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Valuation and Efficient Allocation of GSM Export Credit Guarantees

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Estimates of country-level loan default distributions are developed and used in a loan guarantee model to value the contingent liability of USDA's General Sales Manager (GSM) export credit guarantee portfolio. The results quantify the relationship between increasing guarantee coverage and the resulting actuarial liability to the government. Optimal coverage levels and optimal country-level allocations are determined for given policy objectives and coverage totals. Findings reveal that the government's allocation of country guarantees is risk-inefficient; and guidance is provided for making risk-efficient allocations for any program size.

Key words: contingent liability, export credit, GSM, loan guarantee valuation, risk-efficiency

Introduction

The USDA's General Sales Manager (GSM) credit guarantee program is a key policy instrument used by the federal government in efforts to stimulate exports of U.S. agricultural commodities. According to the *U.S. Code of Federal Regulations* (1998, 1493.2), the stated purposes of the GSM programs are: "(a) to increase exports of U.S. agricultural exports; (b) to compete against foreign agricultural exports; [and] (c) to assist countries, particularly developing countries, in meeting their food and fiber needs. . . ."

In practical terms, GSM programs are used to facilitate exports of commodities to countries that are unable to obtain private financing due to credit risk, currency risk, political instability, or any other impediment to private financing. In addition to the intent to benefit producers of the exported commodity, the programs are also used to satisfy other political and market development motivations. The GSM-102 program can be used to guarantee loans up to three years in length, while the GSM-103 program guarantees loans up to 10 years in length. Both programs typically cover 98% of the principal and interest due. The GSM-102 program dominates allocations, representing over 95% of covered principal since inception. The Commodity Credit Corporation (CCC) is authorized to allocate over $5 billion per year in new guarantees across both programs.

While the goals of increasing exports, supporting development, and other political objectives underpin the loan guarantees, the existing programs have come under increased scrutiny because losses have often been high (at one point reaching nearly 30%),

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and the total authorization seldom used. As a result, existing GSM programs have been simultaneously criticized for being too large and costly, and for failing to exhaust their available guarantee limits, with the implication that they must somehow be failing to generate the maximum benefits (minimum loss) for producers (taxpayers).

Additionally, export credit programs have been criticized during World Trade Organization (WTO) negotiations for their potential trade distortion effects, yet have been largely left in place in some form in most commodity exporting countries. The portion of the program's outlays in excess of fees recovered is recognized under commitments made in the WTO agreements as a subsidy under the notion that "subsidy costs approximate the present value of the estimated net cash outflows at the time the credit guarantees are disbursed by the lender" (CCC, 1998, p. 56). Thus, the calculation of the subsidy rate embedded in the guarantees is central in current trade debates, as is the need for more formal methods for accurately calculating effective subsidy levels in export credit programs for budget scoring purposes. Valuation of the associated loan guarantees is a necessary first step in assessing whether subsidies are embedded in the GSM programs.

The Credit Reform Act of 1990 requires that an estimate of the actuarial value of the GSM guarantees be carried on the federal budget as a current liability, but it does not prescribe an approach for calculating its value. Consequently, widely divergent estimates have been formed. For example, the U.S. General Accounting Office (GAO), using an ad hoc scoring method, computed the costs of the GSM-102 program at 25.1% of exposure, which contrasts sharply with the estimate of 7.4% provided at the same time by the Executive Branch (U.S. GAO, 1994). Internally, the CCC recognized an annual "estimated liability of $43 million for anticipated claims on shipments made"; yet, at the same time, the CCC carried accounting statement balances of unresolved claims paid of more than $2 billion (CCC, 1998, p. 17).

Central to each of the above issues is the valuation of the contingent liability of the government associated with the issuance of export credit guarantees, and the design of efficient methods for the allocation of guarantees across countries with differing risk profiles. In response, this study develops a method to recover complete measures of the underlying repayment capacity distributions for risky-country debt that combines information from the most important commercial sources of country risk ratings. These estimates are used in an economic model of guarantor activity to provide guidance for efficient allocations of guarantees across countries, for any given program size, and with the resulting actuarial costs computed. In this context, inefficiency reflects the condition that higher coverage could be extended with the same expected costs, or that lower expected costs could be achieved at the same coverage levels—each of which could be consistent with other policy objectives, but fail the economic efficiency condition. Equivalently, for any given program size, efficiency can be considered as the lowest expected cost per dollar of guarantee coverage. Thus, to the extent that motivations other than efficiency guide allocations, the difference between the expected costs of the actual allocation and the expected costs of an efficient allocation of the same coverage provides an estimate of the costs of satisfying the non-efficiency motivations.

The methods developed herein can be use to identify optimal allocations across countries that are most risk- and cost-efficient, and to provide estimates of the marginal costs of changing program coverage. The results consider alternative policy objectives and the associated optimal portfolio implications for each case. The findings of this study are thus helpful in providing evidence about the actuarial costs of the existing programs for use in budget scoring and in trade debates as well. Moreover, the results are useful in improving the efficiency of this and other guarantee programs, and can help identify likely impacts of other potential program design changes.

GSM Program Background

The most visible GSM program activity occurs in the form of new guarantees. Beginning with an authorized annual maximum, the CCC offers to guarantee debt to various countries for a wide array of eligible commodities. Fiscal-year totals of new activity are provided in table 1 for the period 1985–1997. As observed, new activity peaked in FY 1992 with just over $5 billion in guarantees. New activity dropped to below $3 billion as recently as FY 1995 as a result of unused allocations. In recent years, GSM-102 activity has continued to guarantee $3 billion to $5 billion in credit. There has also been a shift toward use of the Supplier Credit Guarantee Program, which covers only 65% of the credit for a period up to 180 days.

The only direct inflows to the CCC from the GSM programs come from guarantee fees charged to the exporter. Fee rates were historically set with some discretion in a range based on the terms of guaranteed loans, the risk faced, and other factors. However, there was little variation across countries in actual fee rates, which averaged $0.61 per $100 covered for GSM-102 guarantees in 1997. Current policy sets rates constant across countries except for the term. The rates were last changed on October 1, 2002, and vary by term and repayment interval. Guarantee fee revenue is simply the fee rate times the amount guaranteed [more detail on the fee rate process is provided in the *U.S. Code of Federal Regulations* (1998, 1493.70)]. Also reported in table 1 are the annual claims paid by the CCC, which represent the primary cost of the GSM programs. Most claims represent only small fractions of the outstanding exposures to each recipient, but large claims occasionally occur as well. (Iraq, for example, has not made any payments since the 1991 Gulf War, and is obviously not likely to make any further payments.) Once a claim is paid, the CCC seeks to recover as much of that amount (termed "Paid Claims") as possible from the importer, its bank, or its government. "Paid claims" can thereafter be resolved by being repaid, rescheduled, or forgiven and written off. Until an agreement with the importing country is reached, "paid claims" are carried on the balance sheet of the CCC at the full balance outstanding at the time of default. This amount, as shown in the last column of table 1, exceeded $2 billion at the end of FY 1997.

Valuation Approaches

The bulk of the literature examining credit guarantees utilizes actuarial- or option-valuation methods to assess the expected costs of a loan guarantee. In examining alternative approaches for evaluating guarantee programs in the United States, Mody and Patro (1996) indicate that the U.S. GAO (1994) method is merely a "rule-of-thumb" approach, developed for cases when the market price of debt is unknown or cannot be observed. Therefore, it is not easily generalized to cases beyond the specific set of countries examined. They identify the use of contingent claims or option-valuation methods, which explicitly consider probabilistic information and associated default severities, as preferred over other methods which examine average default experience only.

Table 1. Annual Guarantee Activity and Claims Data, Fiscal Years 1985–1997

Fiscal Year	New Guarantees Extended ($ mil.)	Guarantee Fees ($ mil.)	Paid Claims During FY ($ mil.)	Paid Claims Outstanding ($ mil.)
1985	2,674	16	184	1,037
1986	2,503	15	328	162
1987	2,674	16	184	269
1988	4,557	NA	287	225
1989	3,218	NA	4	234
1990	4,127	NA	17	194
1991	4,360	29	780	971
1992	5,083	39	705	1,465
1993	3,839	34	1,365	2,714
1994	3,219	17	1,168	1,693
1995	2,906	14	737	1,677
1996	3,424	21	223	2,071
1997	3,196	15	31	2,072

Source: *Annual Report of the Commodity Credit Corporation* (various years).

While theoretically appealing, applications employing contingent claims models (e.g., Black-Scholes style models) face a common problem of identifying the underlying asset distribution. Among the previous attempts, Schich (1997) uses two proxies for debt-servicing capacity as the underlying asset in export credit insurance, but is not able to fully identify an underlying debt-servicing capacity probability distribution. Dahl, Wilson, and Gustafson (1999) model GSM guarantees using a contingent valuation framework with the letter of credit as the underlying asset. While their findings show the guarantee value to be about 15% of the value of guaranteed wheat exports to Pakistan, the methods are not easily generalized to a portfolio level. However, although both of these studies utilize a theoretically appealing approach, neither is fully generalizeable because not all GSM recipient countries have debt which is actively traded on secondary markets.

Yang and Wilson (1996a) model allocation decisions of the CCC and find the quantity of credit extended to be negatively related to a measure of country risk and positively related to the imports for a given recipient. In a companion study, Yang and Wilson (1996b) also report a positive relation between GSM allocations and market share. They conclude that because larger allocations result in increased liability, the benefit of increased market share could potentially be more than offset by higher default costs.

Proposed modifications to GSM programs have included the use of "better risk assessment methods" in general, the limiting of exposure to high-risk countries (U.S. GAO, 1994), and a movement toward region-wide allocations rather than county-specific allocations. To offset costs, the GAO (U.S. GAO, 1995a) suggests that Congress eliminate the ban on fees over 1% of exposure, and advocates risk-based fees be instituted by the CCC—but provides no explicit mechanism for doing so. Other GAO suggestions include eliminating the conflicting objectives of having exposure of at least $5.5 billion per year and that recipients be creditworthy enough to repay such exposure, by using a broadened definition of eligibility (U.S. GAO, 1995b). Internally, the Foreign Agricultural Service is looking for ways to increase program activity levels while remaining attendant to risk

(Habenstreit, 1998). Taken together, these studies, and calls for modification in existing methods, provide evidence of a strong need for improved valuation methods, with support for contingent valuation methods where possible. Improved valuation methods could improve allocations and result in cost saving, or alternatively, cover more exports and presumably generate additional surplus at equivalent costs from efficient reallocation.

Theoretical Valuation Model of Export Credit Guarantees

It has been shown that a loan guarantee can be valued equivalently to a put option with a strike price equal to the promised payment on the loan (Merton, 1977). To develop this analogy in the present context, consider a guaranteed loan with promised future payment of k at time T, and a random repayment capacity, c, available to service the loan. If the available repayment capacity realized is greater than the required payment k, the borrower makes a payment equal to k and keeps the remainder $(c - k)$. If, instead, the repayment flow is less than k, then the borrower defaults and pays only c, and the guarantor makes up the difference $(k - c)$ as an indemnity payment to the holder of the guaranteed loan. The probability of default, or equivalently of the guarantor making a payment to the lender, is therefore equal to $F(k)$, or the cumulative probability of the repayment capacity distribution at k. The size of the guarantee payment is $(k - c)$ whenever $c < k$, and zero otherwise. Thus, the resulting expected value of the guarantee payment, $V(k)$, is:

$$V(k) = \int_0^k f(c)(k - c)\,dc, \tag{1}$$

where $f(c)$ is the probability density function of the repayment capacity, and other terms are as defined above. Under specific assumptions about the form of $f(c)$, equation (1) is equivalent to Merton's (1977) option formula.[1]

Use of this approach in an empirical setting requires the estimation of the repayment capacity distribution, $f(c)$.[2] As international finance markets have become relatively well developed, a number of commercial vendors have begun offering a wide array of country risk-rating data products that hold great promise for use in characterizing repayment capacity distributions. The commercially most important sources of these products were purchased and/or collected from public sources as the basic data underlying the estimates that follow. Summary risk-rating scores and composite ratings data were collected on each of the individually reported countries in the GSM programs from 1985 through 1996 from *Institutional Investor*, *Euromoney*, and *International Country Risk Guide* (*ICRG*).[3] The complete set of component data used in the construction of the

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[1] Because the majority of the accumulated interest is also paid under the guarantees, there would be no effect of discounting and adding back the interest accumulated, and thus the effective discount factor is 1 and is omitted.

[2] The U.S. GAO (1994) assumes a uniform parameterization and constructs an estimate by observing single historic default frequencies.

[3] FAS historically used an internally developed method to assess country risk, but has switched to a commercial vendor (Moody's) to provide country risk ratings that are used to assess the repayment likelihoods of recipient countries. However, these are not provided directly in probabilistic form. *Institutional Investor*'s rating is a compilation of ratings by various creditors. It is reported as a single rating for each country on a scale from 0 (least creditworthy) to 100 (most creditworthy). *Euromoney*'s rating is comprised of the summation of scores for different categories. It also ranges from 0 to 100. *ICRG*'s rating is a weighted average of 24 individual components in three broad risk categories. Its composite rating ranges from 0 to 100.

ICRG composite rating was also purchased. In addition, historic GSM defaults by country by year were obtained from the CCC.

Although the commercial ratings are intended to serve as a measure of the relative risks of default across countries, the scales of the ratings differ by source and none purport to be directly calibrated to probability of default measures. Several well-developed methods, commonly referred to as credit scoring models, exist for estimating the probability of default as a function of the ratings at time of loan origination [for a review of alternate methods, see Turvey (1991), or Saunders (1999)]. The most commonly used approach employs a logistic regression model (logit) to relate the incidence of default to variables expected to influence the likelihood of default. The logistic model accounts for the qualitative nature of the default data, accounts for the potential lack of calibration among the ratings data and default likelihoods, and results in forecasts with appropriate probabilistic properties.

In the present case, alternate logit models were fit with each of the rating levels by country at the time guarantees were extended. These rating levels were used to forecast the FY 1996 probability of default for guarantee recipients, P_j. The estimated logit results were significant and similar between the *Institutional Investor* and *Euromoney* variables, and resulted in a slightly poorer fit when estimated using *ICRG* data. The general form of the estimated probability of default from the rating R_j^k for country j, from source k, is:

$$P_j^k = \frac{1}{1 + e^{-(\alpha^k + \beta^k R_j^k)}}. \tag{2}$$

The estimated coefficients (standard errors) were $\alpha = -1.274$ (0.0572), and $\beta = -0.047$ (0.022) using *Institutional Investor* data; and $\alpha = -1.340$ (0.582), and $\beta = -0.032$ (0.016) using *Euromoney* data. Each of the coefficients is significantly different from zero at a 0.05 level of significance. The goodness-of-fit measures are similar when using either the *Institutional Investor* or *Euromoney* ratings, and the coefficients give similar forecasts, but with a slightly greater spread or discrimination among the rated countries when using the *Institutional Investor* ratings data. Thus, the remainder of the results presented here utilize the *Institutional Investor* coefficients.

Next, the severity of GSM defaults was modeled as a function of the individually reported *ICRG* rating components. To do so, observed default magnitudes were first standardized using a Box-Cox transformation to account for disparity of exposure levels and nonnormality. The standardized severity was modeled as a linear function of four statistically significant *ICRG* components: economic expectations vs. reality (*EER*), quality of the bureaucracy (*QOB*), racial and nationality tensions (*RNT*), and foreign trade collections experience (*FCR*) over the same time period as the probability-of-default model. The estimated relationship has an $R^2 = 0.63$ and, with the exception of the intercept, the coefficients on each variable are significant at a 0.05 level with the expected signs. The estimated coefficients (standard errors) are: $\alpha = -0.257$ (0.857), $\beta_{EER} = 0.419$ (0.111), $\beta_{QOB} = -0.616$ (0.229), $\beta_{RNT} = -0.368$ (0.122), and $\beta_{FCR} = -0.482$ (0.199). *ICRG* component levels were then used to forecast the standardized severity for guarantee recipients in FY 1996, which was then converted back to the base units to obtain country-level forecasts of the conditional mean of the repayment capacity distributions in the event a default occurs, C_j^d.

Taken together, the estimated probability of default and conditional mean in cases of default can be used to identify a complete repayment capacity distribution for each country. The cumulative probability of the repayment capacity distribution evaluated at the exposure level for recipient j is specified as:

$$(3) \qquad P_j = \int_0^{k_j} f(c_j)\, dc_j,$$

where k_j is the exposure or guarantee level for recipient j, and $f(c_j)$ is its unknown repayment capacity distribution. Likewise, the forecast of the conditional mean is related to the unknown repayment capacity distribution because:

$$(4) \qquad C_j^d = \int_0^{k_j} c_j \frac{f(c_j)}{F(c_j)}\, dc_j = \int_0^{k_j} c_j \frac{f(c_j)}{P_j}\, dc_j.$$

Thus, equations (3) and (4) together permit identification of recipient j's repayment capacity.

If $f(c)$ is parameterized as a beta distribution with upper bound u, the solution for recipient j is:

$$(5) \qquad f_j(c \mid a, b, u) = \frac{1}{uB(a, b)} \left(\frac{c}{u}\right)^{a-1} \left(1 - \frac{c}{u}\right)^{b-1}, \quad 0 \le c \le u,$$

where a, b, and u are greater than zero, and $B(a, b)$ is the beta function.[4] The distribution is easily standardized to a probability distribution by substituting $z = c/u$. This allows for a characterization in either dollars or probability space. The value $c = k/u$ is the point on the distribution that partitions the probability of default from nondefault, or equivalently, the strike price in an option pricing context.

The parameters of the corresponding beta distributions were estimated for each of the individual countries identified in table 2, with coverage representing over 95% of the GSM programs.[5] Other methods for determining the repayment capacity distributions, including those currently used by FAS or in commercially available credit-scoring packages, could likewise be used if preferred, but this approach has the benefits of making greater use of available risk-rating information and retaining probabilistic information. In contrast to the GAO approach using purely subjective parameterization, this approach estimates the parameters using the commercially most important set of risk forecasts available and corresponding actual historic default measures.

Substituting the beta distribution into the valuation formula in (1) and rearranging, the guarantee value equals:

$$(6) \qquad V_j(k \mid a, b, u) = kI_x(a, b) - u\frac{B(a+1, b)}{B(a, b)} I_x(a+1, b),$$

[4] The beta distribution was selected for flexibility and ease of interpretation. See Zellner (1971), and Patel, Kapadia, and Owen (1976), for properties and analytic descriptions of the beta distribution. Other two-parameter distributions, bounded at zero, could also have been selected, but the results are not likely to be sensitive to the choice. Equations (3) and (4) exactly identify the two parameters of the beta distribution; however, the analytic solution is intractable. Thus, a numeric search coded in *LispStat* was used to solve for the parameters listed in table 2.

[5] The CCC reports a few other countries aggregated into regions. Complete country-specific results of the beta distribution estimates are available from the authors upon request.

Table 2. Exposure and Guarantee Values for FY 1996 GSM Recipient Countries

Guarantee Recipient	Optimal Exposure ($)	Guarantee Value[a] ($)	Exposure Share (ES_j) (%)	Risk Share (RS_j) (%)	Fee Equivalent[b] ($)
Algeria	850,931,486	6,441,075	11.3	67.9	0.757
Brazil	129,004,690	26,705	1.7	0.3	0.021
Egypt	211,998,945	44,625	2.8	0.5	0.021
Indonesia	237,521,815	582,639	3.2	6.1	0.245
Jordan	53,106,993	8,572	0.7	0.1	0.016
South Korea	355,714,139	16,008	4.7	0.2	0.005
Mexico	3,839,226,405	605,474	51.1	6.4	0.016
Morocco	409,248,707	50,983	5.4	0.5	0.012
Pakistan	748,127,835	238,954	10.0	2.5	0.032
Romania	127,915,154	162,816	1.7	1.7	0.127
Russia	47,047,600	511,319	0.6	5.4	1.087
Sri Lanka	92,856,140	500,504	1.2	5.3	0.539
Tunisia	226,410,946	170,857	3.0	1.8	0.075
Turkey	184,425,536	119,638	2.5	1.3	0.065
Total	$7,513,536,390	$9,480,168	100%	100%	

[a] The guarantee is valued for the actual exposure level.
[b] The fee equivalent reflects the actuarial value of the guarantee value per $100 of exposure at the actual size.

where $I_x(a, b)$ is the value of the standardized beta cumulative distribution function, with $x = k/u$ (for demonstration of this result, refer to the appendix). Using this expression, the corresponding guarantee values are calculated and shown in table 2 along with the corresponding exposure levels for the countries individually reported by the CCC.

Also presented in table 2 are the fractions of the total portfolio exposure shares by country (ES_j) and the fractions of the total guarantee liability or risk shares (RS_j) for each country. The exposure share is the fraction of the total guarantee volume committed to a particular country. It is largest for Mexico at over 50% and smallest for Russia at less than 1%. However, the risk share, or equivalently the fraction of the implied total liability represented by a country, can be quite different. For example, Algeria has a relative volume of only 11% (exposure share) of the portfolio, yet its relative portion of the contingent cost is largest at 68% (risk share). The guarantee values, as a percentage of exposure by country, can be compared to the guarantee fees charged by the CCC. These actuarial fee equivalents, reported in table 2, vary widely across countries. The actuarial fee equivalent is the "fair" or average rate which would need to be charged to cover the expected loss for that specific country's case. Note that most of the actuarially fair fees for exposure are less than the average rate charged by the CCC of approximately $0.61 per $100 coverage during the year used in this example. Thus, in the cases where the actual rates charged are greater than the expected costs, there is no implied subsidy.[6] If the fee rates charged were less than the expected costs, the differences could be viewed as a form of subsidized insurance that could be subject to additional scrutiny under WTO rules.

[6] In the vernacular of the WTO agreements, this result would argue that there would be no contribution under the amber box and no charge against the allowable maximum subsidy (AMS) account from the guarantee in this case.

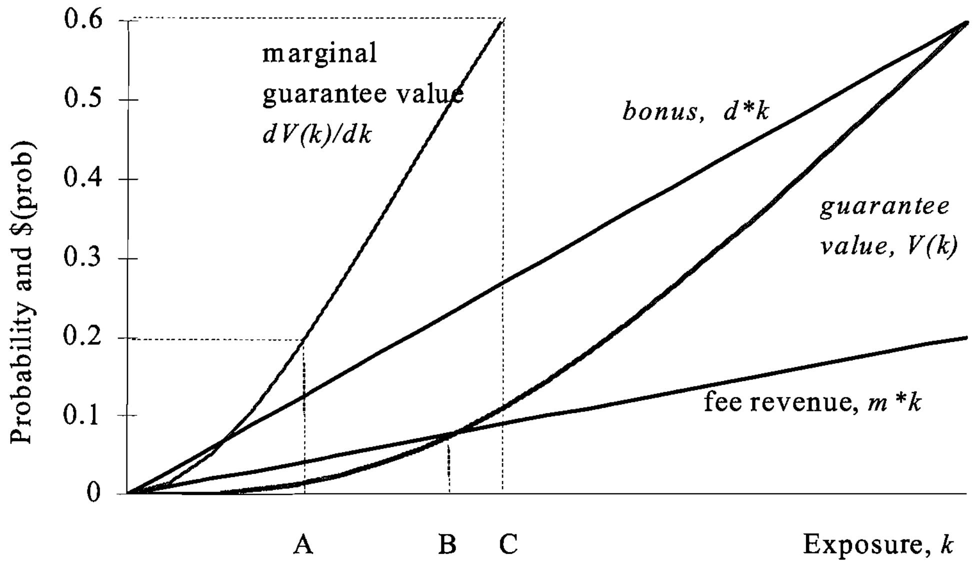

Figure 1. Guarantee program components and optimal exposure level, guarantee value, probability of default, fee revenue, and bonus levels

Framework to Evaluate Optimal Exposure

Determination of the conditions which represent "optimal" exposure requires that the costs and benefits of differing guarantee activities be ascertained. To evaluate optimal exposure, first consider a guarantor with a single recipient country. The guarantor's preferred position depends on the additionality and slippage of the guarantee program, and the impact of changes in prices of commodities sold through non-program channels. For convenience, refer to the net economic benefit per dollar guaranteed as the "bonus rate," or d. The total bonus, analogous to a consumer and producer surplus measure, is the bonus rate, d, times exposure or coverage volume, k. Figure 1 depicts the relationships among the total bonus, the guarantee value, fee revenue, marginal guarantee value, and probability of default. Exposure is on the horizontal axis, and the probability of default is on the vertical axis along with the associated dollar values of the guarantee, bonus, and fees. The total bonus is shown as the upper straight line with a slope, in this example set to 0.6. The lower of the curved lines represents the value of a guarantee as a function of its exposure, $V(k)$, which increases at an increasing rate as the coverage (strike price) increases.

In the single-country case, the guarantor's choice variables are the fee rate and exposure level to set for the recipient. For convenience, assume the fee rate, m, is set constant at some fraction of the exposure level, and is less than the bonus rate. The fee revenue, equal to $m*k$, is depicted by the lower straight line in figure 1, with a slope of 0.2 in this example. For given fee and bonus rates, the choice of optimal guarantee coverage can be reduced to the choice of guarantee coverage amount.

Importantly, different policy objectives lead to different outcomes under a fixed fee rate. If the objective of the guarantor were to maximize net fee revenue, the guarantor would set the exposure at point A in figure 1, where the marginal fee revenue equals the marginal cost of the guarantee. At point A, the slope of the guarantee value equals 0.2, and that exposure level results in the maximum expected profit from fees. Point A also maximizes guarantee program benefits in the absence of any bonus. If the objective were instead to maximize the net bonus from the program, the guarantor would choose exposure at point C in figure 1. At C, the marginal cost associated with the guarantee value equals the marginal bonus of 0.6, and the guarantor would operate at an expected cost that exceeds fee revenues. Such behavior would maximize the welfare distributed back to producers, less any actuarial costs of the guarantees. This case would require a subsidy at the program level using a mechanism similar to that currently employed in the GSM programs. Finally, if the objective were to maximize the bonus subject to operating at the breakeven point with regard to fee revenue and expected cost, the guarantor would operate at B. At point B, the fee revenue equals the expected cost or guarantee value. Point B is also equivalent to the solution a private supplier would reach in a competitive market.

For given fee and bonus rates, there is a unique solution for the single-country case. Changing either the fee or bonus rate does not change the nature of the optimal solutions, but does affect their locations. Thus, by changing the fee rate (bonus rate), a set of points can be found for which the slope of the guarantee value equals the fee rate (bonus rate) across exposure levels. Another set of points exists where the guarantee value equals fee revenue across exposure levels. These facts are useful in the construction of risk-efficient portfolios across multiple countries.

Regardless of the guarantor's objective, the marginal guarantee value is needed to determine the optimal exposure level. From the general guarantee valuation formula in equation (1), it can be shown that the partial derivative of $V(k)$ with respect to k reduces to $F(k)$, the cumulative distribution function at k. Similarly, if the repayment capacity follows a beta distribution, then from equation (6):

$$\frac{\partial V_j(k \mid a, b, u)}{\partial k} = I_k(a, b). \tag{7}$$

In the general case, a guarantor has multiple potential guarantee recipients comprising a portfolio. To be efficient, a guarantor would seek to extend as many guarantees as possible for a given level of liability. The efficient allocation objective function for a guarantor is therefore denoted by:

$$\max_{k_j} \left\{ \pi = \sum_{j=1}^{J} d_j k_j + \sum_{j=1}^{J} m_j k_j - \sum_{j=1}^{J} V_j(k_j) \right\}, \tag{8}$$

subject to the given liability constraint, where d_j is the bonus rate for country j, k_j is the exposure to country j, m_j is the fee rate for country j, and $V(k_j)$ is the guarantee value for country j. Assuming the existence of single fee and bonus rates for each guarantee recipient, m and d respectively, the first-order conditions for the guarantor are:

$$\pi' = d + m - \frac{dV_j(k)}{dk} = d + m - F_j(k_j) = 0, \quad \forall \text{ recipients } j. \tag{9}$$

The second-order condition assuring a maximum is satisfied because $\pi'' = -f_j(k_j)$, which is strictly negative.

The efficient set of portfolios can be located in total guarantee value-total exposure space as the envelope of solutions to the problem of maximizing exposure for every given guarantee value, permitting optimal reallocation of country exposure within each portfolio. In other words, the efficient allocation at a given total liability rate is found, and then the total liability incrementally increased and resolved, and so on, to trace out the set of points defining the efficient frontier.

To describe the guarantee portfolio at a point on the frontier, define total exposure, TE, as the sum of exposure across recipients, or:

$$TE = \sum_{j=1}^{J} E_j, \tag{10}$$

and define guarantee liability, GL, as the sum of the guarantee values across recipients, or:

$$GL = \sum_{j=1}^{J} V_j(E_j). \tag{11}$$

Political preference functions dictate the acceptable tradeoff between total exposure and guarantee liability. These preference functions are assumed to increase in the directions of higher total exposure and of lower guarantee liability. For a given objective, the portfolio frontier is obtained by mapping points where optimal total exposure is measured against its corresponding guarantee liability. For example, if the objective is to maximize fee profits, then there is a set of optimal outcomes corresponding to any given fee rate. The frontier thus constructed gives the most cost-effective combination of total exposure and guarantee liability associated with different fee rates. The point of tangency between the frontier and the political preference functions then identifies the optimal portfolio.

Identification of Country Allocations at Points on the Efficient Frontier

The exposure level for a given country on the risk-efficient frontier can be identified by finding the solution to the inverse of the cumulative distribution function at a given revenue rate. Once the optimal exposure level is known, the guarantee value for that exposure level is computed. The process is repeated for differing levels of the cumulative probability of the repayment capacity across countries, and the risk-efficient frontier of allocations identified.

To demonstrate, consider a revenue rate equal to 1% of the exposure from a 0.5% fee rate, and a 0.5% bonus rate, under an objective to maximize total net revenue (note, this set of values was chosen to generate an efficient allocation example that is of similar, but slightly lower exposure than the actual case). To maximize total net revenue, the guarantor would identify the exposure levels such that the cumulative probability of default for each country equals 1%. Under this example set of conditions, the resulting optimal exposure levels are found for each country at their respective inverse cumulative default functions at 1% default probabilities. The results for the different countries are shown in table 3, along with the corresponding guarantee values. For this set of efficient allocations, each country has the same probability of the loan default, but the costs of the guarantees and exposure levels differ across countries.

Table 3. Portfolio on Risk-Efficient Frontier with 1% Revenue Rate

Guarantee Recipient	Optimal Exposure ($)	Guarantee Value ($)	Exposure Share (ES_j) (%)	Risk Share (RS_j) (%)	Fee Equivalent ($)
Algeria	697,350,502	486,039	9.6	44.7	0.070
Brazil	128,144,890	4,177	1.8	0.4	0.003
Egypt	210,591,942	6,640	2.9	0.6	0.003
Indonesia	214,388,847	214,731	3.0	19.7	0.100
Jordan	52,872,941	926	0.7	0.1	0.002
South Korea	355,844,164	17,265	4.9	1.6	0.005
Mexico	3,817,224,026	135,433	52.6	12.5	0.004
Morocco	407,472,253	9,922	5.6	0.9	0.002
Pakistan	741,112,232	30,099	10.2	2.8	0.004
Romania	123,247,504	19,222	1.7	1.8	0.016
Russia	36,034,584	30,792	0.5	2.8	0.085
Sri Lanka	77,348,709	64,401	1.1	5.9	0.083
Tunisia	220,007,896	42,422	3.0	3.9	0.019
Turkey	180,127,263	25,329	2.5	2.3	0.014
Total	$7,261,767,755	$1,087,398	100%	100%	

On the risk-efficient frontier presented in table 3, Algeria has an optimal exposure level of $697 million. Fee revenue equals $3.4 million with expected bonus of $3.4 million. The expected cost of the guarantee is $486,039. Thus, the net fee revenue (fee revenue less expected cost) would be slightly less than $3 million. Consider also the fee equivalent amounts in the last column of table 3. The fee equivalent is the ratio of the guarantee value to the optimal exposure, and reflects the dollar cost per $100 of exposure. While under the assumed fee rate Algeria would pay $0.50 for every $100 of exposure, the coverage has an expected cost of only $0.07 for every $100 of exposure.

Interestingly, in comparing tables 2 and 3, the relative allocations follow a pattern that generally corresponds to the actual allocations, with reductions to country allocations for which the fee equivalents under the actual allocations were higher. The efficient, and slightly smaller reallocations presented in table 3 result in aggregate guarantee values that drop by about $8.4 million on a reduction in guarantee exposure of about $252 million, or an average rate on the incremental guarantees of just over 3.3%.

Comparing the different countries, the exposure and risk shares (ES_j and RS_j) vary substantially, but are risk-efficient by construction (i.e., no greater coverage at the expected cost is possible, or equivalently, no lower expected cost at the given coverage is possible). Mexico has the largest exposure share at over 50%. At the same time, Algeria has the largest share of the risk in terms of the value of its guarantee relative to guarantee liability, at slightly less than 45%. These results contrast with those found by the U.S. GAO (1995a), which indicate Russia's large share of the GSM risk is undesirable in a portfolio context. However, portfolios on the frontier share the feature that all guarantee recipients represent an equivalent cost at the margin—i.e., for an additional dollar of exposure, the expected guarantee liability is equal, regardless of the recipient.

Once an efficient portfolio is obtained, the given rate can be changed and other efficient portfolios identified. Because the rates equal the slope of the cumulative distribution function of the repayment capacity for each country, the slope of the frontier

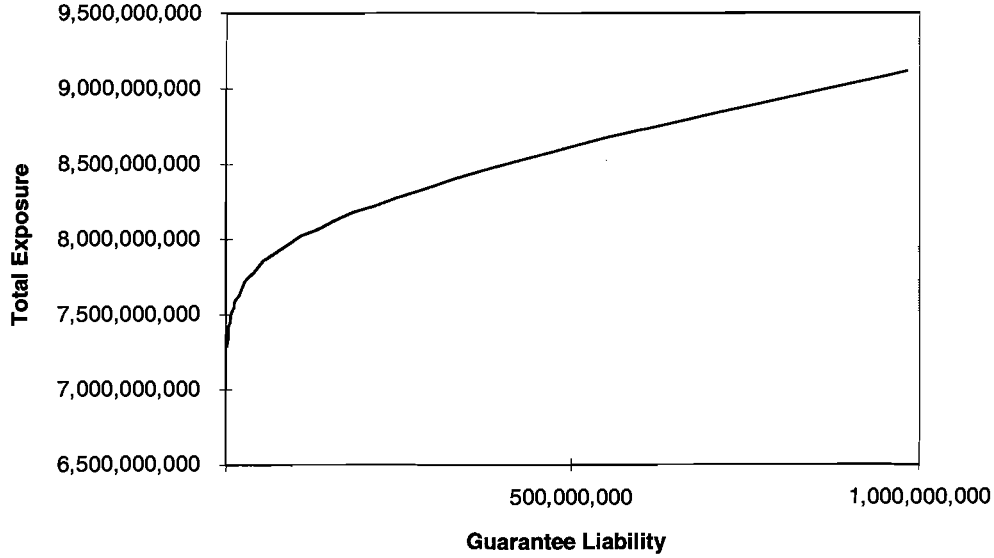

Figure 2. Efficient guarantee portfolio frontier

equals the fee rate at each point. Hence, the frontier also shows the effects on the optimal portfolio for different fee rates. If the fee rate is lower, for instance as a result of a political mandate, then the optimal portfolio would be smaller in terms of total exposure and guarantee liability. If higher fee rates could be charged, then a larger total exposure level could be obtained while remaining risk-efficient. Applied differently, the frontier relationship can be used to set the fee rate (or bonus rate) necessary to be optimal if the guarantor or regulators mandated a specific total exposure level. Guarantee liability is small for low revenue rates, reflecting the low likelihood of any guarantee payments for the corresponding exposure. As the revenue rate or slope increases, total exposure increases at a constant rate, while guarantee liability increases at an increasing rate. At the highest revenue rates, the marginal cost of extending additional guarantees approaches one—a fact that also helps in understanding the sensitivity of the fee rates to size of exposure.

The risk-efficient frontier, shown in figure 2, has the shape expected of a combination of option values. Because all the guarantees are convex in exposure, their weighted average, inverted, is necessarily concave. Evident in the graph is the relation whereby guarantee liability (*GL*) approaches zero as total exposure (*TE*) decreases. The slope approaches one as *TE* increases (although the differing scales on the axes in figure 2 somewhat mask that relationship). All possible portfolios are located on or below this frontier with either a higher guarantee liability or lower total exposure than the portfolios on the frontier.[7]

[7] The data from actual GSM country allocations in 1996 were also examined using equation (6) to assess the efficiency of that year's allocation. The specific allocation was dominated by a section of the efficient frontier which would have permitted approximately \$38 million additional guarantee volume at the same expected loss, or a reduction of the expected loss of approximately \$1.4 million at the same total coverage from optimal reallocation of guarantees in the portfolio. An expansion path can also be constructed by varying the fee rates and computing efficient portfolio allocations at each level. Importantly, the results of this exercise demonstrate that the risk shares are the same as the exposure shares in this case—a feature identified as desirable by the U.S. GAO (1995a), although risk-efficiency was not the GAO's motivation for suggesting that this condition would be preferred. Results of this exercise are available from the authors upon request.

Summary and Conclusions

This study uses the commercially most important sources of country risk-rating data to develop estimates of measures of the repayment capacity distributions for countries in the GSM portfolio, and analyzes the expected costs and risk efficiency of alternative country allocations. The methods demonstrated permit a guarantor to quantify the tradeoff between the exposure and liability of guarantees in a portfolio. The results show that a guarantor seeking to maximize either net fee or bonus revenue would do so by extending guarantees whereby the marginal value of guarantees is equated across recipients. A risk-efficient frontier for guarantee portfolios will result where the cost-effective exposure levels depend on the available fee or bonus rates, and the marginal cost of guarantees is equated across countries. A guarantor may choose to maximize fee revenue in excess of expected default costs, or maximize the total bonus net of expected default costs, or pursue the equivalent of a private guarantor's objective. Regardless of choice, the methods provide a means of identifying the optimal portfolio for the particular objective pursued.

Direct applications include use to demonstrate subsidy levels embedded in the governmentally issued guarantees. As shown in the evaluation, the implied actuarial costs were, in aggregate, slightly below the fee rates being charged, indicating that no trade subsidies were created through the program. As conditions change (either country risks, or fee rates), the methods provide more easily calculated and defended estimates of subsidy levels and other information needed for budget scoring and trade debate purposes.

Both internal and external changes to the guarantee programs would impact the risk-efficient portfolio composition. Different fee and/or bonus rates likewise imply that different portfolios on the efficient frontier would be optimal. And, achieving different levels of total exposure may require a different fee rate, or an acceptance of losses on fees if other benefits are considered. In any case, the methods herein could also assist in assessing the impact of other changes in program design such as changing the subordination method or deductible levels.[8]

Regardless of the guarantor's objective or constraints, rational methods exist to manage guarantees and portfolios in a risk-efficient manner. Hence, the analysis is part of a larger scope of problems associated with managing credit guarantee programs and overall governmental budget exposure.

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[8] For example, the model was used to assess deductible rates with fractional coverage options—both commonly proposed redesigns. As expected, full coverage on a fraction of the loan is less expensive than equivalent fractional coverage on the full loan, consistent with standard option pricing results. Full results on the rates of tradeoff are available from the authors upon request.

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Appendix: Derivation of Text Equation (6), Guarantee Value Form

Let $z = c/u$. Then the unknown repayment capacity distribution can be written as:

$$f(z) = \frac{1}{B(a,b)} z^{a-1}(1-z)^{b-1}, \quad 0 \le z \le 1.$$

The density includes the beta function defined as:

$$B(a,b) = \int_0^1 y^{a-1}(1-y)^{b-1}dy = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}.$$

The standardized beta density function is unimodal if $a, b > 0$, and uniform if $a = b = 1$. The incomplete beta function is defined as:

$$B_x(a,b) = \int_0^x y^{a-1}(1-y)^{b-1}dy,$$

where $a, b > 0$, and $0 < x < 1$. The incomplete beta function ratio is defined as:

$$I_x(a,b) = \frac{B_x(a,b)}{B(a,b)},$$

which is also equivalent to the beta cumulative distribution function. These definitions allow the guarantee value defined for $x = k/u$ to be restated as:

$$V(x) = \int_0^x f(z)(x - z)dz = xF(z) - \int_0^x zf(z)dz = xI_x(a, b) - \int_0^x z\frac{1}{B(a, b)}z^{a-1}(1 - z)^{b-1}dz$$

$$= xI_x(a, b) - \frac{1}{B(a, b)}\int_0^x z^a(1 - z)^{b-1}dz = xI_x(a, b) - \frac{1}{B(a, b)}B_x(a + 1, b)$$

$$= xI_x(a, b) - \frac{B(a + 1, b)}{B(a, b)}I_x(a + 1, b).$$

Substituting k/u for x and rescaling the density values by u results in:

$$V(k) = \frac{k}{u}I_x(a, b)u - \frac{B(a + 1, b)}{B(a, b)}I_x(a + 1, b)u.$$

Canceling the u terms in the first part results in the reduced form given by text equation (6).