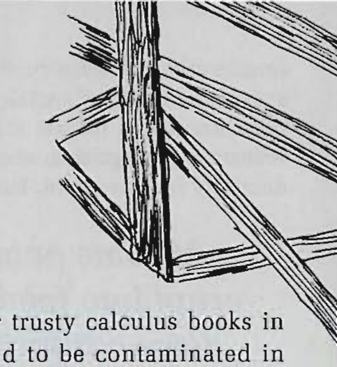
On Farmers Who Solve Equations



by Richard A. Levins

This sense of the word (abstractness) is important, and the logicians are quite right to stress it, since it embodies a truism which a good many people who ought to know better are apt to forget. It is quite common, for example, for an astronomer or a physicist to claim that he has found a 'mathematical proof' that the physical universe must behave in a particular way. All such claims, if interpreted literally, are strictly nonsense. It cannot be possible to prove mathematically that there will be an eclipse to-morrow, because eclipses, and other physical phenomena, do not form part of the abstract world of mathematics; and this, I suppose, all astronomers would admit when pressed, however many eclipses they may have predicted correctly.

G. H. Hardy (p. 47)

When I first read it, I thought this was the most radical analysis of farmer behavior in the history of human thought:

" Farm households, therefore, solve

(1) $$\max_{c, L_1, L_2} \int_0^\infty u\big(c(t), H - L_1(t) - L_2(t)\big)e^{-\delta t}\,dt,$$

subject to

(i) $$\dot{E} = \rho\big(E(t), L_1(t), v\big) + wL_2(t) + y(t) - c(t)$$

(ii) $$E(0) = \bar{E}.$$ "

To my great surprise, I later discovered it is a rather common notion among certain researchers that farmers routinely tackle even the most intractable equations. For example, the 1988 AAEA Outstanding Journal Article award went to the developer of an econometric model which required data "assumed to be generated by farmers solving a single-period maximization problem."

Richard A. Levins is Extension Farm Management Specialist and Associate Professor with the Department of Agricultural and Applied Economics, University of Minnesota, St. Paul.

Nor do only farmers reach for their trusty calculus books in times of crisis. When milk was found to be contaminated in Hawaii, the hapless citizens of Oahu found their problem to be one of solving this beauty:

$$\text{" } \max L = U\big(X_1(Z_1(N)), X_2\big) + \lambda(I - P_1X_1 - P_2X_2 - CN) \text{ "}$$

Even though many agricultural economists assume farmers solve equations, none report names and addresses for those farmers. To make matters worse, the farmers I work with are either not of the equation-solving variety or too modest to admit to being so. This being the case, I am left with little choice but to adopt the working hypothesis that farmers do not really solve these equations. I offer my apologies to those who claim they do, however, and encourage them to keep a sharp eye out for these most interesting of life forms. My task here will be the more modest one of investigating the wisdom of assuming farmers finish up a hard day in the fields with a bout of equation solving.

Friedman's Defense

I suspect that most would say, with a quick tip of the hat to Milton Friedman's defense of "positive economics", that farmers merely act "as if" they solve equations. Most economists have at one time or another spent some time in Friedman's world where leaves on a tree act "as if each leaf deliberately sought to maximize the amount of sunlight it receives, given the position of its neighbors, as if it knew the physical laws determining the amount of sunlight that would be received in various positions and could move rapidly and instantaneously from any one position to any other desired and unoccupied position". This world, too, is one where an expert billiard player acts "as if he knew the complicated mathematical formulas that would give the optimum directions of travel, could estimate accurately by eye the angles, etc., describing the location of the balls, could make lightning calculations from the formulas, and could then make the balls travel in the direction indicated by the formulas."

"As if" turns out to be a powerful concept in the hands of a positive economist. In effect, it frees one from mundane concerns over the truth of assumptions. What matters is only that results coincide with observations of reality. We, therefore, need not worry about whether farmers solve equations, so long as they act as if they do.

Gunfire and Thunder

Let me add one more "as if" to Friedman's collection: "The gunfire pierced the night as if it were thundering". It is often descriptive to compare two unlike things such as gunfire and thunder. Properly applied, "as if" can add richness in normal conversation; beauty in poetry.

But the "as if" belongs to the world of individual perception. While I might think gunfire sounds like thunder, you may disagree.

If so, the value of "as if" in communicating with you is clearly limited. Whether gunfire really sounds like thunder may even become a point of contention and thereby seriously hamper our originally-intended conversation. We must, therefore, choose our words to reflect the common experience of many if clarity is our goal. To say that leaves know physics, that billiard players mimic high speed computers, or that farmers solve equations does little, in my opinion, to foster clear conversation.

While authors may choose "as ifs" as they see fit, they may not use them beyond their conventional descriptive limits. "As if" provides no basis whatsoever for logical analysis. Mathematician G. H. Hardy was moved by a particularly fine piece of poetry to comment: "Could lines be better, and could ideas be at once more trite and more false?" It makes good sense to say that gunfire sounds like thunder, but no sense whatsoever to infer that the presence of gunfire means rainfall is imminent.

Let me consider more closely the argument from gunfire to thunder to rainfall. I write it as follows:

(1) Gunfire sounds as if it is thundering.
(2) Thunder is associated with rainfall.
(3) Gunfire sounds as if it is raining.

What is wrong? The key step in the argument is that "as if" is used to introduce a theory (meteorology) which has nothing to do with gunfire. And it doesn't stop there. All sorts of other weather-related claims about gunfire can be generated. Most will be absurd, even though a few, like "gunfire looks as if it were lightning," may be appealing. But none of these conclusions, absurd or otherwise, can claim validity because meteorology was used. Meteorology has nothing to say about gunfire, and no "as if" is going to change that.

This digression on gunfire now complete, I return to the connection "as if" provides between mathematics and farmers who solve equations. A typical argument might go like this:

(1) Farmers act as if they solve a particular equation.
(2) We can derive some result from the equation.
(3) Farmers act as if the result holds.

In a particularly striking example of this type of reasoning, one article I read claimed that an equation derived therein "implies that farmers work both on and off the farm." That farmers work on and off the farm is obvious; that mathematics has anything to say about where farmers work is far less obvious.

The farmer syllogism has exactly the same structure as the gunfire/thunder example. It begins with "as if", then assumes that a theory appropriate for one part of the "as if" (equation solving)

Author's Note

Some readers of an earlier version of this paper interpreted it as an attack on certain individuals or journals. I had no such intentions. To avoid further confusion concerning my intentions in writing this paper, I have chosen to use some quotes from our professional literature without citing their authors or sources.

Jay Coggins, Earl Fuller, Winston Rego, and Burt Sundquist provided valuable review comments during the development of an earlier version of this paper. In addition, dozens of colleagues made thoughtful remarks when the earlier version was offered as a staff paper. Many of these people did not fully agree with all aspects of this paper, however, and all responsibility for the content of the present paper lies solely with the author.

applies to the second part (farmer behavior). Mathematics, rather than meteorology, is then used to derive new results from the equation farmers "solve." The argument ends by concluding one of the new mathematical results also applies to farmers.

The implications of using mathematics in the farmer example and meteorology in the gunfire example are the same. The conclusion in the farmer syllogism may or may not be true. The fact that it was supposedly derived mathematically tells us nothing except that we should be suspicious of a conclusion drawn so inappropriately.

False Rigor

In short, the mathematics in the farmer argument, no matter how sophisticated, contributes nothing. This conclusion, while perhaps a bit unsettling for research in agricultural economics, would not bother most mathematicians. To quote Bertrand Russell:

> *We are prepared to say that one and one are two, but not that Socrates and Plato are two, because, in our capacity of logicians or pure mathematicians, we have never heard of Socrates and Plato. A world in which there were no such individuals would still be a world in which one and one are two. It is not open to us, as pure mathematicians or logicians, to mention anything at all, because, if we do so, we introduce something irrelevant and not formal. (pp. 196-7)*

I now turn to this question: "If Bertrand Russell was unwilling to use mathematics in mentioning anything at all, why is our profession so hell-bent on using it to mention virtually everything?"

One often hears that mathematics adds rigor to arguments. But I have shown that our use of mathematics depends on the least rigorous of all claims, the "as if". Anything, no matter how absurd, can be shown with the "as if" con game. We can "rigorously" prove absurdities—that gunfire sounds like rainfall or that farmers buy infinitely divisible tractors.

Then, too, mathematics is thought to quantify things. Granted, mathematics is comfortably at home when farmers and their products are counted, when interest rates are calculated, and when budgets are prepared. But this is not what is being done in mathematical modelling. Mathematical models use the language of mathematics to make statements about how the world works. Nothing is quantified in this process; one simply substitutes one language for another.

We also hear that mathematics simplifies analyses. For example, one article assumes that farmers "derive utility not only from consumption during their current lives but also from future descendants future consumption" on into infinity. Why make buying a candy bar such a complicated decision? The answer: "for analytic simplicity." And, too, why does another article assume farmers are solving single-period maximization problems? Again, "to simplify the presentation".

To say that discussing the farm economy in mathematical terms adds simplicity—especially when the mathematics is so complicated that only a very few can participate—is curious, to say the least. There is one sense, however, in which simplicity does arise from the passion for mathematics: the subject matter content of arguments and ensuing journal articles becomes very simple, indeed. Complex mathematics does not make for complex statements about reality. In fact, Russell seems to be saying the opposite. The more rigor we demand from mathematics, the more we sacrifice any connection with reality, and the fewer non-trivial observations we can make.

Take, for example, still another article which leads the reader through many pages bristling with equations. The authors come to the following important (their term) conclusion: "Farmers do not face a perfectly elastic supply of funds or credit upon which they can effortlessly draw to finance their production decisions." Somehow, one supposes, we can now feel more comfortable holding what is perfectly obvious to everyone. But maybe not; the authors feel that "a conclusive resolution of the issue awaits a more thorough empirical study."

As another example, one article challenged the venerable "law of supply" by pointing out that higher product prices may also bring about more price risk. This can cause risk averse producers to diversify into other crops in spite of the higher prices. Particularly since the authors only claim "may" for their statement, the results appear to need no further defense (save for a possible remark or two on why such trivial matters need to be brought to the attention of those interested in the economics of agriculture). But instead of ending their article at the end of the second paragraph, the authors take the reader on a 10-page mathematical steeplechase, only to conclude what was already stated quite nicely in the introduction.

An Alternative

Is there an alternative to mathematics? One alternative, sometimes overlooked, but a good candidate, is plain English. Even the most enthusiastic math-addicts resort to an occasional "intuitive explanation" of their models. These lapses into natural language seem intended to clarify the equations. What, in fact, they are clarifying is what the authors would be saying if they weren't using the mathematics.

Introducing natural language, through the back door of intuitive explanations, has its problems. First and foremost, the intuitive explanations and mathematics are not related in any formal way. For example, one article analyzes farm profits which are "twice continuously differentiable and convex in v, nondecreasing in output prices, nonincreasing in input prices, positively linearly homogeneous in v, and nondecreasing in L1 and K". Later, the same article provides us with the intuition that "for any given level of wealth, farmers maximize their net farm income by choosing an optimal combination of outputs, inputs, and investment." To claim that this intuitive explanation of farmer behavior is true is one thing; to claim it is somehow inherent in the equations is quite another.

But why bother with equations, anyway? The so-called intuitive explanation contains everything necessary to reach the practical conclusions of most analyses. Natural language and a little elementary logic can serve the purposes of agricultural economists quite nicely.

Farmers don't solve equations. Perhaps those who study them shouldn't either. C

For More Reading

Essays in Positive Economics **by Milton Friedman and published by the University of Chicago Press, Chicago, in 1953.**

A Mathematician's Apology **by G. H. Hardy and published by Cambridge University Press, London, in 1941.**

Introduction to Mathematical Philosophy **by B. Russell and published by G. Allen and Unwin, London, in 1967.**