

Evaluating Vector-Virus-Yield Interactions for Peas and Lentils under Climatic Variability: A Limited Dependent Variable Analysis

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This study examines the effects of climatic conditions on interactions among pea and lentil yields, pea aphid *Acyrtosiphon pisum* (Harris) infestations, and outbreaks of PEMV (*Pea enation mosaic*) and BLRV (*Bean leaf roll*) viruses in the Palouse region of eastern Washington. The study analytically and empirically evaluates the effects of aphid outbreaks on per acre yields, implicitly including the effects of adopted pest management activities and explicitly taking into account possible effects of climatic conditions on the severity of pea aphid outbreaks. The results show that aphid outbreaks have historically decreased pea and lentil yields by approximately 5% and 7% on average respectively.

Key words: climate, pest damages, yields

Introduction

The Palouse area, which encompasses parts of eastern Washington and northern Idaho, is one of the leading pea and lentil producing regions in the United States (USDA/NASS, 2007). Food legumes (lentil, pea and chickpea) are principal commodities produced in eastern Washington and part of adjoining northern Idaho (the Palouse). Washington and Idaho produce about 11% and 24% of the total U.S. production of peas and lentils, respectively (USA Dry Pea and Lentil Council, 2007), with a total value of approximately \$160 million. In addition to their direct economic value, these legumes are critical as rotational crops with cereal grains, predominantly wheat. They fix nitrogen for subsequent grain crops, facilitate disease and insect management, support diversification of weed management methods, and increase profitability (Cook and Veseth, 1991; Young et al., 1996; Center, 2000). These crops are critical for agricultural industry in the region.

The viability of pea and lentil industry in the Palouse is threatened by periodic outbreaks of plant viruses, principally *Pea enation mosaic virus* (PEMV) and *Bean leaf roll virus* (BLRV), both of which are spread virtually exclusively by the pea aphid, *Acyrtosiphon pisum* (Harris) (Clement, 2006; Clement, Husebye, and Eigenbrode, 2010). Pea aphid outbreaks were first observed in Oregon in 1917 (Cooke, 1963; Eichmann, 1940) and have been documented in the Palouse since 1983 (Clement, 2006). The most recent infestation occurred in 2005.

PEMV and BLRV are circulative, persistently transmitted viruses (Sylvester, 1980; de Zoeten and Skaf, 2001). The relatively long acquisition, latent, and transmission periods of these viruses (relative to non-persistently transmitted viruses) make vector control a reliable control strategy

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(Perring, Gruenhagen, and Farrar, 1999). Pea aphid, the sole vector of these viruses in the Palouse, does not overwinter in this region due to dry late summers and cold winters. The aphids are known to emigrate seasonally from the lower elevations within the Columbia River Basin where winters are milder and host plants, like alfalfa, are abundant (Clement, 2006). Hence, insecticidal suppression of invading pea aphid is an effective strategy against BLRV and PEMV outbreaks (Davis, McEwen, and Schroeder, 1961; Stoltz and Forster, 1984). The extent of injury to the crop depends upon secondary (within field) spread of these viruses in pea and lentil fields, providing a period after initial introduction during which vectors can be treated successfully.

Our analytical model is based on economic literature that explicitly incorporates damage and production functions into a profit maximization framework (Lichtenberg and Zilberman, 1986; Saha, Shumway, and Havenner, 1997; Norwood and Marra, 2003). Unlike previous studies, which specify damage (or damage abatement) functions in terms of damage control inputs as well as pest populations, we specify our damage function in terms of number of pests only. Unfortunately, we lack appropriate data on historical use of corresponding aphid management activities. This kind of specification precludes us from the ability to explicitly examine the effectiveness of pest management activities. However, the purpose of this study is to evaluate the effects that aphid infestations have had on yields of peas and lentils under historical use of pesticides and other mitigation alternatives, given the influence of climatic variables on yields and aphid outbreaks.

The distinctive feature of our model is that in addition to direct effects of pests on crop yields, the model captures endogeneity of pests as influenced by climate parameters and planted acreage. The analytical model shows that when pests as well as yields are affected by climatic conditions we can decompose the total change in yield into two parts: one is the direct effect caused by pests and the other is the accompanying effect of climatic changes, which also affect pest populations. The analytical formulation shows conditions under which, given that pest population and crop yield functions are concave in climate parameters, the total effect can be unambiguously signed without assumptions on relative magnitudes of component parts.

Our data and empirical model are structured as follows: the annual record of pea aphid outbreaks is categorical, with five levels of severity. The data for virus outbreaks are binary. Consequently, ordered probit and probit specifications are used for the equations where aphid and virus are the dependent variables respectively. The yield functions for peas and lentils are quadratic. In order to account for the endogeneity of aphid infestation, we apply full information maximum likelihood (FIML) estimation to the system of four equations.

Our empirical results are consistent with the relationships examined in our analytical model. Specifically, temperature and precipitation in April display the effects consistent with our analytical model. Results also show that, on average, the marginal effect of aphid outbreaks under adopted pest management activities has been about a 5% reduction in pea yields and a 7% reduction in lentil yields.

Background and Prior Work

Over the past few decades, numerous studies have been published on the economics of pest management incorporating damage function as a key component of the analysis. Unlike previous studies, which typically treated damage control inputs as direct production factors (for example, Carlson, 1777), Lichtenberg and Zilberman (1986) specified damage control inputs within a separate abatement function in conjunction with the production function. They formulated the production function as $Q = F[\mathbf{Z}, G(\mathbf{X})]$, where \mathbf{Z} denotes direct production inputs and $G(\mathbf{X})$ denotes the damage abatement as a function of damage control inputs. This approach has become a standard framework (e.g., Qaim, 2003). The authors proved that an upward bias will be introduced if damage abatement inputs are treated as direct production inputs in a Cobb-Douglas type of production function (e.g., Carlson, 1777). However, when the model is specified as a simultaneous system of output and pesticide demand (Lee and Langham, 1973), then the Lichtenberg and Zilberman's critique does

not apply (see Babcock, Lichtenberg, and Zilberman, 1992). Saha, Shumway, and Havenner (1997) further extend Lichtenberg and Zilberman's model to incorporate possible interaction between damage control and direct production inputs. Formally, the production function is then specified as: $Q = F[\mathbf{Z}, G(\mathbf{X}, \mathbf{z})]$, where \mathbf{z} is a subset of direct input \mathbf{Z} that plays an interactive role with damage control inputs in the damage abatement function. Carpentier and Weaver (1997) prove that in order for the production function above to be multiplicatively separable—that is, $Q = f(\mathbf{z}^D)G(\mathbf{X}, \mathbf{z}^P)$, where \mathbf{z}^D represents direct inputs and \mathbf{z}^P interactive inputs—two conditions must be satisfied. First, $f(\mathbf{z}^D)$ has to exhibit constant return to scale; Second, effectiveness of damage control inputs has to be independent of \mathbf{z}^D .

A potential problem with the damage abatement method is that estimation results are sensitive to the functional form assumed for the abatement function. Fox and Weersink (1995) show that assuming a certain functional form for the damage abatement function is equivalent to imposing assumptions on the marginal return to abatement inputs. They demonstrate that different functional forms may result in contradictory estimates of marginal productivity of inputs. Fare, Grosskopf, and Lovell (1994) provide a nonparametric frontier estimation that relaxes the assumptions on the functional forms for the damage abatement function. In order to deal with the drawback of large data set required by a nonparametric technique and lack of algebraic relationship among variables, Kuosmanen, Pemsil, and Wessler (2006) develop a two-stage, semiparametric approach. They find that such an approach provides a way to combine nonparametric technique with a small sample size to deal with functional form issue.

Norwood and Marra (2003) argue that Lichtenberg and Zilberman's specification may lead to downward bias if pest pressure information is not present. They propose a yield function specification as $Y = \Pi - \delta Z[1 - \psi(D)]$, where Π is the maximum potential yield in the absence of pest damage, δ is the damage caused by a single pest, Z the maximum number of untreated pests, and $\psi(D)$ is the percentage of pest killed by pesticide (D). This formulation explicitly incorporates pest pressure in the yield function via parameter δ . Wossink and Rossing (1998) use a similar formulation that takes pest population into account. Reflecting this logic, our empirical analysis explicitly incorporates virus presence and aphid outbreak severity. Unfortunately, the drawback of our empirical analysis is that we lack information on the historical use of pest control inputs.

In spite of a significant number of studies devoted to the topic of pest damages and pest management, there are still some gaps. First, some pests can be categorized as vectors for viruses that severely damage crops. Such three-way interactions between yields, vectored diseases, and corresponding vectors have not received much attention. A study by Marsh, Huffaker, and Long (2000) is the most recent economic pest management study, to our knowledge, that examines optimal pest management rules under a three-way interaction between yields, viruses, and virus vectors. They use optimal control methodology to illustrate vector-virus-crop interactions and corresponding management options. In our analytical formulation, we follow this logic to examine the roles of viruses and virus vectors as influenced by climatic conditions in a static profit maximization model. Second, if we follow Wossink and Rossing (1998) or Norwood and Marra (2003) to study the effect of pests on yields in the context of vector born diseases, single equation estimation may encounter endogeneity bias. In our context, the endogeneity can be caused by aphid prevalence being a function of climatic conditions and planted acreage. Pea and lentil fields are the primary sources of habitat for the pea aphid. Hence, we suspect that smaller planted acreage of peas and lentils leads to smaller chances that a significant aphid infestation will occur. At the same time, more favorable climatic conditions for yields can also favor pest infestations. As mentioned above, Lee and Langham (1973) is an example of treating output and pesticide demand as a system of simultaneous equations. Other examples of simultaneous equations estimation can be found in Babcock, Lichtenberg, and Zilberman (1992) and Burrows (1983). Babcock, Lichtenberg, and Zilberman (1992) estimate a system of equations to examine the effectiveness of pest control inputs. Burrows (1983) is an empirical study of IPM (Integrated Pest Management) adoption.

While some studies explicitly incorporate pesticide use data (Burrows, 1983; Babcock, Lichtenberg, and Zilberman, 1992), we lack such opportunity. In practice, historical record of pesticide use is typically only available for major crops (USDA/NASS). Moreover, this kind of data is difficult to find on regional or county level. Studies like Burrows (1983) and Babcock, Lichtenberg, and Zilberman (1992) use data collected from samples of representative producers. Unfortunately, county level data on pesticide use in pea and lentil production is not available in our study region. On the other hand, historical data on pea aphid as well as PEMV and BLRV in the Palouse region has been recorded by entomologists (Clement, Husebye, and Eigenbrode, 2010). This article contributes to the existing literature in two important ways. First, we develop a theoretical profit maximization based framework to incorporate the vector-virus-crop interaction. We formulate an analytical model to depict the effects of pea outbreaks on yields under optimal planting and pesticide use decisions, given that the severity of pea aphid outbreaks can depend in part on climate and planted acreage. Second, we use the Full Information Maximum Likelihood method to estimate a system of equations consisting of yields, vector abundance, and virus outbreak equations. To the best of our knowledge, this is the first paper to employ this approach to address the crop-vector-virus interactions with continuous crop yield data, binary virus data, and ordinal multinomial data for vector outbreaks.

Analytical Framework

Consider a competitive producer with a single output (pea and/or lentil) and two inputs (planted acreage and pesticide). Assume that output is given by $Y(\theta)D(A,V)x_1$, where $Y(\theta)$ denotes maximum per acre yield as a function of climate parameter θ , $D(A,V)$ is the damage function denoting the proportion of maximum yield lost due to aphid infestation (A) and virus outbreak (V), and x_1 denotes planted acreage. Damage function D is bounded between 0 and 1, where $D = 1$ indicates that no crop is damaged and $D = 0$ corresponds to all yield being lost. By introducing the endogeneity of aphids, we assume that the number of aphids per acre is determined by planted acreage x_1 , pesticide usage x_2 and the climate parameter θ , or $A = A(x_1, x_2, \theta)$. Additionally, virus is assumed to be a function of the pea aphid, an exclusive vector of the virus, $V = V(A)$.

Let P denote the output price and w_1 and w_2 the factor opportunity cost for x_1 and x_2 respectively. The profit maximization problem is:

$$(1) \quad \max \quad \pi = PY(\theta)D(A, V)x_1 - w_1x_1 - w_2x_2$$

$$s.t. \quad A = A(x_1, x_2, \theta) \text{ and } V = V(A).$$

By substituting A and V functions into the damage control function, we can specify the first order conditions for the optimization problem and define two implicit functions of (x_1, x_2, θ) by equations (2) and (3):

$$(2) \quad F^1 = \frac{\partial \pi}{\partial x_1} = PY(\theta)D(A, V) + PYx_1 \left[\frac{\partial D}{\partial A} \frac{\partial A}{\partial x_1} + \frac{\partial D}{\partial V} \frac{\partial V}{\partial A} \frac{\partial A}{\partial x_1} \right] - w_1 = 0;$$

$$(3) \quad F^2 = \frac{\partial \pi}{\partial x_2} = PY(\theta)x_2 \left[\frac{\partial D}{\partial A} \frac{\partial A}{\partial x_2} + \frac{\partial D}{\partial V} \frac{\partial V}{\partial A} \frac{\partial A}{\partial x_2} \right] - w_2 = 0.$$

If we treat P , w_1 , and w_2 as constants, equations (2) and (3) will define the optimal use of x_1 and x_2 as functions of θ : $x_1^* = x_1^*(\theta)$ and $x_2^* = x_2^*(\theta)$. By implicit function theorem, the comparative statics of $\frac{dx_1^*}{d\theta}$ and $\frac{dx_2^*}{d\theta}$ can be evaluated. Hence, at $A^* = A(x_1^*, x_2^*, \theta)$ and $V^* = V(A^*)$, we have:

$$(4) \quad \frac{dA^*}{d\theta} = \frac{\partial A^*}{\partial x_1} \frac{dx_1^*}{d\theta} + \frac{\partial A^*}{\partial x_2} \frac{dx_2^*}{d\theta} + \frac{\partial A^*}{\partial \theta};$$

$$(5) \quad \frac{dV^*}{d\theta} = \frac{dV^*}{dA} \frac{dA^*}{d\theta} = \frac{dV^*}{dA} \left(\frac{\partial A^*}{\partial x_1} \frac{dx_1^*}{d\theta} + \frac{\partial A^*}{\partial x_2} \frac{dx_2^*}{d\theta} + \frac{\partial A^*}{\partial \theta} \right).$$

Equation (4) shows how aphid infestation level changes with respect to weather conditions under optimal decisions pertaining to planted acreage and pesticide application. In this case, climate conditions affect aphid outbreaks directly via $\frac{\partial A^*}{\partial \theta}$ and indirectly through producers' optimal decisions, which are also influenced by the climate conditions or associated expectations.

Notice that the damage function at the optimal decision point is $D^* = D(A^*, V^*) = D^*(\theta)$. We can express the marginal change in actual yield (or damage adjusted yield), $Y(\theta)D^*$, denoted by $\hat{Y}(\theta)$, with respect to climate variable as:

$$(6) \quad \frac{d\hat{Y}(\theta)}{d\theta} = Y'D^* + Y(\theta) \frac{dD^*}{d\theta} = Y'D^* + Y(\theta) \left(\frac{\partial D^*}{\partial A} + \frac{\partial D^*}{\partial V} \frac{dV^*}{dA} \right) \frac{dA^*}{d\theta}$$

Equation (6) shows how climate affects actual yield. $Y'D^*$ represents the direct effect of climate on the maximum potential yield where D^* accounts for the damage level at optimal decision point. Meanwhile, $Y(\theta) \left(\frac{\partial D^*}{\partial A} + \frac{\partial D^*}{\partial V} \frac{dV^*}{dA} \right) \frac{dA^*}{d\theta}$ shows the indirect effect of climate on actual yield through its effect on the number of pests.

Assuming that increases in aphid population and virus index lead to more damages (decrease in D), and increase in aphid population leads to a larger virus outbreak ($\frac{\partial D}{\partial A} < 0$, $\frac{\partial D}{\partial V} < 0$, and $\frac{dV}{dA} > 0$), the indirect effect in equation (6) and $\frac{dA^*}{d\theta}$ must have opposite signs. Notice also that damage (D^*) and yield ($Y(\theta)$) are positive. Therefore, the total effect of climate on actual yield will be unambiguously positive when yield is increasing in θ and A^* is decreasing in θ . The total effect will be unambiguously negative when yield is decreasing and A^* is increasing with respect to θ . Other than these two scenarios, the total effect is ambiguous and depends on the relative magnitudes of the direct and indirect effects.

To illustrate this relationship graphically, consider case 1 in figure 1 with simplified assumptions that both A^* and $Y(\theta)D^*$ are concave functions and A^* reaches maximum at θ_1 while $Y(\theta)$ reaches maximum at θ_2 . When $\theta < \theta_1$ (or $\theta > \theta_2$), both yield and aphid numbers are increasing (or decreasing), so the total effect is ambiguous. When $\theta_1 < \theta < \theta_2$, the increase in climatic variable decreases aphid infestation and increases crop yield, thus the total effect on actual yield is positive. In case 2, the relative locations of yield and aphid functions are reversed. Using the same logic, total effect can only be determined in the region of $\theta_1 < \theta < \theta_2$.

To examine the relationship between actual yields and aphid infestation, recall that under optimal decisions, the aphid and damage functions become parametric equations $A^*(\theta) = A(x_1^*(\theta), x_2^*(\theta), \theta)$, and $D^* = D(A^*, V^*) = D^*(\theta)$.

By applying the rule of differentiation to the pest damage adjusted yield function, we get:

$$(7) \quad \frac{dY(\theta)}{dA^*} = \frac{d[Y(\theta)D^*]/d\theta}{dA^*/d\theta} = Y(\theta) \left(\frac{\partial D^*}{\partial A} + \frac{\partial D^*}{\partial V} \frac{dV^*}{dA} \right) + \frac{Y'D^*}{dA^*/d\theta}.$$

In equation (7), the first component captures the direct marginal damage due to pest infestation. Since increase in both aphid and virus results in more damages and increase in aphid leads to more virus, this component is clearly negative. On the other hand, the sign of the second component, which depicts the interaction between marginal yield and changes in aphid prevalence due to changes in climatic conditions, remains ambiguous. It is clear that because the first component is negative, the second term will have to be negative for equation (7) to have an unambiguous sign. Otherwise assumptions are needed on the relative magnitudes of the two terms.

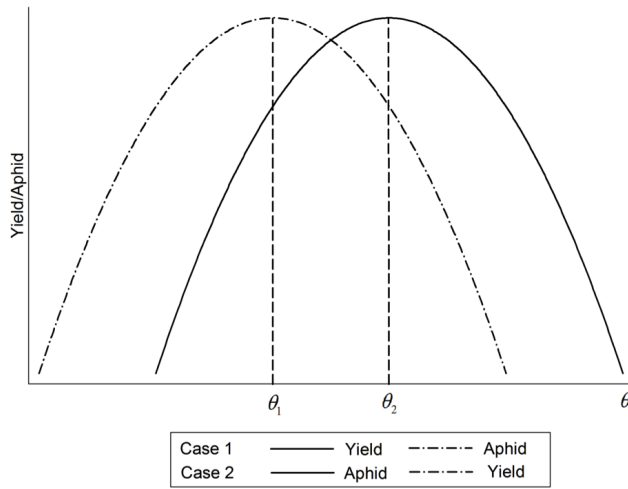


Figure 1. Illustration of Climate Effects on Yields and Pests

It can be shown (see appendix) that without magnitude assumptions, the sign of the total effect (equation 7) can be determined unambiguously in the following four cases:

- (a) $\frac{\partial F^1}{\partial \theta} > 0, \frac{\partial F^2}{\partial \theta} < 0, \frac{\partial F^2}{\partial x_1} < 0, \frac{\partial A^*}{\partial x_1} > 0, \frac{\partial A^*}{\partial \theta} > 0, Y' < 0;$
- (b) $\frac{\partial F^1}{\partial \theta} < 0, \frac{\partial F^2}{\partial \theta} < 0, \frac{\partial F^2}{\partial x_1} > 0, \frac{\partial A^*}{\partial x_1} < 0, \frac{\partial A^*}{\partial \theta} > 0, Y' < 0;$
- (c) $\frac{\partial F^1}{\partial \theta} < 0, \frac{\partial F^2}{\partial \theta} > 0, \frac{\partial F^2}{\partial x_1} < 0, \frac{\partial A^*}{\partial x_1} > 0, \frac{\partial A^*}{\partial \theta} < 0, Y' > 0;$
- (d) $\frac{\partial F^1}{\partial \theta} > 0, \frac{\partial F^2}{\partial \theta} > 0, \frac{\partial F^2}{\partial x_1} > 0, \frac{\partial A^*}{\partial x_1} < 0, \frac{\partial A^*}{\partial \theta} < 0, Y' > 0.$

Here, a) and b) correspond to the second case in figure 1 where the yield function reaches maximum at lower levels of θ than does the aphid function. Similarly, c) and d) correspond to the first case in figure 1 where aphid function reaches maximum at lower levels of θ than the yield function does.

Notice that in all four cases, the total effect (in equation 7) will be negative. Other than these four scenarios, the total effect cannot be unambiguously signed without making further assumptions on the relative magnitudes of component parts (see appendix for further details). It may at first sight appear that the total effect $\frac{dY(\theta)D^*}{dA^*}$ should be negative. However, it is important to keep in mind that the combination of the effect of θ on A , and the effect of θ on Y , will determine how $Y(\theta)D$ changes when A , at least in part influenced by θ , changes at optimal input decisions.

Data

Our choice of exogenous variables for the yield equations primarily follows Burrows (1983). However, unlike Burrows (1983) we don't have the opportunity to include pest management activities explicitly. In addition, we do not have access to producer level data in the Palouse region

Table 1. Summary Statistics

| Variable | Description | Obs | Mean | Std. Dev. | Min | Max |
|------------------|---|-----|----------|-----------|----------|----------|
| <i>A</i> | Aphid Index | 27 | 1.48 | 1.05 | 0.00 | 4.00 |
| <i>V</i> | Virus Index | 27 | 0.19 | 0.40 | 0.00 | 1.00 |
| <i>YL</i> | Lentil Yield (lbs) | 23 | 1,236.09 | 180.50 | 930.00 | 1,600.00 |
| <i>YP</i> | Pea Yield (lbs) | 23 | 1,954.78 | 299.09 | 1,300.00 | 2,500.00 |
| <i>PA</i> | Planted Acreage (thousand acre) | 23 | 171.91 | 23.31 | 130.00 | 223.00 |
| <i>Mar15Pre</i> | Total Precipitation from Mar 1st to Mar 15th | 27 | 0.63 | 0.52 | 0.00 | 2.47 |
| <i>Mar31Pre</i> | Total Precipitation from Mar 16th to Mar 31st | 27 | 0.86 | 0.66 | 0.09 | 2.60 |
| <i>Apr15Pre</i> | Total Precipitation from Apr 1st to Apr 15th | 27 | 0.62 | 0.43 | 0.01 | 1.43 |
| <i>Apr30Pre</i> | Total Precipitation from Apr 16th to Apr 30th | 27 | 0.84 | 0.75 | 0.00 | 3.17 |
| <i>May15Pre</i> | Total Precipitation from May 1st to May 15th | 27 | 0.70 | 0.49 | 0.00 | 1.70 |
| <i>May31Pre</i> | Total Precipitation from May 16th to May 31st | 27 | 0.33 | 0.33 | 0.00 | 1.03 |
| <i>Mar15Temp</i> | Average Temperature from Mar 1st to Mar 15th | 27 | 50.19 | 3.50 | 43.90 | 58.70 |
| <i>Mar31Temp</i> | Average Temperature from Mar 16th to Mar 31st | 27 | 52.83 | 2.35 | 48.40 | 58.70 |
| <i>Apr15Temp</i> | Average Temperature from Apr 1st to Apr 15th | 27 | 56.17 | 3.24 | 50.90 | 64.50 |
| <i>Apr30Temp</i> | Average Temperature from Apr 16th to Apr 30th | 27 | 60.75 | 2.95 | 54.70 | 64.90 |
| <i>May15Temp</i> | Average Temperature from May 1st to May 15th | 27 | 63.45 | 3.17 | 58.70 | 71.50 |
| <i>May31Temp</i> | Average Temperature from May 16th to May 31st | 27 | 66.84 | 2.81 | 61.80 | 74.00 |

Notes: Data come from the National Agricultural Statistics Service, National Climate Data Center, and Clement (2006). Total precipitation is measured in inches and average temperature is in degrees Fahrenheit.

and instead use aggregate region level data. Categorical aphid and virus outbreak data used in this study are the most exhaustive historical data on pea aphid as well as PEMV and BLRV outbreaks qualitatively provided in Clement (2006). These data (figure 3) were collected from 1983 to 2009 in the Palouse region of Washington.

Using pea aphid counts on plants in commercial pea fields, published accounts, and personal communications with other scientists, Clement (2006) devised a numerical index that measures severity of historical aphid outbreaks in the region. The data from Clement (2006) was expanded to cover the period from 1983 to 2009 (Clement, Husebye, and Eigenbrode, 2010). In this index, 0 corresponds to no aphids present; 1 corresponds to ‘few’ counted on plants; 2 corresponds to high counts (peak averages of 10-99 aphids per plant) in 26-50% of the sampled fields; 3 corresponds to very high counts (peak averages >100 aphids per plant) in 51-75% of the fields; and 4 corresponds to peak counts (averaging >100 aphids per plant) in 76-100% of the fields (Clement, 2006). The binary virus variable is 1 if virus outbreak was detected; and 0 otherwise (Department of Plant, Soil, and Entomological Sciences (UI-PSES), 2010).

The U.S. Dry Pea and Lentil Council provided records of planted acreages, yields, and production estimates for peas and lentils for Washington State. The only viable source of county level data for peas and lentils is census data. However, the use of these data would imply only five observations if we performed the analysis at the county level. Fortunately, a major share of peas and lentils produced in Washington State comes from the Palouse region of WA (including Whitman, Spokane and Garfield counties) (see figure 2). Therefore, we use state level data as proxy for the Palouse data. The state level data are given annually from 1986 to 2009.¹

Whitman Mission Station of the National Climate Data Center provides the historical records of temperature and rainfall, which are commonly believed to be the natural factors that impact

¹ 1987 census data for Garfield County is missing, which explains the large gap in that year.

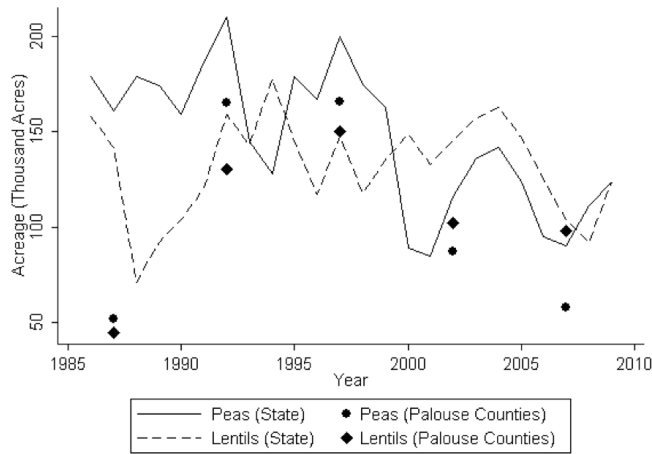


Figure 2. Pea and Lentil Acreages in WA State and the Palouse Counties

crops and insects. In any given period, the temperature variable is coded by averaging the daily mean temperatures in the period; the precipitation variable is generated by summing the daily total precipitation in the period. Our climate data is structured on bi-weekly basis from March 1st to May 31st consistent with ongoing research efforts at the University of Idaho Department of Plant, Soil and Entomological Sciences examining seasonal aphid outbreak dynamics (UI-PSES, 2010). Figure 3 combines the historical trends of pea and lentil yields per acre as well as aphid and virus outbreak indexes.

Table 1 provides summary statistics for the data. Average per acre yields for peas and lentils are 1954 pounds and 1236 pounds, respectively. The region suffered from BLRV and/or PEMV virus outbreaks in approximately 19% of years. Average aphid outbreak index was approximately 1.5, which corresponds to peak averages of 50 aphids per plant. The table also provides temperature and precipitation by periods from March 1 to May 31. This period is chosen because aphid infestations in the Palouse typically take place during this period (Department of Plant, Soil, and Entomological Sciences (UI-PSES), 2010; Clement, Husebye, and Eigenbrode, 2010). Figure 4 provides the normalized average precipitation and temperature from March 1 to May 31 from 1983 to 2009 to show the relative variability of temperature and precipitation. Both variables are divided by their respective means.

Empirical Estimation and Results

We imposed ordered probit and probit specifications on the equations where aphid and virus are the dependent variables respectively to reflect the categorical nature of the data. Roodman (2009) provides a consistent estimator for a broad family of seemingly unrelated regressions models. We follow his procedure to setup a Full Information Maximum Likelihood system of simultaneous equations with two continuous equations (for yields), one ordered probit (for aphid outbreaks) and one probit (for virus outbreaks). A stepwise procedure is used to choose the variables to be retained in the model. The results are provided in table 2.

The results show that temperature and precipitation between April 16 and April 30 follow the pattern illustrated in our analytical model with a quadratic shape for both crop yield and aphid growth. Based on the data used in this analysis, during the period between April 16 and April 30,

Table 2. Estimation Results

| Regressors | Pea Yield Equation | | Lentil Yield Equation | | Aphid Equation | | Virus Equation | |
|---------------------------------|------------------------------|------------------------------|------------------------------|-----------------------------|---|----------------------------|--------------------|--------------------|
| | FIML | OLS | FIML | OLS | FIML | ordered probit | FIML | probit |
| Aphid | -98.90*** (33.47) | -98.78* (47.40) | -81.37*** (17.34) | -81.08** (32.08) | | | 1.81*** (0.43) | 1.21*** (0.45) |
| Mar31Pre | 136.4*** (52.50) | 136.4* (72.49) | 108.8*** (28.76) | 108.8* (51.12) | 1.83*** (0.582) | 1.72*** (0.648) | | |
| Apr15Pre | | | 792.7*** (129.4) | 792.7** (243.8) | | | | |
| Apr15Pre ² | 378.1*** (64.33) | 378.1*** (87.88) | -271.5*** (98.68) | -271.5 (184.6) | | | | |
| Apr30Pre | 919.9*** (205.3) | 919.9*** (282.6) | 516.0*** (115.8) | 516.0** (204.8) | 4.54*** (1.281) | 4.23*** (1.599) | | |
| Apr30Pre² | -295.5*** (54.92) | -295.4*** (75.42) | -133.3*** (31.22) | -133.1** (54.71) | -0.99** (0.407) | -0.93** (0.464) | | |
| May15Pre | 619.9*** (73.83) | 619.9*** (101.2) | 634.0*** (99.15) | 633.9** (205.9) | 1.34* (0.71) | 1.00 (0.73) | | |
| May15Pre ² | | | -218.6*** (64.04) | -218.6 (133.7) | | | | |
| Mar15Temp | -48.37*** (14.72) | -48.31** (20.99) | -806.6*** (152.7) | -806.4** (291.9) | | | | |
| Mar15Temp ² | | | 7.91*** (1.56) | 7.90** (2.99) | | | | |
| Apr15Temp2 | | | | | 0.00* (0.00) | 0.00 (0.00) | | |
| Apr30Temp | 2,182*** (598.1) | 2,182** (832.8) | 1,058*** (318.8) | 1,059 (581.0) | 3.65*** (0.30) | 3.67 (4.33) | | |
| Apr30Temp² | -18.42*** (4.96) | -18.48** (6.91) | -8.77*** (2.65) | -8.80 (4.81) | -0.03*** (0.003) | -0.03 (0.036) | | |
| May15Temp | -1,032*** (376.3) | -1,032* (533.3) | -60.49 (188.1) | -60.15 (353.7) | | | | |
| May15Temp ² | 8.32*** (2.91) | 8.36* (4.12) | 0.75 (1.45) | 0.78 (2.73) | 0.00** (0.00) | 0.00* (0.00) | | |
| May31Temp ² | | | | | -0.00*** (0.00) | -0.00* (0.00) | | |
| Planted Acreage | | | | | -0.01 (0.01) | -0.01 (0.01) | | |
| Constant | -29,213 (21,101) | -29,213 (29,294) | -10,237 (12,510) | -10,237 (22,908) | | | -4.05*** (0.82) | -3.10*** (0.92) |
| R ² | | 0.87 | | 0.88 | | | | |
| Pseudo R ² | | | | | | 0.32 | | 0.50 |
| Adj R ² | | 0.73 | | 0.68 | | | | |
| Likelihood Ratio Test for FIML: | | | | | LR chi ² (36) = 114.35, Prob > chi ² = 0.0000 | | | |

Notes: Numbers in parentheses are standard errors. Single, double, and triple asterisks (*, **, ***) represent significance at the 10%, 5%, and 1% level. Bold variables exhibit statistically significant patterns illustrated in the analytical model.

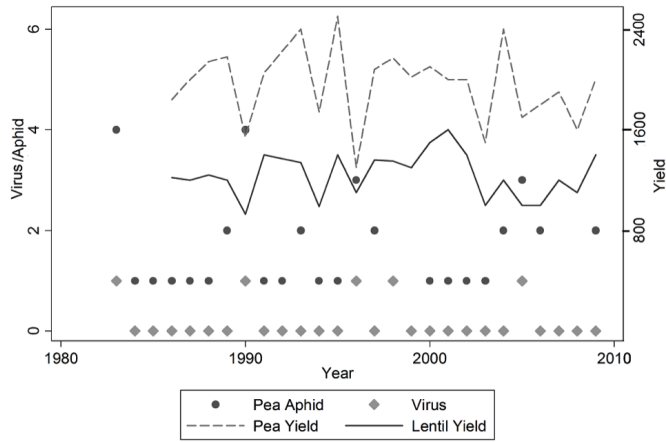


Figure 3. Historical Data on Yields (lbs), Aphid Infestations, and Virus Outbreaks

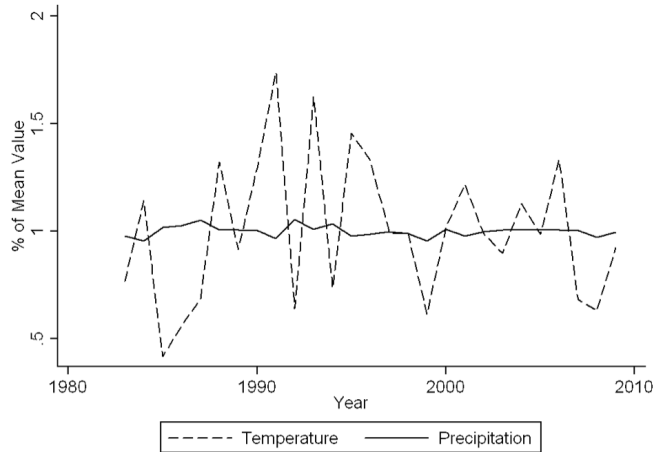


Figure 4. Historical Data on Precipitation and Temperature Variability

optimal precipitation and temperature for pea yields are 1.56 inches and 59.2 degrees respectively. Similarly, optimal precipitation and temperature for lentil yields are 1.94 inches and 60.4 degrees respectively. Optimal precipitation and temperature for aphid during the period are 2.3 inches and 64.5 degrees respectively. This scenario is depicted by case two in figure 1. Hence, increase in average temperature during this period beyond 59.2 degrees (current mean is 60.75 degrees) will have a negative effect on yields of peas. However, beyond 64.5 degrees, aphid population starts to decline which has a positive effect on yields. Similarly, increase in precipitation beyond 1.56 inches (current mean is 0.84 inches) decreases pea yield and increases aphid population. But beyond 2.3 inches, aphid population declines which has a positive effect on yields.

As expected, pea aphid infestations have positive impact on virus outbreaks. The results also show that, given historical use of adopted pest management practices, aphid outbreaks have had a moderate impact on yields (at least 5% for peas and 7% for lentils, on average). Since our empirical

analysis does not control for pesticide use (or any other pest control measure), the effect of aphids on yields estimated in this empirical analysis corresponds to pesticide-use-adjusted effect of aphids. The University of Idaho's Department of Plant, Soil, and Entomological Sciences (UI-PSES) (2010) reports that the effect of untreated aphid outbreaks on per acre yields is a 40% loss under 50 aphids per plant (corresponding to the average aphid severity in our case). Our results show that the effect of pesticide-use-adjusted outbreak of aphids on lentil yields is at least 7%. Hence, pesticide use has mitigated damages caused by the aphids at most by 33% of lentil yields per acre for 1.5 level of infestation severity.

LR test for the simultaneous equations model is significant, justifying the use of simultaneous equations modeling for estimating the inter-relationships among crop productivity, vector insect infestation, and virus outbreaks. As expected, FIML provides estimates with relatively lower standard errors than the single equation estimation.

Concluding Remarks

We derive analytical conditions for unambiguous interactive impacts of climatic conditions and pest outbreaks on crop yields. We empirically demonstrate this relationship in the case of pea and lentil yields as affected by temperature, precipitation and outbreaks of pea aphids. Analytical derivations show how yields are affected by pest outbreaks and climatic conditions when climate also affects pest prevalence. The correlation between pests and yields cannot always be signed unambiguously when pests are at least in part affected by climatic conditions. For example, if increase in pests is due to more favorable climatic conditions (like humidity), which also can have a positive effect on yields, then total change in yield cannot be signed *a priori* without more information about relative magnitudes of the two factors. However, under certain conditions the total effect can be signed. Specifically, if the marginal effects of climate on yield and aphid are opposite in signs, then total effect can be signed unambiguously.

Empirically, we apply the Full Information Maximum Likelihood (FIML) method to examine the interactions among pea and lentil yields, pea aphid infestations, and outbreaks of PEMV and BLRV viruses in the Palouse region of eastern Washington. The results confirm our theoretical analysis with temperature and precipitation from April 16 to April 30 as the climatic variables, which affect both aphid prevalence and yields. For example, between temperature levels of 59.2 and 64.5 degrees (current mean is at 60.8 degree), increase in temperature decreases pea yields and increases aphid infestation. The results also show that aphid infestations have a positive impact on the outbreaks of the viruses and significantly reduce yields even under conventional application of pesticides. For precipitation, our results show that between 1.9 and 2.3 inches (current mean is at 0.84 inches), increase in precipitation decreases pea yields and increases aphid infestation.

The primary limitation of this study is that data on historical use of pest control inputs were not available. Consequently, the results here can only be interpreted in terms of the effects of managed pest outbreaks on yields rather than the effects of unmanaged aphid outbreaks on yields. In other words, the impact of aphid outbreaks on yields estimated in this study corresponds to pesticide-use-adjusted impact of aphid infestations. Nevertheless, results show that under historical use of pesticides to control aphid outbreaks, increase in the severity of aphid outbreaks decrease pea yields by 99 pounds and decrease lentil yields by 81 pounds per acre. It should also be noted that since the data on yields are at the state level, the estimates here are likely to underestimate the historical effect of aphids/virus on per acre yields. If instead per acre yields from only the Palouse region were used, rather than per acre yields in the state, then the estimated effects of aphids/virus would probably be more significant. Hence, estimates in this study correspond to lower bounds of the effects of aphid outbreaks on yields.

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Appendix

In equation (7), $\frac{\partial D^*}{\partial A} + \frac{\partial D^*}{\partial V} \frac{\partial V^*}{\partial A}$ is always negative, assuming that damages increase as A and/or V increase. Since $Y(\theta)$ is positive, the direct effect, $Y(\theta) \frac{\partial D^*}{\partial A} + \frac{\partial D^*}{\partial V} \frac{\partial V^*}{\partial A}$, is negative. Since $D \in [0 : 1]$, the sign of the second term is jointly determined by Y' and $dA^*/d\theta$. Y' can be either positive or negative for a strictly concave function. The sign of $dA^*/d\theta$ can be explored to provide conditions that unambiguously imply the sign of equation (7).

By total differentiation of aphid function $A^*(\theta) = A(x_1^*(\theta), x_2^*(\theta), \theta)$, we have:

$$(A.A1) \quad \frac{dA^*}{d\theta} = \frac{\partial A^*}{\partial x_1} \frac{dx_1^*}{d\theta} + \frac{\partial A^*}{\partial x_2} \frac{dx_2^*}{d\theta} + \frac{\partial A^*}{\partial \theta}.$$

Equation (A.A1) represents the total effect of climate change on the number of aphids at optimal planted acreage and pesticide use.

Taking total derivatives of (1) and (2) produces the following implicit functions:

$$(A.A2a) \quad dF^1(x_1, x_2, \theta) = \frac{\partial F^1}{\partial x_1} dx_1 + \frac{\partial F^1}{\partial x_2} dx_2 + \frac{\partial F^1}{\partial \theta} d\theta = 0$$

$$(A.A2b) \quad dF^2(x_1, x_2, \theta) = \frac{\partial F^2}{\partial x_1} dx_1 + \frac{\partial F^2}{\partial x_2} dx_2 + \frac{\partial F^2}{\partial \theta} d\theta = 0$$

Rewriting the above implicit functions in matrix notation produces:

$$(A.A3) \quad \begin{pmatrix} \frac{\partial F^1}{\partial x_1} & \frac{\partial F^1}{\partial x_2} \\ \frac{\partial F^2}{\partial x_1} & \frac{\partial F^2}{\partial x_2} \end{pmatrix} \begin{pmatrix} \frac{dx_1^*}{d\theta} \\ \frac{dx_2^*}{d\theta} \end{pmatrix} = - \begin{pmatrix} \frac{\partial F^1}{\partial \theta} \\ \frac{\partial F^2}{\partial \theta} \end{pmatrix}.$$

Applying Cramer's rule we get:

$$(A.A4) \quad \frac{dx_1^*}{d\theta} = - \frac{\begin{vmatrix} \frac{\partial F^1}{\partial \theta} & \frac{\partial F^1}{\partial x_2} \\ \frac{\partial F^2}{\partial \theta} & \frac{\partial F^2}{\partial x_2} \end{vmatrix}}{\begin{vmatrix} \frac{\partial F^1}{\partial x_1} & \frac{\partial F^1}{\partial x_2} \\ \frac{\partial F^2}{\partial x_1} & \frac{\partial F^2}{\partial x_2} \end{vmatrix}} \quad \text{and} \quad \frac{dx_2^*}{d\theta} = - \frac{\begin{vmatrix} \frac{\partial F^1}{\partial x_1} & \frac{\partial F^1}{\partial \theta} \\ \frac{\partial F^2}{\partial x_1} & \frac{\partial F^2}{\partial \theta} \end{vmatrix}}{\begin{vmatrix} \frac{\partial F^1}{\partial x_1} & \frac{\partial F^1}{\partial x_2} \\ \frac{\partial F^2}{\partial x_1} & \frac{\partial F^2}{\partial x_2} \end{vmatrix}}.$$

Equation (A.A4) gives the comparative statics of planted acreage and pesticide use with respect to climate conditions at optimal planted acreage and pesticide use.

Substitution of equation (A.A4) into equation (A.A1) yields:

$$(A.A5) \quad \frac{dA^*}{d\theta} = \frac{-\frac{\partial A^*}{\partial x_1} \begin{vmatrix} \frac{\partial F^1}{\partial \theta} & \frac{\partial F^1}{\partial x_2} \\ \frac{\partial F^2}{\partial \theta} & \frac{\partial F^2}{\partial x_2} \end{vmatrix} - \frac{\partial A^*}{\partial x_2} \begin{vmatrix} \frac{\partial F^1}{\partial x_1} & \frac{\partial F^1}{\partial \theta} \\ \frac{\partial F^2}{\partial x_1} & \frac{\partial F^2}{\partial \theta} \end{vmatrix} + \frac{\partial A^*}{\partial \theta} \begin{vmatrix} \frac{\partial F^1}{\partial x_1} & \frac{\partial F^1}{\partial x_2} \\ \frac{\partial F^2}{\partial x_1} & \frac{\partial F^2}{\partial x_2} \end{vmatrix}}{\begin{vmatrix} \frac{\partial F^1}{\partial x_1} & \frac{\partial F^1}{\partial x_2} \\ \frac{\partial F^2}{\partial x_1} & \frac{\partial F^2}{\partial x_2} \end{vmatrix}}.$$

Based on the initial specification of the model in equation (1), $\begin{vmatrix} \frac{\partial F^1}{\partial x_1} & \frac{\partial F^1}{\partial x_2} \\ \frac{\partial F^2}{\partial x_1} & \frac{\partial F^2}{\partial x_2} \end{vmatrix} > 0$ as a second order condition

for profit maximization. Notice also that $\frac{\partial A}{\partial x_2} < 0$, assuming that the number of aphids decreases as pesticide

use increases. Hence, $dA^*/d\theta$ is determined solely by the signs of $\frac{\partial A^*}{\partial x_1} \begin{vmatrix} \frac{\partial F^1}{\partial \theta} & \frac{\partial F^1}{\partial x_2} \\ \frac{\partial F^2}{\partial \theta} & \frac{\partial F^2}{\partial x_2} \end{vmatrix}$, $\begin{vmatrix} \frac{\partial F^1}{\partial x_1} & \frac{\partial F^1}{\partial \theta} \\ \frac{\partial F^2}{\partial x_1} & \frac{\partial F^2}{\partial \theta} \end{vmatrix}$,

and $\frac{\partial A^*}{\partial \theta}$. We can tell that, unless we make further assumptions about relative magnitudes of component terms, (A.A5) is unambiguously greater than zero only if the first term is negative, the second term is positive, and the third term is positive. It is less than zero if the first term is positive, the second term is negative, and the third term is negative.

To demonstrate cases a), b), c), and d), we need to show that the following conditions determine the sign of $\frac{dA^*}{d\theta}$ unambiguously:

- $\frac{\partial F^1}{\partial \theta} > 0, \frac{\partial F^2}{\partial \theta} < 0, \frac{\partial F^2}{\partial x_1} < 0, \frac{\partial A^*}{\partial x_1} > 0, \frac{\partial A^*}{\partial \theta} > 0 \Rightarrow \frac{dA^*}{d\theta} > 0$ (Case 1)
- $\frac{\partial F^1}{\partial \theta} < 0, \frac{\partial F^2}{\partial \theta} < 0, \frac{\partial F^2}{\partial x_1} > 0, \frac{\partial A^*}{\partial x_1} < 0, \frac{\partial A^*}{\partial \theta} > 0 \Rightarrow \frac{dA^*}{d\theta} > 0$ (Case 2)
- $\frac{\partial F^1}{\partial \theta} < 0, \frac{\partial F^2}{\partial \theta} > 0, \frac{\partial F^2}{\partial x_1} < 0, \frac{\partial A^*}{\partial x_1} > 0, \frac{\partial A^*}{\partial \theta} < 0 \Rightarrow \frac{dA^*}{d\theta} < 0$ (Case 3)
- $\frac{\partial F^1}{\partial \theta} > 0, \frac{\partial F^2}{\partial \theta} > 0, \frac{\partial F^2}{\partial x_1} > 0, \frac{\partial A^*}{\partial x_1} < 0, \frac{\partial A^*}{\partial \theta} < 0 \Rightarrow \frac{dA^*}{d\theta} < 0$ (Case 4)

To confirm cases 1 and 2, notice that (A.A5) is unambiguously positive when the following conditions hold

$$\frac{\partial A^*}{\partial x_1} \begin{vmatrix} \frac{\partial F^1}{\partial \theta} & \frac{\partial F^1}{\partial x_2} \\ \frac{\partial F^2}{\partial \theta} & \frac{\partial F^2}{\partial x_2} \end{vmatrix} < 0, \begin{vmatrix} \frac{\partial F^1}{\partial x_1} & \frac{\partial F^1}{\partial \theta} \\ \frac{\partial F^2}{\partial x_1} & \frac{\partial F^2}{\partial \theta} \end{vmatrix} > 0, \frac{\partial A^*}{\partial \theta} > 0.$$

First, consider the situation where $\frac{\partial A^*}{\partial x_1} > 0$ and $\begin{vmatrix} \frac{\partial F^1}{\partial \theta} & \frac{\partial F^1}{\partial x_2} \\ \frac{\partial F^2}{\partial \theta} & \frac{\partial F^2}{\partial x_2} \end{vmatrix} < 0$. In such case the signs of the

determinants imply:

$$(A.A6) \quad \frac{\partial F^1}{\partial \theta} \frac{\partial F^2}{\partial x_2} - \frac{\partial F^1}{\partial x_2} \frac{\partial F^2}{\partial \theta} < 0;$$

$$(A.A7) \quad \frac{\partial F^1}{\partial x_1} \frac{\partial F^2}{\partial \theta} - \frac{\partial F^2}{\partial x_1} \frac{\partial F^1}{\partial \theta} > 0.$$

Profit maximization implies that $\frac{\partial F^1}{\partial x_1}$ and $\frac{\partial F^2}{\partial x_2}$ must be less than zero. Also, by Young's theorem, $\frac{\partial F^1}{\partial x_2} = \frac{\partial F^2}{\partial x_1}$. Notice that (A.A6) can only hold if the first term is negative and the second term is positive, unless we

make further assumptions about relative magnitudes of the two terms. $\frac{\partial F^2}{\partial x_2} < 0 \Rightarrow \frac{\partial F^1}{\partial \theta} > 0$. Also, since the

second term of (A.A6) needs to be positive, $\frac{\partial F^1}{\partial x_2}$ and $\frac{\partial F^2}{\partial \theta}$ must be of the same sign. Notice also that (A.A7) can hold only if the first term is positive and the second term is negative unless we make further assumptions

about relative magnitudes of the two terms. $\frac{\partial F^1}{\partial x_1} < 0 \Rightarrow \frac{\partial F^2}{\partial \theta} < 0$. Also, $\frac{\partial F^1}{\partial \theta} > 0 \Rightarrow \frac{\partial F^2}{\partial x_1} < 0$. Notice that the condition $\frac{\partial F^2}{\partial x_1} < 0$ meets the requirement from (A.A6) that $\frac{\partial F^1}{\partial x_2}$ and $\frac{\partial F^2}{\partial \theta}$ must be of the same sign. Therefore, inequalities (A.A6) and (A.A7) hold simultaneously when conditions in case 1 are satisfied.

Now let's consider the case when $\frac{\partial A^*}{\partial x_1} < 0$ and $\begin{vmatrix} \frac{\partial F^1}{\partial \theta} & \frac{\partial F^1}{\partial x_2} \\ \frac{\partial F^2}{\partial \theta} & \frac{\partial F^2}{\partial x_2} \end{vmatrix} > 0$. This implies that

$$(A.A8) \quad \frac{\partial F^1}{\partial \theta} \frac{\partial F^2}{\partial x_2} - \frac{\partial F^1}{\partial x_2} \frac{\partial F^2}{\partial \theta} > 0;$$

$$(A.A9) \quad \frac{\partial F^1}{\partial x_1} \frac{\partial F^2}{\partial \theta} - \frac{\partial F^2}{\partial x_1} \frac{\partial F^1}{\partial \theta} > 0.$$

By the same logic as above, equations (A.A8) and (A.A9) imply that $\frac{\partial F^1}{\partial \theta} < 0$, $\frac{\partial F^2}{\partial \theta} < 0$, and $\frac{\partial F^2}{\partial x_1} > 0$. These conditions, together with $\frac{\partial A^*}{\partial x_1} < 0$ and $\frac{\partial A^*}{\partial \theta} > 0$ correspond to case 2.

To confirm cases 3 and 4, again notice that (A.A7) is unambiguously negative, without making assumptions

about relative magnitudes of component parts when: $\frac{\partial A^*}{\partial x_1} \begin{vmatrix} \frac{\partial F^1}{\partial \theta} & \frac{\partial F^1}{\partial x_2} \\ \frac{\partial F^2}{\partial \theta} & \frac{\partial F^2}{\partial x_2} \end{vmatrix} > 0$, $\begin{vmatrix} \frac{\partial F^1}{\partial x_1} & \frac{\partial F^1}{\partial \theta} \\ \frac{\partial F^2}{\partial x_1} & \frac{\partial F^2}{\partial \theta} \end{vmatrix} < 0$, and $\frac{\partial A^*}{\partial \theta} < 0$.

First consider the case when $\frac{\partial A^*}{\partial x_1} > 0$ and $\begin{vmatrix} \frac{\partial F^1}{\partial \theta} & \frac{\partial F^1}{\partial x_2} \\ \frac{\partial F^2}{\partial \theta} & \frac{\partial F^2}{\partial x_2} \end{vmatrix} > 0$. The signs of the determinants imply:

$$(A.A10) \quad \frac{\partial F^1}{\partial \theta} \frac{\partial F^2}{\partial x_2} - \frac{\partial F^1}{\partial x_2} \frac{\partial F^2}{\partial \theta} > 0;$$

$$(A.A11) \quad \frac{\partial F^1}{\partial x_1} \frac{\partial F^2}{\partial \theta} - \frac{\partial F^2}{\partial x_1} \frac{\partial F^1}{\partial \theta} < 0.$$

By the same logic as in the cases 1 and 2, the conditions $\frac{\partial F^1}{\partial \theta} < 0$, $\frac{\partial F^2}{\partial \theta} > 0$, and $\frac{\partial F^2}{\partial x_1} < 0$ will ensure that both (A.A10) and (A.A11) will hold. Together with $\frac{\partial A^*}{\partial x_1} > 0$ and $\frac{\partial A^*}{\partial \theta} < 0$, these conditions correspond to case 3.

When $\frac{\partial A^*}{\partial x_1} < 0$ and $\begin{vmatrix} \frac{\partial F^1}{\partial \theta} & \frac{\partial F^1}{\partial x_2} \\ \frac{\partial F^2}{\partial \theta} & \frac{\partial F^2}{\partial x_2} \end{vmatrix} < 0$ we similarly observe that:

$$(A.A12) \quad \frac{\partial F^1}{\partial \theta} \frac{\partial F^1}{\partial x_2} - \frac{\partial F^1}{\partial x_2} \frac{\partial F^2}{\partial \theta} < 0;$$

$$(A.A13) \quad \frac{\partial F^1}{\partial x_1} \frac{\partial F^2}{\partial \theta} - \frac{\partial F^2}{\partial x_1} \frac{\partial F^1}{\partial \theta} < 0.$$

Again, (A.A12) and (A.A13) imply that $\frac{\partial F^1}{\partial \theta} > 0$, $\frac{\partial F^2}{\partial \theta} > 0$, and $\frac{\partial F^2}{\partial x_1} > 0$. Together with $\frac{\partial A^*}{\partial x_1} < 0$ and $\frac{\partial A^*}{\partial \theta} < 0$, these conditions correspond to case 4.

As discussed above, the sign of equation (7) can unambiguously be determined only as negative (unless we make further assumptions on the relative magnitudes of component terms) because the direct effect is always negative. For (7) to have an unambiguously negative sign the indirect effect, $\frac{Y'D^*}{dA^*/d\theta}$, should also be negative which means that Y' and $dA^*/d\theta$ must be of opposite signs. Combined with the four conditions that unambiguously reveal the sign of $dA^*/d\theta$, we obtain conditions a), b), c) and d).