A Technical Efficiency Analysis of Pennsylvania Dairy Farms

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Abstract

This study estimates technical efficiency (TE) using a stochastic production frontier based on a randomly selected cross-sectional sample of Pennsylvania dairy farms[1]. The Cobb-Douglas functional form is a suitable model in the estimation. Estimates of mean technical efficiency and appropriate technical efficiency of individual farms are computed and compared under three distributional assumptions of the efficiency disturbance terms. Maximum likelihood techniques are used for the estimation of the stochastic frontier. The technical efficiency measure is further broken down by different farm size.

Key Words: technical efficiency, dairy farms, stochastic frontier.

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Introduction

Over the past two decades much progress has been made towards refining the frontier function methodology introduced by Farrell in 1957 (Greene, 1980; Jondrow et al., 1982; Kumbhaker, 1987; Kumbhaker et al., 1989; Bravo and Rieger, 1990).

The earlier studies involved the estimation of the parameters of the stochastic frontier production function and the mean technical efficiency for firms in the industry (Battese and Coelli, 1988). It was initially claimed that technical efficiency for individual sample firms could not be predicted. Jondrow et al. (1982) proposed a method of separating the error term of the stochastic frontier model into its two components for each observation. This enables one to estimate the level of technical inefficiency for each observation in the sample. In their paper they presented two predictors for the firm effect for an individual firm on the assumption of the technical inefficiency error term u for the half-normal case and exponential case. Later on Battese and Coelli (1988) presented a more general distribution for firm effects under the assumption that panel data on sample firms are available. Battese and Coelli's paper concludes that the more general model for describing firm effects in frontier production functions accounts for the situations in which there is high probability of firms not in the full technical efficiency. The technical efficiency measures are sensitive to the distributional assumptions. Along with several methodological developments there has been a considerable amount of empirical work, much of it using data from agricultural firms, particularly dairy farms (Bravo and Rieger, 1990). These empirical studies have used a variety of methods and specifications, but there is not enough attention given to exploring the differences of the resulting efficiency measures to the assumptions employed.

Given this perspective, this study uses a stratified random sample of Pennsylvania dairy farms to examine the different technical efficiency measures. Specific objectives of this paper are as follows:

1. To specify an appropriate functional form of the sampled Pennsylvania dairy farms.

2. To estimate the mean technical efficiency under three distributional assumptions: half-normal, generalized truncated normal and normal one-parameter exponential.

3. To predict the technical efficiency for each individual farms under three distributional assumptions.

4. To examine the farm size on the effect of technical efficiency.

The next section contains a brief summary of the theoretical framework used followed by a discussion of the model specification and data. The paper then proceeds with the results and analysis, and ends with some concluding remarks.

Theoretical Framework

Consider the general frontier production function

$$Y_i = x_i \beta + \varepsilon_i \ ,$$

where Y_i denotes the appropriate function (e.g., logarithm) of the production for the *i*th sample firm (i=1,2,…, N); x_i is a vector of appropriate functions of the inputs for observation *i;* β is a vector of coefficients for the associated independent variables in the production function; the ε_i is the error term for observation *i.* The "stochastic frontier" model, introduced by Aigner, Lovell and Schmidt (1977) and Meeusen and Van Den Broeck (1977), postulates that the error term ε_i is made up of two independent components,

$\varepsilon_i = v_i - u_i$,

where v_i are random variables that are assumed to be independent and identically distributed as N$(0, \sigma^2{}_v)$ representing the usual statistical noise and are independent of u_i. On the other hand, u_i are random variables, which are assumed to be independent and identically distributed non-negative random variables representing technical inefficiency. Note that u_i measures technical inefficiency in the sense that it measures the shortfall of output (Y_i) from its maximal possible value given by the stochastic frontier [$x_i\beta + v_i$].

Then consider three cases (Greene, 1998):

I. Assume u_i in the half-normal case (u_i distributed as the absolute value of a N(0, $\sigma^2{}_u$) variable).

$$f(u) = (2/\pi)^2 \exp[-1/2(u/\sigma_u)^2],$$

$$E[u] = \sigma_u \phi\ (0)/\Phi(0) = (2/\pi)^2 \sigma_u$$

$$Var[u] = (1 - 2/\pi)\sigma_u^2$$

II. Assume u_i in the truncation at zero of the N (μ, $\sigma^2{}_u$) case.

$$f(u) = (1/\sigma_u)\phi[(u-\mu)/\sigma_u]/\Phi[\mu/\sigma_u]$$

$$E[u] = \mu + \sigma_u \phi[\mu/\sigma_u]/\Phi[\mu/\sigma_u] = \mu + \sigma_u \lambda_u$$

$$Var[u] = \sigma_u^2[1 - \lambda_u(\mu / \sigma_u + \lambda_u)]$$

III. Assume u_i in the normal one-parameter exponential case.

$$f(u) = \theta \exp(-\theta u)$$

$$E[u] = 1/\theta$$

$$\text{Var [u]}=1/\theta^2$$

When a model of this form is estimated, one readily obtains residuals $\hat{\varepsilon}_i = Y_i - x_i\hat{\beta}$, which can be regarded as estimates of the error term ε_i. Of course, the average technical inefficiency--the mean of the distribution of the u_i--is easily calculated.

Given the frontier production function was stated in logarithmic form in Schmidt and Lovell (1980) and Battese and Coelli (1988), the mean technical efficiency for half-normal case is:

$$\text{MTE} = 2[1 - \Phi(\sigma_u)]\exp(1/2 * \sigma_u{}^2)$$

for the truncated normal case is:

$$\text{MTE} = \{1 - \Phi[\sigma_u - (\mu / \sigma_u)]\} / [1 - \Phi(-\mu / \sigma_u)] * \exp(-\mu + 1/2 * \sigma_u^2)$$

for the exponential case is:

$$\text{MTE} = \theta / (1 + \theta)$$

Besides the mean technical efficiency, it is clearly desirable to be able to estimate the technical efficiency u_i for each observation. Jondrow et al. (1982) proposed that the individual technical efficiency could be considered by the conditional distribution of u_i given ε_i. This distribution contains whatever information ε_i yields about u_i. Either the mean or the mode of this distribution can be used as a point estimate of u_i. In this paper the individual firm technical inefficiency is calculated only by the way of the mean.

For the half-normal case:

$$E[u \mid \varepsilon] = \sigma\lambda(1+\lambda^2)[\phi(\varepsilon\lambda/\sigma)/(1-\Phi(\varepsilon\lambda/\sigma)) - \varepsilon\lambda/\sigma]\text{, where } \sigma = \sqrt{(\sigma_u^2 + \sigma_v^2)}\text{,}$$

$\lambda = \sigma_u / \sigma_v$

For the truncated normal model, change $\varepsilon\lambda/\sigma$ to

$$\mu^* = \varepsilon\lambda/\sigma + \mu(\sigma\lambda)$$

For the exponential model:

$$E[u \mid \varepsilon] = z + \sigma_v\phi(z/\sigma_v)/\Phi(z/\sigma_v)\text{, where } z = \varepsilon - \theta\sigma_v^2$$

Model Specification and Estimation

Although several functional forms can be used to specify the stochastic frontier, desirable forms are linear in the parameters and easily facilitate calculation of individual values for technical inefficiency (TI) and efficiency (TE). Forms that are multiplicative in the inputs and error terms are excellent candidates for the stochastic frontier. In this paper I will assume the Cobb-Douglas functional form is the appropriate model (which has been the practice in most published efficiency papers) and then test the joint significance of the unrestricted model translog functional form. The inefficiency error term is estimated under the three different distributional assumptions: half-normal, truncated normal and normal--one-parameter exponential. Estimation of different models is completed by maximum likelihood estimation (MLE). All estimators begin with ordinary least squares estimates, which are used to obtain the starting values. The OLS estimates are also used to obtain starting values for the variance parameters for the models. With the exception of the constant term, the OLS estimates are consistent, albeit inefficient (Battest and Coelli, 1988; Greene, 1998).

Data and Measurement

The analysis is based on data obtained from a stratified random sample of dairy farms from Pennsylvania. The original data were collected from a mail survey of Pennsylvania and Vermont dairy farms from the population of farms shipping milk in 1996 for the purpose of research in dairy feeding system (Winsten, Parsons and Hanson, 2000). The data in this study is based on a sub-sample of farms, which was drawn randomly following stratification procedures from the mail survey respondents. Since the observations from Vermont are very low (n=24), they were excluded from this study. This left the qualified observations for Pennsylvania dairy farms are 70. Detailed production system technology, farm and family characteristics were obtained from each farms through personal interviews conducted at the farm site (Winsten, Parson and Hanson, 2000).

The frontier production function specified for the dairy farm is defined by

$$\ln Y_i = \beta_0 + \beta_1 \ln x_{1i} + \beta_2 \ln x_{2i} + \beta_3 \ln x_{3i} + v_i - u_i$$

where the subscript i (i =1,2,…, N) refers to the ith sample farm. Since these dairy farms are engaged mainly in milk production, in the present study, the total pounds of milk per cow times number of cows for the year were used as the total output for the milk, which is denoted by Y. The x_1 denotes the labor variable. This variable is measured by total hours of the family and hired labor in the production of milk. In the literature, capital variable is mostly defined as user cost of capital equipment, which includes depreciation, maintenance, insurance, and net interest rate costs for machinery, inventory, and building (Kumbhakar and Heshmati, 1995). Since this data set don't have such kind of variable directly available, in order to capture the most characteristics of this variable, the capital variable is measured by the average of the net asset worth for the year, which is denoted by x_2. The feed variable is calculated by the total consumption of grain and forage per cow times the number of cows per farm, which is denoted by x_3. Considering the diverse farm sizes we added two dummy variables to control the farm size. The number of dairy cows is a convenient proxy for farm size (Kumbhakar et al., 1989). The medium farms are defined as having milking cows between 50 and 100. The large farms are defined as having more than 100 milking cows. This leaves the reference category as small farms, which have less than 50 milking cows. There are total 24 small farms, 30 medium farms and 16 large farms in the data set. Table 1 depicts some summary statistics of the Pennsylvania dairy farms.

Table 1. Summary Statistics of the Pennsylvania Dairy Farms (n=70)

Descriptive statistics	Symbol	Mean	Std. Deviation
Farm assets ($1000)		620.257	455.781
Income from milk ($1000)		24.174	40.415
Farm debts ($1000)		143.651	166.907
Farm land (acres)		234.143	172.426
Number of cows		80.84	55.25
Age of farmers		45.77	11.67
Years of farming of operator		3.04	0.97
Total pounds of milk per farm	Y	1359.09	1055.02
Labor (hours per year)	X1	7055.59	3370.51
Capital ($1000)	X2	552.859	446.679
Feed (lbs)	X3	125.472	101.532

Empirical Results

First, OLS estimation is used to choose the appropriate functional form between Cobb-Douglas function and translog function. The F-test is used to test the joint significance of the coefficients of the unrestricted model in translog for the quadratic terms and interaction terms. The value of F-statistic is small (0.253), which are not significant. It shows that the smaller model is more appropriate. Another realistic reason to choose Cobb-Douglas model is that given the sample size is very small, it saves some degree of freedom. The following analysis is completely based on the Cobb-Douglas functional form.

The parameter estimates for models of different assumptions by way of OLS and MLE are reported in table 2. The estimates for the standard deviations of the OLS and maximum likelihood estimators are presented in parentheses below the OLS and maximum likelihood estimates. Considering the different farm size, I try to estimate the separate model for each small, medium and large farms. However, due to the limit of sample size, it's very difficult to get a good fit. So I combine the medium and large group of farms to estimate the bigger farms (having more than 50 cows) compared to smaller farms (having less than 50 cows). This is also reported in Table 2.

In the all farms model, all variables are significant at the 0.1 level except capital under three different distribution assumptions. These coefficients (elasticities) are found to be all positive, which shows that the marginal product of each input is positive. The

Table 2. Parameter Estimates for Frontier Production Functions for the Pennsylvania Dairy Farms

Estimation Methods With Assumptions	Variable: Intercept	Labor	Capital	Feed	Medium (dummy)	Large (dummy)	Variance Parameters: $\sigma^2(v)$	$\sigma^2(u)$	μ	θ	Log likelihood	RTS RTS	Mean Technical Efficiency
All Farms (n=70)													
OLS (R^2=0.82)	-1.045	0.3504	0.0788	0.538	0.653	1.191	--	--	--	--	--	0.967	--
	(1.56)	(0.127)	(0.035)	(0.203)	-0.916	-0.13							
MLE													
Half Normal	0.133	0.396	0.081	0.36	0.6577	1.162	0.039	0.131			-12.71	0.837	0.86
	(1.77)	(0.166)	(0.077)	(0.198)	-0.097	-0.164							
Truncated Normal	-0.694	0.448	0.0785	0.407	0.665	1.138	0.047	0.145	0.503		-12.78	0.934	0.592
	(1.83)	(0.149)	(0.073)	(0.208)	0.095	-0.155							
Exponential	0.028	0.406	0.0814	0.3498	0.667	1.154	0.048	0.039		5.068	-12.23	0.837	0.835
	(1.53)	(0.148)	(0.068)	(0.172)	-0.09	-0.146							
Smaller Farms[1] (n=24)													
OLS (R^2=0.21)	1.6217	0.442	0.758	0.06	--	--	--	--	--	--	--	1.260	--
	(3.25)	(0.303)	(0.044)	(0.328)									
MLE													
Half Normal	0.913	0.704	0.076	-0.095	--	--	0.000	0.287			-2.44	0.685	0.683
	(6.07)	(0.534)	(0.091)	(0.611)	--	--							
Truncated Normal	1.1	0.68	0.081	-0.095	--	--	0.000	0.240	-0.224		-2.42	0.666	0.749
	(4.76)	(0.637)	(0.11)	(0.115	--	--							
Exponential	1.622	0.442	0.076	0.06	--	--	0.026	0.088		3.366	-5.24	0.578	0.771
	(3.245)	(0.303)	(0.044)	(0.328)	--	--							
Bigger Farms[2] (n=46)													
OLS (R^2=0.58)	-4.99	0.70	0.639	0.782	--	--	--	--	--	--	--	2.121	--
	(2.22)	(0.136)	(0.085)	(0.317)									
MLE													
Half Normal	-3.52	0.718	0.066	0.599	--	--	0.059	0.148			-14.64	1.383	0.754
	(2.26)	(0.135)	(0.105)	(0.373)	--	--							
Truncated Normal	-4.53	0.764	0.527	0.687	--	--	0.063	0.232	0.816		-14.64	1.978	0.461
	(2.72)	(0.137)	(0.104)	(0.4)	--	--							
Exponential	-3.05	0.734	0.07	0.501	--	--	0.064	0.049		4.520	-14.07	1.305	0.819
	(1.93)	(0.125)	(0.096)	(0.322)	--	--							

Note: 1.Smaller Farms are defined as the number of milking cows is less than 50.
2.Bigger Farms are defined as the number of milking cows is greater than 50.

labor coefficient (elasticity) has the highest value compared to feed and capital in each of these models. The size-related dummy variables in all farms model show that, on average, outputs of medium-sized farms are around 93 percent (exp(0.6577)-1) higher and those of large-sized farms are 220 percent (exp(1.162)-1) higher compared to small-sized farms in half-normal model. The truncated normal and exponential model has the similar results. After calculating the returns to scale by the coefficients (elasticity), it was found that all farms have decreasing returns to scale. This finding is consistent with a competitive market structure.

The estimates of mean technical efficiencies (MTE) are also presented in table 2. Comparing these numbers across three different distribution assumption models, the truncated normal has very low mean technical efficiency. The half-normal and exponential normal all have MTE between 0.84-0.86. A test of half-normal and generalized truncated normal is examined. If the parameter μ has value zero, then twice the negative of the logarithm of the generalized likelihood ratio for the restricted (μ =0) and unrestricted ($\mu \neq 0$) frontier models has approximately chi-square distribution with parameter equal to one. The value of this statistic is 0.45, which is not significant at the 0.05 level. So we conclude that the unrestricted truncated normal model is not an adequate representation for the dairy farms in this data.

Now we turn to the estimations compared between the smaller farms and bigger farms. The model for smaller farms does not have a good fit since the observations are very low (n=24). The bigger farms model has a better fit. The R-squared is 0.58. This leaves the comparison between two types of farms with caution. Interestingly, the smaller farms have returns to scale less than unity, while the bigger farms have returns to scale greater than one. If this conclusion is based on “valid” comparisons, this could partly explain why the Pennsylvania dairy farms increased substantially. During the 1950 to 1990 period, the average number of cows on Pennsylvania dairy farms increased from 9 to 50. The average dairy herd size reached 57 cows per farm in 1997 (Pennsylvania Agricultural Statistics Service).

Similarly, the truncated normal is not an appropriate model. The bigger farms have higher mean technical efficiency compared to smaller farms. It is more interesting to compare the technical efficiencies of individual dairy farms.

Table 3 provides the frequencies and percentages of technical efficiencies of individual farms calculated by the formula showed before under three different distribution assumptions and also subdivided by farm size.

The all farm technical efficiencies ranged from 36.34%-93.59% in half normal model, 43.09%-94.5% in truncated normal model and 37.12-94.59% in exponential model. The mean of farm technical efficiencies in half normal model, truncated normal and exponential model is 76.07%, 82.1% and 82.87%. Respectively, the median is 79.27%, 85.26% and 86.27%. The half normal model has consistently lower level of technical efficiencies than truncated model and exponential model. There is no evidence that truncated normal model accounts for high probability of firms not being in the neighborhood of full technical efficiency (Battese and Coelli, 1988).

Comparing the farm technical efficiencies across small farms and bigger farms, the small farms have lower technical efficiencies. The mean of farm technical efficiencies for half normal, truncated normal and exponential is 67.98%, 67.92% and

Table 3. Frequencies and Percentages of Technical Efficiencies Within Decile Ranges For Pennsylvania Dairy Farms By Three Different Distribution Assumptions

All Farms

Technical Efficiency Interval (Percentage)	Half normal		Truncated Normal		Exponential	
	Frequency	%	Frequency	%	Frequency	%
<0.55	3	4.29	2	2.86	2	2.86
0.55-0.6	2	2.86	0	0.00	0	0.00
0.6-0.65	9	12.86	1	1.43	1	1.43
0.65-0.7	5	7.14	3	4.29	3	4.29
0.7-0.75	5	7.14	10	14.29	7	10.00
0.75-0.8	11	15.71	4	5.71	6	8.57
0.8-0.85	21	30.00	15	21.43	13	18.57
0.85-0.9	11	15.71	27	38.57	26	37.14
0.9-0.93	2	2.86	6	8.57	9	12.86
>=0.93	1	1.43	2	2.86	3	4.29
Total	70	100.00	70	100.00	70	100.00

Smaller Farms[1]

Technical Efficiency Interval (Percentage)	Half normal		Truncated Normal		Exponential	
	Frequency	%	Frequency	%	Frequency	%
<0.55	7.00	29.17	7.00	29.17	3.00	12.50
0.55-0.6	3.00	12.50	3.00	12.50	2.00	8.33
0.6-0.65	0.00	0.00	0.00	0.00	1.00	4.17
0.65-0.7	1.00	4.17	2.00	8.33	2.00	8.33
0.7-0.75	4.00	16.67	2.00	8.33	2.00	8.33
0.75-0.8	4.00	16.67	5.00	20.83	0.00	0.00
0.8-0.85	0.00	0.00	0.00	0.00	3.00	12.50
0.85-0.9	0.00	0.00	1.00	4.17	7.00	29.17
0.9-0.93	2.00	8.33	1.00	4.17	3.00	12.50
>=0.93	3.00	12.50	3.00	12.50	1.00	4.17
Total	24.00	100.00	24.00	100.00	24.00	100.00

Bigger Farms[2]

Technical Efficiency Interval (Percentage)	Half normal		Truncated Normal		Exponential	
	Frequency	%	Frequency	%	Frequency	%
<0.55	2	4.35	1	2.17	2	4.35
0.55-0.6	2	4.35	1	2.17	0	0
0.6-0.65	4	8.7	1	2.17	1	2.17
0.65-0.7	6	13.04	2	4.35	2	4.35
0.7-0.75	6	13.04	5	10.87	4	8.7
0.75-0.8	10	21.74	8	17.39	6	13.04
0.8-0.85	10	21.74	8	17.39	10	21.74
0.85-0.9	4	8.7	17	36.96	16	34.78
0.9-0.93	2	4.35	2	4.35	4	8.7
>=0.93	0	0	1	2.17	1	2.17
Total	46	100	46	100	46	100

Note: 1.Smaller Farms are defined as the number of milking cows is less than 50.
2.Bigger Farms are defined as the number of milking cows is greater than 50.

76.17% for smaller farms, and correspondingly 74.74%, 80.56% and 81.14% for bigger farms. From the frequency tables it is obvious that bigger farms are running more technically efficient than small farms. This result is consistent with the findings from Kumbhakar et al. (1989) for Utah dairy farms.

Conclusions

In this study I have considered estimation of technical efficiency (TE) using a stochastic production frontier applied to a sample of Pennsylvania dairy farms under three different distribution assumptions about the efficiency disturbance term. The traditional Cobb-Douglas production function is a suitable model in the estimation. Estimates of mean technical efficiency and technical efficiency of individual farms are computed and presented based on three distributional assumptions of the efficiency disturbance terms. The likelihood ratio test concludes that the generalized truncated normal model is not appropriate for this data set. The mean technical efficiency reviews that generally Pennsylvania dairy farms realizes around 85% technical efficiency in the output of milk production. The result of individual farm technical efficiency indicates that large farms are technically more efficient than small farms.

Future Research

Further work is required on the modeling of stochastic frontier production functions and their technical inefficiencies of production over time for different farms. Additional empirical work is also desirable to include other explanatory variables in the stochastic frontiers and the technical inefficiency models. Consideration of risk issues with a view to their incorporation into production frontier analyses is an area for future work.

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