

# **Urban Sprawl and Obesity**

**Stephanie Bernell**

*Department of Public Health  
Oregon State University*

**Andrew J. Plantinga**

*Department of Agricultural and Resource Economics  
Oregon State University*

**JunJie Wu**

*Department of Agricultural and Resource Economics  
Oregon State University*

May 7, 2003

Paper Prepared for Presentation at the American Agricultural Economics Association Meeting,  
Montreal, Quebec, July 27-30, 2003.

## Urban Sprawl and Obesity

Abstract: In the U.S., urban sprawl and the rise in obesity rates have been two powerful trends during the latter half of the 20<sup>th</sup> century. Previous empirical work has found that obesity rates are influenced by labor market outcomes that are fundamentally shaped by the spatial pattern of developed land. We examine these potential linkages in an urban spatial model augmented to include time allocation and weight. Residents maximize utility defined over housing, weight, and food subject to a fixed time budget allocated to commuting, calorie expenditure, and work. We examine how weight is affected by commuting distance, food prices, and the rate of calorie expenditure; how a reduction in transportation costs affects weight throughout the city; and how initial weight affects location decisions. We identify, and explore the significance of, the conditions under which weight gain is associated with common features of sprawl.

## I. Introduction

Environmental economists have long been concerned with the effects of environmental quality on human health. The benefits of reducing pollution emissions, for example, are often linked to improvements in human health outcomes such as reduced cancer risks. However, the effects of resource use on health have received relatively little attention in the economics literature. In this study, we explore a possible connection between land use and health. Specifically, we investigate how the spatial configuration of land uses can affect weight and, particularly, whether and how “sprawl” patterns of land development are related to obesity. Obesity is a major public health concern. It is associated with heart disease, many types of cancer, type 2 diabetes, and a number of other health problems. According to the Surgeon General, obesity is associated with 300,000 deaths each year in the United States and is second only to tobacco use as a cause of premature death. The annual economic cost of obesity has been estimated at \$99.2 billion (Wolf and Colditz, 1998).

Obesity is defined as an excessively high amount of body fat or adipose tissue in relation to lean body mass (National Research Council, 1989; Stunkard and Wadden, 1993). In 2000, 38.8 million American adults—approximately 20 percent of the adult population—met the classification of obesity, defined as having a body mass index score of 30 or more.<sup>1</sup> This reflects a 61 percent increase in the prevalence of obesity since 1991. An additional 40 percent of adults are overweight.<sup>2</sup> According to data from the U.S. Centers for Disease Control and Prevention (CDC), the obesity epidemic spread rapidly during the 1990s across all states, regions, and demographic groups in the U.S. The highest increase occurred among the youngest ages (18- to

---

<sup>1</sup> The body mass index is computed as weight in kilograms divided by height in meters squared. The authors of this paper have body mass indices of \_\_, 21.7, and \_\_, respectively.

<sup>2</sup> Overweight refers to increased body weight in relation to height, when compared to some standard of acceptable or desirable weight (National Research Council, 1989; Stunkard and Wadden, 1993). Overweight may or may not be due to increases in body fat. It may also be due to an increase in lean muscle.

29-year-olds), people with some college education, and people of Hispanic ethnicity (Table 1). By region, the largest increases were seen in the South, with a 67% increase in the number of obese people (Table 2). Among states, Georgia had the largest increase (101%).

At the most basic level, the cause of obesity is simple. The First Law of Thermodynamics states that energy is conserved, implying a human body that takes in more calories (a measure of energy) than it expends in support of bodily functions must store the additional calories.<sup>3</sup> Far more complicated are the human behavioral factors that result in obesity. A number of recent economic analyses attempt to explain the observed rise in obesity rates.<sup>4</sup> Philipson and Posner (1999) emphasize the role of technological change in lowering the real price of calories and the physical expenditure of calories per hour worked in both market and household production. Ruhm (2000) conducts an empirical investigation of the influence of macroeconomic conditions on obesity rates and other health indicators. He finds evidence of an inverse relationship between obesity and state-level unemployment and suggests the explanation lies with the opportunity costs of time. During economic downturns, the costs of time fall, causing people to allocate more time to exercise and the preparation of healthy meals.

Chou, Grossman, and Saffer (2001) hypothesize that recent labor market trends—the decline in real incomes of some groups and increases in household hours worked and labor force participation rates—have reduced the time spent on food preparation at home and increased the consumption of inexpensive and highly caloric convenience foods (e.g., fast food). In addition, as households have allocated more time to work, less time and energy has been available for active leisure, reducing calorie expenditure. Finally, these authors conjecture that anti-smoking

---

<sup>3</sup> Studies show that genetic factors contribute to overweight and obesity. However, genes do not directly cause obesity but rather increase the susceptibility to obesity in individuals exposed to an environment which promotes behaviors causing weight gain (Hill and Peters 1998).

<sup>4</sup> See Chou, Grossman, and Saffer (2001) for a more complete review of economic studies of obesity.

campaigns and related increases in cigarette taxes have induced substitution of calories for nicotine. An empirical analysis provides general support for these hypotheses. In particular, variables measuring the prevalence of fast-food restaurants, cigarette prices, and weekly hours of work are found to have positive effects on obesity rates.

Sprawl patterns of land development are a common feature of the urban landscape in the U.S. One measure of sprawl is development density—number of people per unit of developed land. In this U.S., development densities have fallen over the past two decades and the data suggest this trend is accelerating.<sup>5</sup> A recent CDC report (Jackson and Kockitzky, 2002) examines the connection between the built environment and human health, and suggests that sprawl has also contributed to the recent rise in obesity rates. The authors emphasize that sprawl, which they define as “uncontrolled, poorly planned, low-density, and single-use community growth,” increases commuting times, requires the use of automobiles at the expense of walking and bicycling, and does not adequately provide facilities such as parks that permit and encourage physical activity. National Center for Health Statistics (2001) reports obesity rates in U.S. adults by gender, region, and urbanization level. Particularly for men, the dominant pattern is for obesity rates to rise as the level of urbanization falls. These findings are at most suggestive of a connection between sprawl and obesity because urbanization levels do not directly correspond to metrics such as development density<sup>6</sup> and other factors that influence obesity rates (income, education level, etc.) are not controlled for in the analysis.

---

<sup>5</sup> Combining data from the U.S. Department of Agriculture’s National Resources Inventory and population estimates from the Bureau of Census, we calculate that people per acre of developed land in the U.S. was 3.11 in 1982, 3.00 in 1987, 2.89 in 1992, and 2.69 in 1997.

<sup>6</sup> The Health and Human Services study classifies U.S. counties into five urbanization categories: large central, large fringe, and small metropolitan areas, and nonmetropolitan areas with and without a city with greater than 10 thousand population. These categories are defined by population levels rather than by developed land area or population densities. A large central metropolitan county, for example, contains all or part of the largest central city of a metropolitan statistical area with greater than 1 million population.

In general, the spatial configuration of developed land affects the tradeoffs that households make between housing, commuting distances, time allocated to work and leisure, and consumption of food and other goods. Thus, we would expect sprawl to influence obesity rates through many of the same channels identified in previous empirical work. The purpose of this paper is to formally examine the relationship between sprawl and obesity in a spatial economic model. Our analysis draws on modeling approaches in the urban economics, labor economics, and public health fields. We analyze a monocentric city model in which households maximize utility defined over housing, weight, and calories. Their choices of housing, food consumption, and calorie expenditure are constrained by a fixed time budget that must be allocated to work, commuting, and leisure. Income, weight, and location are endogenously determined.

The next section describes the basic structure of the model. In Section III, we solve for the static spatial equilibrium and describe how we introduce sprawl into the model. Section IV presents the central results of the paper. We consider how weight is affected by distance to the central business district, food prices, and the rate of calorie expenditure; how sprawl affects weight throughout the city; and how initial weight affects location decisions. Section V presents discussion.

## II. Model Structure

The phenomena of urban sprawl and obesity are analyzed with an urban spatial model augmented to include time allocation and health impacts. Residents of the city consume housing ( $H$ ) and calories ( $F$  for food), taking prices  $p_H$  and  $p_F$  as given. They allocate a fixed time budget ( $T$ ) to work, commuting to and from work, and leisure time. The wage rate is  $w(W_0)$ , where  $W_0$  is initial weight (discussed below). Residents work in the central business district

(CBD) and commute a distance  $x$ . Round-trip commute time is  $\beta x$  and out-of-pocket commuting costs are  $tx$ . Leisure time is used for exercise, household production, relaxation, etc., and is denoted  $\gamma E$  where  $E$  (for expenditure) equals the calories expended during leisure. Thus,  $\gamma$  measures the amount of time required to expend one calorie during leisure time.

The weight of residents ( $W$ ) is determined by their initial weight  $W_0$ , their consumption of calories, and the expenditure of calories through leisure. To satisfy the 1<sup>st</sup> Law of Thermodynamics, we must have  $W \equiv W_0 + F - E$ . In general, one would expect the marginal utility of weight to be positive or negative depending on an individual's initial weight (Philipson and Posner, 1999). If an individual is underweight, for instance, gaining weight contributes positively to utility. We will assume, however, that all residents of our city are at or above their ideal weight and, thus, that the marginal utility of weight is always negative. Summarizing, the utility of each resident is  $U(H, W, F)$ , where  $U_H > 0$ ,  $U_W < 0$ ,  $U_F > 0$ .<sup>7</sup>

For analytical tractability, we assume that utility is additively separable in housing, implying  $U_{HW} = U_{HF} = 0$ . Although it is plausible that the marginal utility of housing is independent of weight and food consumption, the marginal utility of food consumption is likely to depend on weight, and vice-versa. A heavy individual, for instance, may enjoy the marginal calorie less because of guilt arising from societal pressures to be thin, implying  $U_{FW} < 0$ . Alternatively, if obesity is a proxy for a low degree of risk aversion, the obese individual may enjoy the marginal calorie more, implying  $U_{FW} > 0$ . As we will see below, many of our results will depend on the sign and magnitude of  $U_{FW}$ .

---

<sup>7</sup> For analytical tractability, we specify a simple relationship between utility and calorie consumption. See Bouis (1996) for a model of food demand that also includes preferences for tastes and variety.

As noted above, the wage rate is specified as a function of the initial weight, where we assume  $w'(W_0) < 0$ .<sup>8</sup> The inverse relationship between obesity and wages can be explained by several theories of the labor market.<sup>9</sup> The employer discrimination model explains that employers may penalize overweight employees due to prejudice, lack of knowledge about the productivity of the obese, or the belief that obesity serves as a proxy for factors such as higher expected health care costs or lack of discipline (Moon and McLean, 1980; Everett, 1990; Cave, 1992; Martin, 1994). Furthermore, employers may be reluctant to provide firm-specific training to the overweight worker for fear that the worker will provide a low return on the investment (Everett, 1990). According to the occupational crowding model, the obese have fewer employment options because of occupational discrimination, and so must crowd into jobs with low obesity penalties. The crowding effect reduces the relative wages of the overweight (Terrell, 1992). Finally, output differentials could also exist between the obese and nonobese. Nonobese individuals would be more productive in settings in which customers prefer to be serviced by the nonobese or in jobs in which these workers have more favorable interactions with other employees. Pay differentials arise between these two types of workers as they sort themselves across occupations to take advantage of weight related productivity differentials. To the extent that this sorting is less than perfect, wage gaps would arise among the obese, with the wages of the overweight in occupations with high exposure to customers being relatively lower (Hamermesh and Biddle, 1994).

---

<sup>8</sup> While a monotonic relationship between initial weight and wage is unrealistic, we adopt this specification for analytical convenience.

<sup>9</sup> Differences in time preferences offers another explanation. If an individual assigns little value to future events, they may invest less in both their human capital and their health, with low wages and obesity as a potential outcome (Cawley, 2000). In addition, it is possible that obesity is endogenously determined in the labor market. For example, poor job performance may lead to depression and weight gain, low wages, and so forth. While econometrics tests have rejected the latter hypothesis (Pagan and Davila, 1997), a consistent finding in the empirical literature is that among professionals and blue-collar workers, physical attributes, specifically weight, significantly affect the wages of women and have no impact on the wages of men.

### III. Equilibrium and Urban Sprawl in an Open City Model

We begin by analyzing a simple static model. Substituting the weight function into the utility function, the utility maximization problem is given by:

$$\begin{aligned} \max_{H,F,E} U(H, W_0 + F - E, F) \quad s.t. \\ p_H H + p_F F + tx \leq (T - \beta x - \gamma E)w(W_0) \end{aligned} \quad (1)$$

Denoting the Lagrange multiplier on the budget constraint as  $\lambda$  and assuming an interior solution, the first-order conditions are:

$$\begin{aligned} U_H - \lambda p_H &= 0 \\ U_W + U_F - \lambda p_F &= 0 \\ -U_W - \lambda \gamma w(W_0) &= 0 \end{aligned} \quad (2)$$

The second-order sufficient conditions for utility maximization require  $U_{HH} < 0$ ,

$U_{WW} + U_{FF} + 2U_{WF} < 0$ , and  $U_{WW} < 0$ .

In an open city model with costless migration, the equilibrium utility level in the city is exogenous. Specifically, migration between cities occurs until, in equilibrium, gains from migration are exhausted and utility levels are equalized across cities. In addition, there can be no gains, in equilibrium, from changing locations within the city. Residents located farther from the CBD have higher commuting costs, both in terms of resources and time. However, in equilibrium, they are compensated for these costs with lower housing prices. To see this, consider the relationship between equilibrium housing prices and distance to the CBD. To determine the slope of the price gradient, we specify the indirect utility function,

$$\bar{U} = U[H^*, W_0 + F^* - E^*, (T - \beta x - \gamma E^*)w(W_0) - p_H H^* - tx], \quad (3)$$

where  $\bar{U}$  is the exogenous utility level, utility-maximizing values of the choice variables are denoted with a (\*), and the price of food has been normalized to 1. Then, applying the implicit function theorem to (3), we have,

$$\frac{\partial p_H}{\partial x} = -\frac{c}{H^*} < 0, \quad (4)$$

where the numerator is the marginal commuting cost ( $c = \beta w(W_0) + t$ ). The housing price represents the maximum willingness to pay, or the bid-price, by residents at each location. As in standard urban spatial analysis, we find that the bid-price gradient declines with distance to the CBD.

Our primary objective is to analyze the association between weight and urban sprawl. Sprawl is a multi-faceted and, ultimately, subjective phenomenon. While a widely accepted definition of sprawl is unavailable, many associate it with low-density, non-contiguous land development located away from traditional city centers. In this analysis, we will focus on the density of housing and its location relative to the CBD.<sup>10</sup> The simplest way to introduce housing density into our model is to define the housing variable  $H$  to equal lot size per household, or the acres of land occupied by a resident's dwelling. In this case, housing density equals  $1/H$  and greater lot sizes will correspond to lower-density development. Sprawl is also associated with patterns of development in growing urban areas, which motivates an investigation of how expansion of a city affects the weight of its residents. Growth in an open city occurs when current residents increase land consumption and (or) the city becomes relatively more attractive to migrants. Both changes occur in response to declines in transportation costs, increases in the wage level, as well as changes in other model parameters.

---

<sup>10</sup> In our model, land development is contiguous and symmetric with respect to the CBD. Wu and Plantinga (2002) analyze a similar urban spatial model and show that non-contiguous (leapfrog) development can occur with open space in the city.

#### IV. Comparative Statics Analysis

In this section, we analyze the spatial market equilibrium to determine how weight is affected by distance to the CBD, food prices, and the rate of calorie expenditure; how a reduction in transportation costs affects weight throughout the city; and how initial weight affects location decisions. In the spatial equilibrium, the utility level remains constant at  $\bar{U}$ . Thus, for a marginal change in a given parameter  $\alpha$ , the following relationship must hold,<sup>11</sup>

$$U_H \frac{\partial H}{\partial \alpha} + U_W \frac{\partial F}{\partial \alpha} - U_W \frac{\partial E}{\partial \alpha} + U_F \frac{\partial F}{\partial \alpha} = 0. \quad (5)$$

This relationship aids in the development of intuition for the comparative statics results that follow.

##### IV.A. *Weight and distance to the CBD*

We first examine how housing and food consumption, calorie expenditure, and ultimately weight vary with distance to the CBD. Varying  $x$  but holding the model parameters constant, we compute the total differential of the budget constraint and the first-order conditions in (2). The resulting equations, in matrix form, are:

$$\begin{bmatrix} 0 & -p_H & -p_F & -\gamma w(W_0) \\ -p_H & U_{HH} & 0 & 0 \\ -p_F & 0 & U_{WW} + U_{FF} + 2U_{WF} & -U_{WW} - U_{WF} \\ -\gamma w(W_0) & 0 & -U_{WW} - U_{WF} & U_{WW} \end{bmatrix} \begin{bmatrix} d\lambda \\ dH \\ dF \\ dE \end{bmatrix} = \begin{bmatrix} 0 \\ \lambda dp_H / dx \\ 0 \\ 0 \end{bmatrix} dx. \quad (6)$$

---

<sup>11</sup> If we consider a marginal change in  $W_0$ , then the term  $U_W$  also enters additively in (5).

The net effect of distance on the budget constraint is zero since the marginal increase in commuting costs exactly equals the decline in housing expenditures. Applying Cramer's rule, we have:

$$\frac{\partial H}{\partial x} = -\frac{1}{|J|} \left\{ p_F \lambda \frac{dp_H}{dx} [p_F U_{WW} + \gamma w(W_0)(U_{WW} + U_{FW})] + \gamma w(W_0) \lambda \frac{dp_H}{dx} [p_F (U_{WW} + U_{FW}) + \gamma w(W_0)(U_{WW} + U_{FF} + 2U_{FW})] \right\} \quad (7a)$$

$$\frac{\partial F}{\partial x} = \frac{1}{|J|} \left\{ p_H \lambda \frac{dp_H}{dx} [p_F U_{WW} + \gamma w(W_0)(U_{WW} + U_{FW})] \right\} \quad (7b)$$

$$\frac{\partial E}{\partial x} = \frac{1}{|J|} \left\{ p_H \lambda \frac{dp_H}{dx} [p_F (U_{WW} + U_{FW}) + \gamma w(W_0)(U_{WW} + U_{FF} + 2U_{FW})] \right\} \quad (7c)$$

$$\frac{\partial W}{\partial x} = -\frac{1}{|J|} \left\{ p_H \lambda \frac{dp_H}{dx} [\gamma w(W_0)U_{FF} + (p_F + \gamma w(W_0))U_{FW}] \right\} \quad (7d)$$

Second-order conditions require the sign of the determinant of the Jacobian matrix  $J$  to be negative. The signs of the terms in braces depend on the sign and magnitude of  $U_{FW}$ .

If  $U_{FW} \leq 0$ , the comparative statics results in (7) have unambiguous signs:  $\partial H / \partial x > 0$ ,  $\partial F / \partial x < 0$ ,  $\partial E / \partial x < 0$ , and  $\partial W / \partial x > 0$ . Consider, first, the case in which  $U_{FW} = 0$ . At greater distances from the CBD, residents spend more time commuting, leaving less time for work and leisure. As a result, calorie expenditure declines with distance and, all else equal, this raises weight. Residents can offset the utility-reducing effects of greater weight by reducing food consumption. However, because residents also derive utility directly from food, they do not completely offset the weight effects of reduced calorie expenditure (i.e.,  $|\partial E / \partial x| > |\partial F / \partial x|$ ),<sup>12</sup> and the effect of distance on weight is positive. As with commuting costs, residents at greater distances from the CBD are compensated for greater weight with lower housing prices and more

<sup>12</sup> The offset becomes complete as the direct utility effect of food consumption is diminished. Specifically,  $\partial F / \partial x$  converges to  $\partial E / \partial x$  as  $U_{FF}$  and  $U_{FW}$  go to zero.

housing. Thus, the weight term  $U_{ww}$  in (7a) contributes to greater housing consumption, which, from (4), corresponds to larger effects of distance on housing prices.

When  $U_{FW} < 0$ , food increases (makes more negative) the marginal disutility of weight, and this reinforces the comparative statics results discussed above. As before, food consumption is reduced at greater distances from the CBD to offset the weight effects. However, now this has the added effect of reducing the marginal disutility of weight, and there is even greater reduction in food consumption. With lower penalties from weight, residents reduce calorie expenditure such that the net effect of distance on weight is larger. Greater weight also has the effect of reducing the marginal utility of food and so, in equilibrium, residents at greater distances from the CBD are compensated with even more housing.

When  $U_{FW} > 0$ , the comparative statics results have ambiguous signs. There are eight possible cases in which  $\partial F / \partial x$ ,  $\partial E / \partial x$ , and  $\partial W / \partial x$  are either strictly positive or strictly negative (Table 3).<sup>13</sup> The six feasible cases are internally consistent and satisfy the second-order conditions. Two infeasible cases violate the requirement that  $\partial W / \partial x$  have the same sign as  $\partial F / \partial x - \partial E / \partial x$ . Of the six feasible cases, three involve weight increasing in distance to the CBD. Thus, it is possible for food consumption and calorie expenditure to both increase, both decrease, or for food consumption to increase and calorie expenditure to decrease, with a positive net effect on weight in all cases. In the first case, when calorie expenditure increases with distance, weight falls and utility increases. In order to restore utility to its original level, residents increase food consumption to increase weight. For food consumption to increase, we must have  $U_{FW} > |U_{ww}|$ . Because rising weight has a larger effect on the marginal utility of food

---

<sup>13</sup> It is possible for the signs of these partial effects to change with location. We consider only those cases in which the sign of each partial effect is the same at all locations.

than the marginal disutility of weight, the increase in food consumption must be large so that the decrease in utility from weight eventually offsets the increase in utility from food. The net result is greater calorie intake than expenditure and a corresponding increase in weight.

In three of the six feasible cases, weight falls with distance. Food consumption and calorie expenditure can both increase, both decrease, or food consumption can decrease and calorie expenditure increase, with a decline in weight in each case. Consider the case in which both food consumption and calorie expenditure decline. This causes weight to decrease when, from (7d),  $U_{FW}$  is sufficiently larger than  $|U_{FF}|$  and (or)  $p_F$  is relatively large. In contrast to the results for  $U_{FW} < 0$ , lowering food consumption now increases the marginal disutility of weight and so the reduction in food consumption is *greater* relative to the decrease in calorie expenditure. The net result is a decline in weight.

If all residents have the same initial weight ( $W_0$ ), then spatial differences in weight will arise solely from locational differences in the optimizing choices of housing, food, and leisure. When  $U_{FW} \leq 0$ , distance to the CBD has a positive effect on weight and a negative effect on housing densities.<sup>14</sup> Thus, we see that weight gain can be associated in our model with one feature of sprawl—lower density development at greater distances from the city center. The analysis makes clear that, in this setting, sprawl does not cause obesity. Weight and housing density are endogenously determined through the optimizing behavior of residents. Distance from the CBD tends to increase food consumption relative to calorie expenditure, with weight gain being the net effect. Thus, our model offers an explanation for a possible correlation between sprawl and obesity. At the same time, it suggests that the lines of causality do not run simply from one to the other, as some earlier analysts have suggested.

---

<sup>14</sup> Both of these effects can also occur when  $U_{FW} > 0$ .

#### IV.B. *Weight in an expanding city*

Sprawl is associated with the growth of urban areas, and so it is also of interest to examine how expansion of a city affects the weight of its residents. Growth in an open city occurs when current residents increase land consumption and (or) the city becomes relatively more attractive to migrants. We can cause both of these changes to occur by reducing unit transportation costs ( $t$ ). In practice, public provision of transportation infrastructure such as road building, which is often identified as a cause of sprawl, can have a similar effect on transportation costs.

First, we show that reducing transportation costs causes the city to expand in area and examine the conditions under which housing densities decline—two changes in urban structure frequently associated with sprawl. To derive the effect of transportation costs on the housing price, we apply the implicit function theorem to (3), yielding:

$$-\frac{\partial p_H}{\partial t} = \frac{x}{H^*} > 0, \quad (8)$$

Thus, a decline in transportation costs will increase equilibrium housing prices at all locations. We assume that the city's boundary occurs at distance  $\bar{x}$  from the CBD where the housing price equals an exogenously determined price for agricultural land,  $p_A$ . Using  $p_H(t, \bar{x}) - p_A \equiv 0$ , (4), and (8), we have  $-\partial \bar{x} / \partial t > 0$ , implying that the boundary of the city is extended as transportation costs fall.

We apply Cramer's rule, as above, to examine the effect of lower transportation costs on housing densities, given by  $H^{-1}$ . The sign of  $-\partial H / \partial t$  depends on the sign and magnitude of  $U_{FW}$ . If  $U_{FW} \leq 0$ , then  $-\partial H / \partial t < 0$ . Holding location constant, a reduction in transportation

costs relaxes household budget constraints, raising food consumption and the time allocated to leisure, with a net effect of lowering weight at each location. These effects are reinforced by a negative value of  $U_{FW}$ . As food consumption rises to offset the utility gains from more calorie expenditure, the marginal disutility of weight increases and, thus, the increase in food consumption (relative to the larger reduction in calorie expenditure) need not be as great. The result is even greater weight loss. With higher utility from lower weight and higher food consumption, housing consumption must fall to achieve the original utility level. This implies increasing housing densities at each location. When  $U_{FW} > 0$ , it is possible to have  $-\partial H / \partial t > 0$ . In this case, households tend to substitute away from food and leisure (respectively, decreasing the marginal disutility of weight and increasing the marginal utility of food) and toward housing, resulting in lower housing densities at each location.

In addition to expanding the area of the city and affecting housing densities, lower transportation costs increase migration to the city and, hence, the total number of households. To compute the number of households in the city, note that  $H(x)^{-1}$  gives the number of households per unit area at distance  $x$  from the CBD and  $N(x) = 2\pi x dx / H(x)$  equals the number of households in a ring of width  $dx$  at distance  $x$ . The total number of households is then computed as:

$$\tilde{N} = \int_0^{\bar{x}(t)} \frac{2\pi x}{H(t, x)} dx, \quad (9)$$

and the effect of transportation costs on  $\tilde{N}$  is:

$$-\frac{\partial \tilde{N}}{\partial t} = N(\bar{x}) \left( -\frac{\partial \bar{x}}{\partial t} \right) - \int_0^{\bar{x}} N(x) \left( \frac{-\partial H / \partial t}{H} \right) dx. \quad (10)$$

For a small decline in  $t$ , the first term on the right-hand side of (10) measures the increase in households from the expansion in the city boundary and the second term measures the change in households per unit area summed over the area of the city. When  $-\partial H / \partial t < 0$ , a decline in transportation costs increases housing densities and there is an unambiguous increase in the population of the city ( $-\partial \tilde{N} / \partial t > 0$ ). When  $-\partial H / \partial t > 0$ , housing densities decrease throughout the city. There may be a net decline in population if the effect on population of reducing densities is greater in absolute value than the effect from extending the city boundary.

We want to determine how the weight of the city's residents changes as a result of sprawl. To account for changes in the number of households as transportation costs fall, we analyze the total weight of the city's residents, given by:

$$\tilde{W} = \int_0^{\bar{x}(t)} W(t, x) N(t, x) \quad (11)$$

The effect of transportation costs on total weight is given by:

$$-\frac{\partial \tilde{W}}{\partial t} = W(\bar{x}) N(\bar{x}) \left( -\frac{\partial \bar{x}}{\partial t} \right) + \int_0^{\bar{x}} \left\{ \left( -\frac{\partial W}{\partial t} \right) N + W \left( -\frac{\partial N}{\partial t} \right) \right\} \quad (12)$$

For a small decline in transportation costs, the first term in (12) measures the positive effect on average weight of expanding the city boundary and adding residents with weight  $W(\bar{x})$ . The next two terms have ambiguous signs. The second term in (12) measures the change in weight throughout the city due to the decline in transportation costs. The last term measures the change in the weight due to the change in the number of households at each location.

The effects of lower transportation costs on average weight are summarized in Table 4. A reduction in transportation costs expands the city boundary, which increases population and contributes positively to the total weight of residents (the first terms in 10 and 12). Lower transportation costs also affect housing densities, population, and the weight of residents

throughout the city. The effect on total weight through these channels is less clear. When  $U_{FW} \leq 0$ , a decline in transportation costs increases housing densities and population and reduces weight throughout the city. From the last two terms in (12), positive and negative changes in population and weight have opposing effects on total weight. The net effect depends on whether the positive effects of expanding the boundary and increasing population outweigh the negative effects of reduced weight throughout the city. When  $U_{FW} > 0$ , a number of outcomes are possible,<sup>15</sup> most of which have ambiguous implications for total weight. It is possible that the reduction in transportation costs expands the city boundary and reduces housing densities—our indicators of sprawl—and leads to an increase in total weight. This can occur if population declines and weight increases throughout the city.

The central message of this subsection is that drivers of sprawl affect economic and health outcomes not only on the fringe of the city, but also within the existing city. In the previous subsection, we found a correspondence between weight and distance from the CBD. Thus, expansion of the city can contribute to obesity if this involves adding new residents at the city fringe where weights are relatively high. However, we must also recognize the changes that take place within the city. A reduction in transportation costs, for instance, induces households to reallocate their time and income budgets, leading in some cases to a reduction in weight. Thus, we tend to find ambiguous effects of transportation costs on a measure such as total weight. In general, our results suggest that drivers of sprawl can increase obesity rates in some locations, but reduce them in others.

---

<sup>15</sup> We consider cases in which the conditions in Table 4 are either satisfied at all locations or at no locations. It is possible, however, for a condition to be satisfied at some, but not all, locations.

#### IV.C. *Effects of food prices and calorie expenditure on weight*

Philipson and Posner (1999) emphasize the effects on weight of lower real prices for calories and reduced expenditure of calories per hour of work in market and household production. These changes have come with technological advances in modern economies. We consider how weight changes in our model with reductions in food prices ( $p_F$ ) and calorie expenditure through leisure. Recall that  $\gamma$  measures the amount of time required to expend one calorie through leisure. Thus, increases in  $\gamma$  correspond to lower calorie expenditure. Reasons for lower calorie expenditure during leisure time include the adoption of labor-saving appliances in household production and the pursuit of passive activities such as television viewing. One can think of  $\gamma$  as being dependent on  $E$ . For instance, people with little leisure time (i.e., low calorie expenditure) may be more likely to use labor-saving appliances. We have ignored the potential endogeneity of  $\gamma$  for analytical tractability.

Applying Cramer's rule, we derive the following effects of reduced food prices on housing, food consumption, calorie expenditure, and weight:

$$-\frac{\partial H}{\partial p_F} = -\frac{1}{|J|} \left\{ p_F \lambda \left( p_H + p_F \frac{F}{H} \right) U_{WW} + \gamma w(W_0) \lambda \left( p_H + 2p_F \frac{F}{H} \right) (U_{WW} + U_{FW}) \right. \\ \left. + \gamma^2 w(W_0)^2 \lambda \frac{F}{H} (U_{WW} + U_{FF} + 2U_{FW}) \right\} \quad (13a)$$

$$-\frac{\partial F}{\partial p_F} = \frac{1}{|J|} \left\{ p_H^2 \lambda U_{WW} + p_H \lambda \frac{F}{H} [p_F U_{WW} + \gamma w(W_0) (U_{WW} + U_{FW})] + \lambda U_{HH} [\gamma w(W_0)]^2 \right\} \quad (13b)$$

$$-\frac{\partial E}{\partial p_F} = \frac{1}{|J|} \left\{ p_H^2 \lambda (U_{WW} + U_{FW}) + p_H \lambda \frac{F}{H} [p_F (U_{WW} + U_{FW}) \right. \\ \left. + \gamma w(W_0) (U_{WW} + U_{FF} + 2U_{FW})] - p_F \lambda U_{HH} \gamma w(W_0) \right\} \quad (13c)$$

$$-\frac{\partial W}{\partial p_F} = \frac{1}{|J|} \left\{ -p_H^2 \lambda U_{FW} - p_H \lambda \frac{F}{H} [p_F U_{FW} + \gamma w(W_0)(U_{FF} + U_{FW})] \right. \\ \left. + \lambda U_{HH} \gamma w(W_0)(p_F + \gamma w(W_0)) \right\} \quad (13d)$$

A decline in food prices raises equilibrium housing prices (i.e., applying the Implicit Function Theorem, we have  $-\partial p_H / \partial p_F = F / H > 0$ ). The effect on housing consumption depends, as in the results discussed above, on the sign and magnitude of  $U_{FW}$ . When  $U_{FW} \leq 0$ ,  $-\partial H / \partial p_F < 0$  and  $-\partial F / \partial p_F > 0$ . In this case, residents substitute away from more expensive housing and toward cheaper food. However, the effects of lower food prices on calorie expenditure and weight are ambiguous. When  $U_{FW} > 0$ , all of the results in (13) are, in general, ambiguous.

With increasing distance, calorie expenditure unambiguously declines when  $U_{FW} \leq 0$  (equation 7c). The difference here is that a reduction in food price does not directly affect the time budget, and so the effects on calorie expenditure are ambiguous even when  $U_{FW} \leq 0$ . If  $|U_{HH}|$  is relatively small, then calorie expenditure increases ( $-\partial E / \partial p_F > 0$ ) to the extent that weight declines ( $-\partial W / \partial p_F < 0$ ). Consider (13d) in the case where  $U_{FW} = 0$  and  $|U_{FF}|$  is large relative to  $|U_{HH}|$ . In this case, the gain in utility from more food consumption becomes smaller relative to the loss in utility from less housing, and so weight is reduced to restore utility to its original level. When  $|U_{HH}|$  is relatively large, the reduction in food prices can cause weight to increase ( $-\partial W / \partial p_F > 0$ ) in order to offset the utility-increasing effects of higher food consumption. A positive value of  $U_{FW}$  augments this effect since weight gain increases the marginal utility of food consumption.

A reduction in the rate of calorie expenditure, corresponding to an increase in  $\gamma$ , has the following effects on housing, food consumption, calorie expenditure, and weight:

$$\frac{\partial H}{\partial \gamma} = \frac{1}{|J|} \left\{ \frac{p_F^2 \lambda w(W_0) E}{H} U_{WW} + \left( p_F p_H \lambda w(W_0) + \frac{2 p_F \lambda \gamma w(W_0)^2 E}{H} \right) (U_{WW} + U_{FW}) \right. \\ \left. + \left( p_H \lambda \gamma w(W_0)^2 + \frac{\gamma^2 \lambda w(W_0)^3 E}{H} \right) (U_{WW} + U_{FF} + 2U_{FW}) \right\} \quad (14a)$$

$$\frac{\partial F}{\partial \gamma} = \frac{1}{|J|} \left\{ - \left( p_H^2 \lambda w(W_0) + \frac{p_H \lambda \gamma w(W_0)^2 E}{H} \right) (U_{WW} + U_{FW}) - \frac{p_H \lambda p_F w(W_0) E}{H} U_{WW} \right. \\ \left. + p_F \lambda \gamma w(W_0)^2 U_{HH} \right\} \quad (14b)$$

$$\frac{\partial E}{\partial \gamma} = \frac{1}{|J|} \left\{ - \left( p_H^2 \lambda w(W_0) + \frac{p_H \lambda \gamma w(W_0)^2 E}{H} \right) (U_{WW} + U_{FF} + 2U_{FW}) \right. \\ \left. - \frac{p_H \lambda p_F w(W_0) E}{H} (U_{WW} + U_{FW}) - p_F^2 \lambda w(W_0) U_{HH} \right\} \quad (14c)$$

$$\frac{\partial W}{\partial \gamma} = \frac{1}{|J|} \left\{ \left( p_H^2 \lambda w(W_0) + \frac{p_H \lambda \gamma w(W_0)^2 E}{H} \right) (U_{FF} + U_{FW}) + \frac{p_H \lambda p_F w(W_0) E}{H} U_{FW} \right. \\ \left. + \left( p_F \lambda \gamma w(W_0)^2 + p_F^2 \lambda w(W_0) \right) U_{HH} \right\} \quad (14d)$$

These results are symmetric with those for a decline in food price. When  $U_{FW} \leq 0$ , calorie expenditure declines unambiguously as the increase in  $\gamma$  effectively raises its price.

Correspondingly, there is an increase in housing consumption since  $\partial p_H / \partial \gamma = -E w(W_0) / H < 0$ .

Food consumption declines, along with weight, if  $|U_{HH}|$  is relatively small. If  $|U_{HH}|$  is relatively large and  $U_{FW} > 0$ , the decline in calorie expenditure will tend to result in weight gain.

#### IV.D. *Effects of initial weight on location*

Up until now we have assumed that all residents have the same initial weight ( $W_0$ ). Our analysis makes clear, however, that over time we should expect to observe differences in the

weight of residents. For example, residents who live farther from the CBD will tend to gain more weight than residents located near the CBD. A natural question, then, is how does initial weight affects location decisions? Will residents with high initial weight locate near the CBD to avoid weight gain, or will they be outbid at these locations by thin residents trying to avoid weight gain? To examine this issue, we consider the effect of initial weight on the slope of the housing price gradient in (4):

$$\frac{\partial^2 p_H}{\partial x \partial W_0} = \frac{c}{H^* W_0} \left( \frac{\partial H^*}{\partial W_0} \frac{W_0}{H^*} - \frac{\partial c}{\partial W_0} \frac{W_0}{c} \right), \quad (15)$$

where  $c = \beta w(W_0) + t$  are marginal commuting costs. The first and second terms in the parentheses are, respectively, the elasticities of housing consumption and marginal commuting costs with respect to initial weight. For reasons discussed above, we assume  $w'(W_0) < 0$  and, thus, the second term is positive. If the effect of initial weight on housing consumption is positive (or negative and relatively small), then  $\partial^2 p_H / \partial x \partial W_0 > 0$ , and heavy residents outbid thin residents at locations farther from the CBD (and vice-versa). When  $\partial^2 p_H / \partial x \partial W_0 < 0$ , residents with greater initial weight will locate closer to the CBD.

The sign of the expression in (15) hinges on the sign and magnitude of the elasticity of housing consumption with respect to initial weight. The partial effect of initial weight on housing consumption is a complicated expression that cannot be signed even with assumptions about the sign and magnitude of  $U_{FW}$ . An increase in initial weight unambiguously decreases the housing price (i.e.,  $\partial p_H / \partial W_0 < 0$ ) by making the city less appealing to current residents and causing out-migration. However, it also has the effects of reducing income and increasing the marginal disutility of weight. If, in (1), we ignore the effects on the utility function and consider

only the budgetary impacts of an increase in initial weight, we find that  $\partial H / \partial W_0 > 0$  if  $U_{FW} < 0$  and the reduction in housing prices ( $\partial p_H / \partial W_0$ ) is large relative to the wage reduction ( $w'(W_0)$ ). In this case, residents with high initial weight will locate farther from the CBD and will tend to gain more weight.<sup>16</sup> In a spatial market equilibrium, obesity can become a self-reinforcing process.

## V. Discussion

In the U.S. and to a lesser extent in other developed countries, the rise in obesity rates and the prevalence of urban sprawl have been two powerful trends during the latter half of the 20<sup>th</sup> century. In this paper, we explore possible connections between these phenomena. Previous empirical work has found that obesity rates are influenced by a variety of labor market outcomes, including time allocated to work and leisure. Since the spatial configuration of developed land affects commuting distances, time remaining for leisure and work, as well as the consumption of housing, food, and other goods, we would expect sprawl to influence obesity rates through many of the same channels. We recognize that sprawl is only one of many possible factors that may give rise to obesity. Psychological factors such as depression, access to health information, and changes in weight preferences due to technological change (Philipson and Posner, 1999) are likely to be involved as well.

We analyze an urban spatial model augmented to include time allocation and weight. Residents choose utility-maximizing levels of housing, food consumption, and calorie expenditure. Weight, income, and housing prices are determined endogenously in a spatial market equilibrium. A relatively robust result is that weight gain increases with distance to the

---

<sup>16</sup> Recall that weight increases unambiguously with distance when  $U_{FW} \leq 0$ .

CBD. This always happens when food consumption increases the marginal disutility of weight ( $U_{FW} \leq 0$ ) and in one-half of the cases when the opposite is true ( $U_{FW} > 0$ ). In the first case, calorie expenditure declines as distance from the CBD reduces leisure time. Residents partially offset the effect of this on weight by reducing food consumption, but because food contributes directly to utility, residents far from the CBD are willing to gain weight and be compensated with more and cheaper housing. When food consumption reduces the marginal disutility of weight, then the penalty from weight gain can be minimized through food consumption, and the need to compensate weight gain with housing is diminished. As a result, the relationship between weight and distance is no longer unambiguous.

When residents located away from the CBD are compensated for weight gain with more housing, then our model shows that obesity can be associated with one feature of urban sprawl—lower density development away from the city center. Sprawl is often associated with growth in cities and so we also consider expansion of the city resulting from a reduction in transportation costs. In contrast to the distance results, in an expanding city a positive effect of food consumption on the marginal disutility of weight ( $U_{FW} \leq 0$ ) reduces the tendency toward weight gain. The reduction in transportation costs frees up the budget constraint, and residents respond by expending more calories and reducing their weight. The influx of migrants drives up housing prices and so residents consume less housing and development densities increase. When  $U_{FW} > 0$ , it is possible for residents to increase weight and, to offset this, increase housing as well. The result is decreasing development densities accompanied by weight gain.

We evaluate the effects of growth in the city on the total weight of its residents. This allows us to see the effects of declining transportation costs on the population of the city as well as on the behavior of its current residents. Expansion of the city always increases total weight by

adding residents at the city boundary. If the effect of distance on weight is positive, then we are more likely to see obesity associated with this feature of sprawl. The reduction in transportation costs also affects the weight of current residents and the population of the city, in ways that mostly confound the ultimate effect on total weight. The effect on current residents is important to bear in mind, as drivers of sprawl can influence tradeoffs, and hence health outcomes, in the existing city as well as in expanding areas of the city. We assumed that transportation costs fall everywhere in the city. It is also possible for sprawl to adversely affect current residents if it increases congestion and raises commuting times. A rise in unit commute times ( $\beta$  in our model) has the opposite effect of a reduction in transportation costs. In the  $U_{FW} \leq 0$  case, a rise in  $\beta$  would unambiguously increase the weight of current residents.

We also examine the effects of reducing food prices and the rate of calorie expenditure during leisure. These changes have the expected effect of, respectively, increasing food consumption and reducing calorie expenditure. While both changes increase weight gain, residents make other adjustments that render the net effect on weight ambiguous. Part of the reason for the added complexity is that these changes affect the relative prices of housing, food, and calorie expenditure. When we considered increases in distance and reductions in transportation costs, the relative prices of food and calorie expenditure remained constant. Similarly, we found that initial weight has complicated effects on housing consumption, as it affects both income and the marginal disutility of weight. When we ignored the latter effect, we found conditions under which residents with high initial weight would tend to locate far from the CBD. Under the same conditions, weight gain increases with distance to the CBD, suggesting a perpetuating cycle of weight gain.

## VI. References

- Bouis, H. 1996. A Food Demand System Based on Demand for Characteristics: Is There Curvature in the Slutsky Matrix, What Do the Curves Look Like and Why? *J of Development Economics* 51:239-266.
- Cave, D. (1992). Employees are Paying for Poor Health Habits. *HR Magazine* 32: 52-58.
- Cawley, J. (2000) Body Weight and Women's Labor Market Outcomes. NBER Working Paper 7841.
- Chou, S.-Y., Grossman, M., and H. Saffer. 2001. An Economic Analysis of Adult Obesity: Results from the Behavioral Risk Factor Surveillance System. Paper presented at the Third International Health Economics Association Conference, York, England, July 23-25, 2001.
- Everett, M. (1990). Let an Overweight Person Call on Your Best Customer? Fat Chance. *Sales and Marketing Management*, 142, 66-70.
- Hamermesh, D. and Biddle, J. (1994). Beauty and the Labor Market. *American Economic Review* 84: 1174-1194.
- Hill, J.O., and J.C. Peters. 1998. Environmental Contributions to the Obesity Epidemic. *Science* 280:1371-4.
- Jackson, R.J., and C. Kochtitzky. 2002. Creating a Healty Environment: The Impact of the Built Environment on Public Health. Centers for Disease Control and Prevention.
- Martin, C. (1994). Protecting Overweight Workers Against Discrimination: Is Disability or Appearance the Real Issue?" *Employee Relations Law Journal* 20: 133-142.
- Moon, M. and McLean, R. (1980). Health, Obesity and Earnings. *American Journal of Public Health* 70: 1006-1009.
- National Center for Health Statistics. 2001. Health, United States, 2001 With Urban and Rural Health Chartbook. Hyattsville, Maryland.
- National Research Council. *Diet and health: implications for reducing chronic disease risk*. Washington, DC: National Academy Press, 1989.
- Pagan, Jose and Davila, A. (1997). Obesity, Occupational Attainment and Earnings. *Social Science Quarterly*. Volume 78, Number 3, September 1997, 756-770.
- Philipson, T.J., and R.A. Posner. 1999. The Long-Run Growth in Obesity as a Function of Technological Change. National Bureau of Economic Research Working Paper 7423.

- Ruhm, C.J. 2002. Are Recessions Good for Your Health? *Quarterly Journal of Economics* CXV(2):617-50.
- Stunkard AJ, Wadden TA. (Editors) *Obesity: theory and therapy, Second Edition*. New York: Raven Press, 1993.
- Terrell, K. (1992). Female-Male Earnings Differentials and Occupational Structure. *International Labour Review*, v 131, n4-5 (1992): 387-404.
- Wolf, A., and G. Colditz. 1998. Current Estimates of the Economic Cost of Obesity in the United States. *Obesity Research* 6(2):97-106.
- Wu, J., and A.J. Plantinga. 2003. The Influence of Public Open Space on Urban Spatial Structure. *Journal of Environmental Economics and Management*, forthcoming.

Table 1. Obesity Rates of U.S. Adults by Gender, Age, Race, Education, and Smoking Status.

Characteristics	1991	1995	1998	1999	2000
	<i>Percent Obese</i>				
Total	12.0	15.3	17.9	18.9	19.8
Gender					
Men	11.7	15.6	17.7	19.1	20.2
Women	12.2	15.0	18.1	18.6	19.4
Age Groups					
18-29	7.1	10.1	12.1	12.1	13.5
30-39	11.3	14.4	16.9	18.6	20.2
40-49	15.8	17.9	21.2	22.4	22.9
50-59	16.1	21.6	23.8	24.2	25.6
60-69	14.7	19.4	21.3	22.3	22.9
>70	11.4	12.1	14.6	16.1	15.5
Race, Ethnicity					
White, non-hispanic	11.3	14.5	16.6	17.7	18.5
Black, non-hispanic	19.3	22.6	26.9	27.3	29.3
Hispanic	11.6	16.8	20.8	21.5	23.4
Other	7.3	9.6	11.9	12.4	12.0
Educational Level					
Less than high school	16.5	20.1	24.1	25.3	26.1
High school degree	13.3	16.7	19.4	20.6	21.7
Some college	10.7	15.1	17.8	18.1	19.5
College degree or above	8.0	11.0	13.1	14.3	15.2
Smoking Status					
Never smoked	12.0	15.2	17.9	19.0	19.9
Ex-smoker	14.0	17.9	20.9	21.5	22.7
Current smoker	9.9	12.3	14.8	15.7	16.3

Source: CDC, Behavioral Risk Factor Surveillance System, 1991-2000.

Table 2. Obesity Rates of U.S. Adults by Region

Region	1991	1998	2000
	<i>Percent Obese</i>		
New England (CT, MA, ME, NH, RI, VT)	9.9	11.4	17.0
Middle Atlantic (NJ, NY, PA)	12.7	16.7	18.4
East North Central (IL, IN, MI, OH, WI)	14.1	19.1	21.0
West North Central (IA, MN, MO, ND, NE, SD)	12.2	18.0	19.8
South Atlantic (DC, DE, FL, GA, MD, NC, SC, VA, WV)	11.1	18.6	19.5
East South Central (AL, KY, MS, TN)	13.1	20.0	23.0
West South Central (AR, LA, OK, TX)	13.1	20.0	22.2
Mountain (AZ, CO, ID, MT, NM, UT, WY)	9.6	14.1	17.1
Pacific (AL, CA, HI, NV, OR, WA)	10.2	17.0	19.1

Source: CDC, Behavioral Risk Factor Surveillance System, 1991-2000.

Table 3. The Effects of Distance to the CBD on Weight

Cases	Food Consumption	Effects of Distance on Calorie Expenditure	Weight
$U_{FW} \leq 0$	-	-	+
$U_{FW} > 0$			
Feasible Cases	+	+	+
	+	-	+
	-	-	+
	+	+	-
	-	+	-
	-	-	-
Infeasible Cases	+	-	-
	-	+	-

Table 4. Decomposition of the Effects of Lower Transportation Costs on Total Weight

Category	$U_{FW} \leq 0$	$U_{FW} > 0$			
City Boundary ( $-\partial\bar{x}/\partial t$ )	+	+			
		<b>Condition 1</b>			
		<u>Yes</u>		<u>No</u>	
Density ( $-\partial(H^{-1})/\partial t$ )	+	-		+	
Population ( $-\partial N/\partial t$ )	+	-		+	
		<b>Condition 2</b>		<b>Condition 2</b>	
		<u>Yes</u>	<u>No</u>	<u>Yes</u>	<u>No</u>
Weight ( $-\partial W/\partial t$ )	-	+	-	+	-
Total Weight ( $-\partial\tilde{W}/\partial t$ )	+/-	+/-	+/-	+	+/-

Note: Condition 1 is  $p_F^2 U_{WW} + 2p_F \gamma w(W_0)(U_{WW} + U_{FW}) + \gamma^2 w(W_0)^2 (U_{WW} + U_{FF} + 2U_{FW}) > 0$ . Condition 2 is  $p_F U_{FW} + \gamma w(W_0)(U_{WW} + U_{FW}) > 0$ .