ON THE DERIVATION OF INDIFFERENCE MAPS FOR ESTIMATING CONSUMER'S SURPLUS: REPLY

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Kennedy (1980) raises some six questions about the methods I have used to derive indifference maps and estimate benefit values. In order, Kennedy claims (a) that I fail to recognise the assumption of additive utilities in the Ramsey model, (b) that additivity leads to misspecification of the utility scale and (c) that the set of total utility curves should be vertically parallel.[1] Kennedy continues with assertions (d) that the procedure developed by Keeney and Raiffa (1976) could be superior and (e) that indifference curves based on ordinal utility (Kennedy's procedure) are sufficient. Finally (f), he believes I have misquoted Mishan (1971).

His paper is based upon the original journal contribution (Sinden 1974), the preceding monograph (Sinden 1973) with the basic data, the journal article with three further applications (Sinden 1978), a paper reviewing methods of indifference mapping (Sinden and Wyckoff 1976) and my joint book on the valuation of unpriced things (Sinden and Worrell 1979). Kennedy does not cite the basic reports underlying the further applications [as for example Sanderson (1974)] or our recent developments (Findlater 1978, Findlater and Sinden 1980). I draw on all this material and discuss the questions in Kennedy's order — although this is not my order for their importance. Then I consider two basic difficulties that Kennedy ignored and Findlater overcame.

The Basic Methods

The questions about methods concerned the apparent neglect of the additivity assumption and its implications.

The assumption of additive utilities

Kennedy's claim that I ignored the additivity assumption is untrue, both in fact and in perception. The Ramsey model rests on indifference judgments between two prospects. The prospects themselves contain two possible outcomes, each with a probability of occurrence. The probabilities indicate that the outcomes are mutually exclusive. Subjects can reasonably be expected to assume that the expected utilities of each outcome are independent and so to add them for the total utility of a prospect. I understood this to be part of the received wisdom of utility theory and the model and so omitted it from Sinden (1974) — as I omit-

[1] Each total utility curve in the set shows the utility from an increasing quantity of one good (*CSP*) and a fixed quantity of the other (*PL*). The quantity of the other good (*PL*) is increased incrementally in successive curves in the set. Kennedy claims that the slope of each curve should be the same at any particular level of *CSP*. The curves he referred to showed diminishing marginal utility to both goods.

ted several other received wisdoms.[2] In terms of fact, the matter is discussed in section 7.1.2 of Chapter 11 of Sinden and Worrell (1979).[3] I leave the various justifications for the assumption and implicit belief in the expected utility theorem to others such as Yntema and Torgerson (1967), Keeney (1973), Anderson et al. (1977), Keeney and Wood (1977) and Sinden and Worrell (1979).

The scaling of utilities

If utility is additive, Kennedy (1980) claims that the utilities were scaled wrongly, as he explained through his equation (4). The situation arose in the only application he considered because the initial Ramsey game contained two outcomes of zero quantity of recreation activity. An outcome of zero quantity of recreation means staying at home; hence, Kennedy argues, the utility of each of these zero-quantity activities is the same. For general application, Kennedy has incorrectly interpreted the nature of the left-hand side of his equation (3). This left-hand side is in fact a constant utility interval (Sinden 1974, p. 69). Thus, the *difference* between $U(1\ PL)$ and $U(0\ CL)$ is scaled and not the two separate utilities. Further, the problem does not arise at all if both outcomes in the left-hand side [$U(1\ PL)$ and $U(0\ CL)$] are specified differently and are different from the other outcomes in the game. These specifications were followed in the second application in Sinden (1973), in Sanderson (1974) and, in effect, in Findlater (1978). Kennedy cites Sinden (1973).

But what of the specific case in the specific application that Kennedy discusses? His claim holds as long as the utility of staying at home when deprived of a visit to *CSP* is the same as the utility of staying at home when deprived of a visit to *CL*. Conversely, his claim does not hold if (a) the utility of staying at home depends on the utility of what is lost and (b) the utility of gaining a unit of a given activity differs from the utility of losing a unit of the same activity and (c) the utility of a lost unit of *CL* differs from the utility of a lost unit of *CSP*. My recollection of the interviews is that all these three conditions were met. *CSP* is recreation at a nearby state park that all the families visit regularly. *CL* is recreation at Crater Lake, a nearby national park of biologically-unique and aesthetically-spectacular characteristics. Indeed, *CL* was selected just so that its loss was seen as 'more of a problem' than the loss of the '*n*th' visit to *CSP* that season. Kennedy's (1980) recognition of both the compensating and equivalent variations of consumer's surplus seems to support condition (b) above. Kennedy's claim is therefore questionable in its specific case, will not arise with differently-specified quantities of things and may, in general, be suspect.

The scaling question was raised in the context of joint consumption of activities — as usually conceived in an indifference map. This context indicates a different, and apparently superior, way to scale the second total utility curve. Sinden (1974, p. 69) takes a combination of goods (1.25 *CSP*, 1 *PL*) at a point *A* on the first curve. This point was scaled at 10

[2] Two other unstated, received wisdoms were — consumer's surplus is a true social benefit and individual surpluses are aggregated additively and without weights for Pareto comparisons.

[3] Kennedy rests almost entirely on the additivity assumption so I include the following information on timing. Section 7.1.2 was developed while Sinden (1974) was completed.

utiles. The 'base quantity' 1 *PL* was changed to 2 *PL* and the indifference quantity of *CSP* sought in the usual way. The Ramsey game to find this quantity *n* is:

Probabilities	*Prospect I*	*Prospect II*
0.5	1.25 *CSP*	*n CSP*
0.5	1 *PL*	2 *PL*
Combination	*A*	*B*

The scale value of 10 utiles had been taken from a separated, independent utility reading of $U(1.25\ CSP) = 10$. Thus the utility of the combination [$U(1.25\ CSP)$, $U(0\ PL)$] should be 10 and the utility of combination *A* (as the left-hand side of Kennedy's equation (4)) would, presumably, be somewhat higher. Thus a more appropriate Ramsey game to start to identify a second utility curve would be:

Probabilities	*Prospect I*	*Prospect II*
0.5	1.25 *CSP*	*n CSP*
0.5	0 *PL*	1 *PL*
Combination	*A'*	*B'*

Point *A'* is the starting point and should be on a total utility curve labelled '0 *PL*' — and not '1 PL' as in Sinden (1974). Point *B'* is on a total utility curve now to be labelled '1 *PL*'. These games can be repeated in precisely the usual way for a set of points on the second and subsequent utility curves. In similar fashion an entire family of total utility curves can be derived. The points on the arbitrary utility scale should have been different as Kennedy claimed, but possibly not for the reason he suggested.

Should the total utility curves be vertically parallel?

Kennedy (1980) claims that additive utilities should lead to a vertically-parallel set of total utility schedules. Thus the marginal utility from an extra day *CSP*, at a given level of *CSP*, should be constant for all levels of the other activity (*PL*). But this attractive general proposition may not hold empirically.

Empirical applications apply theory to reality and so must specify, *inter alia*, a time period for consumption. Sinden (1974) reported a period of six weekends or twelve days as realistic for his subjects. Consider now a seven-day period and an individual who will already consume 1 day *CSP* in combination with either (a) five days *PL* or (b) two days *PL* as his only other recreation activity. In (a) an extra day *CSP* uses up the entire period in recreation — leaving none of the period at home.[4] In (b) the extra day *CSP* leaves three days at home. The marginal utility of one extra day *CSP* in the Kennedy situation (a given amount of *CSP*) may well vary in cases (a) and (b). One might argue, *a priori*, that the marginal utilities should be different — because the conditions have changed and refer to a given time period. The curves will not be vertically parallel if the marginal utilities differ.

[4] Kennedy (1980) also recognises 'staying at home' as the residual activity.

Alternative Methods

Kennedy goes on to acknowledge the role of indifference mapping in benefit valuation and to suggest two alternative methods to obtain such maps. Consider now his assertions.

The Keeney/Raiffa procedure

Kennedy (1980) notes that Keeney and Raiffa (1976) pioneered multi-attribute utility theory which could provide a method of deriving indifference curves.[5] He continues (p. 295),

'. . . it would be quite possible to use multi-attribute utility functions to calculate consumer surpluses based on derived indifference curves and assumed budgets . . .'

Apparently Kennedy hasn't tried it but then neither have I. The essence of the procedure, as illustrated by Keeney (1973), is the estimation of separate utility functions for each attribute and scaling factors to aggregate the utilities. Both the functions and factors are estimated from games with prospects, outcomes and probabilities – much like the Ramsey model. The procedure for the functions seemed robust and was when I tried it. But the procedure for the scaling factors seemed less robust, resulting in factors which are particularly sensitive to the inherent difficulties of the models.

Kennedy presents only a weak case for this procedure. He states that certain assumptions must hold but fails to say what they are.

The Webster/Kennedy procedure

Kennedy rightly states that indifference curves based on ordinal utility are sufficient for estimating demand curves and consumer's surplus. Webster and Kennedy (1975) used direct questioning to derive discrete, and hence ordinal, indifference curves for the 'goods' of expected income and a given minimum level of income. Their aim was to use the information from the indifference schedules for predictive farm management purposes. He cites other attempts to derive discrete indifference curves and suggests that such methods could be used to derive curves for recreation use and valuation.

Kennedy's suggestion doesn't work. More precisely, it didn't when I tried the same idea in 1972 to derive demand curves. The successful part of this earlier work is reported in Sinden et al. (1972). I obtained a set of ordinally-ranked indifference curves but couldn't derive the necessary demand curve from this map. It proved impossible to obtain enough discrete indifference curves, close enough together, for sufficient points of tangency with the budget lines. There were insufficient points of actual tangency for a 'respectable' demand curve. This problem is overcome if the utility of indifference curves can be scaled because the demand curve can be derived by simple mathematical formulae. This was reported in Sinden and Wyckoff (1976).

[5] Kennedy quotes the 1976 book by Keeney and Raiffa. More relevant to his arguments are Raiffa (1968) and Keeney (1973). These articles preceded my 1974 paper and so were available for consideration for alternative methods.

Discussion

A misquote of Mishan

Kennedy quotes the following part of footnote 5 of Sinden (1978):

'. . . Mishan (1971) promoted the price compensating variation when price falls and the price equivalent variation when price rises'.

He then claims that this is incorrect because 'CV and EV can apply in situations in which price either falls or rises'. This claim seems to rest on Kennedy's misconception of my terminology and my overbrief footnote.[6] Sinden (1978) used the terminology, situations and concepts of the discussion he reviewed. He tried to avoid extra terms, like 'compensating variation' (CV) and 'equivalent variation' (EV) except when he reviewed the discussion of others.

I now flesh out that footnote.[7] Price compensating variation in Sinden (1978) is Mishan's (1971) compensating variation for a fall in price (and extending to a new situation, it is Mishan's equivalent variation for a rise in price). Similarly, price equivalent variation in Sinden (1978) is Mishan's compensating variation for rise in price (and his equivalent variation for a fall in price).

The point of my footnote may have eluded Kennedy. In his discussion of these issues, Mishan relegates the quantity-constrained concepts (quantity compensating and quantity equivalent variations) to a footnote on page 330. Thus Sinden (1978) felt that Mishan had indeed promoted the price compensating and price equivalent variations over the footnoted quantity variations. The latter seem to be particularly relevant to many current land use and environmental problems. So the footnote discussion didn't really relate to whether CV or EV applied when price rises or falls.[8]

Kennedy's extension to this argument seems intuitively unreasonable but empirically verifiable. He states:

'. . . the difference between the two measures in practice is almost certain to be so small as to be swamped by measurement error . . .'

Sinden (1978) reports differences of nearly 2 to 1 and Meyer (1979) reports differences of 20 to 1. Whether they are swamped by measurement error is a question for discussion but, as Sinden (1978) tried to explain, this argument is theoretically testable and seems precisely the wrong way to start an analysis.

Relevance and importance

The importance of these six points seems the reverse of their order of presentation. Incorrect use of readily-available literature — where it

[6] It may also rest on my imperfect understanding of Mishan (1971).

[7] Kennedy (1980) spends some 14 per cent of his manuscript on a point I had relegated to a footnote. Did I underrate the point or does he overrate it?

[8] The debate I reviewed did not rest closely on Mishan (1971) so I tried to avoid introducing his contribution. Another reason for avoiding it concerns an extra element that Mishan had considered. The four surplus concepts of Sinden's (1978) Table 1, and the debate, assumed that changes are actually undertaken and the consumer is subjected to the changes in price and quantity. In contrast, Mishan's equivalent variation assumes the consumer is exempted from the change in price and so some of his surplus concepts are slightly different. Thus Kennedy's points about CV and EV may not be completely relevant.

occurs – is serious and is poor professionalism. Empirical applications and practical methods are seriously needed to increase the relevance of our theory. The additivity assumption must be recognised in the method but seems to me to be justified. But even if it weren't, the method may still be useful. For example, Yntema and Torgerson (1967) have shown that additive effects often swamp non-additive ones and additivity can sometimes be tested (Keeney 1973).

Four minor issues were raised, namely the ease of determining recreation budgets, the possibility of deriving utility from gambling, the relevance of Pareto improvements and the failure of the expected utility theorem. These issues seem, respectively, empirically verifiable, interesting and worth pursuing, interesting but not immediately pursuable and a pedagogically-useful straw man.

Kennedy failed to consider, what I believe to be, the two major drawbacks with the methods – the lengthy but necessary interview and the restriction to two goods. Sinden took three hours for each interview. Sanderson is a better interviewer and halved this time but one-and-a-half hours is still too long for a survey of a large number of people. Through a simplified procedure Findlater (1978) cut the time even further to some 20 minutes.

The simplified interview procedure permits more data to be collected in the earlier interview time of an hour and a half. Sufficient data were collected in this time for a five-good utility function (Findlater and Sinden 1980). We inserted this function as the objective function in a linear program, varied the price of the activity of interest and traced out a demand curve.

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