PROMISING FUTURE RESEARCH RELATED TO CREDIT SCORING

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The evaluation of a farm borrower's credit worthiness has become a timely issue for agricultural lenders. Increasing frequencies of farm failure, declining agricultural loan quality, reduced loan volume and increasingly complex legal and institutional settings are encouraging lenders to more quantitatively assess and monitor a borrower's financial status. Advantages of these quantitative measures include systematic evaluation of borrowers, less reliance on nonprice factors when granting credit, a defensible basis for tailoring credit terms, and improved loan portfolio quality.

Yet, it is my observation that agricultural lenders make only limited use of formal credit scoring models. When questioned, lenders state such models quickly become outdated, are difficult to re-estimate, lack general robustness, and have less than 100 percent accuracy. To encourage adoption, credit scoring models are often promoted as a panacea -- and the initial enthusiasm quickly erodes when the practical limitations of a statistical model become apparent. Obviously, more educational programs must be developed to help lenders implement methods of credit scoring.

However, a need also exists for more research that improves the usefulness of credit scoring models for lender decisionmaking. This paper suggests five such areas of needed research: 1) validate existing models, 2) fine-tune existing models, 3) incorporate more subjective data, 4) acknowledge portfolio effects, and 5) consider dynamics and lender's characteristics in the credit granting decision. The bulk of the paper focuses on the last suggestion in which a stochastic dynamic programming model is estimated with preliminary lender data obtained in an experimental setting.

VALIDATION

Validation of existing models is an important first step towards refinement of credit scoring models. The current performance, accuracy, and sources of error embodied in existing models must be identified if future research is to be logically related to and advance our present knowledge. The product of this endeavor would provide a much more definitive direction to our research than the brief overview presented in this paper.

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Credit scoring models are usually judged on their ability to reproduce lenders' decisions. As lender evaluations are a potential source of error, a more appropriate yardstick maybe the prediction of borrowers' actual financial progress. Thus, one means of validation would be to compare the predictions generated by existing models when they are applied to an identical dataset (Collins). This approach is particularly appealing now that out-of-sample longitudinal data exists.[1] By comparing classification schemes, the results of each model with one another and with borrowers' actual performances over time, important insights into the significance and sign of relevant variables and sources of error may become apparent.

Another approach of validation would utilize an experimental method such as the one developed by Gustafson to determine lender's actual usage of credit scoring models. Possible lender errors in classification of borrowers, in compilation of quantitative data and when they become emotionally involved or swayed by a borrower's personal traits are unknown. By directly observing lender behavior in a controlled setting, the magnitude of these and other error sources could be delineated. Results of such a study may suggest the inclusion of additional explanatory variables in our models.

FINE-TUNING EXISTING MODELS

The process of credit scoring involves, 1) Identifying a set of key variables, characteristics, or factors that best classify a loan according to some predetermined measure of credit-worthiness or quality, 2) Assigning a weight to each of the selected variables based on statistical analysis, prior experience, or conviction, 3) Multiplying the level of each variable by its respective weight and sum to obtain a total score, and 4) Comparing the calculated score with predetermined threshold levels which correspond to various management actions available.

Future research could be directed at improving each of these four steps. For instance, measures of quality developed in past research maybe inappropriate. When operating under greater regulator supervision, perhaps lenders place more emphasis on a loan's repayment status than profitability. Thus a "good loan" (definition of a credit scoring model's dependent variable) occurs when there is a high probability of regular repayments. Specification of lender goals, objectives and concerns when originating a loan is the first step of model development and an important researchable issue.

A related issue concerns the horizon over which model results should be applicable. Clearly, credit scoring models tailored to the 1970's lending environment would lead to poor lending decisions if applied at the present. How robust should models be? Perhaps a single model should not be expected to forecast repayment both during the first year and over the life of mortgage loan.

The array of explanatory variables must be broadened to include more production data warrants further investigation. Prior studies have relied primarily on data maintained by lenders. Such databases rarely contain sufficient information to evaluate borrowers' production and marketing practices which also affect financial management.

SUBJECTIVE DATA

The mathematical techniques that quantify and aid lender decision making can not rely solely on the objective data contained in borrower's financial statements. Although subjective variables are more difficult to specify and measure accurately, credit evaluation is often influenced by (if not completely determined by) factors such as a borrower's character, integrity, management ability, and financial goals. Inclusion of these variables is necessary for complete model specification.

Perhaps here, principles of artificial intelligence and expert systems offer the greatest potential for identifying and specifying the relationships among subjective variables (Winston). These methods are a form of introspection. By evaluating decisionmakers as they perform a specific task, analysts are able to identify a subject's state of knowledge and infer a model of cognitive process that is useful for predicting observed behavior.

The use of standard statistical methods to develop models of credit scoring can be criticized on the grounds that the lender's decision process is to complicated for such simplistic modeling. As lending decisions generally involve multiple variables that interact in complex patterns, complete modeling would require an unreasonable number of cases. Thus, a need exists to investigate these alternative research methods.

PORTFOLIO EFFECTS

Conceptually, credit scoring models predict a borrower's credit worthiness as represented by an index value. In a statistical sense, the score is an expected value. The model does not predict variability nor compare the expected value, variance, and covariance of an individual borrower's score with that of the lender's existing portfolio. In fact, presently estimated statistical models assume classification errors are identical, independent, and normally distributed over all borrowers. As higher moments of score's distribution may affect lender decisions, such comparisons are necessary if credit is to be extended in an optimal manner.

When deciding whether or not to grant credit, lenders should evaluate the contribution of an individual borrower's systematic risk to the loan portfolio. For example, assume a potential borrower is the producer of specialty crops. Initially, credit to the producer may be denied because limited salvage markets exist for the firm's assets, specialty crop returns are highly variable, and lenders may have limited knowledge of their production practices. However, if the borrower's credit worthiness is negatively correlated with the lender's existing portfolio, the granting of credit may be an optimal decision in light of available diversification opportunities.

Robison and Barry develop a portfolio analysis (theory) which illustrates these tradeoffs in the lender's decision and shows the demand for an individual asset q_j as:

$$q_j = \sum_{i=1}^{n}(r_i - r_{n+1})\frac{D_{ij}}{D\lambda} \tag{1}$$

where r_i is the expected net return on the ith asset, D is the determinant of the variance-covariance matrix of n+1 assets under consideration, D_{ij} is the cofactor of the (i,j)th element of D, and λ is a coefficient of risk aversion. In market equilibrium, a decreasing correlation between assets increases their complementarity. Hence, an unfavorable return on the ith loan could be offset by a favorable return on the jth loan -- leading to a stabilization of total lender returns. Thus, rather than only lend to members of a superior class whose returns to the lender maybe highly correlated, an optimal lending decision from a more aggregate perspective may be to grant credit to borrowers of other classes -- as long as their added risks are diversifiable.

Practical implementation of this approach necessitates the inclusion of additional explanatory variables in the credit scoring model representing the excess return, variance and covariance terms shown in equation 1. As with other variables in the model, estimation may occur with either historical data or lender's subjective estimates.

DYNAMICS AND LENDER CHARACTERISTICS

An apparent paradox exists in agricultural credit. Despite high levels of default and negative rates of return, lenders continue to grant operating credit to farmers with whom they have no prior business experience. Although the granting of operating credit appears irrational in light of pricing and credit rationing options available, lenders receive information that sufficiently improves decision-making in subsequent periods. Hence, extension of credit over a period of time is profit maximizing if it is evaluated in a dynamic rather than static setting.

If credit extension is profitable in the long run but not in the short run, single period credit scoring decision rules maybe too myopic. Lenders appear willing to incur a significant short run cost in an effort to acquire long term customers. Factors affecting these tradeoffs in the lender's decision need to be included in scoring models and are mathematically illustrated with a stochastic dynamic programming (DP) model.

Stochastic Optimization Model

Optimization problems with separable objective functions and discrete decision variables are readily solved by DP and yield optimal decision rules which are in closed-loop form (Dreyfus and Law). The following model is similar to one formulated by Bierman and Hausmann for commercial trade credit in that it accounts for dynamics of repayment, but differs because it accounts for greater detail including partial repayment and is empirically estimated.

Extending credit to unfamiliar farmers is a risky decision for lenders as repayment is uncertain and loan losses are costly to a financial institution (Gustafson et al.). Assume one of the following mutually exclusive repayment states i is likely: (a) full repayment of principal and interest [i=1], (b) repayment of interest only [i=2], and (c) default [i=3]. Expected profit (π) in period n is defined as:

$$\pi(n,i) = \sum_{j=1}^{J} p(n,i,j)\,[REV(n,i,j) - CF(n) - AO(n,i,j) - LL(n,i,j)] \qquad (2)$$

where the probability of transition from state (n,i) to state (n+1,j) is p(n,i,j), REV(n,i,j) is uncertain gross revenue from lending, CF(n) is a lender's cost of funds which is known is advance, AO(n,i,j) are administrative and overhead expenses, and LL(n,i,j) is a loan loss charge for unrecovered principal. Gross revenue is equal from borrowers who repay fully or interest only on outstanding debt and zero from borrowers who default. Administrative and operating expenses vary with repayment status. Loan losses arise when borrowers default.

Lenders are assumed to maximize expected monetary values. The fundamental recurrence relation of DP for this application is:

$$f(n,i) = 0 \qquad\qquad \text{for } n=N$$
$$f(n,i) = \max\ [\text{extend credit, deny credit}] \tag{3}$$
$$= \max\ [\pi(n,i) + \alpha\delta\sum_{j=1}^{J} p(n,i,j)\ f(n+1,i,j),\ 0] \quad \text{for } n<N$$

where $f(n,i)$ is the expected value of an optimum policy of credit extension from period n to the horizon, α is the probability a borrower will apply for credit again in n+1 and δ is a discount factor.

Initially, lenders must decide whether to extend or deny credit to a new applicant. If losses from credit extension are expected to exceed returns, credit is denied and the firm's return is zero.[2] If expected returns are positive, credit is extended and a likelihood α exists that the customer will apply for credit in future periods. Hence, the firm realizes two returns, a current return and the discounted value of future credit extensions. Each return considers the expected profits and costs of full repayment, partial repayment and default. If credit is granted, lenders must again decide whether to extend or deny credit one period later; as long as credit is granted, the problem recurs in subsequent periods, and returns from those future periods must be considered in solving the present decision.

The second term of equation 2 tends to zero. The probability ($@_{t+n}$) that the customer still applies for credit declines as n increases. Further, the discounted value of those returns also falls to zero. These relationships thereby permit a finite analysis and define ending conditions. Horizon year N is the point where the value of the recursive function is zero. Terminating before this date could change the initial decision, although any change is likely to be insignificant for most practical problems.

The credit granting function above has a number of desirable characteristics. It allows for prior probabilities of payment, includes the potential for future profit and permits systematic revision of repayment probabilities based on past experience (Bierman and Hausman).

Transition probabilities from one state to another can be either estimated with historical data or subjectively specified. To keep the above DP model manageable, a traditional Markovian relationship for repayment is postulated:

$$p(n,i,j) = \text{Prob}\ (x_{n+1}=j \mid x_n=i) \tag{4}$$

indicating the probability of transiting to state j is conditional upon the current state i. Transition probabilities p(n,i,j) have the usual statistical properties:

$$0 \leq p(n,i,j) \leq 1 \tag{5}$$

$$\sum_{j=1}^{J} p(n,i,j)=1 \tag{6}$$

Experimental Method and Data Collection Procedures

Data to estimate the model were collected in an experimental setting during which lender responses to a simulated borrowing situation were elicited. This approach was selected over other survey methods because it: (a) provided the necessary quantitative and probabilistic information for model estimation; (b) obtained lender's responses to a specific management problem; and (c) minimized the possibility of extraneous variables influencing the lender's decision. In addition, the method has been successfully used in the study of Illinois cash grain farmers' investment behavior (Gustafson).

Two representative farm situations, one located in the Red River Valley and the other in the East Central region of North Dakota, were constructed to reflect diverse areas of cash grain production in the state. Data were obtained from adult vocational agriculture farm business summaries (Watt, et al.). The Valley farm consisted of 1,385 acres while the East Central farm involved 2,855 acres. Crops representative of each region (continuous and fallow wheat, barley, sunflowers - East Central farm only, and sugarbeets - Valley farm only) were raised; no livestock was produced, crop sales occurred at harvest; participation in government programs was assumed; no off-farm income was available; The Valley farm cash rented 290 acres whereas the East Central farm share rented 1,640 acres. Financial statements for each farm were prepared with the aid of a simulation model.[3]

Financial characteristics of the farms were structured to represent an established borrower who was seeking a lender with lower cost financing. Debt-to-asset ratios were set to .40 for each farm. A panel of farm lenders located outside of each region considered these ratios representative and served as a pretest mechanism for the study.

The first situation was presented to five randomly selected lenders who granted farmers credit in the geographic region surrounding Wahpeton, ND and Breckenridge, MN while the second situation was introduced to six farm lenders in the

Jamestown and Valley City, ND areas.[4] Each lender was from a unique commercial bank or Farm Credit Service's office. These two areas were selected because of the high concentration of financial institutions in predominately rural areas of homogeneous farm production.

During the experiment, lenders described the characteristics of their institution; were provided with a biographical sketch of the borrower, historical and projected financial statements from the simulation model; and asked if they would grant the operating loan request (fig.1). If the initial request was denied, the experiment was terminated.

If operating credit was granted, lenders were asked to specify credit terms, subjectively estimate the likelihood the case farm borrower would transit to one of the three possible repayment states and the administrative, operating and loan loss expenses associated with each state. After these data were elicited, the financial performance of the case farm was simulated again for each resulting repayment state.[5] One at a time, updated financial statements (illustrating the case farm's possible financial position and credit application one year hence) were provided to the lender and the experimental process repeated.

To minimize respondent burden, the experiment was only conducted for two consecutive periods. After the second session was completed, lenders were informally asked if third period expectations would significantly differ from those of the second period given that additional information (more trials) would be available. All of the surveyed lenders stated additional information would not alter their expectations.

A main disadvantage of the experimental method is the abstraction from actual decision situations. In an effort to validate the experimental approach, a research assistant made a incognito formal application for operating credit to one of the financial institutions selected for pretest. The supervisor of the loan officer (who was informed of the trial) was instructed to casually elicit the loan officer's subjective estimate of the applicant's probability of full, partial and no repayment if the loan application was forwarded for review and processing. Similar data to that of the case farm was used to complete the loan application.

One week later, the same loan officer was asked to participate in the experiment. In both instances, the loan officer granted the operating loan request and provided identical probability estimates. Although the loan officers may have offered wrote responses, they did so in both real world and experimental settings.

Results

Data collected during the experiment are summarized in table 1. All institutions surveyed had assets of less than $100 million. The average number of agricultural operating loans granted annually per institution ranged from 42 to 250. The size of these operating loans averaged $84,636. Loan size was the only variable that differed statistically by region. Operating loans in the Red River Valley averaged $120,400, while operating loans granted to farmers in the East Central region averaged $54,833. This difference reflects the varying capital requirements of farms in each

Table 1. Characteristics of Financial Institutions Surveyed

Item	Mean	Standard Deviation
Number of Operating Loans Outstanding	120	55
Average Operating Loan Size (dollars)	84,636	60,858
Current Interest Charged on Operating Loans (percent)	11.71	1.02
Average Cost of Funds (percent)	7.85	.92
Administrative Costs and Loan Losses (percent)	3.09	.72
Average Length of Time Farmers Remain Customers of Institution (years)	19.5	6.9

region. Assets of the representative Valley farm totaled $1.362 million versus $.566 million for the East Central farm. Profit margins on lender's operating loans averaged .77 percent after cost of funds, administrative costs and loan loss charges were deducted.

Farmers with operating loans at these institutions were expected to remain customers for nearly 20 years. Lenders explained that even in light of the recent financial crisis, farmers still used available profits to purchase additional assets and expand the size of their business as opposed to reducing existing debt levels.

Elicited Repayment Probabilities

Subjectively estimated conditional probabilities of repayment elicited from the lenders are shown in table 2. After evaluating the representative new customer, all of the lenders decided to grant the case farm's operating loan request. Lenders expected full repayment with 87.8 percent probability, payment of interest only with 5.5 percent probability and default with 6.7 percent probability.

After granting operating credit for one period, lenders have more information to appraise the case farm's credit worthiness. Lenders believe that if the case farm borrower repaid the previous operating loan, the farm is more likely to do so in the future as the expected probability of default drops from 6.73 percent to 1.00 percent. Similarly, if the farm defaults, it is expected to do so again in the future. The probability of default

Table 2. Conditional Operating Loan Repayment Probabilities Elicited From Survey Lenders[a]

Status of Case Farm Borrower	Probability of: Full Repayment	Partial Repayment	No Repayment
	(percent)		
New Customer	87.82 (5.27)	5.45 (2.58)	6.73 (3.85)
Existing Customer that Repaid Previous Operating Loan:			
Fully	96.36 (1.92)	2.64 (1.57)	1.00 (1.04)
Partially	69.82 (19.71)	23.36 (15.98)	6.82 (7.40)
No Repayment	20.00 (22.58)	25.55 (19.93)	54.45 (28.83)

[a]Standard deviations are shown in parentheses.

given the borrower only pays interest on a previous operating loan, is not statistically different from that of a new borrower -- although probability of full repayment is less. Unlike the uniform expectations lenders have for a case farm borrower that fully repays past loans, lender estimates of future repayment status are highly variable for a borrower that either partially repaid or defaulted on previous loans. The probabilities elicited are stationary with respect to time. This is consistent with lender's statements that farmer's leverage positions and susceptibility to financial risks remain stable over time. For the population as a whole, expected probabilities of full, partial and no repayment in the second period are 89.8, 5.3 and 4.9 percent, respectively -- not statistically different from first period expectations.

Optimal Decision Rules

Given the case farm's expected probability of repayment, an average operating loan size of $84,636 and profit margins described above, a myopic decision rule which does not consider the value of future credit extensions is to deny the loan request. Single period expected gross returns are $611.85 but expected costs including those of default are $622.59 resulting in an expected payoff of $-10.74.

Optimal decision rules for granting operating credit over a finite horizon are obtained when the DP credit-granting model is estimated. The expected payoff of following such a policy and granting operating credit to the case farm borrower is $1189.77. This value includes the present value of all future credit extensions and the possibility borrower patronage ceases.

At the end of the first period, expected future payoffs of granting operating credit another period to the case farm borrower that fully, partially and did not repay credit in the last period are $1882.35, $1178.28 and $-4584.98, respectively. Hence, an optimal policy at this stage is to deny credit if the borrower defaulted on previous operating loans. As operating margins are small and costs of default high, defaulting borrowers are not given a second chance.

Lenders continue to grant the case farm credit until year 20 as long as farmers fully and partially repay. At that time, credit will only be granted if he fully repaid in year 19. Future payoffs from extending credit to borrowers that only partially repaid are insufficient to warrant credit extension during the last period.

A borrower's characteristics, including net worth and income-generating capacity, primarily determine whether an operating loan is granted in a static single-period analysis

(Sonka et al.). When the credit granting decision is evaluated in a dynamic setting, a lender's discount rate and subjective estimate of a borrower's conditional probability of default and patronage in future periods become equally important factors. These variables, in addition to varying costs of funds, administrative expenses and profit margins explain why farmers may be granted credit by one lender and not another.

The repetitive utilization of credit affects the initial credit-granting decision. One reason the myopic and optimal decision rules could differ is if probabilities of repayment for the population as a whole were not stationary with respect to time. However, as noted above, this is not the case. Granting credit to the case farm is only profitable if the borrower continues to patronize the institution in the future.

The value of the optimal policy is sensitive to changes in a lender's discount rate and assessment of a borrower's patronage (fig. 2). As a lender's discount rate increases or expectations of customer patronage decrease, the value of the optimal policy declines. These variables likely differ by lender. Hence, a lender's characteristics, in addition to those of a borrower, determine whether operating credit is granted.

Value of Credit Scoring

There is a second application of the DP credit-granting model. The recursive relationship f(n,i) provides the present value of an optimal credit-granting policy from n to the horizon given repayment probabilities p(n,i,j). The value of techniques employed by lenders when evaluating a borrower's credit worthiness, such as credit scoring and discriminate analysis, which lead to improved estimation of p(n,i,j) can be ascertained with the recursive relationship.

After evaluating the representative new borrower, lenders in the survey expected default on the first year operating loan with 6.7 percent probability. Figure 3 illustrates how improved credit scoring techniques can influence the present value of an optimal credit granting policy. Such methods allow lenders to identify and deny credit to marginal borrowers -- increasing the odds remaining customers will repay.

If improved evaluation techniques had led lenders to expect half the default rate, 3.4 rather than 6.7 percent, the value of the optimal policy would have risen $414 to $1604. This value would increase further if probability estimates of repayment beyond the first period were also revised upward. Given these payoffs, lenders, either individually or cooperatively with peer institutions, could devote more resources to the development of improved credit-scoring models and place less emphasis on the ad hoc methods of evaluation.

CONCLUSION

This paper has shown that prediction of whether an agricultural loan is ultimately successful or not requires far more complex analyses than previous studies of borrower financial data. In particular, the subjective elements, portfolio effects, and longrun profitability of credit extension need to be acknowledged.

Credit scoring does not imply that the lending decision can eventually be reduced to a single index number. Hopefully, a properly constructed and used credit scoring model will identify the risk/return characteristics of a loan on a consistent basis, providing additional information on which to base lending decisions. The index can neither replace nor substitute for good loan judgment. The credit granting decision will still continue to be based on factors beyond the index.

FOOTNOTES

[1]An alternative method of model validation is for researchers to divide an original data set -- using the first portion for model estimation and the second half for parameter verification. However, sufficient data are rarely available to implement this approach. In addition, when model validation occurs at a time period which is different from when the original analysis is performed, any biases which may have influenced results may become apparent.

[2]Costs associated with credit analysis are considered sunk costs because they are incurred regardless of the lending decision.

[3]The selected model was the Farm Financial Simulation Model (FFSM) developed by Schnitkey, Barry and Ellinger. FFSM is a multiyear spreadsheet of a farm's financial performance that reports results in terms of a set of coordinated financial statements.

[4]One additional lender in the Wahpeton area and two in the Valley City - Jamestown area were contacted but removed from the sample because they did not grant operating credit to farmers.

[5]Following Gustafson, yields, commodity prices, farm income, and asset values of the case farms were randomly varied between the first and second year decision situations in order to add an element of uncertainty to the simulation.

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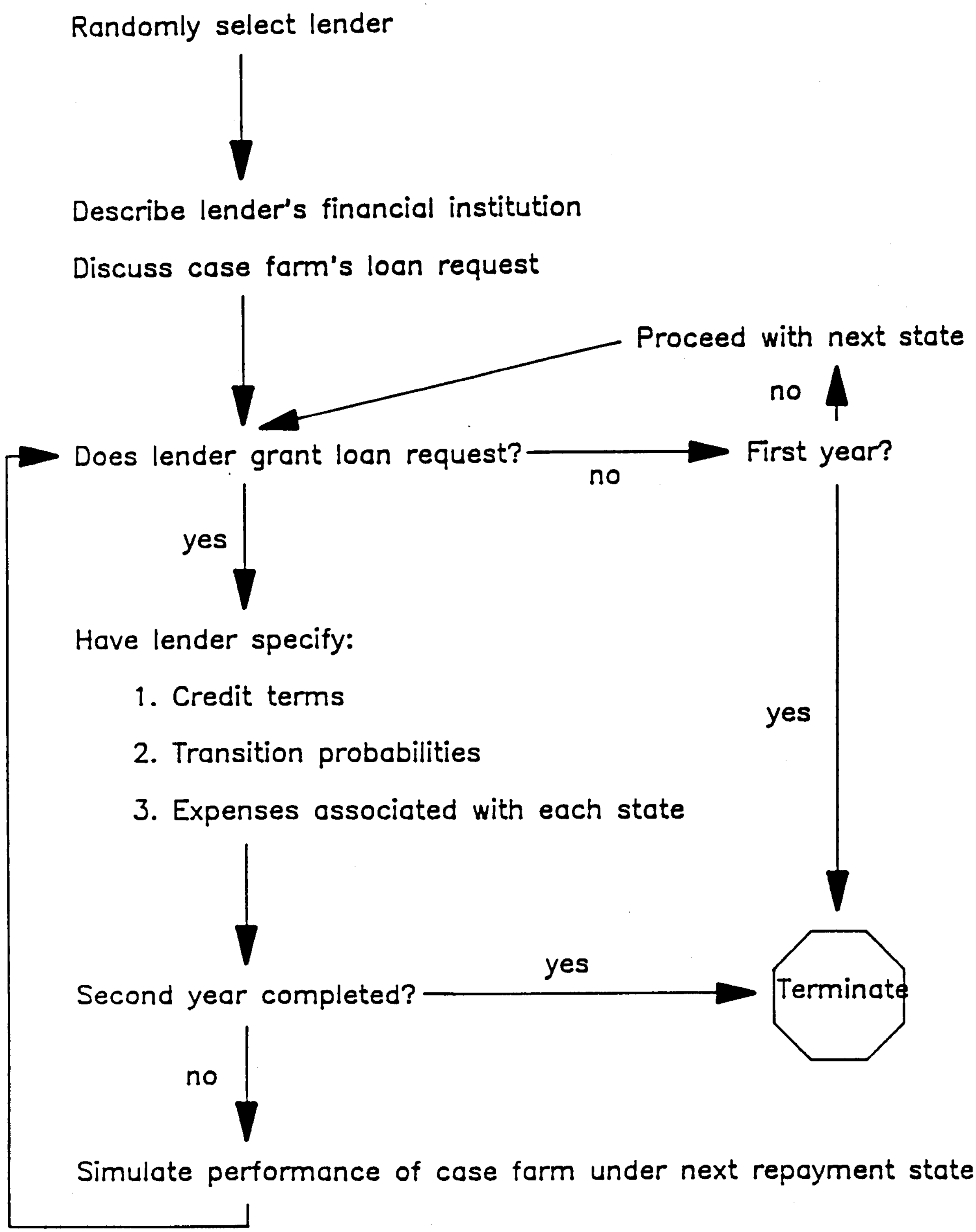

Figure 1. Experimental Procedure

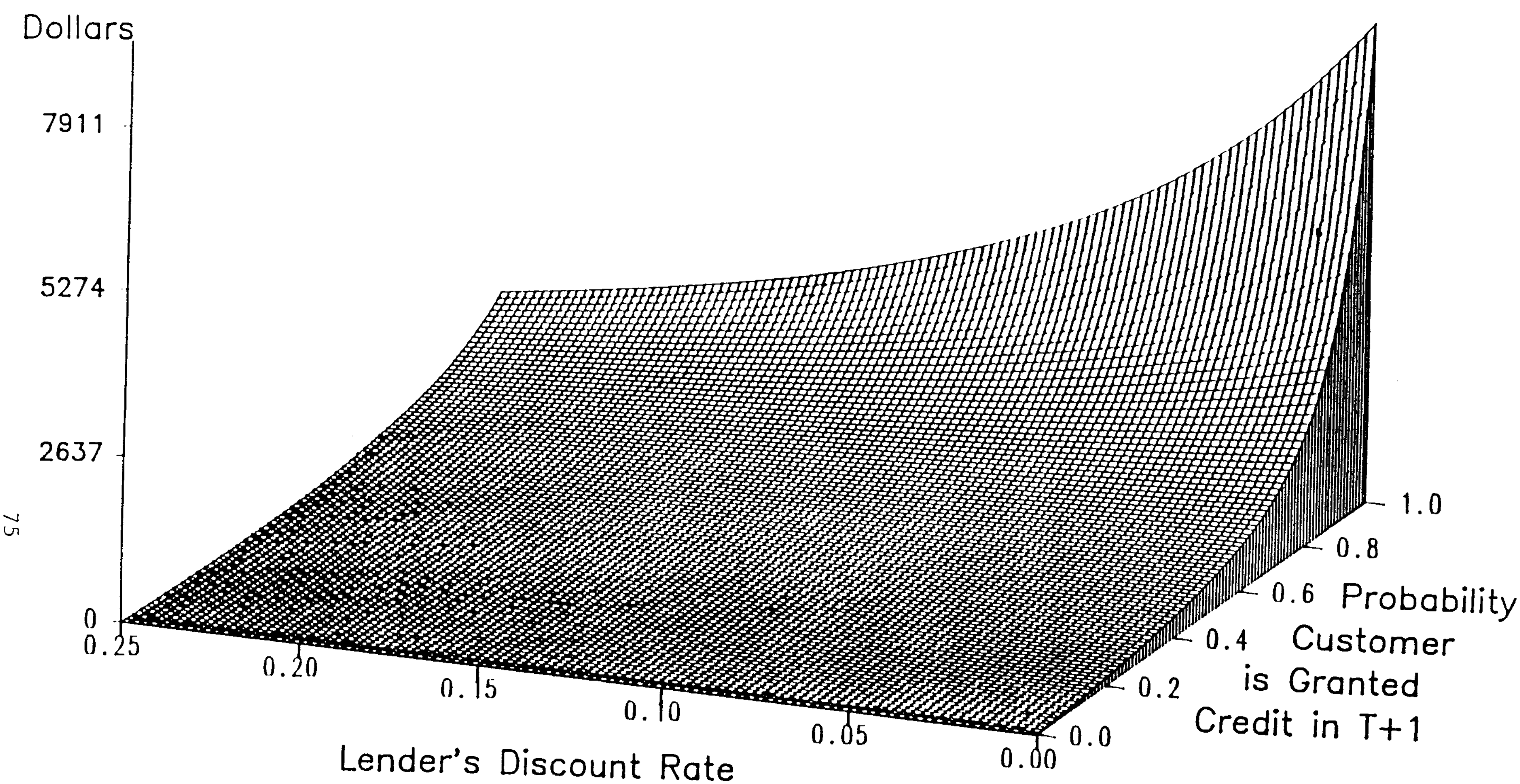

Figure 2. Value of Optimal Credit Policy When Lender's Discount Rate and Customer Patronage Vary

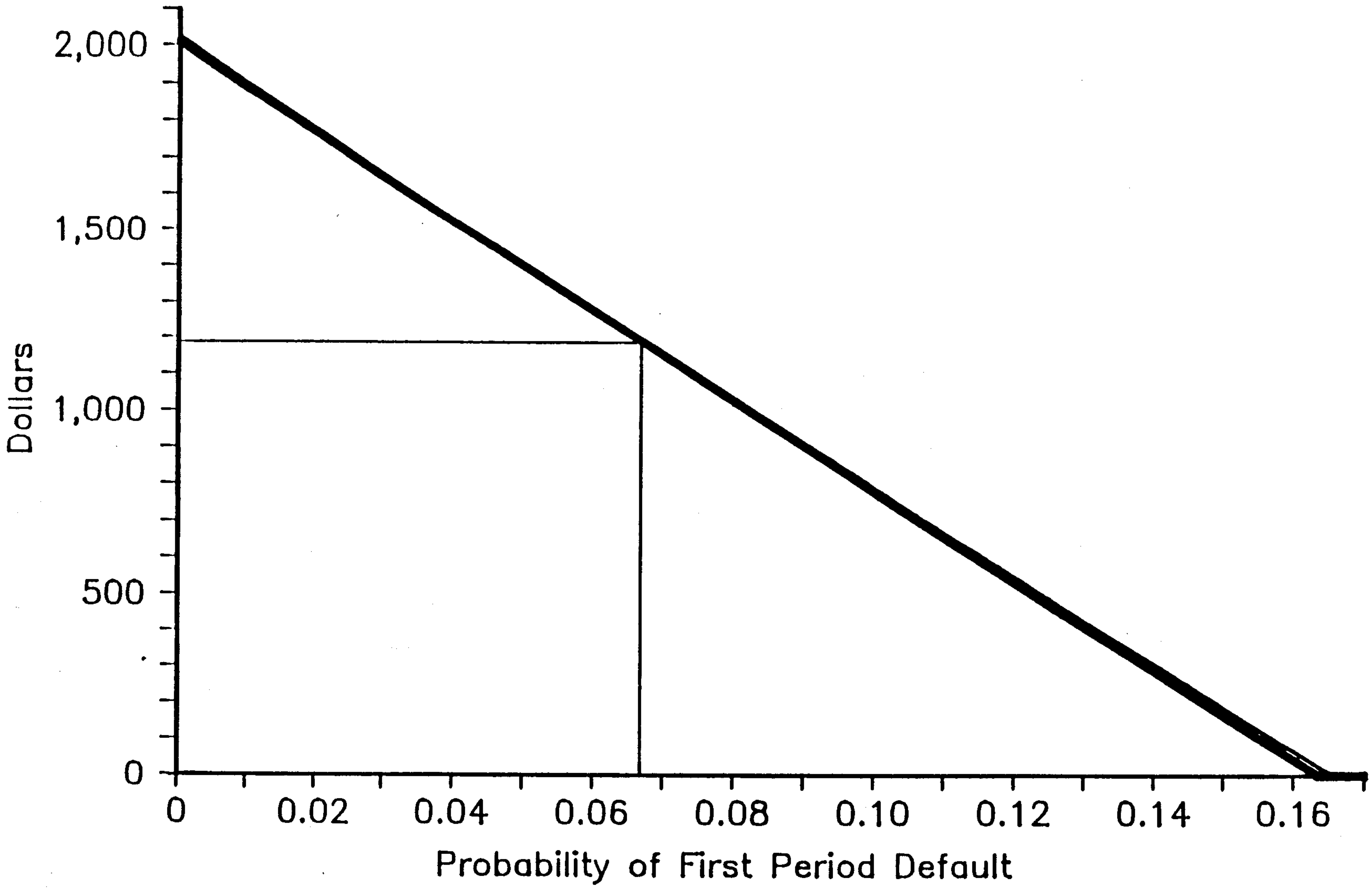

Figure 3. Value of Optimal Credit Granting Policy Given Varying Probabilities of Default.