementation of a spatially uniform input tax would lead to a lower level of input use compared to the socially optimal input use. Consequently, if $\tau^{DU}(t)$ were greater than $\tau^{*}(t,\varepsilon_m^{*})$, the generated emissions in the presence of a spatially uniform tax would be lower than the optimal level of emissions. Mathematically we have

$$z^{DU} \equiv \int_{\varepsilon_m^U}^{\varepsilon_1} \gamma(\varepsilon)u^{DU}(\varepsilon)x^{DU}(\varepsilon)l(\varepsilon)d\varepsilon < z^{*} \equiv \int_{\varepsilon_m^{*}}^{\varepsilon_1} \gamma(\varepsilon)u^{*}(\varepsilon)x^{*}(\varepsilon)l(\varepsilon)d\varepsilon,$$

where the superscript *DU* denotes the corresponding values of the variables in the presence of the optimal spatially uniform tax. As this is a contradiction with the assumption that aggregate emissions have to be identical for both policies, one can conclude that $\tau^{DU}(t)$ has to be smaller than $\tau^{*}(t,\varepsilon_m^{*})$. □

Corollary: *The value of the function V(z) of problem (2) is equivalent to the sum of the Aggregate Net Income of the farmers,* $ANI(\tau)$, and the collected taxes, $T(\tau)$, *where τ indicates a technologically and spatially differentiated tax on the variable input u. The intertemporal maximization of V(z), with respect to z, or of* $V(\tau) = ANI(\tau) + T(\tau)$ *with respect to* τ *leads to the same optimal trajectories of the variables in stage 1 and 2. Hence, the optimization process of V can either be formulated as a function of z or as a function of τ.*

Proof of the Corollary

The equivalence of $V(z)$ and $V(\tau) = ANI(\tau) + T(\tau)$ can be seen by observing that the first-order conditions of (3) - (4) and (11) – (12) provide the same optimal values of the variables $u(\varepsilon)$ and $x(\varepsilon)$ granted that the tax τ leads to the pollution z. Consequently, the necessary condition (5) will be also be satisfied. Taking into account that taxes revert to society, the social net benefit of farming is given by the aggregate net income of the farmers, *ANI*, plus the collected taxes, $T(\tau)$. In this way the functions $V(z)$ and $ANI(\tau) + T(\tau)$ are identical, and its evaluation at the optimal values of the decision variables produces the same values for $V(z)$ and $V(\tau)$. In the second stage we have to take into account that there is a functional relationship between z and τ as shown in the proof of proposition 2. Therefore, $V(z)$ turns into $V(z(\tau))$, and the formulation of the social planner's decision problem in the second stage, problem (6), turns into

(A1) $$W(\tau^*(t)) \equiv \max_{\tau(t)} \int_0^\infty \exp^{-\delta t} \left(V(z(\tau(t))) - m(s(t)) \right) dt,$$

subject to $\dot{s}(t) = z(\tau(t,\varepsilon)) - \zeta s(t), \quad s(0) = s_0.$

As the first order conditions of problem (A1) coincide with those of (6), we have that $W(z^*(t)) = W(\tau^*(t))$. □

Analysis of the steady state

For a sustainable environmental policy, the social planner is particularly interested in the achievement of a steady state, defined by equations (8) and (9) with $\dot{s} = \dot{\varphi} = 0$. For any initial value of s within the neighborhood of s^∞ where the superscript ∞ indicates the steady-state equilibrium value, it is possible to find a corresponding value of the shadow cost, which assures that the optimal environmental abatement policy leads toward the long-run optimum. Assuming an interior solution, equation (7) can be solved for $z = \hat{z}(\varphi, s)$. By the implicit function theorem we obtain

(A2) $$\frac{\partial \hat{z}}{\partial \varphi} = \frac{1}{V_{zz}} \le 0, \quad \frac{\partial \hat{z}}{\partial s} = 0.$$

Substituting into $z = \hat{z}(\varphi, s)$ (8) and (9) we obtain

(A3) $$\dot{\varphi} = \left(\delta + \zeta\right)\varphi - m_s,$$
$$\dot{s} = \hat{z}(\varphi, s) - \zeta s, \quad s(0) = s_0.$$

A linearization of the canonical system of differential equations around the steady-state values of φ and s results in

(A4)
$$\begin{pmatrix} \dot{\varphi} \\ \dot{s} \end{pmatrix} = \begin{pmatrix} \frac{\partial \dot{\varphi}}{\partial \varphi} & \frac{\partial \dot{\varphi}}{\partial s} \\ \frac{\partial \dot{s}}{\partial \varphi} & \frac{\partial \dot{s}}{\partial s} \end{pmatrix} \begin{pmatrix} \varphi - \varphi^{\infty} \\ s - s^{\infty} \end{pmatrix}.$$

The implicit function theorem is also used to calculate the elements of the Jacobian matrix evaluated at the steady-state equilibrium with $\dot{\varphi} = \dot{s} = 0,$ leading to

(A5)
$$\tilde{J} = \begin{pmatrix} \frac{\partial \dot{\varphi}}{\partial \varphi} = \delta + \zeta > 0 & \frac{\partial \dot{\varphi}}{\partial s} = -m_{ss} < 0 \\ \frac{\partial \dot{s}}{\partial \varphi} = \frac{1}{V_{zz}} < 0 & \frac{\partial \dot{s}}{\partial s} = -\zeta \leq 0 \end{pmatrix}.$$

Thus, the determinant of the Jacobian matrix is negative. Moreover, since the trace of the Jacobian matrix is equal to $\delta > 0,$ the eigenvalues have opposite signs. Therefore, the steady-state equilibrium is locally characterized by a saddle point. The isoclines of the phase diagram in the (s, φ) space are given by

(A6)
$$\frac{d\varphi}{ds}\Big|_{\dot{\varphi}=0} = -\frac{\frac{\partial \dot{\varphi}}{\partial s}}{\frac{\partial \dot{\varphi}}{\partial \varphi}} > 0, \qquad \frac{d\varphi}{ds}\Big|_{\dot{s}=0} = -\frac{\frac{\partial \dot{s}}{\partial s}}{\frac{\partial \dot{s}}{\partial \varphi}} < 0,$$

The resulting phase diagram is depicted in the figure A1. It shows that the stable path leading to the steady state is upward sloping, while the unstable path is downward sloping and, thus, the pollution stock and its shadow cost evolve in the same direction.

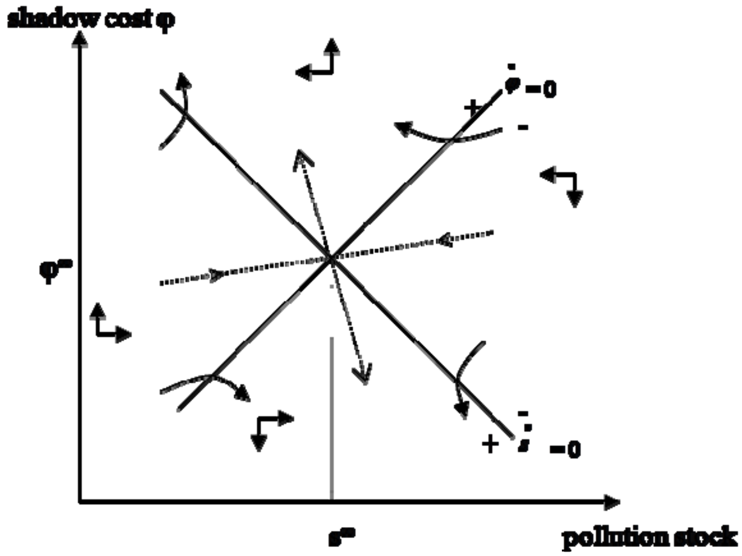

Figure A1: The Phase diagram in the (s, φ) space.

To find the optimal intertemporal path of $\hat{z}(t)$, we totally differentiate equation (7) with respect to time and obtain

(A7) $$\frac{d\hat{z}}{dt} = \frac{1}{V_{zz}}\frac{d\varphi}{dt}.$$

Taking into account that $V_{zz} \leq 0$, the aggregate emissions and the shadow cost evolve over time in opposite directions and therefore the aggregate emissions and the pollution stock also evolve in opposite directions.

Proof of Proposition 3

The solution of problem (A1) provides the optimal differentiated tax $\tau^*(t,\varepsilon)$, or the optimal static but technologically and spatially differentiated tax $\tau^{SS}(\varepsilon)$. Totally differentiating the Hamiltonian, H, associated with problem (A1) with respect to time and using the dynamic envelope theorem, we obtain $\frac{dH}{dt} = -\frac{dm}{ds}\dot{s} - \dot{\varphi}\dot{s} - \varphi\left(\dot{z} - \zeta\dot{s}\right)$. Making use of equation (8), we get

$\frac{dH}{dt} = -\varphi(\delta\dot{s} + \dot{z})$. The efficiency losses of a dynamic policy in comparison with a static policy can be measured by $\frac{dH}{dt} - \left.\frac{dH}{dt}\right|_{SS}$, where the subscript *SS* indicates the evolution of the Hamiltonian in the presence of a static tax. In the case of a static tax, the changes of *s* and *z* over time will be small, if not even zero, and therefore the losses in efficiency can be related with the magnitude of the slope of *H* with respect to time. In other words when $\frac{dH}{dt}$ in absolute terms is large, it is important to implement a dynamic policy. As shown in the previous analysis of the steady state, $\dot{s} = z - \zeta s$ and $\dot{z} = \frac{1}{V_{zz}}\dot{\varphi}$ have opposite signs, and therefore the efficiency losses of a static policy will be high when $\dot{s}$ and $\dot{z}$ do not compensate each other. This situation may arise as a result of two different circumstances.

Case 1:

We find that $\dot{s} = z - \zeta s$ is large and $\dot{z} = \frac{1}{V_{zz}}\dot{\varphi} = \frac{1}{V_{zz}}\left((\delta + \zeta)V_z - m_s\right)$ is small in absolute terms, if *z* is large, *s* is small, $|V_{zz}|$ is large, and $\dot{\varphi}$ is small. This case may occur if the current stock of pollutant is relatively small compared to the long-run equilibrium stock ($s < s^{\infty}$). In this situation the optimal pollution stock and its shadow cost increase over time and, consequently, the optimal spatially and temporally differentiated tax also increases over time. Therefore, the static tax will be higher than the initial value of the optimal spatially and temporally differentiated tax.

Case 2:

We observe that $\dot{s} = z - \zeta s$ is small and $\dot{z} = \frac{1}{V_{zz}} \dot{\varphi} = \frac{1}{V_{zz}} \left((\delta + \zeta) V_z - m_s \right)$ is large if z is small, s large, $\left| V_{zz} \right|$ small, and $\dot{\varphi}$ large. Case 2 is opposite to case 1 and characterizes the situation of a "restoration policy"; $s > s^{\infty}$. It requires that the tax decrease over time as a result of the decrease in the stock. Hence, the static tax is lower than the initial value of the optimal dynamic tax.

In both cases the static tax is in between the initial and the final value of the dynamic tax. Therefore, the efficiency losses of a static policy are smaller, the closer the initial value stock of the pollutant is to the steady-state value of the stock; vice versa, we obtain the opposite result.

In both situations, a large V_z and m_s, and a small decay rate, ζ, will make that $\dot{s}$ and $\dot{z}$ do not compensate each other, contributing to high efficiency losses.

Sensitivity analysis

Since the depth of the water table in the west part of San Joaquin Valley is usually between 5 and 15 feet, we also conducted a sensitivity analysis to evaluate how the initial severity of the waterlogging problem affects the efficiency of the second-best policies. Figure A2 shows the efficiency loss of the second-best policies for different initial water-storage capacities. The efficiency loss of the static but spatially differentiated tax increases with the water-storage capacity. This development can be explained by the fact that an initially higher water-storage capacity allows extending the duration of agricultural production, which, in turn, amplifies the error of a static tax. On the other hand, the efficiency loss of the dynamic but spatially uniform policy decreases with an increase in the water-storage capacity. In the case of an early intervention, when the initial water-storage capacity is high (25 feet), the dynamic spatially uniform policy outranks the static but spatially differentiated policy. Therefore, comparing second-best policies, in the case of an early intervention, it is more important to differentiate the tax over time than over space. If the initial water-storage capacity is low (5 feet), the ranking of these two policies is reversed, i.e., the static but spatially differentiated policy outranks the dynamic spatially uniform policy. In this case, a tax policy that discriminates according to the heterogeneity of the land quality will be the preferred second-best policy. This analysis shows that: a) for the same level of heterogeneity, the efficiency losses of the different policies depend on the severity of the initial environmental problem, and b) the ranking of the instruments can be reversed over time. Finally, figure A2 also shows for any initial water storage capacity that the efficiency loss of the static and spatially uniform tax is always higher than of any other considered policy.

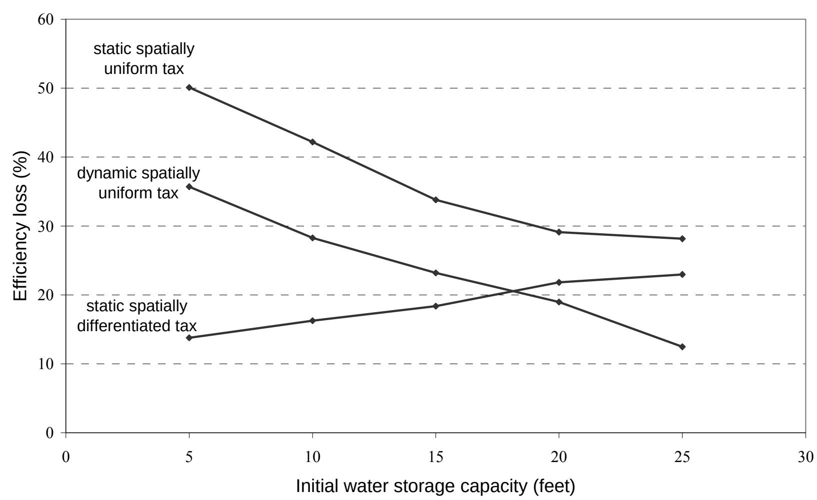

Figure A2. Efficiency loss of the second-best uniform policies as a function of the water-storage capacity