A REAPPRAISAL OF THE NEOCLASSICAL APPROACH
TO MODELLING BUSINESS INVESTMENT

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A REAPPRAISAL OF THE NEOCLASSICAL APPROACH

TO MODELLING BUSINESS INVESTMENT

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ABSTRACT

Empirical studies of investment behaviour are typically based on the three models of investment available in the literature: the Jorgenson neoclassical model, the adjustment cost model, and Tobin's q theory. A case is put for a reappraisal of the empirical implementation of these models.
I. Introduction

The explanation of variations in the level of business investment is a major challenge for economic theory and for applied econometrics. Theories of investment that have formed the basis for empirical work may be characterised (in roughly chronological order) as: profits, accelerator, neoclassical, costs of adjustment, and Tobin's q. The latter three of these, and the relationship among them, are not well understood. For example, a recent paper which appeared in the Economic Record (Kohli and Ryan (1986)) contains a number of confusing and misleading statements about these models of business investment. To a lesser extent, the paper by McKibbin and Sieglof (1988) is also potentially confusing. The purpose of this paper is to clarify some aspects of the neoclassical model that seem to be not well known in the applied literature, and to relate these aspects to the other models.

Section II provides a brief summary of the development of investment theories, and Section III presents a number of criticisms of the standard presentation of neoclassical theory, and some alternative interpretations. Section IV contains some comments on the Kohli-Ryan, and McKibbin-Sieglof papers in the light of these results.

The use of the adjective neoclassical is potentially rather confusing. In a sense, all of the models of investment considered below are neoclassical (which is not surprising, since they are all closely related). In the literature, the term neoclassical is usually used to refer narrowly to the Jorgenson model. However, Hayashi also includes the adjustment cost models within this term, sometimes referring to them as modified neoclassical. I will attempt to be precise where there is potential for ambiguity. Many of the ideas presented below already
exist in the literature, for example in Abel (1979, 1980), Hayashi (1982), Junankar (1972), McLaren (1971, 1978), Nickell (1978) and Precious (1987), but a more concise and integrated presentation appears to be warranted.

II. Investment Theories

The 1960's were without doubt a watershed decade in the development of empirical models of investment behaviour. While the 1950's had been dominated by the "regression races" between the accelerator models and the profits models, the 1960's saw the publication of three papers that laid the foundations for developments that have continued until the present. These three papers were by Jorgenson (1963), Eisner and Strotz (1963) and Tobin (1969).

The Jorgenson model became known as the "neoclassical model of investment behaviour" and the initial work was extensively elaborated upon in Jorgenson (1965, 1967). A selection of empirical applications by Jorgenson and his co-workers would include Jorgenson and Stephenson (1967a, 1967b) and Hall and Jorgenson (1967). This neoclassical theory found its way into many countries' econometric models, including Australia's RBA1 (Mackrell, Frisch and Roope (1971)). Modifications suggested by authors such as Eisner and Nadiri (1968, 1970) and Bischoff (1969) have been absorbed into the model. Applications are now too numerous to list, but the fact that the neoclassical model of investment behaviour is alive and well is borne out by, for example, the recent collection of essays edited by Weiserbs (1985), Australian studies by Higgins et. al. (1976) and Hawkins (1979), and the empirical comparisons of models by Clark (1979) and Bernanke, Bohn and Reiss (1988).
The model introduced by Eisner and Strotz has since become known as "the adjustment cost model", and was developed by Lucas (1967), Gould (1968), Treadway (1971), Cooper and McLaren (1980) and Epstein (1981). While the formal structure of this model is deceptively similar to the (Jorgenson) neoclassical model (both models aim to choose the path of investment in order to maximize the present value of future revenue flows to the firm) the resulting investment models are quite different.

The Tobin model has since become known as Tobin's "q" theory of investment, and proposes that the rate of net investment should be related to the ratio of the market value of a unit of capital to its replacement cost - this ratio is defined as q. In this model, investment will take place whenever q > 1. The q theory is not based on any formal model, and Hayashi (1982) has shown that the adjustment cost model can be given a q interpretation.

III. A Reappraisal of the Neoclassical Model

The standard Jorgenson neoclassical model can be characterised as follows: for a given discount rate r, and for given paths of price of output, p(t), wage rate w(t) and price of new capital goods s(t), choose paths of output Q(t), labour input L(t), capital stock K(t) and gross investment I(t) in order to maximize the present value of the flow of funds to the firm

\[ V = \int_0^\infty e^{-rt}[pQ - wL - sI]dt \] (1)

subject to the technology of the form as captured in the production
function

\[ Q = f(K, L) \]  \hspace{1cm} (2)

the capital accumulation identity

\[ \dot{K} = I - \delta K \]  \hspace{1cm} (3)

and the initial condition

\[ K(0) = K_0 \]  \hspace{1cm} (4)

where \( K_0 \) is given at time 0.

In (1) - (3) the variables \( p, w, s, Q, L, K, I \) are all functions of time \( t \), but such dependence is suppressed for notational clarity. This specification corresponds to the typical Jorgenson specification of the model (see, for example, Jorgenson (1965)), except that the production function has been written explicitly rather than implicitly. The implicit specification would be useful if we were to allow for a multi-output production function, but the usual applications allow for only one output, and hence the simpler form (2) is chosen. Jorgenson invariably chooses to set the problem up as one of constrained optimization, introducing time dependent multipliers for constraints (2) and (3), but it is far easier to eliminate \( Q \) and \( I \) from the problem by substituting (2) and (3) into (1). The optimal paths for \( Q \) and \( I \) can then be derived from the paths for \( K \) and \( L \) via (2) and (3). Thus the firm's problem is to choose paths for \( K \) and \( L \) to maximize

\[ V = \int_0^\infty e^{-rt}[pf(K, L) - wL - sK - s\delta K]dt \]  \hspace{1cm} (5)

subject to (4).
This is now a fairly simple problem in the calculus of variations (see, for example, Kamien and Schwartz (1981)). Denoting the integrand of (5) by $F(t,L,K,\dot{K})$, the necessary conditions (Euler equations) for $L$ and $K$ are:

$$F_L = 0 \quad \text{i.e.} \quad pf_L(K,L) = w \quad (6a)$$

$$F_K = \frac{d}{dt} F_K \quad \text{i.e.} \quad pf_K(K,L) = c \quad (6b)$$

where $c$ is the "implicit rental price of capital" $c = s(r + \delta) - \dot{s}$. The pair of simultaneous equations (6) are exactly the same as those that arise in the static profit maximizing theory of the firm where two inputs $K$ and $L$ are available at prices $c$ and $w$, but now these equations are to hold for all $t$. The only feature that distinguishes capital from labour is that while the firm "rents" labour, it owns capital, and a unit of capital provides services over more than one period. Thus the appropriate price of capital per unit per period is $c(t)$ rather than $s(t)$, and the only role that posing the problem as an intertemporal optimization problem has played is to derive the appropriate construction for $c$, although this definition of $c$ has a fairly intuitive economic interpretation ($rs$ is the opportunity cost of owning a unit of capital for one period, $\delta s$ is the value of a unit of capital that is used up over one period, and $\dot{s}$ is the capital gain (or loss) per unit of time).

The equations (6a) and (6b) can in principle be solved to give the solutions
and substitution in (2) and (3) gives

\[ Q = Q^*(w/p, c/p) = f(K^*, L^*) \]

\[ I = I^*(w/p, c/p, (w/p), (c/p)) = K^* + \delta K^* . \]

To pursue the illustration, choose a particular functional form for the production function. Following Jorgenson, let

\[ f(K, L) = K^\alpha L^\beta \]  \hspace{1cm} \text{(8)}

except that, unlike Jorgenson, set

\[ \alpha + \beta < 1 . \]

Then

\[ L^* = \left[ \left( \frac{w}{\beta p} \right)^{(1-\alpha)} \left( \frac{c}{\alpha p} \right)^\alpha \right] \gamma \]  \hspace{1cm} \text{(9a)}

and

\[ K^* = \left[ \left( \frac{w}{\beta p} \right)^{\beta} \left( \frac{c}{\alpha p} \right)^{(1-\beta)} \right] \gamma \]  \hspace{1cm} \text{(9b)}

where \( \gamma = 1/(\alpha + \beta - 1) . \)

The role of the constraint \( \alpha + \beta < 1 \) is now clear; if, as Jorgenson assumes, \( \alpha + \beta = 1 \), then (6a) and (6b) will not, in general, have a solution for arbitrary values of \( p, w \) and \( c \). For any values of \( \alpha + \beta \geq 1 \) the maximization problem is undefined. The Jorgenson solution is to

\[ \text{However, Peter Wilcoxen has pointed out to me that, even if } \alpha + \beta = 1, \text{ for any values of } K \text{ and } L \text{ there will exist prices } w \text{ and } c \text{ at which (6a) and 6(b) are satisfied.} \]
combine (6b) with (8) to give

\[ K^* = \alpha p Q / c. \]  \hspace{1cm} (10)

This equation is then used in empirical applications by treating \( Q \) as exogenously determined, and allowing \( K \) to adjust to \( K^* \) according to a distributed lag that is imposed arbitrarily from outside the model. This procedure is inadmissible on many counts, including: (i) (10) has been derived from a paradigm in which \( Q \) is jointly determined and hence cannot be treated as exogenous - clearly, if \( Q \) is exogenous the appropriate model is cost minimization (see below) and there can be no role for the price of output in the equation for desired capital; (ii) any estimates based on (10) alone will be inconsistent because of simultaneity bias; (iii) once actual capital stock is allowed to deviate from desired capital stock the model is invalid - for example, (6a), (6b) and (8) can no longer hold if \( K \neq K^* \). (If there is a delivery lag, this should be included in the optimization model.) To make the point in another way, it is a rather odd "neoclassical" model that allows no role for the wage rate in the demand for capital. (Note that this result is not due to the use of a Cobb-Douglas function with unitary elasticity of substitution - an analogous result holds, for example, for a CES production function.)

Another interesting point about the solution paths (7) is that there is no necessity for (7b) to satisfy the constraint (4). It is normally the case that in maximizing a criterion such as \( \int F(t,K,\dot{K}) dt \) the Euler equation generates a second-order differential equation in \( K \) (via the term \( \frac{d}{dt} \frac{d}{dt} F_2 \)). Then two constants of integration are needed to solve the Euler equation, and these are usually provided by the initial condition \( K(0) = K_0 \) and a transversality condition. This is not the
case when $F$ is linear in $\dot{K}$ - then the Euler equation merely provides the static-type condition $F_K = 0$ which allows solution directly without the use of endpoint conditions. The implications of this point, and some of the other points above, can be brought out in an alternative presentation which may also be helpful to those who are not comfortable with the use of the calculus of variations.

Recalling the criterion (5), the term $s\dot{K}$ can be integrated by parts to give

$$\int_0^\infty e^{-rt} s\dot{K} \, dt = e^{-rt}sK\bigg|_{t=0}^\infty + \int_0^\infty e^{-rt}(rs - s)K \, dt.$$ 

Assuming $sK$ is dominated by $e^{-rt}$ for large $t$, the first term is simply $-(s(0)K(0))$ and substitution in (5) allows the criterion to be written as

$$\int_0^\infty e^{-rt}[p_f(K,L) - wL - cK] \, dt + s(0)K(0).$$

The $t = 0$ value of the optimal path $K(t)$ was written as $K(0)$ to allow for the fact (noted previously) that the path may not satisfy $K(0) = K_0$. If the path for $K$ does have an initial jump of $(K(0) - K_0)$ then $s(0)$ times this jump must be subtracted from the above value to give the equivalent representation to (5):

$$V = \int_0^\infty e^{-rt}[p_f(K,L) - wL - cK] \, dt + s(0)K_0.$$  \hspace{1cm} (11)

Maximization of $V$ so defined is not a problem in the calculus of variations at all - it is merely a sequence of static optimizations indexed on $t$, for which the solutions for each $t$ are equations (7), and $V$ is merely the present value of this flow of maximized profits, plus the value of the initial stock of capital. When written in the form of (11), it is clear that maximization of the integral is equivalent to the
integral of the maxima, and hence that equations (7) are the appropriate solutions. This is a general result: whenever the integrand $F(t,x,\dot{x})$ (where above $x' = (K,L)$) is linear in $\dot{x}$ a seemingly dynamic optimization problem will collapse to a sequence of static optimizations, and the necessary conditions from the calculus of variations, when correctly applied, will merely reproduce conditions for static optimization. It is thus transparent why conditions (6) should give the static profit maximizing first-order necessary conditions.

This derivation also clarifies the relationship between the neoclassical models and the cost of adjustment models. In the adjustment cost models the integrand of the criterion function is usually assumed to be strictly concave in $\dot{K}$, and a true dynamic optimization problem results. To illustrate, consider the simple case in which $p$, $w$ and $s$ are constant in planning time. Then the cost of adjustment model will generate an optimal path for $K(t)$ which approaches $K^*(w/p, c/p)$ through time. The less concave is $F$ in $\dot{K}$ the quicker does $K(t)$ approach $K^*$, until in the limiting case when $F$ is linear in $\dot{K}$ the optimal result is to adjust $K$ to $K^*$ immediately, which is the Jorgenson "neoclassical" solution. In this degenerate case, the cost of adjustment model distinction between variable factors $L$ and quasi-fixed factors $K$ disappears, and there is no theory of investment, just a theory of the stationary value of capital, and its appropriate "rental price", $c$. If the prices do vary over planning time, the stationary value of $K^*$ is replaced by a target path $K^*(w(t)/p(t), c(t)/p(t))$. While a discrete approximation to the changes in this path could generate a model of investment (as in Brechling (1975)), this is not the usual neoclassical model.

This alternative specification of the problem is also useful to
handle the paradigm of cost minimization. If the path of output \( Q(t) \) is specified as given, then maximizing (1) subject to (2), (3) and (4) is equivalent to minimizing

\[
C = \int_0^\infty e^{-rt} \left[ wL + cK \right] dt \tag{12}
\]

subject to \( Q = f(K,L) \), which is equivalent to the discounted integral of a series of static constrained minimization problems. First order conditions are

\[
\frac{\mu f_L(K,L)}{\mu f_K(K,L)} = \frac{w}{c} \tag{13a}
\]

\[
Q = f(K,L) \tag{13c}
\]

where \( \mu \) is a Lagrange multiplier, allowing solutions

\[
\hat{K} = \hat{K}(Q, w/c) \tag{14a}
\]

\[
\hat{L} = \hat{L}(Q, w/c) \tag{14b}
\]

Again, using the Cobb-Douglas as an illustration, the solutions are

\[
\hat{K} = \left[ (\beta c/\alpha w)^\beta Q \right]^{1/(\alpha+\beta)} \tag{15a}
\]

\[
\hat{L} = \left[ (\beta c/\alpha w)^{-\alpha} Q \right]^{1/(\alpha+\beta)} \tag{15b}
\]

As indicated above the Jorgenson model can be thought of as a limiting case of the adjustment cost model: if, after substitution of constraints such as (2) and (3) the integrand of (1) is linear in the \( \dot{K} \) term, then the dynamic optimization problem collapses to a sequence of static optimization problems. The adjustment cost models avoid this linearity in at least two ways: by incorporating a dependence on \( \dot{K} \) (or
I) in the production function (2), or by introducing a non-linear relationship between $\dot{K}$ and $I$ in (3). Following Hayashi, replace (3) by

$$\dot{K} = \psi(I,K) - \delta K$$

(16)

where $\psi$ is concave and increasing in $I$. Maximization of (1) subject to (2) and (16) is a simple problem in optimal control, for which it is useful to introduce the (current value) Hamiltonian

$$H(K,I,L,\lambda) = pf(K,L) - wL - sI + \lambda[\psi(I,K) - \delta K]$$

(17)

First-order conditions for optimality then correspond to the two first order conditions for maximizing $H$ with respect to $I$ and $L$, the transition equation (16) for $K$, and a transition equation for $\lambda$. The key to this problem is the co-state variable, $\lambda$, which from (17) can be interpreted as the shadow price of a unit of capital. If the maximized present value of the firm corresponding to an initial stock of capital $K$ is defined as $V(K)$, then $\lambda = \partial V / \partial K$, the marginal value to the firm of an extra unit of capital. Tobin's $q$ may be defined as the ratio of $\lambda$ to the price of a new unit of capital, $q = \lambda / s$. Solution of the intertemporal problem is essentially equivalent to the derivation of $\lambda$ (or $q$). To make the (heroic) assumption that $q$ is observable is to assume away any intertemporal optimization problem. If the shadow price of a unit of capital is known, the firm's problem is again a sequence of static optimization problems - the maximization of (17) with respect to $I$ (and $L$, but by now it should be clear that the choice of $L$ is always a static optimization problem in this type of problem).

It may be of interest to interpret the Jorgenson model within this framework - it simply corresponds to $\lambda = s$ and $q = 1$ at all points in time. This follows immediately from (11).
While the theoretical relationship of the cost of adjustment model to the q theory has been noted by, for example, Abel (1979) and Hayashi (1982), the empirical implications of this relationship seem to have been missed. For ease of exposition, again assume that prices \( p, w, s \) and the discount rate \( r \) are constant in planning time. If the firm acts, by assumption, to maximize present value (1), subject to (2), (16) and (4), then the maximized value of \( V \) will be a function of the givens \( K_0, p, w, s, r \) and can be represented as

\[
V(K_0, p, w, s, r)
\]  

(18)

corresponding to the evaluation of (1) along the optimal paths for \( K(t) \) and \( L(t) \). As derived in Cooper and McLaren (1980) and McLaren and Cooper (1987) there will be a duality between the functional form of \( f(K, L) \) in (2) plus \( \psi(I, K) \) in (16) and \( V \) in (18). In particular, given a specification of \( f(K, L) \) and \( \psi(I, K) \), the optimization then determines optimal paths for \( I \) and \( L \):

\[
\hat{I}(t; K_0, p, w, r, s)
\]  

(19a)

\[
\hat{L}(t, K_0, p, w, r, s)
\]  

(19b)

and the optimal value of the firm \( V(K_0, p, w, r, s) \). The optimal level of investment at time 0 is related to \( V \) by an analogue of Hotelling's Lemma:

\[
\bar{I} = r(V_{sK_0} - 1)^{-1}V_s + \delta K_0 .
\]  

(20)

While it is obvious that \( I \) and \( q \) are related - note that in this notation \( q \) is defined by
q(K₀, p, w, r, s) = \left(\frac{V}{K₀}\right) / s, \hspace{1cm} (21)

it would not be a very sensible econometric specification to regress I on q. If one were to believe that one could obtain useful observations on V, then a sensible procedure would be a systems estimation of (consistently specified) equations (18), (19a) and (19b), in which V, I and L were jointly endogenous. While one could possibly replace (18) by (21) (using q instead of V) this also would not seem sensible, since observations on q are usually derived from observations on V, by using extra structure on the functional form of V (and hence f and ψ) such as is implied by constant returns to scale in f and ψ.

IV. Some Comments On Two Recent Papers

The recent paper by Kohli and Ryan (1986) is typical of a paper that takes the Jorgenson derivation as given, and interprets empirical results within its framework. Based on the results presented in Section III, the main comments will relate to the presentation and interpretation of the neoclassical model and empirical results in sections I and II of Kohli and Ryan. Note first that Kohli and Ryan (p.452) state that ... "Assuming cost minimization, Jorgenson derives the desired stock of capital from a Cobb-Douglas production function as" ... (equation (10) in our notation). But (10) is a first order condition based on profit maximization, not cost minimization, and in any event is not a solution of the necessary condition. The suggestion that (10) is the result of cost minimization probably stems from the fact that in applications of (10), output is treated as exogenous. But once output is given there can be no role for the price of output in an optimizing relationship. The appropriate cost minimizing demand for
Kohli and Ryan then lament the empirical performance of equations such as (10) in capturing the response of Australian investment to the substantial increase in real wages in the 1970s. But of course this is hardly surprising given that wages do not enter (10). (They do, of course, enter the correct equation (9b)). The statement in footnote 8 on p.452 that the accelerator is a limiting case of the neoclassical model is correct, but for the wrong reason. It is the analogue of (15a) (corresponding to a CES function) that collapse to the flexible accelerator as the elasticity of substitution goes to zero.

The next step in Kohli and Ryan is to make the sensible suggestion that perhaps the stock of capital should be treated as fixed, and its first order condition used to determine the rental price of capital. But the appropriate relationship is then either the inverted form of (9b), or the inverted form of (15), but certainly not the inverted form of (10), their equation (2). (The inverted form of (9b) in fact corresponds to their equation (7) under a Cobb-Douglas variable profit function.)

Kohli and Ryan then state that "... properly applied neoclassical theory suggests that for a given capital stock, an exogenous increase in real wages leads to a reduction in the real rental price of capital. This makes the ownership of capital less attractive, it decreases its shadow price and, by the same token, it reduces incentives to produce and install capital goods. Besides decreasing investment, the exogenous increase in real wages also tends to reduce output and employment ..." (p.453) and illustrate this argument by Figure 1 (p.454). But this argument and diagram correspond exactly to the results implied by
equations (15) (for cost minimization, moving from $P^0$ to $P^1$ on the fixed isoquant) or by equations (9) (for profit maximization, leading to lower demands for labour and capital). However, it should be noted that these qualitative results depend critically on the assumed form of the production function. It is possible to construct an example based on a CES production function in which the substitution effect outweighs the output effect of the real wage increase. The net effect is an empirical question.

It should be pointed out that none of these points impinge on the model derived and estimated by Kohli and Ryan in Sections III and IV. In fact that model has much more in common with the "correct" interpretation of a neoclassical model presented above than it does with the Jorgenson model. Even the rate of return on capital (their (16)) which corresponds to the inversion of the expression for the implicit rental price of capital is not, in fact, well known from neoclassical theory, because the neoclassical theory does contain an implicit portfolio model in which the rate of return on capital is equalized at the margin with the opportunity cost of funds, $r$. To the extent that the Kohli and Ryan model does have a theory of investment (as distinct from a theory of portfolio allocation) it is arbitrarily imposed by the assumption of a partial adjustment mechanism.

In McKibbin and Siegloff (1988) the investment equation is presented as though it is the solution to an intertemporal optimization problem. While in a sense this is true, it is also misleading given that they will use $q$ as exogenous data. As indicated in Section III, knowledge of $q$ is equivalent to avoiding an intertemporal optimization problem. Given $q$, investment is merely chosen to maximize the Hamiltonian (17), so that in the notation of McKibbin and Siegloff,
their equations (7), (9) and (11) are quite irrelevant. As also indicated in Section III, within the usual optimization paradigm adopted in such models, single equation estimation with q exogenous is inadmissible.

V. Conclusion

If the term "neoclassical model of investment behaviour" were to be interpreted as a model based on maximization of present value of the firm, this would include the various adjustment cost models, and the Jorgenson model specification as a degenerate special case. This would then highlight why the usual Jorgenson type specification of investment demand equations is not consistent with the neoclassical paradigm. The Tobin q model can be thought of as an alternative qualitative statement of the implications of the neoclassical model, but the positive relation between investment and q relates to a positive correlation between two jointly determined endogenous variables, and is not the basis for single equation estimation.

* I have benefited from discussions with Russel Cooper, Mark Upcher and Peter Wilcoxon on various points in this paper.


