ON RISK MODELING AND ITS IMPLICATIONS FOR ECONOMIC ANALYSIS

Jean-Paul Chavas *

I - Introduction

The influence of risk on economic decisions has been the subject of much research. The fact that farm production is typically risky and that agricultural markets tend to be unstable has stimulated much interest in the economics of risk in agricultural production (e.g. Halter and Dean; Anderson et al.). Now, a fairly common view among economists is that the riskless profit maximization approach (as in standard production theory) gives an imperfect characterization of farmer behavior. In that context, the expected utility maximization approach under uncertainty is often proposed as a better alternative (e.g. Lin et al.).

Much traditional analysis of agricultural production has focused on the characterization of farm technology and its implications for resource allocation (e.g. Heady and Dillon). However, with the incorporation of risk in production models (e.g. in mean-variance analysis or stochastic dominance), model results typically depend on risk preferences (e.g. the agent’s risk aversion coefficient). Although many attempts have been made to measure the risk preferences of decision makers (e.g. see Young for an overview), the reliability of the measures remain mostly unproven. This implies that much of agricultural decision analysis under risk is made conditional on particular risk preferences. And the fact that the degree of risk aversion could vary significantly across individuals and across time periods decreases sharply the normative usefulness of our risk models. In other words, if different behavior can be explained by a difference in (risk) preferences between individuals, then few of the research results on agricultural decisions under risk can be easily translated into farm management, marketing or policy recommendations. This is somewhat undesirable.

An alternative approach would be to try to minimize the role of preferences in economic behavior and to maximize the role of other factors that are more amenable to measurement and analysis. This is the argument presented by Stigler and Becker who treat preferences as stable over time and across individuals and propose to explain differences in behavior by differences in prices and (household) production technology. In the context of risk analysis, this suggests that an attempt to model risk behavior in terms of technology and the institutional environment of the decision maker may be useful. First, technology may be fairly easy to measure and characterize. Second, it would imply that risk behavior is a by-product of the economic environment of the decision maker; this would help generate useful hypotheses about some of the factors influencing risk behavior.

*Associate Professor of Agricultural Economics, University of Wisconsin-Madison.
Third, by providing more precise information on how risk can affect different individuals, it would strengthen a normative analysis of economic decisions under risk and make economists more effective in proposing solutions to real world problems. This suggests the need for a new look at risk behavior and some of the factors that may generate it.

The objective of this paper is to explore briefly the implications of temporal uncertainty for risk behavior. We consider the case where current decisions are made under risk while other decisions are made in the future as uncertainty is being resolved. This allows an analysis of how technology and the temporal resolution of uncertainty can influence production behavior under risk. It helps provide some insights on the impact of information and technology on economic decisions under uncertainty.

A general two period model representing a decision-making process under temporal uncertainty is developed in section II. It is argued that, in the absence of good a priori information about the nature of the objective function, the effect of risk aversion on decisions may not be identifiable under temporal risk. The reason is that the valuation of information and its impact on decision may lead to behavior similar to the one obtained under risk aversion. It suggests a need to refine our understanding of the role of information in economic analysis. A few examples are given in section III as illustrations of the usefulness of our modeling approach. Implications for welfare analysis are discussed in section IV. It is shown that traditional welfare measures are in general biased under temporal uncertainty. Also, when possibilities of risk sharing exist, it is argued that an individualistic ex-ante approach to welfare measurement tends to underestimate aggregate willingness-to-pay. Finally, section V concludes the paper.

II - A two period Model

Consider an economic agent facing a two period planning horizon \((t=1,2)\) and a preference function \(U[w,x_1,x_2,e]\) where \(U\) is a twice continuously differentiable von Neumann Morgenstern utility function satisfying \(U'_w = \partial U/\partial w > 0\), \(w\) is initial wealth, \(x_t\) is the vector of decision variables at time \(t\), and \(e\) is a random vector representing temporal uncertainty about the state of the world. We assume that \(e\) is not known at time \(t=1\) and has a given subjective probability distribution, but becomes observable by the agent before the second period decisions are made. In this context, if the agent maximizes expected utility, then economic decisions are made according to the following dynamic programming problem

\[
V(w) = \max_{x_1 \in A_1} \max_{x_2 \in A_2} \mathbb{E} U[w,x_1,x_2,e] \tag{1}
\]

where \(\mathbb{E}\) is the expectation operator over the random variables \(e\), \(A_1\) is the feasible region for \(x_1\), \(A_2(x_1)\) is the feasible region for \(x_2\), and \(V(w)\) denotes the indirect objective function of the agent. From backward induction, denote by \(x_2^\ast(w,x_1,e)\) the optimal second period decisions (conditional on \(x_1\)), and by \(x_1^\ast(w)\) the optimal first period decisions. Note that \(x_2^\ast(w,x_1)\) are ex-post decisions in the sense that they are made after \(e\) becomes observable, while \(x_1^\ast(w)\) are ex-ante decisions since they are made
while e are still uncertain. If the decisions $x_2$ were made at time $t=1$, they
would correspond to the following problem

$$\max_{x_2 \in A_2} \text{EU}[w, x_1, x_2, e]$$

which has for solution $x_2(w, x_1)$ the ex-ante choice functions for $x_2$,
conditional on $x_1$.

From the literature on decision theory (e.g. Lavalle), define the
conditional value of information about $e$ as the certain monetary value
$D(x_1, w)$ which satisfies

$$E\max_{x_2 \in A_2} U[w, x_1, x_2, e] = \max_{x_2 \in A_2} E\left[U[w + D(x_1, w), x_1, x_2, e]\right]$$

or $E\left[U[w, x_1, x_2(w, x_1, e), e]\right] = E\left[U[w + D(x_1, w), x_1, x_2(w + D(x_1, w), x_1), e]\right]$.

$D$ is the amount of money that must be paid to the agent at time $t=1$ in
order to make him indifferent between making the second period decisions $x_2$
knowing $e$ versus deciding about $x_2$ without learning about $e$. From (2), it
follows that $D(x_1, w)$ is the conditional selling price of information about $e$:
it is conditional on the first period decisions $x_1$; and it is a selling price
in the sense that it measures the monetary value of doing away with the
information, using the informed situation as the reference point. It is well
known that the value of information is always non-negative, i.e., that

$$D(x_1, w) \geq 0$$

Indeed, obtaining costless information can never make a decision maker
worse-off. Furthermore, if the information is relevant to the
decision-making process, obtaining it will usually make him better off. In
this case, the agent would be made worse off by "selling" relevant
information and would need to be compensated by a lump sum payment. This
lump sum payment $D(x_1, w)$ is the smallest amount of money which would make the
agent willing to choose $x_2$ during the second period ($t=2$) without learning
about $e$. It represents the valuation of the ability to maintain flexible
plans and revise decisions as new information becomes available.

The definition of the value of information in (3) is useful in the sense
that it can be combined with (1) to reformulate the dynamic programming
problem as

$$V(w) = \max_{x_1 \in A_1} E\left[U[w + D(w, x_1), x_1, x_2, e]\right]$$

Denote by $x_1(w)$ and $x_2(w)$ the optimal solutions to (5). Clearly, $x_1(w) = x_1(w)$ since (5) is simply a reformation of (1): it corresponds to the
ex-ante decisions of the first period as before. However, $x_2(w)$ is different
from $x_2(w, x_1^*(w), e)$. First, it is now an ex-ante decision since the decision
$x_2(w)$ is made based on the information available at time $t=1$ (i.e. before the
agent learns about $e$). Second, $x_2(w)$ is a compensated choice function since
it can be influenced by the wealth compensation $D(x_1, w)$. In other words,
$x_2(w)$ is the decision that would be made if the agent had to decide $x_2$ at time $t=1$ (i.e. before knowing $e$) while he is compensated for not being able to take advantage of the information that becomes available between $t=1$ and $t=2$. A special case of interest may be associated with the absence of a wealth effect in the decisions $x_2$, i.e. $\delta x_2/\delta w = 0$ (e.g. the case of constant absolute risk aversion for a firm maximizing the expected utility of terminal wealth; see Sandmo; Chavaš (1985)). In this case, the compensation $D(x_1,w)$ will have no influence on $x_2$, and $x_2$ would become simply the decision that would be made if the agent had to decide $x_2$ at time $t=1$, i.e. $x_2$ could be interpreted as the ex-ante plans of the agent for $x_2$.

Note that, with $D$ set equal to zero, (6) would correspond to an open-loop solution, all decisions being made ex-ante (before $e$ is known). Given $\delta U/\delta w > 0$ and $D \geq 0$ in (5), it follows that open-loop models are always inferior to closed-loop models (such as (1)) as they fail to capture the value of flexibility associated with the revision of plans. This suggests that open-loop (or static) models of production are not appropriate tools of analysis whenever new information has a significant influence on economic decisions.

At this point, it will be useful to consider the special case where $U(w,x_1,x_2,e) = U(w+f(x_1,x_2,e))$, $f(.)$ being the return function (e.g. discounted profit) and $[w+f(x_1,x_2,e)]$ representing the present value of terminal wealth. Then (5) takes the form

$$V(w) = \max_{x_1 \in A_1, x_2 \in A_2} E[U[w+D(w,x_1) + f(x_1,x_2,e)]]$$

Using the Arrow-Pratt definition of the risk premium $R(w,x_1,x_2) = w + Ef(x_1,x_2,e) + D(w,x_1) - U$ of $E(U(.)$, expression (6) can be alternatively written in terms of its certainty equivalent as

$$\max_{x_1 \in A_1, x_2 \in A_2} Ef(x_1,x_2,e) + D(w,x_1) - R(w,x_1,x_2)$$

where $R > (\rightarrow)0$ under risk aversion (risk neutrality).

Expression (7) indicates that current decisions are made by maximizing the sum of the three terms in (7). It suggests that it desirable to have a) a high expected return $E(.)$, b) a low risk premium $R(.)$ if the agent is risk averse ($R>0$), and c) a high value of $D(.)$, i.e. a good ability to adapt to temporal uncertainty. In the riskless case, $R=D=0$, and (a) yields the familiar case of profit maximization. Characteristic (b) corresponds to the introduction of risk and risk behavior in static models (e.g. Arrow; Pratt). An extensive amount of research has focused on the influence of risk behavior on farmer’s decisions (e.g. Anderson et al.; Newbery and Stiglitz;Binswanger; Roumasset et al.). Finally, characteristic (c) reflects the ability of the agent to modify production plans as new information becomes available. With a few exceptions (e.g. Baquet et al.; Antle), little work has been done on the empirical implications of this concept for technological choice and farm production decisions. If flexibility of production and
marketing plans is an important way for farmers to deal with uncertainty, then additional research on this topic may have high potential payoffs.

Note that D and R are clearly different since D tends to increase the value of the objective function while R would decrease it under risk aversion. Also, under risk neutrality, then R=0 but in general D>0. Thus, although R=D=0 in the riskless case, under temporal uncertainty D>0 is not due to risk preferences as the flexibility to respond to new information remains important even under risk neutrality.

To further illustrate this, consider the following local measure of the risk premium in (6) (Pratt)

\[ R = \frac{1}{2} r \cdot \text{Var}(\cdot) \]  \hspace{1cm} (8)

where \( r = \frac{U_{x_2}(w+D+E(f))}{U_{x_2}(w+D+E(f))} \cdot \text{Var}(\cdot) \) denotes the variance and subscript letters denote derivatives (e.g. \( U_w = \frac{\partial U}{\partial w}, U_{ww} = \frac{\partial^2 U}{\partial w^2} \)). Similarly, using the definition of the value of information, then a local measure of D for unconstrained problem (3) can be shown to be

\[ D = \frac{\text{COV} \left( U_{x_2}(\overline{x}_2), x_2^* \right) + \frac{1}{2} \text{E} \left( [x_2^* - x_2'] \cdot U_{x_2x_2}(\overline{x}_2) \cdot [x_2^* - x_2'] \right)}{\text{E} U_w(\overline{x}_2)} \geq 0 \]  \hspace{1cm} (9)

A comparison of (8) and (9) indicates that, although R and D are different, they both depend on \( x_1 \) and on the subjective probability distribution of \( e \). In the absence of precise information about the nature of the objective function (1), this suggests that it is difficult to isolate the effect of R from the effect of D on current choices \( x_1 \). In other words, it is possible that some type of risk behavior is attributed to risk aversion when in fact it is due to the valuation of information (or vice versa). In order to solve this identification problem, good a priori information must exist on the nature of the objective function, the technology and the characterization of the uncertainty. As argued in the introduction, good a priori information on preferences may be difficult to find. On this basis, it may be reasonable to focus our attention on how other factors (beside risk preferences) can influence economic behavior under risk. In the next section, we briefly illustrate our argument in the context of a few examples.

III - Some Examples

If the value of information \( D(x_1,w) \) could be easily obtained, then expression (5) would provide a convenient basis for analyzing current economic decisions. Unfortunately, the value of information as defined in (3) can be rather complex, as the influence of some parametric change on D is not obvious in the general case. For example, although it may seem intuitive, it is not necessarily true that increasing uncertainty (as measured from a mean preserving spread) always increases the value of information (see Gould; Hess). This suggests that the discussion of the properties of the value of information and its implications can best proceed in the context of some specific examples.
1 - An irreversible decision:

Consider the case of a fixed resource initially used in a particular way. At time $t=1$, the resource can be left in its initial state ($x_1=0$) or transformed into some alternative use ($x_1=1$). A similar choice is possible at time $t=2$, i.e. $x_2$ (0 or 1). However, choosing $x_1=1$ is an irreversible decision in the sense that $x_1=1$ implies $x_2=1$, i.e. a loss of the option of choosing $x_2$. Alternatively, choosing $x_1=0$ implies $x_2$=(0 or 1), leaving the option of choosing $x_2$ opened. Under temporal uncertainty, this problem can be formulated as

$$\text{Max } \mathbb{E} \text{ Max } (U(w,x_1,x_2,e) : x_t = (0 \text{ or } 1), t=1,2; x_1 + x_2 \leq 1)$$

(10)

which is a special case of (1). Denote by $D(w,x_1)$ the conditional value of information in (10). Clearly $D(w,1)=0$: irreversibility implies no value of information for the second period decision when $x_1=1$. Alternatively, the conditional value of information $D(w,0)$ is non-negative. In this case, $D(w,0)$ has also been called the quasi-option value (see Arrow and Fisher; Henry).

Using formulation (5), the first period decision in (10) can be alternatively expressed as

$$\text{Max } (\mathbb{E} U[w+D(w,x_1),x_1,x_2,e] : x_t = (0 \text{ or } 1), t=1,2; x_1+x_2 \leq 1)$$

(11)

It follows that the optimal choice is $x_1=1$ if

$$\mathbb{E} U[w,1,1,e] \geq \text{Max } (\mathbb{E} U[w+D(w,0),0,1,e], \mathbb{E} U[w+D(w,0),0,0,e])$$

Otherwise, choose $x_1=0$. This decision rule suggests that the conditional value of information $D(w,0)$ (the "quasi-option" value) plays an important role in the evaluation of optimal decisions under irreversibility. For example, because future information can be of value only if $x_1=0$, the prospect of more information in the future can be shown to discourage the adoption of an irreversible decision ($x_1=1$) in period 1 (see Epstein, 1980). This may help explain the cyclical nature of some form of economic behavior (e.g. Bernanke).

2 - The case of capacity constraint:

Consider an investment decision that consists in the purchase of a durable asset $x_1$ (a scalar) at time $t=1$ (e.g. building a plant of size $x_1$). At time $t=2$, the asset $x_1$ is available for productive use, but the decision-maker can choose $x_2$ the capacity utilization of the asset, with $x_2 \leq x_1$. In this case, the rate of capacity utilization is $x_2/x_1 \leq 1$, $x_2=x_1$ corresponding to full capacity utilization at time $t=2$. Under temporal uncertainty and risk neutrality, the profit function is denoted by $f_1(x_1)+f_2(x_2,e)$, yielding the following problem:

$$\text{Max } \mathbb{E} \text{ Max } (f_1(x_1) + f_2(x_2,e) : x_2 \leq x_1)$$

$x_1\geq 0 \quad x_2\geq 0$
which is a special case of (1). Assuming the concavity of the profit function in \((x_1, x_2)\), the conditional value of information then is obtained from the following saddle point characterizations

\[
D(x_1) = E \max_{x_2 \geq 0} \min_{\lambda \geq 0} (f_2(x_2, e) + \lambda [x_1 - x_2]) \\
- \max_{x_2 \geq 0} \min_{\lambda \geq 0} (E f_2(x_2 + e) + \lambda [x_1 - x_2]) \tag{11}
\]

where \(x_2^*, \lambda^*\) are the solutions of the first optimization problem in (11) while \(x_2, \lambda\) are the solutions of the second optimization problem. In this context, it follows from the envelope theorem that the marginal value of conditional information is \(\frac{\partial D}{\partial x_1} = E(\lambda^*) - \lambda^*\).

From (5), the first period decision can then be characterized by the saddle point problem

\[
\max_{x_1 \geq 0} \min_{\lambda \geq 0} (D(x_1) + f_1(x_1) + f_2(x_2, e) + \lambda [x_1 - x_2]) \\
= f_1(x_1) + f_2(x_2, e) + \lambda [x_1 - x_2] \tag{13}
\]

where \(S(x_1)\) denotes the salvage value of \(x_1\) at time \(t=2\). Thus, \(r_1'x_1\) constitutes a fixed cost at time \(t=2\). Moreover, if \(r_1'x_1 > S(x_1)\), then \(r_1'x_1 - S(x_1)\) constitutes a sunk cost at time \(t=2\), i.e. a cost that cannot be recovered even by going out of business. Such a sunk cost would contribute to asset fixity, entry barriers, exit barriers and could lead to low return on fixed assets (see Johnson and Quance; Johnson and Pasour; Baumol et al.).
In the context of (13), the conditional value of information is

\[ D(x_1) = E \max_{x_2 \geq 0} \left\{ f(x_1, x_2, e) - r_2 x_2, S(e) \right\} \]

\[ - \max_{x_2 \geq 0} E(I(x_1, x_2, e).[f(x_1, x_2, e) - r_2 x_2] + [1-I(x_1, x_2, e)].S(e)) \]

where \( I(x_1, x_2, e) \) is an indicator variable taking the value 1 if \( f(x_1, x_2, e) - r_2 x_2 > S(e) \), and 0 otherwise.

In general, the value of information \( D(x_1) \) can be expected to be positive. For example, even in the case of a Leontief technology where \( x_1 \) and \( x_2 \) are scalars and output is \( y = \min(a(e)x_1, b(e)x_2) \), then \( D(x) \) may not be zero. In particular, given \( x_1 \) and a Leontief technology, the choice of \( x_2 \) in general would satisfy \( x_2 \leq \frac{a(e)}{b(e)} x_1 \), which reduces to a case similar to the capacity constraint problem just discussed.

From (5), the first period decision can be characterized by

\[ \max_{x_1 \geq 0, x_2 \geq 0} \left\{ D(x_1) + E[I(x_1, x_2, e).[f(x_1, x_2, e) - r_2 x_2] + [1-I(x_1, x_2, e)].S(e)] - r_1 x_1, 0 \right\} \] (14)

Expression (14) indicates that the conditional value of information will influence entry decisions in the industry as well as the choice of the production decisions \( x_1 \).

In order to illustrate further the influence of temporal uncertainty on production decisions, assume that the firm will be actively producing with probability one during period 2. Express the revenue of the competitive firm as \( f(x_1, x_2, e) = p(e).y(x_1, x_2, e) \) where \( p(e) \) is the uncertain output price and \( y(x_1, x_2, e) \) is the uncertain production function. The analysis of firm behavior under temporal price uncertainty has been presented by Hartman and Epstein (1978). Here, we focus on a risk neutral case that can include both temporal price and production uncertainty and may be of interest in empirical work. Consider the following specification for the production function

\[ y(x_1, x_2, e) = a(x_1, e) + x_2 b(x_1, e) + 1/2 x_2 A(x_1)x_2 \]

where \( A(x_1) \) is negative definite matrix corresponding to the strict concavity of the production function in \( x_2 \). Assuming that \( I(x_1, x_2, e) = 1 \), then the value of information is given by

\[ D(x_1) = E \max_{x_2 \geq 0} \left\{ p(e).y(x_1, x_2, e) - r_2 x_2 \right\} - \max_{x_2 \geq 0} \left\{ E p(e).y(x_1, x_2, e) - r_2 x_2 \right\} \]

and (14) takes the form

\[ \max_{x_1 \geq 0, x_2 \geq 0} \left\{ D(x_1) + E[p(e).y(x_1, x_2, e)] - r_1 x_1 - r_2 x_2, 0 \right\} \]
Under competition, the first order necessary conditions for an interior solution for \( x_1 \) are

\[
\frac{\partial D}{\partial x_1} + E[p(e) \frac{\partial y(x_1, x_2, e)}{\partial x_1}] - r_1 = 0 \tag{15}
\]

indicating that the marginal valuation of information \( \frac{\partial D}{\partial x_1} \) can play an important role in production decisions.

Under production uncertainty (with \( p(e) = p \text{ known} \)), the value of information \( D(x_1) \) takes the simple form

\[
D(x_1) = -\frac{p}{2} \text{trace} \left( A(x_1)^{-1} \text{Var}[b(x_1, e)] \right) \geq 0
\]

and the marginal value of information \( \frac{\partial D}{\partial x_1} \) in (15) can be positive, zero, or negative depending on how \( A(x_1) \) and the variance of \( b(x_1, e) \) varies with \( x_1 \). This illustrates the fact that second order approximations of a production function are not flexible in modeling the impact of temporal production uncertainty on economic decisions. These results could be used in the modeling of many dynamic agricultural production processes. For example, the effectiveness of pest management strategies appear to depend heavily on the information available at the time of the decision. This suggests that the economics of pest control may benefit from a detailed analysis of the role of information in the cost-benefit evaluation of alternative strategies.

Under price uncertainty \( p(e) \) (with a known production function \( y(x_1, x_2) = a(x_1) + x_2 b(x_1) + \frac{1}{2} x_2 A(x_1) x_2 \)) and assuming that \( x_2 \) is a scalar, the marginal value of information in (15) takes the form

\[
\frac{\partial D(x_1)}{\partial x_1} = \frac{1}{2} \left[ \frac{r_2}{A(x_1)} \right]^2 \frac{\partial A(x_1)}{\partial x_1} \left[ \frac{E(\frac{1}{p}) - \frac{1}{p}}{E(p)} \right]
\]

Since \( \frac{1}{p} \) is a convex function of \( p \), it follows from Jensen's inequality

\[
\frac{1}{p} - \frac{1}{E(p)} > 0, \text{ implying that } \frac{\partial D(x_1)}{\partial x_1} = \text{sign} \left[ \frac{\partial A(x_1)}{\partial x_1} \right]
\]

Again, it shows that second order approximations of a production function are not flexible in modeling the impact of temporal price uncertainty on economic decisions. Indeed, third order terms are needed in the production function if one wants to avoid imposing strong restrictions on the way information influences production decisions in a dynamic context (see Epstein, 1978). It illustrates that revenue uncertainty can either stimulate or dampen production choice depending on the nature of the production technology. Further empirical work on the characterization of agricultural production technology and its influences on information valuation is clearly needed to help shed more light on this issue.
IV. Implications for Welfare Analysis

In this section, we briefly discuss some of the implications of our approach for welfare analysis. Previous literature has distinguished between ex-ante and ex-post welfare optimality (e.g. Hammond; Ulph). We follow this distinction and discuss successively ex-ante and ex-post welfare evaluation.

A - Ex-Ante welfare:

We focus our attention on the case of a competitive agent where the decision variables \( x_t \) are quantities of commodities either purchased or sold at (possibly uncertain) market prices. Let \( p_t \) be the present (discounted) value of these prices.\(^6\) We adopt the convention that the quantities sold are positive while the quantities purchased are negative, implying that the present value of terminal wealth can be written as \( (w + p_1 x_1 + p_2 x_2) \). In this context, consider the case where equation (1) takes the form:

\[
V(w, \bar{p}_1, \bar{p}_2) = \max_{x_1 \in A_1} \max_{x_2 \in A_2} E \left[ U[w + p_1 x_1 + p_2 x_2, x_1, x_2, e] \right] \tag{16}
\]

with solutions \( x_1^*(w, \bar{p}_1, \bar{p}_2) \) and \( x_2^*(w, \bar{p}_1, \bar{p}_2, x_1) \), where \( \bar{p}_t = E(p_t) \). The formulation (7) appears to be fairly general as a model of economic behavior. It corresponds to the maximization of expected utility of terminal wealth typically used in the modeling of production and investment decisions under uncertainty. Also, it includes as a special case consumption, household production and portfolio selection models; for example, a typical model of household behavior would take a form similar to (16) after substituting the budget constraint in the utility function of the household.\(^8\)

Using the conditional value of information, an equivalent representation of the indirect objective function in (16) is given by

\[
V(w, \bar{p}_1, \bar{p}_2) = \max_{x_1 \in A_1} \max_{x_2 \in A_2} E[U[w + \delta(w, \bar{p}_1, \bar{p}_2, x_1) + p_1' x_1 + p_2' x_2, x_1, x_2, e]] \tag{17}
\]

with solutions \( \bar{x}_1(w, \bar{p}_1, \bar{p}_2) = x_1^* \) and \( \bar{x}_2(w, \bar{p}_1, \bar{p}_2) \).

In the Hicksian tradition, we define welfare measures as the certain amount of money that must be received (or paid if negative) by the agent facing a change in some parameter \( \theta \) in order to keep him on a particular level of utility. We focus here on an ex-ante analysis where the payment of the compensation is made at time \( t=1 \). Also, to simplify the discussion, we will assume in this section that the \( x \)'s are continuous decision variables. In that context, the compensation associated with a change in the parameters \( \theta \) is defined by

\[
V[w + C, \theta^1] = \bar{V} \tag{18}
\]

where \( \bar{V} \) is a reference level of utility \( \bar{V} = V[w, \theta^0] \). In that context, \( C(w, \theta^1, \bar{V}) \) is the ex-ante willingness of the agent to pay for the difference between two situations \( \theta^0 \) and \( \theta^1 \), using \( \theta^0 \) as the reference point.
Differentiating (18) will respect to \( \Theta \) implies that the compensation \( C \) can be measured as

\[
C = \int_{\Theta_0}^{\Theta_1} \frac{\partial c}{\partial \theta} d\Theta = -\int_{\Theta_0}^{\Theta_1} \frac{\partial V}{\partial \theta} d\Theta \tag{19}
\]

Expression (19) can provide a convenient way of obtaining ex-ante welfare measures. Of particular interest is the case where welfare change can be expressed in terms of price changes. For example, if \( \Theta \) is the expected price of the \( i^{th} \) commodity at time \( t=1 \) (\( \Theta = p_{1i} \)), then, from (16) and using the envelope theorem, (19) becomes

\[
C = -\int_{\Theta_0}^{\Theta_1} x_{1i}^c dp_{1i} \tag{20}
\]

where \( x_{1i}^c \) is the compensated choice function \( x_{1i}^c[w,p_1,p_2,V] = x_{1i}^c[w+C(w,p_1,p_2,V), p_1,p_2] \). Expression (20) gives the well-known result that welfare change due to shifts in a current expected price is given by the "agent surplus" (or welfare triangle) measured by the area under the compensated choice function and between the two expected prices \( \Theta_0 \) and \( \Theta_1 \) (e.g. Pope and Chavas).

Alternatively, if \( \Theta \) is the expected price of the \( i^{th} \) commodity at time \( t=2 \) (\( \Theta = p_{2i} \)), then, from (16) and the envelope theorem, welfare change (19) becomes

\[
C = -\int_{\Theta_0}^{\Theta_1} \frac{E U_w}{E U_w} x_{2i}^c dp_{2i} - \int_{\Theta_0}^{\Theta_1} E(x_{2i}^c) dp_{2i} - \int_{\Theta_0}^{\Theta_1} \frac{COV(U_w, x_{2i}^c)}{E U_w} dp_{2i} \tag{21}
\]

where \( x_{2i}^c \) denotes the compensated choice functions \( x_{2i}^c = x_{2i}^c[w+C(.), p_1,p_2,x_{1i}^c(w+C(.), p_1,p_2), e] \). The first term on the right hand side of (21) is similar to (20) except that it involves expected future decision \( E(x_{2i}^c) \); it is the expected value of the welfare triangle as measured by the expected value of the area under the compensated choice function \( x_{2i}^c \) and between the two expected prices \( \Theta_0 \) and \( \Theta_1 \). It follows that the second term on the right hand side of (21) is the difference between the willingness to pay \( C \) and the expected value of a welfare triangle. Thus, it is a "correction factor" reflecting the bias of the traditional welfare triangle measure in the sense that it is the amount of money by which the expected value of the triangle (the first term on the right hand side of (21)) must be adjusted in order to provide an exact measure of ex ante willingness-to-pay (C). The properties of this correction factor have been investigated in detail in the consumer case (see Chavas, Bishop and Segerson).

In an attempt to provide some insight in the nature of the correction factor, consider the formulation in (17) (instead of (16)). Using (17) and
the envelope theorem, then the compensation associated with a change in $p_{2i}$ in (19) becomes

$$C = \int_{\Theta_0}^{\Theta_1} \frac{x_i^c}{1 + \partial D/\partial w} \, dp_{2i} - \int_{\Theta_0}^{\Theta_1} \frac{\partial D/\partial p_{2i}}{1 + \partial D/\partial w} \, dp_{2i} \tag{22}$$

where $x_c$ is the compensated choice function $x_c = x(w+C, p_1, p_2)$. This provides an equivalent representation of $C$ in (21). It indicates that the value of information plays a direct role in the valuation of economic activities using welfare triangle measures. For example, from the equivalence of (21) and (22), it is clear that the "correction factor" depends on the value of information and can be written in general as

$$\frac{\text{COV}(U_w, x_{2i})}{EU_w} = \frac{x_{2i}^c + \partial D/\partial p_{2i}}{1 + \partial D/\partial w} - E(x_{2i}^c)$$

**B - Ex-Post welfare:**

In the ex-ante approach just discussed, welfare evaluation depends on the nature of uncertainty (i.e. the probability distribution of $e$) but not on the actual state of the world (i.e. the actual realization of $e$). While this is appropriate when all decisions are made ex-ante, it neglects the possibility of ex-post welfare evaluation (where compensation could depend on $e$). There are a number of cases where an ex-post evaluation appears more appropriate. Examples include the study of liability rules, risk sharing schemes and insurance contracts (e.g. see Groves). Also, any discussion of "safety nets" in the design of public policy suggests the necessity of an ex-post welfare approach. In such cases, an ex-post analysis would be required where welfare compensation is conditional on the state of nature.

In order to formalize ex-post welfare analysis, consider the case of $N$ individuals facing $m+1$ possible states of nature $e=(e_0, e_1, \ldots, e_m)$, $P_{ij}$ denoting the subjective probability that the $i^{th}$ individual faces the $j^{th}$ state of nature; $i=1,\ldots,N; j=0,1,\ldots,m$. Also, denote by $z$ a public good (e.g. a government farm program) that affects the $N$ individuals. If the indirect objective function of the $i^{th}$ decision maker in the absence of the public good ($z=0$) is

$$\bar{U}_i = \max_{x_{1i} \in A_{1i}} \max_{x_{2i} \in A_{2i}} \sum_{j=0}^{m} P_{ij} \max_{x_{1i}} U_{ij}[w_i, x_{1i}, x_{2i}, e_j, 0],$$

then there exists a set of compensation schemes $C_i=(C_{i0}, \ldots, C_{im})$ which satisfy

$$\max_{x_{1i} \in A_{1i}} \max_{x_{2i} \in A_{2i}} \sum_{j=0}^{m} P_{ij} \max_{x_{1i}} U_{ij}[w_i+C_{ij}, x_{1i}, x_{2i}, e_j, z] = \bar{U}_i, \ i=1,\ldots,N, \tag{23}$$
where $C_{ij}$ is the present value of the income compensation to the $i^{th}$ individual in the $j^{th}$ state of nature. Expression (23) implies that the compensation scheme $C_i = (C_{i0}, \ldots, C_{im})$ is the amount of money the $i^{th}$ agent is willing to pay (or to receive) in each state of the world so that he is indifferent between a "public project" $z$ and no project at all ($z=0$). In general, (23) implies the existence of an infinite number of possible schemes since it involves the solution of a single equation for $(m+1)$ variables, $C_{ij}, j=0, \ldots, m$.

Expression (23) is useful in the formulation of potential Pareto improvement tests. Assume that the cost of the public project is given by $F(z,e)$ where $F(0,e)=0$. The potential Pareto improvement criterion says that a public project $z$ should be implemented if society (here the group of $N$ individuals) is willing to sacrifice resources sufficient to cover actual costs in each possible state of the world (e.g. Graham). Then, the net aggregate willingness-to-pay for $z$ in state $j$ is

$$\sum_{i=1}^{N} C_{ij} - F(z, e_j)$$

where $C_{ij}$ is defined in (23). In this context, it is of interest to find the maximum sure payment (net of cost) that society would be willing to make for $z$. The amount of sure aggregate net payment is the amount

$$\sum_{i=1}^{N} C_{io} - F(z, e_0) = \sum_{i=1}^{N} C_{ij} - F(z, e_j), j=1, \ldots, m \quad (24)$$

The aggregate net payment in (24) is sure since it is the same for all states of nature. In this context, the maximum sure net payment of society for the public project is given by

$$W = \text{Max} \left( \sum_{i=1}^{N} C_{io} - F(z, e_0) \right) \text{ s.t. (23) and (24)} \quad (25)$$

where $C = (C_1, \ldots, C_N)$. Denote by $C^*$ the optimum compensation scheme and by $z^*$ the optimum public project in (25). If $W>0$ in (25), then the public project $z^*$ with compensation scheme $C^*$ is said to pass the potential Pareto improvement test. Alternatively, if $W<0$ in (25), then there does not exist a Pareto improving way of providing the public project $z$.

It should be noted that if the solution $(C^*, z^*)$ to (25) were actually implemented, then a Pareto efficient distribution of risk would result. In particular, if the cost of the project is non-random, the taxation scheme would be equivalent to establishing competitive markets for contingent claims against the states of the world (see Graham, p. 718). Thus, the above model should be useful in the analysis of risk sharing schemes (e.g. implicit or explicit insurance contracts) or in the design of "safety nets" in public policy. For instance, risk sharing strategies among members of a group can be attractive if each agent faces different and imperfectly correlated risks. Or, risk sharing strategies can be beneficial if different members of a group...
are differentially affected by risk, or if they have access to different qualities of information.

Also, note that the ex-ante approach discussed earlier is a special case of the ex-post approach presented here: they become equivalent (for a given $z$) when $C_{i0} = C_{ij}$ for all $i = 1, \ldots, m$, i.e. when the compensation to the $i^{th}$ individual is the same for all states of nature. When the possibilities of risk sharing are important, this suggests that, by imposing the above restriction, the ex-ante approach may significantly underestimate the aggregate willingness to pay for a project.

Finally, how can we obtain measurements of appropriate ex-post compensations? First, as in the ex-ante welfare measures, traditional welfare triangles can be shown to be biased measures of the willingness to pay $C_{ij}$ because of the existence of a "correction factor" which depends on the value of information. Further investigation of this correction factor appears to be needed. Second, when some form of risk sharing takes place, then an individualistic approach to welfare measure is no longer appropriate. In this case, the evaluation of the $C_{ij}$'s that solve (25) depend on all the individuals in the group. This means that a group (society) is in general more than just the sum of its members. By suggesting that there may be economic returns to group action (beyond purely individual choices), this argument may provide a basis for economists to become more effective in the analysis of collective choices (e.g. government farm policy).

V - Conclusion

Temporal uncertainty appears to be the rule for basically all economic agents. Assuming that agents are expected utility maximizers, we explored some avenues refining the role of information and technology in decision-making. By trying to avoid making risk behavior a matter of taste and preference, we presented a framework that can help sharpen our analytical insights in the economics of uncertainty. Also, we explored some of the implications of our approach for welfare analysis. Although we limited our discussion to two period models for the sake of simplicity, our approach could be easily generalized to n-period models of decision-making under temporal uncertainty.

Note that our discussion was based on economic behavior associated with the maximization of expected utility. In this context, the expected utility model has been criticized by some for not providing an accurate representation of behavior (e.g. Allais; Kahneman and Tversky). While there is strong evidence that this is the case for "timeless risk" models, the introduction of temporal uncertainty appears to have the potential of resolving many of these concerns (see Machina). It suggests that additional work on temporal uncertainty could improve significantly the quality of our models: it might help us obtain a better understanding of the real world and make us more effective in proposing solutions to economic problems.

Finally, the nature of temporal uncertainty raises the question of learning. While we have focused our attention on "passive learning", the case of "active learning" (where the subjective probability distribution of e
may depend on the actions of the decision maker) needs to be further investigated. However, this is beyond the scope of this paper.

References


Footnotes

1/ This follows from the assumption that $U_w > 0$ and the fact that

$$E \max_{x_2} U(\cdot) \geq \max_{x_2} E U(\cdot)$$ (e.g. see Lavalle).

2/ Using the envelope theorem, note that

$$\frac{\partial D}{\partial w} = \left[ \frac{\partial E U(x^*)}{\partial w} \right] - 1.$$ 

In the constant absolute risk aversion case where

$$U(w+f(\cdot)) = e^{-R[w+f(\cdot)]}, R \text{ being the Arrow-Pratt absolute risk aversion coefficient},$$

it follows that $\partial D/\partial w = 0$, i.e. that initial wealth has no impact on the conditional value of information $D$.

Also, in the unconstrained case, we have (from the envelope theorem)

$$\frac{\partial D}{\partial x_1} = E \left[ \frac{\partial U(x^*_2)}{\partial x_1} - E \frac{\partial U(\bar{x}_2)}{\partial x_1} \right] - E \frac{\partial U(\bar{x}_2)}{\partial w},$$

a result that can be useful in the evaluation of the first-order conditions associated with (5).

2/ Expanding $U(w,x^*_2)$ around $\bar{x}_2(w,x_1,T)$ gives

$$U(w,x^*_2) = U(w,\bar{x}_2) + U_{x_2}(w,\bar{x}_2)(x^*_2 - \bar{x}_2) + 1/2 (x^*_2 - \bar{x}_2)'U_{x_2x_2}(w,\bar{x}_2)(x^*_2 - \bar{x}_2)$$

Similarly, expanding $U(w+D,\bar{x}_2,\cdot)$ around $D=0$ yields

$$U(w+D,\bar{x}_2(w+D),\cdot) = U(w,\bar{x}_2(w),\cdot) + [U_w + U_{x_2} \frac{\partial \bar{x}_2}{\partial w}]_{w,\bar{x}_2} D$$

Using these two expressions, taking expectations, noting that

$E_{\bar{x}_2} (\bar{x}_2) = 0$ from (2) (assuming an interior solution), and using (3)

yields expression (9).
Here, we assume that entry or exit is voluntary. See Chavas, Pope and Leathers for an analysis of production decisions under free entry and timeless risk. For a case of forced exit (through foreclosure), see Leathers and Chavas who argue that if foreclosure is a punishment for breach of a loan contract, the risk neutral firm can behave as if it were risk averse.

For a discussion of the influence of fixed factors on production decisions under timeless risk, see Chavas (1987).

We assume the existence of a riskless asset, implying that the discount rate is chosen to be the rate of return on the riskless asset (see Graham).

Throughout this section, we assume that the feasible sets $A_1$ and $A_2$ are independent of prices.

The prices $p$ would then be relative prices deflated by the price of the commodity used for the substitution.