Valuing Asset Insurance in the Presence of Poverty Traps: A Dynamic Approach

Sarah A. Janzen  
*University of California Davis*  
sajanzen@ucdavis.edu

Michael R. Carter  
*University of California Davis*  
mrcarter@ucdavis.edu

Munenobu Ikegami  
*International Livestock Research Institute, Nairobi, Kenya*  
M.Ikegami@cgiar.org


Copyright 2012 by Janzen, Carter and Ikegami. All rights reserved. Readers may make verbatim copies of this document for non-commercial purposes by any means, provided that this copyright notice appears on all such copies.
Valuing Asset Insurance in the Presence of Poverty Traps: A Dynamic Approach

Sarah A. Janzen, Michael R. Carter and Munenobu Ikegami

June 2012

Abstract:

Ample evidence exists to suggest that nonlinear asset dynamics can give rise to an environment of poverty traps. When dynamic asset thresholds matter, risk not only affects households ex post, but it also influences ex ante behavior. In this environment some households may have much to gain from a productive safety net which prevents asset levels from dipping below the Micawber threshold. Insurance is a market-based mechanism that can act as a safety net, improving the risk management strategies available to vulnerable households. In this paper we use dynamic programming methods to assess whether vulnerable households will ‘self-select’ into an asset insurance scheme. We show that while such households optimally insure at low levels, insurance serves to crowd in additional investment, causing a shift in the Micawber threshold. This investment comes from the hope of reduced vulnerability that insurance offers in the future. Finally, we use our model to make predictions about the value of index-based livestock insurance (IBLI) in Marsabit district of northern Kenya. Our results suggest that the behavioral changes brought about by insurance may result in decreased poverty levels over time.

Sarah Janzen is a Ph.D. candidate in the Department of Agricultural and Resource Economics at the University of California, Davis. Michael Carter is a Professor in the Department of Agricultural and Resource Economics at the University of California, Davis. Munenobu Ikegami is a post-doctoral scientist at the International Livestock Research Institute in Nairobi, Kenya. We thank the BASIS Research Program on Poverty, Inequality and Development and UKaid (Department for International Development) for financial support.
1 Introduction

Across the globe, observations of widespread persistent poverty are common. The Chronic Poverty Research Center supplements observational data to suggest that nearly half a billion people are chronically poor (Shepherd, 2011). These numbers suggest that it is not uncommon for poor households to remain poor for years, decades, and even generations. Such persistent poverty is decidedly different from transitory poverty, where households may fall into poverty for a brief period of time, before they are able to “pull themselves up by their own bootstraps” and transition out of poverty once again. For these two different types of poverty, persistent and transitory, the appropriate policy intervention will vary.

Determining the appropriate policy intervention begins with a study of the structural foundations behind persistent poverty, which has culminated into an interest by many economists in the study of poverty traps. Economists typically define poverty traps as “any self-reinforcing mechanism which causes poverty to persist” (Azariadis and Stachurski 2004). The study of poverty traps therefore focuses on identifying and explaining the existence of low well-being “basins of attraction” within an economy (Barrett and Carter, forthcoming).

In this paper we focus on a multiple equilibrium poverty trap. This type of poverty trap is characterized by multiple stable states, with at least one equilibrium associated with low levels of welfare, and another associated with high welfare. The existence of multiple stable steady states implies also the existence of at least one “threshold” or “tipping point” at the boundaries between the two regions. Most recent studies of poverty traps use an asset-based approach building on Carter and Barrett (2006), who suggest that an asset poverty line can readily be used to distinguish stochastic from structural transitions. Furthermore, identification of a dynamic asset poverty threshold allows researchers to distinguish between persistent and transitory poverty by understanding the underlying patterns of asset dynamics.

For a simplistic example in asset space, consider two households with similar asset levels near the asset threshold. Such seemingly similar households may end up on divergent paths if they begin on opposite sides of the dynamic asset threshold. Moreover, in a risky setting a single asset shock can have permanent consequences if it shifts households onto an alternative path. In particular, a shock which drops a household to a level of assets below the dynamic asset threshold inevitably sends them into a poverty trap, destined for the low-welfare steady state equilibrium.

This ex post effect of shocks is not the only way risk affects poverty dynamics when a poverty trap exists. Evidence exists to suggest that ex ante vulnerability may cause households to limit their exposure to risk by choosing lower risk activities at the cost of higher expected returns. If perceptions of asset thresholds induce a risk response, as both theoretical (Lybbert and Barrett 2007 and Lybbert and Barrett 2010) and empirical evidence (Carter and Lybbert 2010 and Santos and Barrett 2011, Hoddinott 2006) suggest, then ex ante coping strategies can actually influence the locations of the relevant dynamic asset threshold. In this way, the dynamic asset threshold that matters is based on optimal behavior. It is obvious then that the behaviorally-based asset threshold, commonly referred to as the Micawber threshold, depends on the choices available to the consumer. In addition, the threshold can actually shift as the environment around it changes.

Empirical evidence of poverty traps to date are mostly based on a test of multiple dynamic equilibria, where at least one equilibrium falls below some pre-determined poverty live. While
evidence of nonlinearities in income and asset dynamics are common, not all studies find
evidence of a low equilibrium poverty trap. Some studies suggest strong empirical evidence
of poverty traps (Adato, Carter and May 2006, Barrett et al. 2006, Barrett and Swallow
2006, Van Campenhout and Dercon 2009, Lybbert et al. 2004), while others do not (Antman
and McKenzie 2007, Jalan and Ravallion 2004, Lokshin and Ravallion 2004, Quisimbing and
Baulch 2009). Barrett and Carter (forthcoming) provide a discussion of the many significant
shortcomings inherent in such direct tests of poverty traps, and offer an alternative behavioral
approach as a test of poverty traps.

In general, it seems likely that nonlinear asset dynamics which give rise to poverty traps
exist in some places even if they do not exist elsewhere. In particular, geographically isolated
communities which suffer from a lack of available credit and insurance markets (such as is
common in Africa) are most likely to exhibit nonlinear asset dynamics indicative of a poverty
trap. For this paper we take as given that a poverty trap exists, and explore a particular
threshold-targeted policy which accounts for poverty traps.

The moral imperative to assist those suffering from persistent poverty is strong. Assuming
that poverty traps do indeed exist, the obvious question facing concerned individuals follows:
what is the appropriate policy to address such dynamics? Traditional policies of food aid
and cash transfers can only alter dynamics if the transfers are large enough to lift households
above the threshold. But in the absence of additional social protection policies which stake
out a safety net below which households cannot fall, such households remain vulnerable
to future collapse. This suggests the importance of social protection policy which directly
accounts for the threshold.

Barrett, Carter and Ikegami (forthcoming) use a numerical dynamic programming simu-
lation to compare needs-based social protection with a budget-neutral “threshold-targeted”
policy that prioritizes expenditures as follows: 1. Offset impacts of (costlessly observed)
shocks that push individuals below the Micawber Threshold. 2. Transfer assets to lift the
(costlessly identifiable) “potentially productive poor” above the threshold. 3. Needs-based
transfers to the poor. The details of the model can be found in their paper, but we sum-
marize here an important inter-temporal paradox which stems from their analysis. While a
purely needs-based distribution of aid is initially more favorable to the poorest, over time
they must compete with the vulnerable but initially non-poor for transfers. If the aid bud-
get remains constant, then individual transfers shrink as more people collapse into poverty,
unable to graduate from poverty without larger transfers. Over time the initially poorest
would have fared better under the threshold-targeted scheme. In essence, the model shows
that threshold-targeted policies can work by eliminating unnecessary deprivation.

Obviously, the Barrett, Carter and Ikegami model depends on large assumptions of per-
fected information and targeting. Moreover, the policy paradox they present (everyone is dy-
namically better off if limited funds are allocated first to those marginally better off rather
than the poorest of the poor) represents a conundrum for those who truly seek to help those
suffering from devastating poverty. In this paper we want to see if we can generate the bene-
fits of threshold-targeted social protection policy using a self-targeting risk transfer contract,
insurance, for which beneficiaries pay a market, or near-market price. In this way, limited aid
budgets can still be used to assist the least well off, but vulnerable households benefit from
social protection against collapse. The crucial question is whether vulnerable households
will self-select into such a contract. In this paper, we explore the answer to this question.
In doing so, we are also able to explore the degree to which benefits of threshold-targeted social protection policy can in fact be realized through a market-based mechanism.

This paper is not the first to consider the benefits of insurance in the presence of poverty traps. Chantarat et al. (2010) and Kovacevic and Pflug (2010) both consider whether an active insurance market can work as a safety net for vulnerable households. A critical limitation of these studies is that they both restrict behavioral choice, focusing instead on a state variable which follows a stochastic, albeit deterministic path to determine each household’s future welfare. In doing so, the models ignore the endogenous ex ante effect of the risk reduction brought about by insurance. Cai et al. (2010) find empirical evidence of an endogenous ex ante effect of insurance in China, where formal insurance increases farmer’s tendency to invest in risky sow production. Alderman and Haque (2007) argue that it “is more a matter of the degree to which behavior is modified rather than if it changes,” suggesting that the Chantarat et al. (2010) and Kovacevic and Pflug (2010) models may be overlooking an important component in the value of insurance. Our paper builds on the intuition established in both of the papers mentioned, and then takes each analysis one critical step farther by allowing greater flexibility of behavioral choice.

The rest of this paper is structured as follows: Section 2 develops a dynamic model of investment, consumption and asset insurance in the presence of a structural poverty trap. To give the model greater traction, we consider a specific insurance contract available to pastoralist households in northern Kenya (Chantarat et al. 2007, Chantarat et al. 2011, Mude et al. 2009), a setting where poverty traps have been well established (Lybbert et al. 2004, Santos and Barrett 2011, Barrett et al. 2006, McPeak and Barrett 2001). The theoretical model provides the crucial intuition for understanding optimal behavior by Micawber households (those holding assets around the Micawber threshold): that such households have a high shadow price of liquidity. This high shadow price will present a stark tradeoff for households between using limited available cash on hand for building up additional assets through investment and preserving current assets through insurance.

We use dynamic programming techniques to solve for the optimal investment, consumption and insurance policy functions. Section 3 provides details regarding the numerical implementation. Before presenting the derived policy functions in Section 4, we restrict the insurance decision to zero or full insurance while iterating over heterogeneous prices. This allows us to construct a measure of heterogeneous households’ willingness to pay for asset insurance, which can be cautiously interpreted as a static value function for insurance.

In Section 5 we compare the optimal behavior policy functions derived by the model before and after the introduction of an insurance market. While Micawber households optimally insure in small increments, we show that many of the benefits of insurance come from crowding in additional investment. One of our unique contributions stems from recognizing the importance of this behavioral change induced by insurance in altering poverty dynamics. We develop these findings further in Section 6 by looking at household asset dynamics with and without the opportunity to insure.

Our results suggest that the welfare gains from asset insurance for a given household stem from four primary features. First, we find that an insurance market can act as an effective safety net, reducing the fraction of the population vulnerable to collapsing to a low level equilibrium. Second, by allowing for behavioral change, the Micawber threshold (the asset level below which households fall to a poverty trap) shifts downward. This threshold
shift opens the door to the high welfare equilibrium to a number of households otherwise bound for the low welfare state. Third, the reduction in risk actually shifts the equilibrium outcomes upward. That is, both the average high and low level equilibriums are higher when insurance is available. Finally, as would be expected with any insurance contract, households are better able to smooth both assets and consumption.

Since the shadow price of liquidity is high for Micawber households, we might expect that these households exhibit extreme price sensitivity to the cost of insurance. One obvious way to insure a greater number of highly vulnerable households, without taking away from critical investment, is to offer subsidies on insurance. In Section 7 we conduct an analysis of the price elasticity of demand, and then analyze demand for insurance under various subsidy schemes, including a threshold-targeted subsidy scheme.

To see what these results mean in a real context, Section 8 uses the empirical herd distribution in Marsabit district of northern Kenya to consider the implications our model poses for poverty dynamics in a specific context. Our results suggest that the behavioral changes brought about by insurance have the potential to result in decreased poverty levels over time. In so doing, we show that many of the benefits of threshold-targeted social protection policy may indeed be realized through a market-based mechanism. Section 9 closes with some concluding remarks.

2 A Dynamic Model of Asset Insurance

In this section we present a dynamic household model of investment, consumption and asset insurance in the presence of risk and a structural poverty trap. In order to give the theoretical model some context, we consider a particular index insurance contract available to pastoralist households in northern Kenya, but note that our results can be applied more generally to other asset insurance products offered in a context where nonlinear asset dynamics apply.

The model assumes a single numeraire good, livestock, which can be insured. Each household has an initial endowment in the form of a livestock herd, $H_0$, where the subscript denotes time. In order to aggregate a herd of mixed livestock which is common in this region, all assets are expressed as tropical livestock units (TLU) so that a herd can consist of cattle (1 TLU), camels (1.4 TLU), goats or sheep (.1 TLU each).

Because the problem is dynamic in nature, we begin by modeling herd dynamics over time. In each period herds can experience both growth and loss. Following Dercon (1998) we model the livestock growth function as $f(H_t)$, which can be thought of as a livestock production function. More than just growth, this production function encompasses livestock births as well as “flows” such as milk products, which are a primary staple for people in this area. The loss function is modeled as $m(\theta_{t+1}, \epsilon_{t+1}, H_t)$. This mortality function depends on both a random aggregate shock ($\theta$) which is the same for all households, and an idiosyncratic shock ($\epsilon$) specific to the household’s own herd. Both shocks are stochastic, exogenous, and realized for all households after decision-making in the current period, and before decision-making in the next period occurs. In other words, the relevant mortality is unknown whenever a household’s decisions are to be made.

We assume a simple structure for the mortality function, such that either shock can be interpreted as proportional herd loss, and $m(\theta_{t+1}, \epsilon_{t+1}, H_t)$ is simply a linear function of each
shock multiplied by herd size:

\[ m(\theta_{t+1}, \epsilon_{t+1}, H_t) = \theta_{t+1} H_t + \epsilon_{t+1} H_t \]  

(1)

Households face a tradeoff between consumption today and investing in the herd for future consumption. The tradeoff is particularly stark in our model since credit markets are assumed to be absent. Assuming a lack of available credit markets implies that consumption must not be greater than the current herd and its flows. That is:

\[ c_t \leq H_t + f(H_t) \]  

(2)

It is also straightforward to impose a non-negativity constraint on herd size at any time \( t \):

\[ H_t \geq 0 \]  

(3)

Under these assumptions, and in the absence of formal insurance, herd dynamics are captured by the following equation of motion:

\[ H_{t+1} = H_t + f(H_t) - c_t - m(\theta_{t+1}, \epsilon_{t+1}, H_t) \]  

(4)

The tradeoff is captured in this: a household can consume all the “growth” in a given period, but then the herd will be smaller in the next period whenever mortality is greater than zero. Similarly, the household can consume more than the flows. Divestment occurs if the household consumes all the flows and part of the herd. Whatever portion of the flows is leftover after consumption can be thought of as an investment back into the herd.

It has been shown that herd dynamics seem to follow a particular growth path where growth is negative if a herd falls below a certain threshold (i.e. if \( H_t \leq \hat{H} \)), growth is approximately constant for medium levels of herd size (i.e. for \( \hat{H} < H_t \leq \bar{H} \)), and then positive growth is observed for large levels of herd size (i.e. for \( H_t > \bar{H} \)) (see Lybbert et al. 2004, Santos and Barrett 2011, and Sieff 1998). To capture these dynamics we allow households to choose between two different production technologies: a low return and a high return technology. The low return technology is analogous in this context to sedentarism, whereas the high return technology can be thought of as the more productive pastoralist production technology. Pastoralism offers higher returns because livestock are brought to better pastures, whereas in sedentarism livestock are constrained to lower quality forage close to the village.\(^1\)

With sedentarism, we assume that households are able to supplement their incomes with petty trade in the village (for example by selling milk or handicrafts) or by collecting food aid. This supplemental fixed income is denoted as \( f \). It can also be interpreted as the transaction costs of pastoralism saved by choosing the low technology. Equation 5 below thus defines the structural form assumed for the production technologies:

\[ f(H_t) = \begin{cases} 
\alpha_L H_t^{\gamma_L} + f & \text{if } H_t \leq \hat{H} \\
\alpha_H H_t^{\gamma_H} & \text{if } H_t > \hat{H}
\end{cases} \]  

(5)

\(^1\)Toth (2010) offers some evidence that the incentive to engage in mobile pastoralism determines whether a household will become trapped; he posits that households who optimally choose a sedentary lifestyle will fall into a poverty trap whereas those who optimally choose a mobile herding lifestyle will remain above a poverty threshold. We follow the same logic, though the focus of this paper is not why the poverty trap exists.
where $0 < \gamma_L < \gamma_H < 1$. Figure 1 shows the general shape of $f(H_t)$ under the assumptions set forth. Note that households with smaller herd sizes will optimally choose sedentarism whereas households with larger herds will choose pastoralism. This feature creates non-convexities in the implicit production function (defined by the outer envelope of the two production technologies). These nonconvexities coupled with borrowing constraints and risk drive the poverty trap mechanisms.

We can now specify the household’s objective function in the absence of a working insurance market. The household is assumed to be risk averse and will maximize the expected present discounted value of consumption by choosing consumption for each time period, with expected utility at time $t$ denoted as $u_t$. Implicitly, the household is also deciding how much to invest back in the herd for future benefits. We assume the standard constant relative risk aversion (CRRA) utility function where $\rho$ is the coefficient of relative risk aversion:

$$u_t(c_t) = \frac{c_t^{1-\rho}}{1-\rho}$$

(6)

In the absence of insurance or credit markets, the household’s optimization problem is characterized by the standard Bellman Equation. We consider the simple case where the shocks are distributed i.i.d., so that the most recent shock, either covariate or idiosyncratic, does not give any information about the next period’s shock. In this case, there is only one state variable, $H$. Under these assumptions, the problem can be expressed as follows:

$$V(H_t) = \max_{c_t > 0} u(c_t) + \beta E_{\theta, \epsilon}[V(H_{t+1}|c_t, H_t)]$$

(7)

2If instead the shocks are serially correlated, the agent would use the most recent shock to forecast future herd size. The state space would then include current and maybe past realizations of $\theta$ and $\epsilon$ in addition to $H$. While this may more closely reflect reality, doing so only clouds our understanding of the mechanisms at work in this particular problem.
subject to the equation of motion for herd dynamics (Equation 4), the budget constraint (Equation 2) and the non-negativity constraint on herds (Equation 3). β is the time discount rate and the expectation depends on beliefs about the structure of shocks θ and ϵ.

To derive the first order condition we take the derivative of the value function with respect to \( c_t \). Doing so requires using the chain rule to obtain the derivative of the expected value of all future utility (which can be written as \( \mathbb{E}_{\theta, \epsilon}[V(H_{t+1}(c_t, H_t, \theta, \epsilon))] \)) with respect to \( c_t \). Formally, we write the expectation in summation form where \( \phi_i \) represents the probability of observing \( \theta_{t+1} = \theta_i \) and \( \psi_j \) represents the probability of observing \( \epsilon_{t+1} = \epsilon_j \). Setting the derivative of the value function with respect to \( c_t \) equal to zero yields:

\[
u'(c_t) + \beta \sum_{i} \sum_{j} \left( \phi_i \psi_j \frac{\partial V(H_{t+1})}{\partial H_{t+1}} \bigg|_{\theta_i, \epsilon_j} \frac{\partial H_{t+1}}{\partial c_t} \bigg|_{\theta_i, \epsilon_j} \right) = 0 \quad (8)
\]

From Equation 4 we know that \( \frac{\partial H_{t+1}}{\partial c_t} \) equals -1, and obviously doesn't depend on the stochastic variables \( \theta \) or \( \epsilon \). Pulling that term out of the double summation and then realizing that the remaining summation is simply the expected value of the partial derivative of the value function with respect to \( H_{t+1} \) yields a much simpler form for the first order condition:

\[
u'(c_t) = \beta \mathbb{E}_{\theta, \epsilon}[V'(H_{t+1})] \quad (9)
\]

This equation says the marginal utility of current consumption must be equated to the discounted expected value of assets carried forward to the future, and makes clear the intertemporal tradeoff between consumption and investment.

The imposition of a structural poverty trap means the value function is not a monotonic function, and the shadow price of assets, \( [V'(H_{t+1})] \), will be relatively high for Micawber households (those households situated near the asset threshold). That is, the marginal benefit of carrying an asset forward to the future is especially high for those households located near the behavioral switching point. The reason is intuitive. A small change in assets around the Micawber threshold can have path-altering implications. For example, giving an additional asset to a household just below the threshold allows them to escape the poverty trap, completely altering their dynamic path. On the contrary, taking 1 asset from a household just above the threshold drops the household asset level below the threshold toward ultimate herd collapse. This heterogeneity of the shadow price of assets, or liquidity, is captured by \( V'(H_{t+1}) \), and will therefore also be exhibited in optimal behavior. Since the marginal benefit of assets is relatively high for Micawber households this first order condition implies higher levels of investment by such households, and lower levels of consumption.

Let us now suppose households are given the opportunity to insure their livestock. If the household wants insurance, it must pay a premium equal to the price of insurance (\( p \)) times the number of assets (TLU) insured (\( I_t \)). In theory, the household can choose how many livestock to insure, but it should not be allowed to insure more livestock than it owns. That is, the number of tropical livestock units (TLU) insured for the next season must be less
than or equal to the current period herd size:

\[ I_t \leq H_t \] (10)

Because of the rapid expansion and development of index insurance contracts in developing countries, we consider index insurance rather than traditional insurance. The basic index insurance contract specifies that an indemnity payout will be made if the index exceeds a certain strike point \( s \). If the index is such that a payout is made, then the household also receives the indemnity payment \( \delta \) for each livestock unit insured. In this way, the indemnity payment for each unit of insurance purchased at time \( t \) can be written as:

\[ \delta = \max((i(\theta_{t+1}) - s), 0) \] (11)

The household’s decision today is whether or not to insure the herd against future loss. Timing is critical here. Note that \( s \) is known by the household in advance of the decision and assumed to be constant over time for this problem. However, when the household chooses consumption and how much insurance to purchase, they don’t know what kind of mortality their herd is about to experience. This also means households don’t know in advance if the insurance index \( i \) will cause the insurance to pay out before moving to the new period. This risk enters through the random covariate shock \( \theta_{t+1} \) which is realized for all households after optimal choices have been made. Hence, \( \delta \) can be written as a function of the index \( i \) which depends on future \( \theta_{t+1} \).

A notable feature of index insurance is that the insurance contract and indemnity payments are based on an aggregate index, rather than individual outcomes, a feature made clear by the definition of \( \delta \). In this case, both the mortality function and the index depend on the covariate shock. While they are positively correlated, they need not be perfectly correlated. The difference between individual livestock mortality and the index (which can be thought of as predicted livestock mortality) represents basis risk. Hence, risk enters the problem in three related ways: the covariate shock \( \theta_t \), the idiosyncratic shock \( \epsilon_t \), and basis risk \( (i(\theta_t) - m(\theta_t, \epsilon_t, H_t)) \). To simplify the problem we assume the index perfectly predicts the covariate shock, so that \( i(\theta_t) = \theta_t \). In this case basis risk is simply captured by \( \epsilon \).

We can now rewrite the Bellman Equation to reflect the additional choice variable, \( I_t \).

\[ V(H_t) = \max_{c_t > 0, 0 \leq I_t \leq H_t} u(c_t) + \beta \mathbb{E}_{\theta, \epsilon}[V(H_{t+1}|c_t, I_t, H_t)] \] (12)

The solution is restricted by the non-negativity constraint on \( H \) in every period (Equation 3), the equation of motion for herd dynamics (Equation 13 below), and the budget constraint (Equation 14 below). The latter two are revised as follows to reflect the available insurance market:

\[ H_{t+1} = H_t + f(H_t) - c_t - pI_t - m(\theta_{t+1}, \epsilon_{t+1}, H_t) + \delta(i(\theta_{t+1}))I_t \] (13)

\(^3\)Note that in practice this constraint is extremely difficult to enforce, and preliminary empirical evidence suggests that this constraint is not binding. Alderman and Haque (2007) point out that laborers and merchants whose incomes are indirectly linked to (livestock) production could, in principal, choose to purchase insurance at a level commensurate with the laborer’s perceived exposure to a given shock.

\(^4\)One possible extension is to relax this assumption to better understand how basis risk affects insurance demand by households affected by a poverty trap threshold. We leave this to future analysis.
There are now two first order conditions. As before, to derive the first order conditions we write the expectation in summation form and then use the chain rule to obtain the derivative of $E_{\theta, \epsilon}[V(H_{t+1}(c_t, I_t, H_t, \theta, \epsilon))]$. The first equation looks the same as Equation 9 except that insurance actually changes the expectation of future asset holdings and therefore future well-being so that the term on the right hand side of Equation 9, $E_{\theta, \epsilon}[V'(H_{t+1})]$, is actually altered by insurance. For households that choose to insure in a given period, index insurance essentially removes the worst possible outcomes from the distribution, while increasing the probability of experiencing a covariate shock equal to the strike point $s$. Using our parameterization, the relevant distributions are plotted in Figure 2, where relevant refers to covariate mortality (assuming no idiosyncratic shocks) adjusted by the insurance payout.

Households make a choice between facing the probability distribution of shocks with and without insurance. The cost of facing the payout-adjusted distribution of shocks with insurance is the price you pay for insurance today, $p$. This results in fewer assets for the future. The second first order condition requires that households will purchase additional units of insurance until the expected marginal benefit of insurance equals the expected marginal cost of purchasing insurance, where both the marginal benefit and marginal cost of insurance depend on where you end up on the asset spectrum in the following period.

Formally, we first take the derivative of the value function with respect to $I_t$ and set it equal to zero:

$$c_t + pI_t \leq H_t + f(H_t)$$

(14)

$$\beta \sum_{\forall i} \sum_{\forall j} \left( \phi_i \psi_j \frac{\partial V(H_{t+1})}{\partial H_{t+1}} \bigg|_{\theta_i, \epsilon_j} \frac{\partial H_{t+1}}{\partial I_t} \bigg|_{\theta_i, \epsilon_j} \right) = 0$$

(15)

From Equation 13 we know that $\left( \frac{\partial H_{t+1}}{\partial I_t} \bigg|_{\theta_i, \epsilon_j} = -p + \delta(\theta_i) \right)$. Substituting in and rearranging
terms we arrive at the following form of the first order condition:

\[
\sum_{\forall i} \sum_{\forall j} \left( \phi_i \psi_j V'(H_{t+1}) \big|_{\theta_i, \epsilon} p \right) = \sum_{\forall i} \sum_{\forall j} \left( \phi_i \psi_j V'(H_{t+1}) \big|_{\theta_i, \epsilon} \delta(\theta_i) \right) \quad (16)
\]

Notice that \( p \) is independent of \( \theta \) and \( \epsilon \) so that it can be pulled outside the double summation, whereas the indemnity payment \( \delta \) depends on the random covariate shock. Before pulling \( p \) outside the summation, however, leaving it makes it clear that in order to arrive at the expected marginal cost of insurance, the premium is weighted by two terms: the probability of experiencing a given set of shocks and the shadow price of assets given the observed shocks. This second weighting term \( V'(H_{t+1}) \big|_{\theta_i, \epsilon} \) is largest when the shock experienced pushes a household to the Micawber threshold, because it is at that point along the asset spectrum where the marginal benefit of an additional asset is largest. The total weighting function depends additionally on the probabilities \( \phi_i \) and \( \psi_j \) associated with the shocks on which \( V'(H_{t+1}) \big|_{\theta_i, \epsilon} \) depends. Equation 16 thus implies that the marginal cost of insurance is highest when asset shocks are likely to send a households to the Micawber threshold.

The weighting function is the same for the marginal benefit of insurance. The difference, of course, is that while the premium paid for insurance is constant, \( \delta \) varies depending on the covariate shock \( \theta \). Moreover, with index insurance we know that \( \delta \) will be zero as long as the index is less than the strike point \( (i(\theta) \leq s) \). This means we can write the term on the right hand side of Equation 16 as follows:

\[
\sum_{\forall i \leq s'} \sum_{\forall j} \left( \phi_i \psi_j V'(H_{t+1}) \big|_{\theta_i, \epsilon} \delta(\theta_i) \right) + \sum_{\forall i > s'} \sum_{\forall j} \left( \phi_i \psi_j V'(H_{t+1}) \big|_{\theta_i, \epsilon} \delta(\theta_i) \right) \quad (17)
\]

But the first term is zero because for all \( i \leq s' \), we know that \( i(\theta) \leq s \) so that \( \delta(\theta_i) = 0 \). This brings up an important limitation of index insurance when considering threshold households. If the strike point is high, then insurance only covers catastrophic losses. But for households situated near the threshold small losses can have catastrophic implications for which they would receive no insurance payout. These households may place a high weighting function for events \( \theta_i \leq s \), but the benefit from insurance in these events is zero.

Equation 16 simplifies by pulling \( p \) outside the summation, making the following substitution, \( \sum_{\forall i} \sum_{\forall j} \phi_i \psi_j V'(H_{t+1}) \big|_{\theta_i, \epsilon} \bigg|_{\theta_i, \epsilon} = E_{\theta, \epsilon}[V'(H_{t+1})] \), and using the cancellation discussed in Equation 17. Doing so we arrive at the following form for the first order condition:

\[
p E_{\theta, \epsilon}[V'(H_{t+1})] = \sum_{\forall i > s'} \sum_{\forall j} \left( \phi_i \psi_j V'(H_{t+1}) \big|_{\theta_i, \epsilon} \delta(\theta_i) \right) \quad (18)
\]

which says households will insure until the marginal cost of insurance is equal to the marginal benefit of insurance, where both the marginal cost and marginal benefit depend on the marginal benefit of assets after a given shock. As asset levels near the Micawber threshold, the marginal cost of insurance increases, whereas the marginal benefit of insurance depends largely on whether the payout will be able to keep the household from falling into a poverty trap. Moreover, the closer a household is to the threshold ex ante, the more likely a small
shock is capable of sending a household below the threshold into a poverty trap. If such small shocks are less than or equal to $s$, then the marginal benefit of insurance for such a shock is zero, and insurance cannot help to prevent against collapse. In such cases, households will optimally choose to allocate funds toward investment in an attempt to move away from the threshold rather than insure.

The solution to this problem finds the optimal investment, consumption, and insurance decisions in each period. We use dynamic programming techniques to find a policy function for each behavior as it depends on asset levels. The next section outlines the assumptions made in order to arrive at a solution.

3 Numerical Implementation

To solve the problem using numerical methods, we first assume a heterogenous population with identical preferences and uniformly distributed initial asset levels. In Section 8 we extend the analysis to consider the dynamic implications our findings hold for the observed empirical distribution of asset levels.

In order to realistically reflect the risky environment that pastoralists find themselves in, the parameters used for the numerical analysis must be calibrated to data collected in the local setting. We use a rough discretization of the estimated empirical distribution of livestock mortality in northern Kenya reported in Chantarat et al. (2011) to establish a vector of covariate shocks. Since mortality rates have been shown by the same study to be highly correlated within the geographical clusters upon which the index is based, we assume relatively small idiosyncratic shocks. Using the empirically-derived discretization allows expected mortality to be 8.46% with the frequency of events exceeding 10% mortality an approximately one in three year event. These two features both reflect observed mortality characteristics in the region. The assumed probability distribution of combined shocks are displayed in Figure 2.

From the distribution of covariate shocks we calculate the actuarially fair premium using the same strike point as is found in the actual IBLI contract. Parameters for the utility function ($\rho$ and $\beta$) are homogenous across the population, and specified using plausible values known from economic theory, and then subjecting the model to specification tests.

The production function has been specified to follow the dynamics outlined in the model. Rather than explicitly calibrating the parameters of the production function based on empirical data (which may be impossible), we instead use general knowledge about the dynamics of the pastoral system in this region based on Lybbert et al. 2004 and Santos and Barrett 2011. This quasi-calibration allows us to set the threshold and equilibrium levels approximately where they have been shown to exist. The link between non-linear asset dynamics and the production technology is made complete by the empirically-derived distribution of the shocks. The parameters used to solve the dynamic programming problem are reported in the Appendix in Table 1.

The solution to the problem can be found by solving a stochastic dynamic programming problem. We use value function iteration, by which it follows that the Bellman equation has a unique fixed point as long as Blackwell’s Sufficient Conditions (monotonicity and discounting) are satisfied. Notice that the timing of events is as follows:

11
1. In period $t$ households choose optimal $c_t$, $I_t$ and (implicitly) $i_t$ (where $i_t$ denotes investment) based on state variable $H_t$ and the expectation of future livestock mortality and insurance payout.

2. Households observe exogenous shocks $\theta_{t+1}$ and $\epsilon_{t+1}$ which determine livestock mortality $m(\theta_{t+1}, \epsilon_{t+1}, H_t)$ and insurance payout $\delta(i(\theta_{t+1}))$.

3. These functions, $m(\theta_{t+1}, \epsilon_{t+1}, H_t)$ and $\delta(i(\theta_{t+1}))$, together with the optimal choices from period $t$ determine $H_{t+1}$ through the equation of motion for herd dynamics (Equation 13).

4. In period $t+1$ households choose optimal $c_{t+1}$, $I_{t+1}$ and (implicitly) $i_{t+1}$ based on the newly updated state variable $H_{t+1}$ and the expectation of future livestock mortality and indemnity payment...

The critical timing assumption is that the shocks happen post-decision and determine $H_{t+1}$ given your choices of $c_{t+1}$, $I_{t+1}$ and $i_{t+1}$, and then once again all the information needed to make the next period’s optimal decision is contained in $H_{t+1}$.

Before looking at optimal behavior, it is useful to compare the value functions (and more specifically the derivative of the value functions) defined by the optimal consumption and investment decisions with and without the option to insure. This comparison provides a way to see how insurance alters the optimization decision made by households.

Figure 3 plots the derivative of the discounted expected value function, interpreted as the shadow price of assets (or liquidity), which as we saw from the first order conditions is a critical component determining optimal choice. This figure demonstrates three critical insights. First, Micawber households (with between 10 and 15 initial assets) have a high shadow price of assets. As was previously discussed, the intuition lies in the fact that a small change in assets around the Micawber threshold can have path-altering implications. Second, the shadow price of an additional asset increases with insurance for Micawber households. This shift occurs because insurance removes the worst possible outcomes from the distribution, while increasing the probability of experiencing a covariate shock equal to the strike point $s$. In addition, insurance improves the ability of households to cope with risk not only now, but also in the future. Third, because optimal behavior depends on the shadow price of assets, optimal behavior will also change when the shadow price of assets changes. As will be shown, this actually shifts the Micawber threshold, the relevant behavioral threshold above which households strive toward the high equilibrium.

4 Willingness to Pay for Full Insurance

We know that vulnerable households situated near the Micawber households have a lot to gain from social protection policy which acts as a safety net preventing against future collapse into a poverty trap. While insurance is a promising form of social protection policy, it requires that households pay a price. A primary goal of this analysis is to define a measure of how households value insurance in the presence of a structural poverty trap. One critical question is thus: What is a household’s willingness to pay for insurance? If vulnerable households
are willing to pay the market price of insurance, then insurance can act as a market-based social protection mechanism. However, if the market price is above the willingness to pay for Micawber households, who have a particularly high shadow price of liquidity, then such households will not “self-select” into the program and some of the gains offered by insurance may not be observed.

In this section we therefore construct a rudimentary measure of households’ willingness to pay for insurance. To do so, we first solve the household’s optimal decision problem, allowing households to choose consumption, investment, and whether or not they would like to insure all owned assets for a given price. In this section we limit the insurance choice set to zero or full insurance in order to derive a willingness to pay for full insurance measure. To arrive at the WTP measure, we iterate over optimal insurance purchase decisions for a vector of premiums centered around the actuarially fair premium. This allows us to see clearly for whom it is optimal to buy insurance at various prices.

Specifically, we denote the value function when choosing to fully insure at time $t$ as $V(H_t; I_t = H_t)$, and the value without insurance as $V(H_t; I_t = 0)$. Because we are iterating over different prices, we add a mark up rate to the value function to express the price of insurance whenever the household purchases insurance. Denoting the mark up rate on the actuarially fair insurance premium $p$ by $\lambda$, we define the willingness to pay for an agent with a herd size of $H_t$ as the amount, $(1 + \lambda)p$ which satisfies the following:

\begin{align}
V(H_t, \lambda ; I_t = H_t) &\geq V(H_t ; I_t = 0) \quad \text{for all } \lambda \leq \overline{\lambda} \quad (19) \\
V(H_t, \lambda ; I_t = H_t) &< V(H_t ; I_t = 0) \quad \text{for all } \lambda > \overline{\lambda} \quad (20)
\end{align}

We compute $\overline{\lambda}$ as follows. First, we discretize $\lambda$ into $\{-0.4, -0.3, \ldots, 0.6\}$. Second, for each value of $\lambda$, we compute the optimal consumption, investment and insurance purchase decisions for all possible levels of the state variable, $H_t$. Third, for each value of the state

\footnote{We later relax the restriction of $I_t = [0, 1]$ in Section 5, where we consider optimal decisions when given the option of partially insuring the herd.}
variable, we search the value of $\lambda$ at which the agent switches the optimal insurance purchase decision and thus conditions (19) and (20) hold.

Figure 4 shows $\lambda(H_t)$ using a smoother. We observe a willingness to pay that is greater than the actuarially fair price for households safely above the Micawber threshold, and approximately equal to the actuarially fair price for households below the threshold. However, households in the neighborhood of the Micawber threshold exhibit a markedly lower willingness to pay for full insurance in the current period. In fact, it seems that when faced with the actuarially fair price, these households prefer no insurance to full insurance. Why? Micawber households have a high shadow price of assets.

To build intuition, let us return to the first order conditions, Equations 9 and 18. Equation 9 states that the marginal benefit of consumption today must be equal to the marginal benefit of carrying an additional asset into the future. But as we saw in Figure 3, the shadow price of assets is high for Micawber households. This means households along this part of the asset spectrum are willing to give up some consumption in order to carry a greater number of assets into the future. For these households, moving away from the Micawber threshold by accumulating assets is crucial. This movement offers reduced vulnerability of collapse and increased probability of reaching the high welfare state.

Insuring assets obviously offers a reduction in vulnerability as well, but at the cost of asset accumulation by payment of the premium. Equation 18 suggests that households will derive greater benefits from insurance if it keeps them above the threshold. Hence, the decision to insure depends largely on the probability of collapse, and if collapse remains likely even with insurance (as would be the case if a shock too small to trigger a payout actually causes collapse), then Micawber households may actually be willing to take on greater risk in order to accumulate assets through greater investment. This seems to suggest two things about insurance. First and most obvious, Micawber households have a lower willingness to pay for insurance in the short term. However, insurance may actually serve to crowd in additional
investment. This second insight stems from the fact that insurance actually increases the shadow price of assets for these households.

In fact, we can test whether this second feature of insurance is true by comparing the optimal investment policy function with and without insurance. Before doing that, it is worth noting that the willingness to pay measure reported here is static. If the availability of insurance actually alters the expected path for some Micawber households, and we will soon show that it does, then dynamically Micawber households may value insurance quite highly, even if they choose not to purchase insurance today. In essence, the promise of reduced vulnerability in the future might be reflected in a dynamic value function, which would be overlooked by the static WTP value measure presented here.

It should also be noted that we are not the first to demonstrate that Micawber households may not want to purchase insurance. The same result has been previously captured in the theoretical models put forth by Chantarat et al. (2010) and Kovacevic and Pflug (2010), both of which suggest that paying the premium can send households below the threshold, making them worse off. This is especially true when paying the premium takes away from critical investment. In the next section we highlight other behavioral changes induced by the availability of insurance to show that this low WTP reveals only one aspect of the story. To get the full picture, we must look at optimal behavior not only as it relates to insurance but also consumption and investment. Doing so is one of the unique contributions we seek to make in this paper.

5 Optimal Behavior

The WTP measure presented in Section 4 is derived from optimal insurance choices to the problem presented in Equation 12, where insurance is limited to zero or full insurance. In reality, households are given the option to partially insure assets. That is, households can insure a subset of their assets. In this section we relax the assumption of zero or full insurance and allow households to incrementally insure at the actuarially fair price. By using dynamic programming techniques we obtain a policy function for each behavioral choice: investment, consumption, and insurance.

Figure 5 plots the optimal partial insurance policy function. The results closely match the shape of the willingness to pay (for full insurance) curve, with one important distinction. While households can choose zero insurance, it is optimal for all households, regardless of their proximity to the asset threshold, to insure at least some portion of the herd. The policy function for Micawber households dictates a low level of insurance (only 20% of the herd), whereas households above the threshold insure closer to 90% with proportion insured increasing as asset levels move away from the threshold. Threshold households clearly benefit from some protection, but the total behavioral impact of insurance requires taking a deeper look at the budget constraint and optimal behavior.

Remember that a household must choose to allocate their cash on hand between consumption, insurance, and investment back into the herd. Remember also that Micawber households have a high shadow price of liquidity. In the absence of insurance, these households could choose to forgo consumption in order to build up the herd. But in a risky environment, it may not seem worth it. Even if they are able to get above the threshold, a
bad shock can send them right back to where they started. Because the shadow price of assets increases for Micawber households with the availability of insurance, Equation 9 dictates that those households will reduce consumption and increase investment when insurance becomes available. The promise of a safety net which prevents against future collapse actually incentivizes investment. If herds can be protected once they reach the asset threshold, then it’s more rewarding to attempt to rebuild the herd. If they are have good luck and are able to achieve a higher herd size, it then becomes optimal to insure.

This is exactly what we see in the optimal investment and consumption policy functions. Figures 6 and 7 show the optimal investment and consumption choices under autarky and in the presence of an insurance market. Threshold households insure only a small portion of their herd, but their optimal consumption and investment also change. They consume less and invest more. These households benefit dynamically from the very presence of an insurance market, even if they barely insure today. The possibility of insuring more once their herd gets big enough offers enough incentive to take on the extra risk of increased investment. These households may choose to suffer through some tough low-consumption years as a result, but in the long run they can be made better off.

An opposite behavioral effect results for households above the asset threshold. In the absence of a functioning formal insurance market, these households informally “insure” by investing more in their herds, while forgoing consumption. When formal insurance becomes available, households instead choose to use their cash on hand to purchase insurance, forgoing additional investment. Such households continue to invest, but they invest less than if they were uninsured. This finding supplements findings by Francesca de Nicola (2011) who also predicts a reduction of investment when insurance is introduced. This behavioral effect is especially pronounced once households reach the high level equilibrium. At that point, the marginal benefit of investing is low, so households prefer to allocate their resources toward
Figure 6: Optimal Investment Decisions for various herd sizes

Figure 7: Optimal Consumption Decisions for various herd sizes
consumption. In addition, these households find it optimal to insure a larger proportion of assets.

These results suggest the importance of considering the asset poverty trap in analyzing demand for insurance. If such an asset threshold actually exists, and households are able to perceive the threshold and its implications, then such a threshold will have huge implications for the optimal insurance purchase decision. Whether the policy function can be observed empirically is the subject of forthcoming research.

6 Dynamic Implications at the Household Level

Understanding how optimal behavior changes with the introduction of an insurance market is interesting. However, what really matters is how an active insurance market can alter asset dynamics and result in new equilibrium outcomes. To see how asset dynamics are altered, we use the optimal consumption, investment and insurance policy functions to simulate asset dynamics by subjecting agents to a different series of random shocks for each simulation.

We show three primary impacts of actuarially fair insurance on dynamic asset accumulation when we account for a structural poverty trap:

1. **Vulnerability Effect**: Insurance acts as a safety net, offering a reduction in the vulnerability of collapse to the low level equilibrium.

2. **Shifting Threshold Effect**: The relevant asset threshold below which households collapse to a low level equilibrium is reduced. That is, when assets can be insured, fewer assets are necessary to have a positive probability of reaching the high equilibrium.

3. **Shifting Equilibrium Effect**: An insured herd is likely to reach a higher terminal herd size regardless of initial endowment than its uninsured counterpart.

4. **Smoothing Effect**: The path to accumulation involves fewer ups and downs; it is smoother.

We begin with the first effect. As we would expect, relative to an uninsured population, a much smaller proportion of the population can be identified as vulnerable (likely to fall to a low level equilibrium) when households are able to purchase insurance. This “vulnerability effect” of insurance is most clearly depicted in Figure 8 which shows the probability of arriving at a low level equilibrium with and without the opportunity to insure. The assumption of a poverty trap inherent in our model is clearly observed in this figure which shows a 100% probability of approaching the low level equilibrium for herds that are already below some critical asset threshold. This asset threshold, which appears to be around 13.5 TLU in the absence of insurance, is what we have been calling the Micawber threshold. Because most households below the Micawber threshold do not experience any change in the probability of arriving at a low level asset equilibrium, they are essentially trapped with or without insurance. Other than those very near the threshold, these households do not gain via the vulnerability effect.

The truly vulnerable population is that which has a positive, but less than 100% chance of falling to the low level equilibrium, where a higher probability of collapse indicates greater
vulnerability. These households are not trapped, but are at risk of experiencing asset collapse which could send them into a trap. Using our parameterization and in the absence of insurance, all households above the Micawber threshold can be classified as vulnerable under this definition.

As we would expect, the presence of insurance sharply reduces the number of households which can be classified as vulnerable. As asset levels increase, households autarkic vulnerability also decreases, and therefore the reduction in vulnerability decreases. Nonetheless, households with a high asset endowment actually shift from approximately 10% vulnerability to 100% protection against catastrophic losses which would otherwise result in a poverty trap. This demonstrates the effectiveness of insurance acting as a safety net against collapse for households at the upper end of the asset spectrum.

If we characterize the vulnerability effect as a change in vulnerability, then the vulnerability effect is largest for households with asset levels between 12.3 and 17 TLU who experience a 25% to 75% reduction in vulnerability. Some of these households shift from a 30-40% chance of ending up at a low welfare equilibrium to an almost sure chance of reaching the high equilibrium. Other households shift from near 100% probability of collapse to having the ability to reach the high equilibrium in at least two out of three cases.

It is these households in particular, who move from trapped to vulnerable, who experience a second effect of insurance, what we coin the “shifting threshold effect.” For these households, the high equilibrium isn’t attainable in autarky, but the availability of insurance actually induces a large behavioral response which makes the high equilibrium achievable. In autarky, households need at least 13.5 TLU to have any chance of reaching the high equilibrium. Households with assets below this threshold exhibit drastically reduced investment and increased consumption recognizing that they are trapped. However, the introduction of insurance actually causes a shift in the Micawber threshold. When households have the opportunity to insure, the asset threshold below which households are trapped drops to 12.3
assets. Any household with 12.3-13.5 TLU exhibits dramatically different behavior when an insurance market is introduced because they are able to access a previously inaccessible avenue which could potentially lead them out of the trap. For them, insurance offers path altering benefits which are overlooked in the static value function presented in the previous section which shows these same households exhibiting the lowest WTP for insurance.

One way to present asset dynamics, and the change in asset dynamics induced by insurance, is to consider a terminal asset outcome relative to an initial endowment of assets. The terminal outcome will obviously depend on the shocks a household experiences, but over a large number of simulations it can be useful to compare a number of representative outcomes. Figure 9 plots the 10th, 25th, 50th, 75th and 90th percentiles of the terminal herd size across simulations as a function of initial asset endowment under autarky and with insurance. Presenting asset dynamics in this way makes clear our assumption of a multiple equilibrium poverty trap, since households below an asset threshold tend toward a low asset equilibrium, and households above an asset threshold tend toward a slightly more variable high asset threshold whether insurance is available or not.

Figure 9 demonstrates both the Vulnerability Effect and Shifting Threshold Effect. In the absence of insurance, even a large herd isn’t completely safe from collapsing to a poverty trap. Figure 8 showed that large herds face just over 10% probability of collapse. This vulnerability is demonstrated by the low asset outcome experienced by the 10th percentile herd. Moreover, a herd seemingly far from the Micawber threshold with 30 TLU actually still ends up at the low asset equilibrium 10% of the time. This experience is sharply contrasted by the 10th percentile high level asset outcome when households have the opportunity to insure. Here we see once again the effectiveness of insurance as a safety net against collapse.

Although perhaps difficult to observe, the shifting threshold effect is also visible in Figure 9. It is, however, more readily apparent when we plot the median outcomes on the same graph. It then become more obvious that the asset endowment necessary to reach a high
Figure 10: Median Herd Transition: Initial to Terminal

terminal asset outcome is reduced. We do this in Figure 10.

Figure 10 also brings out the third primary effect of actuarially fair insurance on dynamic asset accumulation when we account for a structural poverty trap: the “shifting equilibrium effect.” A cursory glance at the median terminal herd size shows a distinctly elevated high level equilibrium, and a smaller but noticeable positive shift in the low level equilibrium when insurance becomes available. Based on the assumptions set forth by the model, the median uninsured household has a hard time accumulating a herd greater than 36 TLU. This is in contrast to the average high level equilibrium attainable with insurance: 40 TLU after 50 years. That’s an approximately 10% increase. This higher level is reached because the effect of negative shocks is reduced.

We can combine these three effects onto a single graph by expressing the vulnerability effect as a change in probability of collapse, and the shifting equilibrium effect as a change in median equilibrium outcomes. Plotting both of these changes, Figure 11 highlights the importance of the shifting threshold effect, and the benefits of insurance to Micawber households, for it is these households who experience the largest changes when an insurance market is introduced. Even though these households choose limited levels of insurance, insurance alters expectations about the future, which causes Micawber households to alter their behavior dramatically. Destined for the low equilibrium without insurance, this change in behavior makes the high equilibrium attainable for Micawber households. This dynamic benefit of insurance is missed in any static measure of willingness to pay, and any study which ignores behavioral change.

The final effect of insurance demonstrated by our model is really a more general effect of insurance that we expect to see whether or not a poverty trap exists. This final effect, the “smoothing effect,” simply implies that insurance should help households to better smooth income and assets. This means their path to accumulation will involve fewer ups and downs,
it will be smoother. Since this effect is well established and doesn’t rely on an assumption of poverty traps, we do not dwell on it here, rather we simply note that it exists.

7 Pricing of Insurance: Optimal Subsidies

Our model provides a clear framework for thinking about demand for asset insurance in the presence of a poverty trap. Theory suggests that insurance could be used as a safety net to prevent households from falling into a poverty trap, however, our model suggests that households who have the most to gain from insurance may not ‘self select’ into an insurance scheme because the shadow price of liquidity is high. In fact, we show that households existing near the Micawber threshold may find it optimal to insure very few assets, using limited cash on hand for investment instead. This limits the ability of insurance to act as a safety net, even if it crowds in additional investment and offers many potential dynamic benefits.

However, besides our construction of a willingness to pay measure where insurance purchases were limited to zero or full insurance, we have thus far only considered actuarially fair insurance. As the price changes, demand for insurance will also change. If demand for insurance is elastic, then a small change in prices can have a large impact. Thus, in this section we analyze the price elasticity of demand and show how subsidies can actually shift demand and alter poverty dynamics.

It is simple to extend the analysis presented in Section 4 in which we iterate over heterogeneous pricing while giving households the option to partially insure. Doing so, we obtain an optimal policy function for consumption, investment, and insurance using various insurance price schemes. Figure 12 plots the optimal insurance policy function for various discount and loading prices. Based on the assumptions in the model, households seem willing to tolerate
some loading. However, it is clear that demand is not entirely inelastic. For example, the optimal choice under 10% loading seems to reduce demand by those near the threshold by about half. However, for large herds the change in quantity insured is relatively small. As the percentage loading increases however, we see that demand for insurance reduces dramatically, with even large herds insuring very little unless they exist above the high equilibrium (where the marginal benefit of investing is quite low.)

Alternatively, if a government were to offer a 30-40% subsidy on the actuarially fair price we find that almost all households choose near full insurance. The apparent dip for threshold households is nearly wiped out. Furthermore, the dip actually shifts left, indicating that the behaviorally relevant Micawber threshold also moves with the price.

The shift in threshold is more readily apparent if we look directly at the reduction in vulnerability of collapsing into a poverty trap under different price regimes. Figure 13 shows that each additional 10% subsidy can actually shift the level at which households collapse in probability by approximately .5-1 TLU. This means a 40% subsidy shifts the threshold by approximately 3 TLU, allowing a greater number of households to reach the high equilibrium.

It should be obvious by now that the threshold shift is induced by a change in expectation about the future, which affects the first order conditions, and thereby results in altered behavior. This is made evident in the optimal investment policy function. In addition to the optimal investment policy functions under autarky and with actuarially fair insurance, as was shown in Figure 6, we now plot the optimal investment when insurance is offered at 40% less than than the actuarially fair price. Here it’s clear that the lower price induces more households to alter their behavior and attempt to reach the high equilibrium. In this way, not only does insurance act as a safety net, but it can also pull a greater number of households out of a poverty trap by inducing behavioral change.

8 Poverty Dynamics Beyond the Household

Up to this point, our model develops intuition toward understanding how asset insurance will influence behavior when dynamic asset thresholds induce poverty traps, and how such asset thresholds affect demand for asset insurance. This is useful in a broad context if we believe structural poverty traps exist. Moreover, while modeling the insurance decision in the context of livestock insurance for pastoralists, we have considered what the benefits will mean for asset dynamics of a specific household. In this section we take that one step further by considering the impact of insurance on poverty dynamics of an entire population.

Doing so requires adding an additional assumption to our already lofty set of assumptions because we need to specify a distribution of asset levels. We use empirical data of the distribution of herd sizes in Marsabit district of northern Kenya, from a dataset that includes a random sample of households in that region in 2011. While empirical data for asset levels is available, we are still forced to use the basic assumptions of production technologies, even though the precise structure of the poverty trap mechanism becomes much more important in this analysis.

Despite the lofty assumptions, there may still be some benefit to an attempt toward defining the implications of this model in a specific context, especially with regard to poverty dynamics. We therefore use the empirical livestock distribution across households in Marsabit,
Figure 12: Optimal Insurance Decisions for Various Prices
Figure 13: Probability of Collapse to Low Equilibrium under Various Price Schemes

![Probability of Collapse with Heterogeneous Subsidy Schemes]

- Autarky
- Actuarially Fair
- 10% Discount
- 20% Discount
- 30% Discount
- 40% Discount

Figure 14: Optimal Investment Decisions for Various Price Schemes

![Optimal Investment Decisions]

- Autarky
- Actuarially Fair Insurance
- 40% Discount Insurance
and conduct simulations of the optimal consumption, investment and insurance decisions for households who are subjected to a series of random shocks. Doing so allows us to also consider the evolution of various indicators of economic performance with and without insurance. For this analysis we focus on 3 common indicators: the poverty gap, poverty headcount and GDP.

The estimate of GDP is fairly straightforward. It is simply the sum of individual production. In our case we use:

\[ GDP_t = \sum_{i=1}^{n} f(H_{i,t}) \]  

where \( n \) is the total number of individuals in the sample population.

The other two measures, poverty gap and headcount, are in the family of Foster-Greer-Thorbecke (FGT) measures, and are calculated as follows:

\[ P^y_\gamma = \frac{1}{n} \sum_{y_j < y_p} \left( \frac{y_p - y_j}{y_p} \right) \gamma \]  

Here, the income poverty line \( y_p \) is the income generated by the asset level at which the kink in the implied production technology occurs. Individual \( j \)'s income \( y_j \) is estimated using \( f(H_t) \), and \( \gamma \) is the FGT sensitivity parameter. For the poverty headcount, \( \gamma \) is equal to zero, and for the poverty gap \( \gamma \) equals 1.

The predicted evolution of these three economic indicators in Marsabit with and without insurance are presented in Figure 15. Looking first at the poverty gap, we see that the gap reduces with livestock insurance. This is a result of the shifting equilibrium effect. Because the low level equilibrium shifts upward, households are closer to the poverty threshold. In fact, the estimated gap is biased upward if we consider also the shifting threshold effect which reduces the asset level, and thereby the income level necessary to escape the poverty trap. Our calculation instead holds the threshold income level \( y_p \) constant. If we allowed \( y_p \) to shift once insurance was made available then we would expect that the gap would be even smaller.

The poverty headcount decreases and then flattens out with insurance, whereas it steadily increases under the autarkic setting. Because households just below the asset threshold are able to move out of the poverty trap once insurance is available, we observe a reduction in the number of households below the poverty income threshold \( y_p \). Furthermore, once out of the trap, these households are no longer vulnerable to collapse because they can insure. On the contrary, uninsured households remain vulnerable to collapse. This susceptibility to collapse, combined with an inability to escape once collapsed, is why we see a higher poverty headcount in the absence of insurance.

Finally, we consider the sum of household production, to obtain an estimate of economy GDP which we measure in TLU rather than monetary value. Here again we see benefits due to asset insurance. These results suggest GDP approximately 3% higher after 10 years, and approximately 5% higher after 25 years.
Figure 15: Evolution of Poverty Measures

Evolution of Poverty Gap

Evolution of Poverty Headcount

Evolution of GDP
9 Conclusion

In this paper we take as given that a poverty trap exists. Under an assumption of nonlinear asset dynamics, we sought to determine whether the benefits of social protection policy can be generated through insurance for which households pay a market, or near-market price. Our results suggest that insurance offers many dynamic benefits at the household level even though the most vulnerable households demonstrate the lowest willingness to pay for insurance. This low willingness to pay, and the decision to insure in general, depend on a high shadow price of assets for Micawber households. Such households only find it optimal to insure if they think it likely that the payout offered with insurance will keep them above the threshold.

One unique contribution of this paper is the emphasis we place on ex ante behavior when insurance is introduced. We show that even though Micawber households choose low levels of insurance, the promise of reduced vulnerability in the future actually crowds in additional investment for these households. This behavioral change actually induces a shift in the Micawber threshold, opening the door for a greater number of households to reach the high welfare equilibrium. Furthermore, we show that by subsidizing insurance we can crowd in higher levels of both insurance and investment, shifting the Micawber threshold to an even lower asset level.

Finally, we demonstrate the implications our model holds for poverty dynamics in a region. This analysis suggests that behavioral changes and reductions in risk brought about by insurance may result in decreased poverty levels over time. In this way we show that insurance can act as a safety net for many households, even though households at the threshold choose low levels of insurance.
References


## Appendix A: Tables

### Table 1: Parameters used in Numerical Simulations

<table>
<thead>
<tr>
<th>Production Technology Parameters</th>
<th>Utility Function Parameters</th>
<th>Insurance Contract Parameters</th>
<th>Random Shocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma_L = 0.25 )</td>
<td>( \beta = 0.95 )</td>
<td>( p = 0.0275 )</td>
<td>( \theta = {0.0, 0.05, 0.10, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40, 0.45, 0.50, 0.55, 0.60} )</td>
</tr>
<tr>
<td>( \gamma_H = 0.56 )</td>
<td>( \rho = 1.5 )</td>
<td>( s = 0.15 )</td>
<td>( \epsilon = {0.0, 0.005, 0.010, 0.015} )</td>
</tr>
<tr>
<td>( f = 2.725 )</td>
<td></td>
<td></td>
<td>( P(\theta) = {0.3226, 0.3226, 0.1489, 0.0744, 0.0546, 0.0298, 0.0124, 0.0099, 0.0050, 0.0050, 0.0050, 0.0050} )</td>
</tr>
<tr>
<td>( \alpha_L = 1.6 )</td>
<td></td>
<td></td>
<td>( P(\epsilon) = {0.55, 0.15, 0.15, 0.15} )</td>
</tr>
<tr>
<td>( \alpha_H = 1.33 )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\( P(\theta) \) and \( P(\epsilon) \) represent the probability distributions for the parameters \( \theta \) and \( \epsilon \), respectively.