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A Bayesian Approach to Optimal Cross-Hedging of Cottonseed Products Using Soybean Complex Futures

Shaikh Mahfuzur Rahman, Jeffrey H. Dorfman, and Steven C. Turner

Cottonseed crushers face substantial risk in terms of input and output price variability and they are limited in their planning by the lack of a viable futures contract for cottonseed or cottonseed products. This study examines the feasibility of cross-hedging cottonseed products using the soybean complex futures. Different cross-hedging strategies are evaluated for eight time horizons relative to the expected profit and utility of the crusher. A Bayesian approach is employed to estimate both model parameters and optimal hedge ratios, allowing consistency with expected utility maximization in the presence of estimation risk. The results reveal that both whole cottonseed and cottonseed products can be successfully cross-hedged using soybean complex futures. The profitability of cross-hedging cottonseed products depends on the size of the contract, the optimal choice of strategy, the time of hedge placement, and the hedging horizon.

Key words: Bayesian decision science, cottonseed, cross-hedging, risk management

Introduction

With each hundred pounds of fiber, the cotton plant produces approximately 155 pounds of cottonseed. At present production levels, the national average is around 990 pounds of cottonseed produced per acre of cotton grown [National Cottonseed Products Association (NCPA), 2002]. Less than 5% of the seed must be set aside to plant the following year's crop. The remaining seed is used in the cottonseed processing industry or is fed to cattle. A small amount is exported. When raw cottonseed moves from the gin to a cottonseed oil mill, it is composed of three parts: linters, which are short fibers still clinging to the seed; hulls, a tough, protective coating for the kernel; and the protein- and oil-rich kernel itself. In recent years, industry-wide yields of products per ton of cottonseed have averaged about 320 pounds of oil, 900 pounds of meal, 540 pounds of hulls, and 160 pounds of linters, with manufacturing loss of 80 pounds per ton (NCPA, 2002).

Thus, the value of cottonseed is determined by the value of the products produced. Of the four primary products produced by cottonseed processing plants, oil is the most valuable. On average, it accounts for about 40–50% of the total value of all four products. Approximately 1.3 billion pounds of cottonseed oil are produced annually, making

Shaikh Mahfuzur Rahman is Ph.D. student, Department of Agricultural and Resource Economics, University of Maryland; Jeffrey H. Dorfman is professor, Department of Agricultural and Applied Economics, The University of Georgia; and Steven C. Turner is professor and department head, Department of Agricultural Economics, Mississippi State University.

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cottonseed oil the third leading vegetable oil in the United States (NCPA, 2002). Cottonseed meal is the second most valuable product of cottonseed, usually accounting for over one-third of total product value. It may be sold in the form of meal, cake, flakes, or pellets. Cottonseed meal is used principally as feed for livestock and is generally sold at a 41% protein level. Its major value is as a protein concentrate.

Cottonseed hulls are used primarily as feed for livestock. Hulls differ from meal in that they are roughage rather than a protein supplement. In feeding value, hulls are comparable to good quality grass hay and can serve as a practical supplement to pastures. Cottonseed linters, the short fibers removed from seed as the first step in processing, are sometimes referred to as "the fabulous fuzz." Through mechanical and chemical conversion, they enter a wider variety of end-use products than any of the other products of cottonseed.

Cottonseed products enter markets that are highly competitive. Soybean oil, corn oil, peanut oil, sunflower and safflower oil, and some of the animal fats are competitors of cottonseed oil. Cottonseed meal encounters a similar degree of competition from other protein concentrates, like peanut meal and sunflower meal, but especially soybean meal. Other cottonseed products face similar numbers of potential substitutes. As a result, cottonseed crushers face substantial price risk. With no viable futures market existing for cottonseed oil, meal, hulls, and linters,[1] cross-hedging offers an opportunity to mitigate this risk.

The central hypothesis of this study is that even though no active futures market exists for whole cottonseed and the cottonseed crush, processors can reduce input and output price risk through cross-hedging. The input price risk can be reduced by cross-hedging the whole cottonseed with soybeans, and the output price risk can be reduced through cross-hedging cash cottonseed products with soybean products, commodities having established futures markets.

By definition, cross-hedging is the pricing of a cash commodity position by using futures for different commodities. Simple cross-hedging uses futures of one commodity to offset a cash position, and multiple cross-hedging uses two or more different commodities. However, cross-hedging is more complicated than a direct hedge. Difficulties arise in selecting the appropriate futures contracts as cross-hedging vehicles and determining the size of the futures position to be established. Potential cross-hedging vehicles must be commodities that are likely to demonstrate a strong direct or inverse price relationship to the cash commodity. This study is concerned only with simple cross-hedging. Soybean, soybean oil, and soybean meal are selected as cross-hedging vehicles for this analysis because soybean complex products are often the closest substitutes for cottonseed products. Furthermore, there are active futures markets for soybeans, soybean oil, and soybean meal.

When cross-hedging their inputs and outputs, cottonseed crushers must make two very important decisions. First, they must decide how much of the commodities to hedge. Second, they must decide when to place and lift the hedge. In this study, we present a set of time-specific cross-hedging strategies for cottonseed and cottonseed products which are easy to manage. Employing Bayesian decision science, expected utility and profit-maximizing cross-hedge ratios are estimated for eight different hedging horizons.

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[1] The Minneapolis Grain Exchange traded a cottonseed futures contract for two years starting in May 2000. The contract never achieved a sustainable volume and was therefore de-listed.

These optimal Bayesian cross-hedging strategies are then applied in a simulated cross-hedge of cottonseed, meal, and oil in the crop year 1998–99 for different risk-aversion levels. The simulation results support the optimal estimated hedge strategies with few exceptions.

A Brief Review of the Literature

Previous Works on Cross-Hedging

An extensive theoretical description of cross-hedging for a commodity for which no futures market exists is provided by Anderson and Danthine (1981). Assuming a non-stochastic production process (no yield risk), Anderson and Danthine considered the problem of hedging in a single futures market but with many possible trading dates. Their cross-hedging model used a mean-variance framework to derive an optimal hedging strategy, assuming the agent had knowledge of the relevant moments of the probability distribution of prices. Kahl (1983) illustrated the derivation of optimal hedging ratios under different assumptions about the cash position, arguing that, when the futures and cash positions were endogenous, the optimal hedging ratio was independent of risk aversion. In her comparison of studies by Heifner (1972) and Telser (1955/56), Kahl showed that the optimal hedging ratio was not dependent on the risk parameter. Following Wilson (1989), the optimal hedge ratios obtained from minimizing the variance of revenue were equivalent to parameters estimated from ordinary least squares (OLS) regression of cash price changes on future price changes.

Ames, Shumaker, and Myneni (1992) investigated the possibility of cross-hedging canola with a complex of soybean meal and soybean oil futures contracts. Adopting a minimum variance method as a measure of hedging effectiveness, they found canola could be effectively hedged using soybean oil and meal futures. Fackler and McNew (1993) examined the derivation and estimation of optimal hedge positions for firms that deal in multiple commodities and have multiple relevant futures contracts available for hedging. They note that soybean crush hedging must account for the existing relationships among the cash prices of soybeans, soybean oil, and soybean meal. Further, they showed that hedge ratios derived from separate single commodity estimates are also suboptimal. Based on their results, multi-product optimal hedge positions provide significant risk reductions relative to simpler approaches.

In a cross-hedging consulting study performed for a cottonseed crusher, Dahlgran (2000) examined how futures markets should be used to hedge cottonseed crushing. He applied a soybean crushing spread in a cross-hedging context with a portfolio risk-minimization objective to develop the desired hedge ratios for a variety of cross-hedging portfolios with a wide selection of commodities and for several hedge horizons. Dahlgran reported that the effectiveness increased the longer the term of the hedge. He also documented actual experiences of businesses applying his recommendations, and found that the economics of hedge management might be as important as the underlying risk aversion in determining hedging behavior.

Previous Works Examining Estimation Risk

Whenever economic analysis involves incorporating estimated parameters into theoretically derived decision rules, the optimal outcome depends on the estimation procedure.

This problem is called estimation risk (Bawa, Brown, and Klein, 1979). Estimation risk is ever-present in economic problems. Typically it is ignored, and sample parameter estimates are directly substituted for the true but unknown parameters in theoretical decision rules.

As shown by Lence and Hayes (1994), the optimal futures position estimated by means of the parameter certainty equivalent (PCE) approach lacks normative value because it is generally suboptimal when there is uncertainty regarding the actual parameter values. They provided a model based on a Bayesian decision criterion which can be used to obtain an optimal futures position in the realistic situation where the decision maker has sample information and prior beliefs regarding the relevant parameters. Lence and Hayes claimed their model nested both the theoretical model with perfect parameter information and the PCE formula, and yielded the perfect parameter information paradigm when the decision maker is completely confident about his or her prior information relative to the sample information. The PCE formula was nested within their model when the sample (data) information completely dominated the prior information and the sample size is infinite. Lence and Hayes also presented the results of some simulations regarding the futures position obtained by means of the Bayesian criterion, the PCE approach, and the perfect parameter information (PPI, i.e., assuming the priors equal the true parameters) case. The simulations demonstrated the sensitivity of the optimum futures position to the method which was used. The authors inferred that the differences in the optimal futures position implied a large monetary value to investors using the proposed method. Our study applies a very similar methodology to the situation of cross-hedging.

Decision Making Under Risk for Cottonseed Crushers

Assume a risk-averse cottonseed crusher has a utility function $u(\pi)$, where π is the total profit from crushing, characterized by two important properties: nonsatiation and decreasing marginal utility of returns [$u'(\pi) > 0$, and $u''(\pi) < 0$]. These two properties imply the utility function is concave. Because of the shape of the utility function, a risk-averse individual prefers a sure amount to taking a risk with the same expected payoff; i.e., $u[E(\pi)] > E[u(\pi)]$. Risk-averse individuals are willing to pay an insurance premium to avoid the uncertainty involved in risky decisions.

Typically, a risk-averse decision maker is assumed to have a negative exponential utility function of the form:

$$u(\pi) = -e^{-\phi\pi}, \tag{1}$$

where ϕ denotes the Arrow-Pratt coefficient of absolute risk aversion. Constant absolute risk aversion (CARA) is commonly used to analyze producers' decisions under risk (e.g., Antle and Goodger, 1984; Babcock, Choi, and Feinerman, 1993; Buccola, 1980; Chalfant, Collender, and Subramanian, 1990; Lee and Brorsen, 1994; Lence and Hayes, 1994; Yassour, Zilberman, and Rausser, 1981). The value of ϕ generally lies between 0.001 and 0.000001, with smaller values implying less risk aversion.

Following Kallberg and Ziemba (1979), the risk-aversion level can be defined as $\alpha = \phi\omega$, where α is the level of risk aversion and ω is initial wealth. If gross returns per dollar of initial wealth are around unity, then moderate risk aversion corresponds to α

Table 1. Possible Risk Aversion Coefficient (ϕ) Values

	Level of Risk Aversion		
Wealth at Risk	Moderate-to-Low ($\alpha = 1$)	Moderate ($\alpha = 2$)	Moderate-to-High ($\alpha = 3$)
ω = \$50,000	$\phi = 0.00002$	$\phi = 0.00004$	$\phi = 0.00006$
ω = \$100,000	$\phi = 0.00001$	$\phi = 0.00002$	$\phi = 0.00003$

values between 2 and 4. In the present case, it is difficult to calculate the actual wealth at risk. It was found that the operating capital of a plant with annual crushing capacity of 1,000 tons of cottonseed is approximately \$200,000. Approximately, 50–75% of this amount is typically borrowed capital (Right, 2000). Therefore, ω was set equal to \$50,000 and \$100,000. Defining risk-aversion levels as moderate-to-low ($\alpha = 1$), moderate ($\alpha = 2$), and moderate-to-high ($\alpha = 3$), the corresponding coefficients of absolute risk aversion can be solved for and used in the empirical model to follow, allowing for some sensitivity analysis of the risk-aversion level assumed and ensuring the values entertained are plausible. Values for ϕ are shown in table 1. While calibration of the coefficient of absolute risk aversion is not new, it is rare; given the simplicity of calibrating it by this method, we are unsure why such an exercise is not done more often.

In equation (1), π is the total profit from cottonseed crushing. The profit function of a cottonseed crusher who cross-hedges using soybean complex futures includes futures prices of soybean and soybean products along with the cash prices of cottonseed and cottonseed products, crushing cost, and the corresponding hedge ratios. Consider a cottonseed crusher with a crushing plant capable of crushing 1,000 tons (2,000,000 pounds) of cottonseed cost-effectively. According to the National Cottonseed Products Association (NCPA), crushing 1,000 tons of cottonseed produces 900,000 pounds of cottonseed meal, 320,000 pounds of cottonseed oil, 540,000 pounds of hulls, and 160,000 pounds of linters. This leads to the profit from crushing 1,000 tons of cottonseed, given by:

$$(2)\qquad \pi = -(2{,}000{,}000 \times P_c) + \beta_c \times 2{,}000{,}000(P_{sl} - P_{sp}) + (900{,}000 \times P_{cm}) + \beta_m \times 900{,}000(P_{smp} - P_{sml}) + (320{,}000 \times P_{co}) + \beta_o \times 320{,}000(P_{sop} - P_{sol}) + 540{,}000 \times P_{ch} + 160{,}000 \times P_{cl} - C_c \times 100,$$

where P_c is the cash price of cottonseed per pound at time of purchase by the mill, P_{cm} is the cash price of cottonseed meal per pound at time of sale, P_{co} is the cash price of cottonseed oil per pound at time of sale, P_{ch} is the cash price of hulls per pound at time of sale, P_{cl} is the cash price of cottonseed linters per pound at time of sale, P_{sp} is the soybean futures price at the time of placing hedge, P_{sl} is the soybean futures price at the time of lifting hedge, P_{smp} is the soybean meal futures price at the time of placing hedge, P_{sml} is the soybean meal futures price at the time of lifting hedge, P_{sop} is the soybean oil futures price at the time of placing hedge, P_{sol} is the soybean oil futures price at the time of lifting hedge, β_c is the hedge ratio for soybeans, β_m is the hedge ratio for soybean meal, β_o is the hedge ratio for soybean oil, and C_c is the crushing cost per ton.

In equation (2), profits from hedges are calculated by considering the differences between the futures prices at the time of placing and lifting hedge, according to the futures position taken by the crusher. A risk-minimizing crusher establishes a long position by buying soybean futures contracts and offsets it by selling the same number of contracts at the time of buying cash cottonseed. Therefore, the profit from soybean futures transactions is determined by the difference of soybean futures price at the time of lifting hedge from the price at the time of placing hedge. On the other hand, the crusher establishes short positions by selling soybean meal and oil futures contracts. She offsets the short positions by buying the futures contracts at the time of selling cash cottonseed meal and oil. So, the profits from soybean meal and oil futures transactions are the differences of the futures prices at the time of placing hedge from the prices at the time of lifting hedge. It should also be mentioned here that the costs of rollovers are hidden in the hedge profit terms.

The average cash price for hulls ($0.0605 per pound) in Atlanta was obtained from *Feedstuffs* magazine. The approximate average price of linters is $0.15 per pound, and the average crushing cost is approximately $50 per ton (Right, 2000). Using these data, the above profit function can be reduced to:

(3) $$\pi = 2{,}000{,}000 \times \left[-P_c + \beta_c(P_{sl} - P_{sp})\right] + 900{,}000 \times \left[P_{cm} + \beta_m(P_{smp} - P_{sml})\right] + 320{,}000 \times \left[P_{co} + \beta_o(P_{sop} - P_{sol})\right] + 6{,}670.$$

Data

The data used in this analysis are constructed from three sources. The cash cottonseed and cottonseed meal price data for three locations—Los Angeles, Memphis, and San Francisco—are obtained from various issues of *Feedstuffs* magazine. The observations are Wednesday closing prices from July 6, 1994 through September 15, 1999. To avoid bias in the empirical results, data from the brief period when a cottonseed futures contract existed are not used. Cottonseed oil market prices were not available on a local or regional basis. Monthly average prices for cottonseed oil are obtained from *Oil Crops Situation and Outlook Report*, published by the U.S. Department of Agriculture's Economic Research Service. The soybean complex, soybean meal, and soybean oil futures prices are obtained from the Chicago Board of Trade (CBOT). The soybean and soybean meal futures prices are also the Wednesday closing prices for the same time period and are always for the contract nearest to maturity. The soybean oil futures prices are monthly averages for the contract nearest to maturity.

For the market efficiency tests (discussed below), the available five years of weekly data are used for cottonseed and cottonseed meal, while 10 years of monthly average price data are employed for cottonseed oil and soybean oil. For the hedging simulations to follow, monthly data are used, as we must revert to the least frequent observation on any series involved. Most monthly data are for the last Wednesday of the month; the exception is cottonseed oil prices for which only a monthly average is available. Table 2 provides a summary of the data.

Preliminary Testing for Market Efficiency

To establish whether cross-hedging can increase profit and/or reduce risk, we test for market efficiency in both the cottonseed spot markets and the soybean complex futures markets. If the markets in question are efficient (in the sense that one cannot forecast

Table 2. Data Summary Statistics ($/ton)

Description	Location	No. of Observ.	Mean	Std. Dev.	Minimum	Maximum
Cottonseed	Fort Worth	272	169.33	25.85	99.00	230.00
	Los Angeles	272	200.83	20.91	129.00	250.00
	Memphis	272	144.40	23.46	92.50	200.00
	San Francisco	272	191.52	21.11	143.00	240.00
Soybean futures		272	214.63	38.02	87.83	298.67
Cottonseed meal	Fort Worth	272	162.79	38.69	97.50	239.00
	Los Angeles	272	190.92	35.54	132.00	254.00
	Memphis	272	150.07	40.32	85.00	292.50
	San Francisco	272	181.68	35.35	117.00	315.00
Soybean meal futures		272	194.59	48.05	121.70	304.50
Cottonseed oil		120	513.64	79.08	348.20	683.20
Soybean oil futures		120	463.15	64.57	310.20	591.80

Sources: *Feedstuffs*; U.S. Department of Agriculture/Economic Research Service; Chicago Board of Trade (CBOT).
Notes: Cottonseed and cottonseed meal cash prices and soybean and soybean meal futures are Wednesday closing prices from July 6, 1994 to September 15, 1999. Cash price of cottonseed oil and soybean oil futures are monthly averages from October 1989 to September 1999.

future prices in a profitable manner), successful cross-hedging still can result in risk reduction. In a simple sense which will serve here, market (in)efficiency will be assumed (roughly) equivalent to the presence (absence) of a unit root in the time series of the prices being studied. Market efficiency is examined by performing Bayesian unit root tests on all cash and futures prices, using the procedures developed in Dorfman (1993). This test is chosen because the low power of frequentist tests such as the Dickey-Fuller test, combined with their assignment of the null hypothesis to market efficiency, leads to a large number of incorrect findings of efficient markets; i.e., too many unit roots are claimed to exist (cf. DeJong and Whiteman, 1991).

The Bayesian test for market efficiency used here compares the probability of a nonstationary root to the probability of a stationary dominant root, assuming an autoregressive time-series model for the data series being tested [here we use an AR(3) model]. After setting the prior distribution on the roots of the time series and specifying a likelihood function, Monte Carlo integration techniques are employed to numerically approximate the posterior probabilities in favor of and against stationarity. In the tests performed here, two different prior specifications are used—a beta distribution on each root and an uninformative prior. Two different likelihood functions are also investigated—one nonparametric and a standard Gaussian (normal) one. Posterior probabilities are calculated numerically by Bayes' theorem which states the posterior is proportional to the prior times the likelihood. Posterior odds ratios can then be formed from the posterior probabilities by dividing one posterior probability by the other; an odds ratio greater than one shows posterior support for the hypothesis placed on the top of the odds ratio. The posterior odds ratios were computed for all combinations of prior distribution and likelihood assumptions, adding robustness to the procedure and serving to check the results' sensitivity to the assumptions involved in the prior and likelihood distributions chosen.

The test results are reported in table 3. An odds ratio greater than one implies an efficient market, while an odds ratio less than one implies an inefficient market. The

Table 3. Results of Bayesian Tests for Nonstationarity

	Odds Ratios			
Sample	K_{nb}	K_{nf}	K_{gb}	K_{gf}
Cottonseed (Fort Worth)	1.2751*	1.3706*	0.2912	0.2917
Cottonseed (Los Angeles)	0.0681	0.1194	0.0430	0.0232
Cottonseed (Memphis)	0.0632	0.0526	0.0843	0.0645
Cottonseed (San Francisco)	0.2328	0.1844	0.0688	0.0336
Cottonseed meal (Fort Worth)	0.6960	0.7734	0.1724	0.1634
Cottonseed meal (Los Angeles	0.3752	0.4051	0.3602	0.3768
Cottonseed meal (Memphis)	0.0018	0.0018	0.2343	0.2375
Cottonseed meal (San Francisco)	0.0298	0.0327	0.2736	0.2715
Cottonseed oil	0.0941	0.0264	0.0994	0.0204
Soybean futures	0.3368	0.3076	0.2348	0.2508
Soybean meal futures	1.3453*	1.0753*	0.3376	0.3423
Soybean oil futures	0.1675	0.0618	0.1576	0.0539

Notes: K is the posterior odds ratio in favor of a nonstationary dominant root, where the subscripts represent innovation density and prior, respectively. The subscript n stands for the nonparametric density, g for the Gaussian (normal) density, b the beta prior, and f the flat prior. Odds ratios marked by an asterisk (*) support efficient markets.

test results reject market efficiency (the presence of nonstationary roots) except for the cottonseed price series of Forth Worth and soybean meal futures price series. The test on cottonseed cash prices for Fort Worth strongly supports an efficient market when employing the nonparametric density. The test on the soybean meal futures contracts also favors an efficient market under nonparametric density. But when assuming a normal distribution for price changes, the tests show very little posterior support for unit roots and the corresponding market efficiency. Of the 48 odds ratios, only four are greater than unity. These results are somewhat contradictory to most standard efficiency tests of futures markets, but are in keeping with previous Bayesian efficiency test results (cf. Dorfman, 1993) which treat unit root and stationary processes equally. The difference is assumed to be caused by the low power of the standard tests.

The Bayesian Decision Science Approach

The utility function for the crusher can be obtained by substituting the profit function in (3) into the negative exponential utility function in (1). Bayesian decision science is a method for finding the optimal decision (hedge ratio) to maximize the expected value of an objective function (utility function) while optimally accounting for the parameter uncertainty associated with estimation (Klein et al., 1978; DeGroot, 1970; Berger, 1985). The Bayesian optimal hedge ratios, or decision vector $(\beta_c, \beta_m, \beta_o)'$, are the choices that maximize the crusher's expected utility where the expectation is taken over the posterior probability distribution of all estimated model parameters, thereby incorporating the parameter uncertainty. The basic idea is to select a strategy that has the best weighted average performance under different parameter values, where the most likely parameter values (like the point estimates for the regression coefficients) get the highest weights (weights are equal to the probability of a parameter value being "true").

The methodology begins with a model of the spot and future prices involved, in the form of regression models. A Bayesian adds a prior distribution on the unknown regression parameters (denote these by the vector γ) and can then derive a posterior distribution for the unknown parameters which optimally combines this prior information with the information in the collected data set. The decision science approach to selecting an optimal decision vector (set of hedge ratios) then uses the posterior distribution of the model parameters γ to evaluate the expected utility of each possible decision vector considered. This is easily accomplished by some form of search algorithm such as a grid search or iterative quasi-Newton search similar to those used in maximum likelihood estimation; specifically, one searches for the hedge ratios that maximize the crusher's expected utility. The decision vector (hedge ratios) having the highest expected utility is the optimal choice (optimal hedge ratios). For further details, see Berger (1985); Dorfman (1997); or Lence and Hayes (1994).

In the modern, numerical approach to Bayesian statistics, the expected utility of each hedge ratio under consideration is computed by (*a*) generating a large number of random draws of the γ vector from the posterior distribution, (*b*) computing the expected utility of each considered hedge ratio conditional on that value of γ, and (*c*) taking a simple average of these expected utility values to arrive at the expected value for that set of hedge ratios. This yields an optimal set of hedge ratios which account for the uncertainty inherent in using an estimated model for price movements by considering all outcomes whether or not the estimate of γ is equal to γ. Using the entire posterior probability distribution of the γ vector in choosing the optimal hedge ratios acknowledges that the model is imperfect and accounts for this uncertainty in a logically coherent and optimal way.

Application to Optimal Cross-Hedging Strategies

To estimate the optimal cross-hedge ratios, simulations were performed using eight data sets. For hedges placed at the end of May, four data sets were constructed with different durations: 4, 8, 12, and 24 weeks. Four similar data sets were constructed for hedges placed at the end of October. May and October were chosen as the times for placing hedges because cotton is typically planted throughout March and early April and harvested in September through November (NCPA, 2002). Thus, by the beginning of May, a cottonseed crusher would have an estimated amount of cottonseed production. To protect herself from fluctuations in cottonseed, meal, and oil prices, she would like to place cross-hedges around May-June. By the end of October, the cottonseed crusher should know the actual amount of cottonseed produced. She would also have an estimated production of meal, oil, hulls, and linters. So, there may be some potential for placing cross-hedges during the end of October.

Each of the eight data sets was constructed with the Memphis cash prices of cottonseed, cottonseed meal, and cottonseed oil, along with the Chicago Board of Trade (CBOT) futures prices of soybeans, soybean meal, and soybean oil. Both cash and futures prices at the time of placing and lifting hedges were obtained for 10 consecutive years, 1988–89 through 1997–98. Employing Bayesian decision science, simulations were performed with each of the data sets. The prior belief here is that the hedge ratios lie between 0 and 1.2, but the prior belief is uniformly distributed within this range.

Hedge ratios smaller than zero and greater than one imply speculation. With cross-hedging, ratios above 1.0 can be risk-reducing depending on the relative price volatility, contract sizes, substitution rate between products, and correlation between the two prices involved. Thus, setting the upper limit of the (cross-)hedge ratio to 1.2 allows for some "conversion" between the two commodities.

Simulations were performed in six steps. First, using the observations of cash and futures prices and the hedge ratios to be selected (β_c, β_m, β_o), a profit function was constructed according to equation (3). For a fixed set of hedge ratios, 10,000 values for all the unknown parameters and random elements of the models presented are drawn from their posterior distributions. Profit and expected utility of profit are then computed for each of those draws. The expected values of profit and utility of profit are calculated by taking the mean value over the 10,000 calculated values (one for each drawn set of parameters). This completes the simulation for a fixed set of hedge ratios.

The above steps were repeated for all possible combinations of hedge ratios from 0 to 1.2, with 0.1 increments for each of the hedge ratios. Resulting expected profits and expected utility of profits were saved in a matrix along with the corresponding values of the parameters. The optimal hedge ratios, which gave the maximum expected profit and the maximum expected utility, were then separated from the saved matrix.

Cross-Hedging Strategy Results

Using the procedures described above, the eight different cross-hedging horizons were evaluated for five different values of the coefficient of absolute risk aversion. Simulation results are summarized in tables 4, 5, and 6. The expected profit-maximizing Bayesian cross-hedge ratios, along with the corresponding optimum profits for all the marketing alternatives, are shown in table 4. The expected utility-maximizing Bayesian cross-hedge ratios and corresponding optimal utilities under different levels of risk aversion are presented in tables 5 and 6. It is evident that the choice of a cross-hedging strategy based on expected profit maximization is insensitive to risk preference. However, optimal cross-hedge ratios based on expected utility maximization vary with the risk aversion coefficient.

Table 4 shows the Bayesian cross-hedge ratios for the whole cottonseed (β_c), cottonseed meal (β_m), and cottonseed oil (β_o) for the eight alternative hedging horizons. The estimated cross-hedge ratios are either 1.2 or zero.[2] The expected profit-maximizing simulation procedure gives the extreme values of the parameters based on the historical patterns in prices. The empirical results suggest cross-hedging cottonseed is always expected to be profitable if the hedge is placed by the end of October, and by the end of May only for four weeks. Cross-hedging cottonseed oil is always expected to be profitable if the hedge is placed by the end of May, and never profitable if placed by the end of October. The May 8-week, October 4-week, and October 8-week cross-hedges of cottonseed meal do not give any profit on average.

The results also indicate cross-hedging cottonseed, cottonseed meal, and cottonseed oil at the same time is not profitable on average unless the hedge is placed by the end

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[2] The results show that if we had allowed for a wider range of possible hedge ratios (negative numbers or greater than 1.2), the optimal ratios would have changed to be negative or larger than 1.2. However, we did not expand the allowable range for hedge ratios, as we wanted to stay with our initial goal of optimal hedging, not speculation.

Table 4. Profit-Maximizing Cross-Hedge Ratios of Cottonseed, Meal, and Oil

	Cross-Hedge Ratios			
Cross-Hedging Horizon	Whole Cottonseed (β_c)	Cottonseed Meal (β_m)	Cottonseed Oil (β_o)	Optimum Profit ($)
May 4-week	1.2	1.2	1.2	3,436
May 8-week	0.0	0.0	1.2	2,475
May 12-week	0.0	1.2	1.2	19,247
May 24-week	0.0	1.2	1.2	34,946
October 4-week	1.2	0.0	0.0	31,359
October 8-week	1.2	0.0	0.0	26,627
October 12-week	1.2	1.2	0.0	24,132
October 24-week	1.2	1.2	0.0	19,543

of May only for four weeks. The corresponding optimum profits for all the alternative cross-hedging strategies are also presented in table 4. It is clear that the May 24-week cross-hedging of cottonseed meal and oil ($\beta_c = 0.0$, $\beta_m = 1.2$, and $\beta_o = 1.2$) gives the maximum expected profit among the eight marketing strategies. The May 4-week cross-hedging of cottonseed, meal, and oil ($\beta_c = 1.2$, $\beta_m = 1.2$, and $\beta_o = 1.2$) gives the minimum expected profit among all of the strategies.

Considering five different risk aversion coefficients, the expected utility-maximizing Bayesian cross-hedge ratios and the corresponding optimal expected utility levels for alternative May cross-hedging are presented in tables 5 and 6. As observed from table 5, the cross-hedge ratios are identical to the expected profit-maximizing ones when the absolute risk aversion coefficient is very low ($\phi = 0.00001$). However, the May 8-week cross-hedge ratio for cottonseed meal increases with the coefficient of absolute risk aversion. On the other hand, the May 24-week cross-hedge ratio for cottonseed meal decreases as ϕ rises. Such a result can only arise if that hedge has an expectation of increasing both profit and risk. Consequently, as a crusher becomes more risk averse, the extra profit anticipated is not enough to offset the additional risk. An abrupt decrease in the May 4-week cross-hedge ratio for cottonseed oil is also observed when $\phi = 0.00006$.

In contrast, an increase in the October 4-week cross-hedge ratios for cottonseed meal (0.0 to 0.8) and oil (0.0 to 0.4) is observed when $\phi = 0.00006$. Therefore, this is a case where the hedge must reduce expected profit and risk; so risk-neutral crushers are not interested in the position (thus the optimal hedge ratio of 0). The October 8-week cross-hedge ratio for meal increases gradually with the risk aversion coefficient up to ϕ = 0.00004, but falls abruptly (1.2 to 0.1) when $\phi = 0.00006$. A similar abrupt decrease (1.2 to 0.3) is also observed in the October 8-week cross-hedge ratio for the whole cottonseed. These cases are ones where anticipated profit is worth the anticipated risk in a hedge position at low risk-aversion levels, but not at higher ones. The October 12-week and 24-week cross-hedge ratios for cottonseed fall with the increase in the risk aversion coefficient. However, an increase in ϕ shows strong support in favor of cross-hedging cottonseed oil (0.0 to 1.2) using the October 8-week, 12-week, and 24-week terms.

The rationale for these results is clear in most cases. When the crusher is risk neutral, we obtain optimal hedge ratios at the edges of our allowable range. These positions are

Table 5. Expected Utility-Maximizing Cross-Hedge Ratios for Different Risk Aversion Coefficients

Risk Aversion Coefficient	Cross-Hedge Ratios											
	May 4-week			May 8-week			May 12-week			May 24-week		
	β_c	β_m	β_o	β_c	β_m	β_o	β_c	β_m	β_o	β_c	β_m	β_o
$\phi = 0.00001$	1.2	1.2	1.2	0.0	0.0	1.2	0.0	1.2	1.2	0.0	1.2	1.2
$\phi = 0.00002$	1.2	1.2	1.2	0.0	0.1	1.2	0.0	1.2	1.2	0.0	1.2	1.2
$\phi = 0.00003$	1.2	1.2	1.2	0.0	0.4	1.2	0.0	1.2	1.2	0.0	0.9	1.2
$\phi = 0.00004$	1.2	1.2	1.2	0.0	0.5	1.2	0.0	1.2	1.2	0.0	0.6	1.2
$\phi = 0.00006$	1.2	1.2	0.3	0.0	0.8	1.2	0.0	1.2	1.2	0.0	0.4	1.2
Risk Aversion Coefficient	Cross-Hedge Ratios											
	October 4-week			October 8-week			October 12-week			October 24-week		
	β_c	β_m	β_o	β_c	β_m	β_o	β_c	β_m	β_o	β_c	β_m	β_o
$\phi = 0.00001$	1.2	0.0	0.0	1.2	0.0	0.0	1.2	1.2	0.0	1.2	1.2	0.0
$\phi = 0.00002$	1.2	0.0	0.0	1.2	0.4	1.2	1.2	1.2	0.7	1.2	1.2	1.2
$\phi = 0.00003$	1.2	0.0	0.0	1.2	1.0	1.2	0.8	1.2	1.2	1.2	1.2	1.2
$\phi = 0.00004$	1.2	0.0	0.0	1.2	1.2	1.2	0.3	1.2	1.2	1.0	1.2	1.2
$\phi = 0.00006$	1.2	0.8	0.4	1.2	0.1	1.2	0.0	1.2	1.2	0.8	1.2	1.2

Table 6. Resulting Utility from Alternative Cross-Hedging Strategies Under Different Risk Aversion Coefficients

Risk Aversion Coefficient	Expected Utility, by Cross-Hedging Strategy			
	May 4-week	May 8-week	May 12-week	May 24-week
$\phi = 0.00001$	-0.9843	-0.9886	-0.8372	-0.7247
$\phi = 0.00002$	-1.0075	-1.0064	-0.7247	-0.5523
$\phi = 0.00003$	-1.0751	-1.0529	-0.6515	-0.4367
$\phi = 0.00004$	-1.1976	-1.1321	-0.6099	-0.3507
$\phi = 0.00006$	-1.6721	-1.4247	-0.6017	-0.2332
Risk Aversion Coefficient	Expected Utility, by Cross-Hedging Strategy			
	October 4-week	October 8-week	October 12-week	October 24-week
$\phi = 0.00001$	-0.7484	-0.8037	-0.8313	-0.8727
$\phi = 0.00002$	-0.5887	-0.7045	-0.7863	-0.8459
$\phi = 0.00003$	-0.4870	-0.6621	-0.8181	-0.8849
$\phi = 0.00004$	-0.4223	-0.6695	-0.9009	-0.9890
$\phi = 0.00006$	-0.3527	-0.7866	-1.2266	-1.3706

motivated by the profit opportunity; the crusher either wants to take no position in the futures market or as much as possible depending on the anticipated price change (expected profit from the position). Profit dominates here because a risk-neutral agent doesn't care about reducing risk. When risk aversion is introduced, some optimal hedge ratios are inside the allowable range, not just at the edges. The movement toward the center must be due to a trading of risk reduction against profit potential, such as the May 24-week cottonseed meal cross-hedge.

Table 6 summarizes the resulting expected utilities from the eight alternative May and October cross-hedging strategies under different levels of risk aversion. Table 6 shows that expected utility increases with hedge length for the May hedges, with a 24-week cross-hedging strategy giving the highest level of expected utility among the four alternatives under all levels of risk aversion. For October hedges, expected utility decreases the longer the term of hedge, and a 4-week cross-hedging strategy gives the highest level of expected utility among the four alternatives under all five risk aversion coefficients. This is the exact opposite case to that experienced under May cross-hedging.

These results could help cottonseed crushers protect themselves against input and output price risks. An expected profit-maximizing crusher can meet her objectives by cross-hedging whole cottonseed using an October 4-week strategy ($\beta_c = 1.2$, $\beta_m = 0.0$, and $\beta_o = 0.0$), and cottonseed meal and oil employing a May 24-week cross-hedging strategy ($\beta_c = 0.0$, $\beta_m = 1.2$, and $\beta_o = 1.2$). A risk-averse crusher who tries to maximize expected utility would reach her goal by choosing the same hedge horizons, but she must determine the optimal hedge ratios corresponding to her risk aversion coefficient. For example, a moderate-to-high risk-averse cottonseed crusher ($\phi = 0.00006$) would choose the strategy of cross-hedging cottonseed meal and oil using a May 24-week strategy with hedge ratios of $\beta_c = 0.0$, $\beta_m = 0.4$, and $\beta_o = 1.2$, and using an October 4-week strategy with hedge ratios of $\beta_c = 1.2$, $\beta_m = 0.8$, and $\beta_o = 0.4$.

Simulation of the Bayesian Cross-Hedging Strategies' Effectiveness

The results reported in the previous section provide the optimal cross-hedge ratios and other information needed to design an optimal cross-hedging strategy. A cottonseed crusher would establish a long position by buying soybean futures contracts and a short position by selling soybean meal and soybean oil futures contracts.

Establishing the appropriate size of the futures position to be taken, the number of contracts of the cross-hedging vehicle required to equate to a specific cash position needs to be multiplied by the cross-hedge ratio. Suppose a cottonseed crusher in Georgia is planning to process 1,000 tons (2,000,000 pounds) of cottonseed from which approximately 900,000 pounds of meal and 320,000 pounds of oil would be produced. In order to protect herself from the fluctuations of prices in the cash markets, she would like to place cross-hedges against cottonseed, cottonseed meal, and cottonseed oil using soybean, soybean meal, and soybean oil futures, respectively.

The soybean futures trading unit at the Chicago Board of Trade is 5,000 bushels, which is equivalent to 300,000 pounds (60 pounds/bushel). Thus, the number of soybean contracts equivalent to 1,000 tons of cottonseed is 6.67 (2,000,000 pounds/300,000 pounds). To cross-hedge 1,000 tons of cottonseed, the crusher must take a long position of $\beta_c \times 6.67$ soybean futures contracts. On the other hand, the respective trading units of soybean meal and soybean oil futures contracts at CBOT are 100 tons (200,000 pounds) and 60,000 pounds. Hence, in order to cross-hedge cottonseed meal and oil, the crusher has to short $\beta_m \times 4.5$ soybean meal futures contracts, and $\beta_o \times 5.33$ soybean oil futures contracts, respectively. Using the Bayesian cross-hedge ratios presented in the previous section, the results of cross-hedging for all of the eight alternative strategies under different levels of risk aversion can be evaluated. Throughout the simulations, only integer contract positions are allowed, to be as realistic as possible.

Table 7. Resulting Profit and Utility from Cash Pricing and Cross-Hedging

Risk Aversion Coefficient	Cross-Hedging Strategy: May 4-week Profit	May 4-week Utility	May 8-week Profit	May 8-week Utility	May 12-week Profit	May 12-week Utility	May 24-week Profit	May 24-week Utility
$\phi = 0.00001$ Cash Pricing	-6,206.0	-1.0640	9,400.0	-0.9103	17,692.0	-0.8378	14,402.0	-0.8659
$\phi = 0.00001$	-1,084.4	-1.0109	13,496.8	-0.8727	32,245.6	-0.7244	22,744.4	-0.7966
$\phi = 0.00003$	-1,084.4	-1.0331	18,428.8	-0.5753	32,245.6	-0.3801	22,204.0	-0.5137
$\phi = 0.00006$	2,351.6	-0.8684	17,780.8	-0.3441	32,245.6	-0.1445	20,656.4	-0.2896

Risk Aversion Coefficient	Cross-Hedging Strategy: October 4-week Profit	October 4-week Utility	October 8-week Profit	October 8-week Utility	October 12-week Profit	October 12-week Utility	October 24-week Profit	October 24-week Utility
$\phi = 0.00001$ Cash Pricing	-14,146.0	-1.1520	-30,150.0	-1.3519	-44,676.0	-1.5632	-33,612.0	-1.3995
$\phi = 0.00001$	-6,706.0	-1.0694	-33,750.0	-1.4041	-49,608.0	-1.6423	-55,704.0	-1.7455
$\phi = 0.00003$	-6,706.0	-1.2228	-26,450.0	-2.2111	-40,344.0	-3.3546	-55,704.0	-5.3181
$\phi = 0.00006$	-7,714.0	-1.5886	-24,560.0	-4.3649	-34,104.0	-7.7387	-47,224.0	-17.0038

Note: Total profit and expected utility from cash pricing are calculated assuming a moderate-to-low risk-averse crusher ($\phi = 0.00001$) who does not use the futures market.

The resulting expected profit and utility from the May and October cross-hedging alternatives for different sets of cross-hedge ratios under the three risk aversion coefficients are presented in table 7 using data from 1998 to illustrate and evaluate the alternative strategies. Assuming a moderate-to-low risk-averse crusher ($\phi = 0.00001$) who buys and sells on the same dates as the analyzed cross-hedge actions, the expected profit and utility from simple cash pricing is also computed and provided in table 7 for comparison. As observed from the upper portion of table 7, with respect to the resulting expected utility, all of the May cross-hedging alternatives are superior to cash pricing. With respect to expected profit, all of the May cross-hedging strategies are superior to cash pricing except for the May 4-week (with $\beta_c = 1.2$, $\beta_m = 1.2$, and $\beta_o = 1.2$). It is also evident that the May 12-week cross-hedging is superior to the May 24-week, the May 24-week cross-hedging is superior to the May 8-week, and the May 8-week cross-hedging is superior to the May 4-week cross-hedging in general. Particularly, the May 12-week cross-hedging strategy (with $\beta_c = 0.0$, $\beta_m = 1.2$, and $\beta_o = 1.2$) is the most preferable among the May marketing alternatives. The May 24-week cross-hedging is inferior to the May 12-week because of an unusually abrupt decrease in cottonseed meal cash price near the end of 1998. Thus, with a small exception, 1998 May cross-hedging results confirm the findings of the Bayesian cross-hedging method described in the previous section.

The profit and utility from different October cross-hedging scenarios under the three risk aversion coefficients are presented in the lower portion of table 7. Results confirm that utility decreases the longer the term of cross-hedging, without exception. The October 4-week cross-hedging (with $\beta_c = 1.2$, $\beta_m = 0.0$, and $\beta_o = 0.0$) is found to be the most effective. Table 7 also shows that only the October 4-week cross-hedging (with $\beta_c = 1.2$, $\beta_m = 0.0$, and $\beta_o = 0.0$), the October 8-week cross-hedging (with $\beta_c = 1.2$, $\beta_m = 1.0$, and $\beta_o = 1.2$, or with $\beta_c = 0.3$, $\beta_m = 0.1$, and $\beta_o = 1.2$), and the October 12-week cross-hedging (with $\beta_c = 0.8$, $\beta_m = 1.2$, and $\beta_o = 1.2$, or with $\beta_c = 0.0$, $\beta_m = 1.2$, and $\beta_o = 1.2$) are superior to cash pricing. All other October cross-hedging strategies are inferior to cash pricing.

Conclusions

This study has shown that soybean, soybean meal, and soybean oil futures can be used successfully as cross-hedging vehicles for cottonseed, cottonseed meal, and cottonseed oil, respectively. The empirical results imply a cottonseed crusher must be careful about choosing the proper time for placing hedges, the appropriate size of the positions, and the hedge length. The Bayesian cross-hedging rules suggest that hedges for cottonseed meal and oil should be placed by the end of May for longer terms, i.e., for 12 to 24 weeks, and the hedge for cottonseed should be placed by the end of October for shorter terms, e.g., 4 weeks. Above all, May 12- and 24-week cross-hedging of cottonseed meal and oil and October 4-week cross-hedging of the whole cottonseed are the most effective marketing strategies. While these recommended strategies are obviously contingent on the time period we studied and the cross-hedges we considered, a general conclusion of this study is that soybean complex futures can serve as a satisfactory hedging instrument for cottonseed, reducing any need to (re)start futures markets in cottonseed or its related products.

In contrast to Dahlgran's (2000) analysis, the present study of cross-hedging cottonseed and its products uses only soybean complex futures contracts, which are not as difficult and costly to manage as the huge hedge vehicle pool used by Dahlgran. This research also shows that the superiority of one hedging horizon over the other depends not only upon the appropriate size of the futures contracts but also on the time the hedge is placed. Thus, this approach to cross-hedging of cottonseed and its products eliminates some difficulties reported in Dahlgran's study and may be more amenable to use by actual crushers.

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