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William K. Jaeger
Department of Economics, Williams College

February 1, 2001

Abstract

This paper compares the optimal environmental tax with two alternative definitions of marginal environmental damages. One definition reflects the social marginal rate of substitution between income and the environment; the other reflects the sum of households’ marginal willingness to pay. The analysis finds that the definition based on the social marginal rate of substitution provides a consistent benchmark for setting environmental taxes that is compatible with both the Pigouvian principle and the double dividend hypothesis. The definition based on the sum of households’ marginal willingness to pay, however, is found to be incompatible with optimal taxation and an unreliable benchmark for making welfare inferences.

JEL Classification: H2, Q2

Key words: environmental tax, second best, double dividend, optimal taxation,

Contact information: Fernald House, Economics Department, Williams College, Williamstown, MA 01267
413-597-3213 (phone) 413-597-4045 (fax) Wjaeger@williams.edu
I. Introduction

The question of whether an environmental tax should be set higher or lower than marginal environmental damages has been a focal point for theorists and policymakers alike. For a very long time, the Pigouvian Principle provided an accepted rationale for equating the two values. Recently, however, two conflicting strands of literature have emerged suggesting that in a second-best world with revenue-motivated taxes, the optimal environmental tax will be: a) higher than marginal damages because of the "double dividend" effect, or b) lower than marginal damages because of a "tax interaction" effect. Both strands of literature have involved (contradictory) inferences about the expected welfare gains from environmental tax reform.

Beginning with Tullock (1967), an initial strand of literature suggested that revenues from environmental taxes could be used to finance reductions in preexisting revenue-motivated taxes, an approach that could both improve the environment and reduce the welfare costs associated with the overall tax program (see also Terkla 1984; Lee and Misiolek 1986). This notion, now widely referred to as the "double dividend hypothesis," carried with it the intuitive inference that the optimal environmental tax would generally exceed marginal environmental damages when the revenues are used in this way (Pearce 1991, Oates 1995).

More recently, however, a second strand of literature has concluded that in a second-best setting where revenues must be raised with distortionary taxes, the optimal environmental tax will generally lie below marginal environmental damages, even when the revenues are used to finance reductions in pre-existing revenue motivated taxes (Bovenberg and de Mooij 1994; Parry 1995; Bovenberg and Goulder 1996). To explain this seemingly counterintuitive result, the authors of this literature claim that in addition to the "revenue recycling" effect associated with the double dividend, there exists an opposing "tax interaction" effect which exacerbates the
distortionary effects of environmental taxes. In contrast to the earlier double dividend literature, this recent “tax interaction” literature concludes that the net welfare gains from environmental tax reform will generally be lower than previously thought, and that an increase in revenue requirements will make it more costly, rather than less costly, to protect the environment (Bovenberg and de Mooij 1994; Fullerton 1997; Parry and Oates 2000). ¹

These recent contributions have attached considerable weight to comparisons between the optimal environmental tax and marginal environmental damages, although the definition of marginal environmental damages has not come under close scrutiny. To address this question, the current analysis considers two alternative definitions of marginal environmental damages, and examines the relationship that each has with the optimal environmental tax, and their suitability as a guide for setting policy or making inferences about the welfare effects from environmental tax reform.

The first definition reflects the social marginal rate of substitution between income and the environment—derived directly from the social planner’s problem. The second definition reflects the household’s marginal rate of substitution between income and the environment summed across households—thereby reflecting their aggregate willingness to pay. The two definitions differ in that the first definition includes the revenue consequences for an incremental change in either income or the environment, and reflects their Pareto efficient use.

For a given model, the value of marginal environmental damages differs significantly between these two definitions, and the differences are found to vary in important and illuminating ways for two different types of externalities; one affecting factor productivity and the other involving an environmental amenity that affects welfare directly. Hence, the analysis

¹ This now-extensive literature includes Goulder 1995; Fullerton 1997; Schöb, 1997; Bovenberg and de Mooij 1997; Goulder, Parry, Burtraw 1997; Parry, Williams, Goulder 1999; and Goulder, Parry, Williams and Burtraw 1999, Parry and Bento 2000, among others.
below will evaluate both definitions in the context of models which represent each type of externality, allowing two-way comparisons to highlight their differences.

In a second-best setting with revenue-motivated taxes, the definition based on the social marginal rate of substitution—referred to hereafter as “marginal social damage”(MSD)—is found to be consistent with the expressions from which optimal taxes are derived analytically, and the relationship between the optimal environmental tax and marginal environmental damages is found to be independent of the type of externality. For realistic parameter values the optimal environmental tax is found to exceed marginal social damages. By contrast, the definition of marginal environmental damages based on the private marginal rate of substitution between income and the environment, but summed across households—referred to hereafter as “marginal private damages”(MPD)—is found to be incompatible with the expressions from which optimal environmental taxes are derived analytically. Indeed, marginal private damages does not reflect the marginal rate of substitution between income and the environment for the social optimization problem. As a result, the relationship between the optimal environmental tax and marginal private damages varies across types of environmental externalities (holding other parameters constant), and produces results that in some cases appear incongruent with the Pigouvian principle and the double dividend hypothesis.

The remainder of the paper presents the analysis both analytically and with the aid of numerical models. In section II definitions of marginal social damages and marginal private damages are derived analytically, for both productivity and amenity externality cases. In section III, expressions for the optimal environmental tax are derived and compared to both definitions of marginal environmental damage, for both externality types. Section IV presents numerical models and quantitative comparisons for both externality types. Section V concludes.
II. Theoretical model

In this section both definitions of marginal environmental damage are evaluated for two different models: one in which the environment affects productivity, and the other involving an amenity which affects welfare directly. Both settings can be characterized in a basic model where $m$ identical households maximize utility by allocating time, $T$, between leisure, $L$, and labor supply, $T-L$. Production is assumed to take place according to a linear production technology with labor productivity, $h$. Output takes the form of two private consumption goods, a non-polluting good, $C$, and a polluting good, $D$. Units are normalized so that marginal rates of substitution between $C$ and $D$ are unity in the absence of taxes.

Exogenously determined revenues, $mG$, are collected using excise taxes on $C$ and $D$, and these can be interpreted as payments out of labor income. For simplicity these revenues are returned to households in the form of lump-sum transfers, rather than introducing an explicit and detailed representation of the public sector or provision of public goods. Environmental quality, $E$, is characterized by the function $E = e(mD)$, where $\frac{de}{d(mD)} < 0$.

A. Production externalities.

For broad classes of models, economy-environment interactions can be characterized as affecting factor productivity such that aggregate production depends on environmental quality (Pezzey 1989; Mäler 1991). To reflect this, let labor productivity be a function of $E$, $h = h(E)$ where $\frac{dh}{dE} > 0$. The model can represent settings where pollution reduces labor productivity.
directly, for example due to health effects, or it may reduce labor productivity indirectly as with soil degradation or air pollution effects on farming, or marine pollution on fishing.

The household’s maximization problem can be represented as

\[
\begin{align*}
\text{Max:} & \quad u(C,D,V) \\
\text{s.t.} & \quad (1+t_c)C + (1+t_d)D = h(E)(T-L) + G
\end{align*}
\]

where environmental quality, \(E\), and government transfers, \(G\), are taken as exogenous. Let \(\lambda\) denote the Lagrange multiplier on the household budget constraint, or the private marginal utility of income. Denoting this capital theoretic formulation with subscript \(k\) and letting superscript \(p\) denote private or household valuation, the marginal rate of substitution between income and the environment for a representative household can be derived from (1) as

\[
MRS^k = \frac{\partial U}{\partial E} = \frac{\lambda \frac{dh}{dE}(T-L)}{\frac{\partial U}{\partial L}}.
\]

Let \(V(\cdot)\) represent the social optimization problem for provision of the environment and collection of revenues, \(mG\). For the current model this problem is

\[
\begin{align*}
\text{MAX:} & \quad \max_{C,D,V} u(C,D,V) \\
\text{s.t.} & \quad (1+t_c)C + (1+t_d)D = h(E)(T-L) + G \\
\text{s.t.} & \quad m_t C + m_t D = mG \\
& \quad E = e(mD)
\end{align*}
\]

where \(\mu\) denotes the Lagrange multiplier on the revenue constraint, interpreted as the social cost in terms of utility of raising an additional dollar of revenue. From (3), the social marginal rate of substitution between income and the environment is
\[
MRS_s = \frac{\partial V}{\partial E} \bigg|_{\partial L} = \frac{\lambda m d(h(T-L))}{\partial E} + \mu m \left( t_c \frac{\partial C}{\partial (h(T-L))} \frac{\partial (h(T-L))}{\partial E} + t_D \frac{\partial D}{\partial (h(T-L))} \frac{\partial (h(T-L))}{\partial E} \right) + \lambda + \mu \left( t_c \frac{\partial C}{\partial L} + t_D \frac{\partial D}{\partial L} \right)
\]

(4)

Given the marginal effect on \(E\) from consumption of \(D\), defined by \(E = e(mD)\), the marginal social damage (MSD) for consumption of \(D\) is therefore

\[
MSD_k = MRS_s \frac{de}{d(mD)} - \frac{de}{d(mD)}
\]

(5)

Let \(\alpha\) denote the social marginal utility of income reflected by the denominator in (4) and (5). The value of \(\alpha\) differs from \(\lambda\) because, in the presence of distortionary taxes, a unit of income increases utility directly by \(\lambda\) and indirectly by the increased revenue resulting from additional expenditure (Diamond 1975; Auerbach 1985). Thus, the value of \(\alpha\) reflects a Pareto efficient use of a unit of income, or the utility difference between \textit{ex ante} and \textit{ex post} social optima where both households and the social planner have re-optimized.\(^2\)

The definition of marginal private damages (MPD), can be characterized as the sum across all representative households of their marginal willingness to pay for a unit increase in \(E\).

Combining (2) with the relations \(E = e(mD)\) and \(h = h(E)\), \(\text{MPD}_k\) is thus defined as

\[
\text{MPD}_k = \frac{m \frac{\partial U}{\partial E} - \frac{de}{d(mD)}}{\frac{\partial U}{\partial L} d(mD)} = \frac{m \lambda d(h(T-L))}{\partial E} - \frac{de}{d(mD)}
\]

(6)

\(^2\) Generally we expect that \(\mu > \alpha\), so that \(\mu - \alpha\) is the marginal excess burden of the tax, or the difference between the shadow cost in terms of utility of raising an additional unit of revenue and the social marginal utility of income (Auerbach 1985).
Comparing (5) and (6) we see that these two definitions differ in ways that depend on tax rates as well as the responsiveness of labor supply to changes in real wages. By inspection we see that both the numerator and denominator will be larger in (5) than in (6), assuming labor supply responds positively to wages (implying that the terms involving $\mu$ are positive). Moreover, we expect the numerator in MSD$_k$ to exceed the numerator in MPD$_k$ by more than the difference between their denominators. This will be true if $L(dh/dE) < \partial(h(T-L))/\partial E$, which is expected since the latter expression reflects both income and substitution effects. It follows that MSD$_k >$ MPD$_k$, implying that when marginal environmental damage is defined from the social planner's perspective, it will be higher relative to the optimal environmental tax than the sum of household's marginal private damages.

We now turn to the model of an amenity externality.

B. Amenity externalities.

For the case where environmental damage affects welfare directly, the household's maximization problem is

$$\text{Max}_{C,D,L,E} u(C,D,L,E)$$

subject to

$$(\lambda) \quad (1+t_c)C + (1+t_D)D = h(T-L) + G$$

where once again environmental quality $E$ and government transfers, $mG$, are again assumed by households to be exogenous. Letting subscript $a$ denote the amenity model and superscript $p$ denote the household maximization problem, the private marginal rate of substitution between income and the environment based on the model in (7) is

$$MRS^p_a = \frac{\partial U/\partial E}{\partial U/\partial L} = \frac{\partial U/\partial E}{\lambda}$$
The corresponding welfare maximization problem facing the social planner is

\[
\text{MAX } \int_{C,D} \left[ \max \left( u(C, D, L, E) \right) \right] \quad \text{s.t.} \quad (1 + t_c)C + (1 + t_d)D = h(T - L) + G
\]

\[
\begin{align*}
\text{MRS}^S &= \frac{\partial U/\partial E}{\partial U/\partial L} = \frac{m \frac{\partial U}{\partial E} + m \mu \left( t_c \frac{\partial C}{\partial E} + t_d \frac{\partial D}{\partial E} \right)}{\lambda + \mu \left( t_c \frac{\partial C}{\partial L} + t_d \frac{\partial D}{\partial L} \right)} \\
\text{MSD}_a &= \frac{\partial V/\partial E}{\partial V/\partial L} d(mD) - de = \frac{m \frac{\partial U}{\partial E} + m \mu \left( t_c \frac{\partial C}{\partial E} + t_d \frac{\partial D}{\partial E} \right)}{\lambda + \mu \left( t_c \frac{\partial C}{\partial L} + t_d \frac{\partial D}{\partial L} \right)} \cdot d(mD)
\end{align*}
\]

Given (9) and using \( E = e(mD) \), marginal social damages (MSD\(_a\)) is defined as

\[
\text{MSD}_a = \frac{\partial U/\partial E}{\partial U/\partial L} d(mD) - de = \frac{m \frac{\partial U}{\partial E} + m \mu \left( t_c \frac{\partial C}{\partial E} + t_d \frac{\partial D}{\partial E} \right)}{\lambda + \mu \left( t_c \frac{\partial C}{\partial L} + t_d \frac{\partial D}{\partial L} \right)} \cdot d(mD)
\]

From the numerator in (10), we see that the marginal value of the environment in utility units includes the indirect gains or losses from its affect on the tax base. This effect may be positive or negative depending on the sign of the second term in the numerator of (9). To the extent that environmental quality is a substitute for leisure, environmental damage will discourage labor supply, thereby narrowing the tax base. Conversely, if environmental quality is a complement to leisure, environmental damage will encourage labor supply, thereby broadening the tax base.

Applying the definition of marginal private damages from the recent literature to the amenity case—denoted here as \( \text{MPD}_a \)—implies summing household's marginal willingness to pay for the environment, and using \( E = e(mD) \) and \( \text{MRS}^p_a \) to define \( \text{MPD}_a \) as

\[
\text{MPD}_a = \frac{m \frac{\partial U}{\partial E} - de}{\frac{\partial U}{\partial L} d(mD)} = \frac{m \frac{\partial U}{\partial E} - de}{\lambda}
\]
Comparing (10) and (11), the relative magnitude of the numerators is ambiguous given that the second term in the numerator of (10) may be positive or negative. Thus, we cannot say for certain whether \(\text{MSD}_a\) will be larger or smaller than \(\text{MPD}_a\). However, if we restrict preferences to ensure that the term involving \(\mu\) in the numerator of (10) is zero (a common practice in the recent literature (Bovenberg and de Mooij 1994; Bovenberg and Goulder 1996)), then the two measures differ only in their denominators, where \(\alpha > \lambda\) implying \(\text{MSD}_a < \text{MPD}_a\).

We conclude that the relative values of the two definitions are reversed in the amenity case compared to the production externality case. This inconsistent relationship between these two measures can be explained simply: MSD is based on the marginal rate of substitution between income and the environment derived directly from the social planner’s problem, while MPD is somewhat ad hoc. It takes account of one aspect of the social optimization problem (non-rivalry and market failure in \(E\)), while overlooking the revenue-raising aspect of the problem. Thus, by omitting the revenue consequences of a unit change in either environmental quality or income, MPD will overstate or understate the social marginal rate of substitution between income and the environment in the presence of distortionary taxes.\(^3\)

III. Comparisons with the optimal environmental tax: analytical results

We now want to compare both definitions of marginal environmental damage to the optimal environmental taxes. In the first-best case with a Pigouvian tax and no revenue-motivated taxes, these two definitions differ analytically but will have the same value because, in

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\(^3\) The issue is similar to well-known procedures in benefit-cost analysis where accounting or shadow prices define “the value of the contribution to ... socioeconomic objectives made by any marginal change in the availability of commodities or factors of production” (Squire and van der Tak 1975, p. 26; see also Mishan 1971).
the absence of a revenue constraint, the private and the social definitions of the marginal utility of income are equal. Recall that the private marginal utility of income does not reflect either environmental damage (due to the marginal propensity to pollute) or the lump-sum transfers returning environmental tax revenues. Thus at the first-best optimum with only a Pigouvian tax 

\[ t^* = -m \frac{\partial U}{\partial E} \frac{de}{d(mD)} \]

and revenues returned lump-sum, we have the identity

\[ \alpha = \lambda + m \frac{\partial U}{\partial E} \frac{de}{d(mD)} \frac{\partial (mD)}{\partial L} + t^* \frac{\partial (mD)}{\partial L} = \lambda. \]  

(12)

Thus, the social marginal utility of income will equal the private marginal utility of income.

In a second-best setting, as we have seen, the utility measures of marginal social damage differ from marginal private damages. It is straightforward to show that the environmental component of the optimal tax is derived analytically using expressions corresponding to the marginal rates of substitution in the social optimization problem: the numerators in (5) and (10) emerge directly. In the interest of notational convenience, and to generalize for many goods, detailed derivations for a general model with \( n \) goods are presented in Appendix A, where \( X_z \) is the notational equivalent to \( D \). For both the productivity and amenity models, the social planner’s first-order conditions for the dual problem are found in [A4] and [A13], respectively. In both cases, the relevant terms on the left-hand side correspond to the numerators in \( MSD_a \) and \( MSD_b \).

The relationship between the optimal environmental tax and marginal environmental damages can be evaluated based on the optimal tax expressions derived in Appendix A. Noting the notational equivalence between \( \pi \) and \( \alpha MSD \) comparing (5) to [A5] and (10) to [A14], we can write these as

\[ \frac{t^* c}{(1+t^* c)} = \left( \frac{\mu - \alpha + \theta}{\mu} \right) R \]  

(13)
and

\[
\begin{align*}
\frac{t^*_D}{(1 + t^*_D)} &= \left(\frac{\mu - \alpha + \theta}{\mu}\right)^R + \frac{\alpha \cdot \text{MSD}}{\mu(1 + t_D)}
\end{align*}
\]  

(14)

where $\theta$ represents the direct and indirect income effects on the environment (see Appendix A), and where $R$ is the "Ramsey term" reflecting the revenue generating potential for a marginal change in the tax on any good due to the direct and indirect effects on consumption for all goods. We have dropped the subscript on MSD since the tax expressions for both types of externalities are otherwise identical.

These expressions, which are similar to those derived by Sandmo, are difficult to interpret by inspection, in part because of the lack of transparency in $R$. Nor can the two components of the optimal tax rule for $D$ be evaluated separately by inspection, since the denominator is a function of both terms.\(^4\) The problem can be simplified by assuming that $C$ and $D$ are similar from a revenue raising perspective. This has also been done in the recent literature by restricting preferences so that utility is homothetic in consumption, and weakly separable in leisure, environmental quality, and government consumption (Bovenberg and de Mooij 1994; Bovenberg and Goulder 1996). Here, however, we assume only that the Ramsey terms for both goods are equal, which allows us to derive an expression for the environmental component of the optimal tax.\(^5\)

\(^4\) Fullerton (1997), Schöb (1997), Bovenberg and de Mooij (1997) and Bovenberg (1997) have suggested that Sandmo's formula indicates that the optimal pollution tax should be less than marginal social damages, but this interpretation overlooks the endogeneity of the denominator on the left-hand side: one cannot infer by inspection that the differential between the optimal tax on a polluting good versus a similar non-polluting good will simply equal the value of the second term (since the first term equals the tax on non-polluting goods).

\(^5\) Of course, because the differential between the optimal taxes on polluting and non-polluting goods is an empirical question, we can draw no precise conclusions for any specific case. Thus, the recent literature and the current analysis should be interpreted as reflecting what will be true for a typical good, or on average.
For the polluting good, D, we rearrange (14) to get

\[
t_{D}^{*} = \frac{\left(1 - \frac{\alpha + \theta}{\mu}\right)R}{\left(1 - \frac{\alpha + \theta}{\mu}\right)R + \frac{\alpha MSD}{\mu \left(1 - \frac{\alpha + \theta}{\mu}\right)R}}. \tag{15}
\]

From (13) we can express the Ramsey term as

\[
R = \frac{1 + t_{C}^{*}}{1 - \frac{\alpha + \theta}{\mu}}. \tag{16}
\]

To evaluate the optimal tax \(t_{D}^{*}\) relative to MSD, we substitute (16) into the second term of (15) and rearrange to get

\[
t_{D}^{*} = \frac{\left(1 - \frac{\alpha + \theta}{\mu}\right)R}{\left(1 - \frac{\alpha + \theta}{\mu}\right)R + \frac{\alpha MSD}{\mu}}. \tag{17}
\]

We can evaluate the relationship between the optimal environmental tax and MSD in one of two ways; either evaluating the difference between the optimal tax expressions for C and D or by differentiating \(t_{D}^{*}\) with respect to MSD. Taking the differentiation approach, we obtain

\[
\frac{\partial t_{D}^{*}}{\partial (MSD)} = \frac{(1 + t_{C})\alpha}{\mu}. \tag{18}
\]

indicating that the optimal tax may rise by more or less than MSD depending on the relationship between the revenue-motivated tax rate \(t_{C}^{*}\) and \(\alpha/\mu\).\(^6\)

These expressions are computed for explicit numerical models in section IV. However, as a first approximation, estimates in the literature for the U.S. economy (for example, Browning

\(^6\) We can also confirm from (17) that the optimal environmental component of the tax on D will be zero when marginal damages are zero, thereby rejecting the possibility that the optimal environmental tax might be positive even in cases of zero environmental damage (see Goulder 1995).
place the marginal income tax rate at about 40% (implying an equivalent expenditure tax rate, \(t_c = .667\)), and the marginal excess burden at about 0.25 (implying \(\alpha/\mu = 0.8\)). By these estimates the value of (18) is 1.33, suggesting that the optimal environmental tax should exceed (or rise by more than) marginal environmental damages.

The relationship between the optimal environmental tax and MPD cannot be evaluated directly because the analytical expressions for the optimal tax do not correspond to MPD, except for the special case in the amenity model where \(E\) is separable in utility so that \(\partial V/\partial E = m \partial U/\partial E\). Let \(MD_a\) denote marginal private damage for this special case, so that we have the identity \(\lambda MD_a = \alpha MSD_a\). Substituting this expression into (17) and differentiating with respect to \(MD_a\) we have

\[
\frac{\partial t^*_D}{\partial (MD_a)} = \frac{(1 + t_c)\lambda}{\mu}
\]

(19)

By inspection we see that the value of this expression will be less than in (18) since \(\lambda < \alpha\), which raises the possibility that, for this special case, the optimal tax will rise by more than MSD, while at the same time it will rise by less than \(MD_a\). 7

Thus, the main finding in this section is that the optimal environmental tax is derived from expressions compatible with MSD. This should come as no surprise since both are derived directly from the social welfare maximization problems in (3) and (8). In addition, the relationship between the optimal environmental tax and MSD is found to be independent of the type of environmental externality; by contrast, the relationship between the optimal environmental tax and MPD is not compatible with these derivations, and the divergence between the two will depend on the type of externality and other parameter values.

7 These optimal tax expressions, like Sandmo’s (1975), are equivalent to expressions derived in Bovenberg and Goulder (1997) for a model with an income tax. This alternative tax normalization does not alter any real variable or outcome; results will differ only in terms of units (Fullerton 1997; Schöb 1997). The equivalence of these results with those under the alternative normalization are demonstrated in Appendix B.
IV. Comparisons with the optimal environmental tax: numerical results

The relationship between the optimal environmental tax and (either definition of) marginal environmental damages is an empirical question, dependent on preferences, technologies and tax levels all of which influence the parameters in (18) and (19). To provide quantitative estimates for these relationships, numerical results are presented here for two models similar to the model proposed by Bovenberg and de Mooij (1994). We first consider the production externality case in which labor productivity is dependent on the environment, utility is homothetic in C and D and separable in leisure. These preferences imply that the optimal revenue-motivated taxes on C and D will be equal, so that the optimal environmental tax will equal the difference between the two optimal taxes, \( t_D^* - t_C^* \).

For purposes of numerical estimation, nested models such as (3) can be represented as a single maximization model by introducing the household’s first-order conditions as constraints on social maximization, similar to the first-order approach used in principle-agent problems (Jewitt 1988). Thus we have

\[
\begin{align*}
\text{Max} & : \quad m[u(C, D, L)] \\
\text{s.t.} & \quad \alpha \quad (1 + t_c) C + (1 + t_D) D = h(E)(T - L) + G \\
& \quad \mu \quad mt_c C + mt_D D = mG \\
& \quad \lambda_1 \quad U_C(1 + t_D) = U_D(1 + t_c) \\
& \quad \lambda_2 \quad U_L(1 + t_c) = U_C h \\
& \quad \phi \quad E = e(mD)
\end{align*}
\]

(20)

By representing the problem in this way, the shadow value of the Lagrange multiplier on income will reflect the social value of a unit of income because all adjustments are Pareto efficient. Indeed, the private marginal utility of income, \( \lambda \), does not appear directly in the model.
because it does not represent a Pareto efficient use of a unit of income, but rather a movement
from a Pareto efficient state to a non-Pareto efficient state.

Taking this approach, our numerical version of the production externality problem is

\[ L = \left(C^{0.5} D^{0.5} + 7L^{0.5}\right) + \alpha(hL + 400 - (1 + t_c)C - (1 + t_D)D) \]
\[ + \mu(t_c C + t_D D - G) \]
\[ + \lambda_1(0.5(1 + t_D)C^{-0.5}D^{0.5} - 0.5(1 + t_c)D^{-0.5}C^{0.5}) \]
\[ + \lambda_2((1 + t_c)3.5L^{-0.5} - 0.5C^{-0.5}D^{0.5}) \]
\[ + \phi(h - 1/(1 + 0.0005D)) \] (21)

and the household's maximization problem is

\[ L = \left(C^{0.5} D^{0.5} + 7V^{0.5}\right) + \lambda(hL + 400 - (1 + t_c)C + (1 + t_D)D) \]

The model is calibrated so that the uncompensated elasticity of labor supply is similar to
empirical estimates. In this case the value is 0.23.

The optimality conditions are summarized in table 1. We can immediately compare the
optimal environmental tax \(t^*_D - t^*_C = 0.442\), with both measures of marginal environmental
damage. In this case, MSD = 0.3477 so that the optimal environmental tax is higher than
marginal social damages, the ratio being 1.27. MPD is lower than MSD in this case, so that the
ratio of this same optimal environmental tax to MPD produces a ratio that is higher still, 1.407.

For the amenity case, we construct a model that is similar to the production externality
model but where E enters utility directly, but separably. Our numerical model is

\[ L = \left(C^{0.5} D^{0.5} + 7L^{0.5} + E\right) + \alpha(hL + G - (1 + t_c)C - (1 + t_D)D) \]
\[ + \mu(t_c C + t_D D - G) \]
\[ + \lambda_1(0.5(1 + t_D)C^{-0.5}D^{0.5} - 0.5(1 + t_c)D^{-0.5}C^{0.5}) \]
\[ + \lambda_2((1 + t_c)3.5L^{-0.5} - 0.5C^{-0.5}D^{0.5}) \]
\[ + \phi(E + 0.1D) \] (22)

where the household maximization problem is
\[ L = (C^{0.5}D^{0.5} + L^{0.5}) + \lambda(L + 300 - (1 + t_c)C + (1 + t_D)D). \]

Given the separability of \( L \), this represents the special case where \( \lambda \text{MPD} = \alpha \text{MSD} \).

The optimality conditions are presented in table 2. In this model the ratio between the optimal environmental tax and MSD is the same as in the productivity model, 1.27. This consistency is expected based on (18), so long as the parameter values for \( \mu \), \( \alpha \) and \( t_c \) are similar. By contrast, the ratio of the optimal environmental tax to marginal private damages is now less than one, at 0.95. In this particular case, we observe that the relationship between the optimal environmental tax and marginal private damages varies by as much as 50 percent depending on the type of externality being considered.

Note that in each of these two models there is only one optimal environmental tax. Whether that tax appears to be higher or lower than marginal damages is a function only of the definition of marginal environmental damages being used. The correspondence between these numerical results and the analytical expressions derived above is confirmed by plugging \( t_c \) and the values of \( \mu \) and \( \alpha \) from table 2 into (18) and (19) to get 1.27 and 0.95, respectively.

Looking at the relationship between the optimal environmental tax and MPD for the amenity case in table 2, one might conclude that the optimal environmental tax is lower in the second-best setting than in the first-best (Pigouvian) case, and that the optimal environmental tax actually declines with increasing revenue requirements. For the current model, such an interpretation would be mistaken. The optimal environmental tax and both measures of marginal damages are plotted in figures 1 and 2 for productivity and amenity models, respectively. In the case of the amenity model (figure 2), we see that once revenue requirements cannot be satisfied with the first-best Pigouvian tax alone, distortionary taxes are necessary and taxes on both goods rise, with the \( t^{*}_D \) rising by more than \( t^{*}_C \) so that the optimal environmental tax increases. This
occurs despite the fact that the marginal environmental damage in utility units is constant, and the social marginal utility of income varies only negligibly (due to the lower overall utility levels with increasingly distortionary taxes). Indeed, what becomes clear in figure 2 is that with rising revenue requirements, the optimal environmental tax increases, but the value of marginal private damages rises even faster due to the declining private purchasing power of a unit of income as tax rates rise: as $\lambda$ declines MPD rises.

The recent literature has concluded, however, that “in the presence of preexisting distortionary taxes, the optimal pollution tax typically lies below the Pigouvian tax…” (Bovenberg and de Mooij 1994) based on a point observation where MPD is greater than the optimal environmental tax. This observation has in turn been interpreted as invalidating the double dividend hypothesis when it is defined as the notion that “any additional revenue requirements should be met by raising the tax on the dirty good by more than taxes on clean goods” (Fullerton 1997). These interpretations notwithstanding, the results presented here offer support for that particular definition of the double dividend hypothesis.

In the productivity externality case depicted in figure 1, marginal private damages declines with rising revenue requirements, in absolute terms as well as in relation to marginal social damages. This is expected since the divergence between the two expressions is greater in the numerators (due to substitution effects) than in the denominators.

A consistent relationship emerges from these numerical models between the optimal environmental tax and marginal social damages, compatible with the analytical results presented in the previous section. This result is important because it provides a consistent basis for setting

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8 In the production externality case, marginal social damage rises slightly with rising revenue requirements given increasing distortionary taxes but rising labor productivity as environmental damage declines.
policy targets which is independent of the type of environmental externality that might be at issue.

V. Concluding comments

Comparisons between the optimal environmental tax and marginal environmental damages have been used as a benchmark comparison for policymaking, as well as for making inferences about the welfare effects from environmental tax reform, the validity of the double dividend hypothesis, and the existence of a tax interaction effect. Marginal environmental damages, however, can be defined in more than one way. The analysis presented here finds that defining marginal environmental damages based on the social marginal rate of substitution between income and the environment yields a consistent benchmark for setting optimal environmental taxes, one that is congruent with both the Pigouvian Principle and the double dividend hypothesis. Using this benchmark, the optimal environmental tax will equal marginal social damages in the first-best case, and will generally exceed marginal social damages in a second-best setting.

By contrast, when marginal environmental damages are defined as the sum of household’s marginal private damages, no consistent relationship with the optimal environmental tax exists: MPD will likely be lower than the optimal environmental tax for a production externality, and higher than the optimal environmental tax for an environmental amenity. The essential shortcoming of the marginal private damages measure is that it does not reflect the marginal rate of substitution between income and the environment for the social welfare maximization problem. In particular, the private marginal utility of income does not reflect a
Pareto efficient use in a second-best setting. Pigou recognized the importance of defining values in social terms when calling for equating the ‘marginal social net products’ across all resources, where the value of the marginal social net product refers to values when the effect of any government intervention on price has been removed (Pigou 1952, p. 135n).

It may be that the inclination toward using marginal private damages reflects a view that empirical methods tend to produce estimates of marginal private damages, as with stated and reveal preference techniques. However, other widely-used techniques produce estimates of marginal social damages, as with production function methods, replacement cost methods, or shadow project methods.

Both analytical and numerical results presented above indicate that for realistic estimates of tax rates and labor supply, the optimal environmental tax rises with an increased revenue requirement, implying that environmental quality will improve for two reasons. First, a higher level of revenue-motivated taxes will tend to discourage consumption as well as the pollution associated with consumption, in which case revenue-motivated taxes contribute to a cleaner environment by themselves (see Gaube 2000). Second, in addition to higher revenue-motivated taxes, the pollution component of the tax on the polluting good is also shown to rise relative to marginal social damages.
Appendix A: Detailed derivation of the optimal taxes

Implicit optimal tax rules are derived here for a general model with \( n+1 \) goods, taking an approach parallel to Sandmo (1975). The essential problem can be stated as one in which \( m \) identical individuals maximize utility \( U = u(X_0, X_1, \ldots, X_n, E, G) \) for goods \( j = 0, \ldots, n \), where leisure is \( X_0 \) and where labor supply is taken out of a time endowment normalized to equal one so that labor supply equals \( 1-X_0 \). Units are chosen for goods and income so that all pre-tax prices equal one, and where there are \( n-1 \) non-polluting \( X \) goods (excluding leisure) and one good \( X_z \) which produces an environmental externality. The consumption of \( X_z \) is assumed to erode the environment, \( E \), where \( E = e(mX_z) \) and where \( \frac{de}{d(mX_z)} < 0 \). Two sets of optimal tax expressions are derived, one for each types of environmental externality.

Production externality

In this model, labor productivity, \( h \), is a function of environmental quality such that \( h = h(E) \). We define aggregate output as \( mh(E)L = \sum mX_j + mG \), where \( mG \) is financed through collection of tax revenues. Each household’s maximization problem can be stated as

\[
\begin{align*}
\text{Max}_{X_0, X_1, \ldots, X_n} & : \quad u(X_0, X_1, \ldots, X_n) \\
\text{s.t.} & \quad hL = \sum_{j=1}^{n} (1 + t_j)X_j
\end{align*}
\]

[A1]

so that individuals maximize utility subject to their budget constraint while ignoring both the environmental consequences of their own consumption choices and government behavior. The Lagrangian expression for each household is thus

\[
L = u(X_0, X_1, \ldots, X_n) + \lambda \left[ h(1-X_0) + G - \sum_{j=1}^{n} (1+t_j)X_j \right] \quad \text{for } j = 1, \ldots, z, \ldots, n.
\]

[A2]
Consumer prices are given as \( p_j = 1 + t_j \) for \( j = 1 \) to \( n \), but where income is untaxed, so that \( p_0 = 1 \).

The first-order conditions for each household take the form

\[
U_j = \lambda(1 + t_j) \quad \text{for } j = 1, \ldots, n
\]

and

\[
U_o = -\lambda h(E) \quad \text{for } X_0.
\]

The social planners problem is then

\[
\max_{\lambda, t_j} m \left[ u(X_0, X_1, \ldots, X_n) \right] \quad \text{s.t.} \quad h(1 - X_0) + G = \sum_{j=1}^{n} (1 + t_j)X_j
\]

\[
s.t. \quad m \sum_{j=1}^{n} (1 + t_j)X_j = G
\]

\[
\lambda = h(E(mX_2)) \quad \text{[A3]}
\]

Taking the dual approach, we define the household’s indirect utility function as \( v(p_0, p_1, \ldots, p_n) = u(x_0, p_1, \ldots, p_n) \), so we can state the social optimization problem as the Lagrangian equation

\[
L = mu(v(p_0, p_1, \ldots, p_n)) + \mu \left[ m \sum_{j=1}^{n} t_jX_j - G \right]
\]

In the presence of environmental effects on labor productivity, the first-order conditions for the social optimization problem are

\[
-\lambda X_j + \mu \left[ \sum_{j=1}^{n} \frac{\partial X_j}{\partial p_j} + X_j \right] + \left[ \lambda \left( m(1 - X_0) \frac{\partial h}{\partial E} \frac{de}{d(mX_2)} \right) + \mu \sum_{j=1}^{n} t_j \frac{\partial X_j}{\partial h(1 - X_0)} \frac{de}{d(mX_2)} \right] \frac{\partial h}{\partial p_j} = 0 \quad \forall j \neq 0 \quad \text{[A4]}
\]

where \( \lambda = \frac{dV}{d(h(1 - X_0))} = \frac{\partial U^*}{\partial h(1 - X_0)} \) is the household’s marginal utility of income.

Let \( \pi_k \) denote marginal social damages in utility units for the production externality case, or

\[
\pi_k = \lambda \left( mL \frac{\partial h}{\partial E} \frac{de}{d(mX_2)} \right) + \mu \sum_{j=1}^{n} t_j \frac{\partial X_j}{\partial h(1 - X_0)} \frac{de}{d(mX_2)} \quad \text{[A5]}
\]
where marginal social damage includes both the direct loss of income to households as well as the loss in revenues due to changes in labor income, \( h(l-X_0) \), due to the change in \( h \) as well as the resulting change in labor supply.

Simplifying the notation in [A4] using [A5] we have

\[
-\lambda X_j + \mu \left( \sum_{i=1}^{\infty} t_i \frac{\partial X_i}{\partial p_j} + X_j \right) + \pi_k \frac{\partial X_z}{\partial p_j} = 0 \quad \forall j \neq 0 \tag{A6}
\]

Derivations of optimal tax rules often include substitution of the Slutsky equation in such a way that the social marginal utility of income, \( \alpha \), is represented along with the shadow cost of raising an additional dollar of revenue (Auerbach 1985). Diverging from the approach taken by Sandmo, we rearrange the planner's first-order conditions and use the Slutsky equation to split the cross-price effects into compensated effects (superscript \( U \)) and effects on income, \( Y \), as

\[
\frac{\partial X_z}{\partial p_i} = \frac{\partial X_z^U}{\partial p_i} - X_i \frac{\partial X_z}{\partial Y}.
\]

We substitute \( \alpha \) to obtain

\[
-\alpha X_j + \mu \sum_{i=1}^{\infty} t_i \frac{\partial X_i^U}{\partial p_j} + \mu X_j + \pi k \left( \frac{\partial X_z^U}{\partial p_j} - X_i \frac{\partial X_z}{\partial Y} \right) = 0 \quad \forall j \neq 0 \tag{A7}
\]

We define \( S \) as the determinant of the Slutsky matrix of compensated demands, so that \( S_{ij} \) is the cofactor of the element for the \( j \)th row (price) and \( i \)th column (quantity). Using Cramer's rule we can solve for the optimal taxes

\[
t_j = \frac{(\mu - \alpha) \sum_{i=1}^{\infty} X_i S_{ij}}{\mu S} + \pi_k \sum_{i=1}^{\infty} \left( \frac{\partial X_z^U}{\partial p_i} - X_i \frac{\partial X_z}{\partial Y} \right) S_{ij} \tag{A8}
\]

where the second term on the right-hand side is the environmental component of the tax. From theorems about the expansion of determinants, we know that

\[
\sum_{i=1}^{\infty} \frac{\partial X_z^U}{\partial p_i} S_{ij} = \begin{cases} 0 & \text{for } j \neq Z \\ S & \text{for } j = Z \end{cases}
\]
Let $R$ denote the “Ramsey term” for compensated demands or $R = \sum_{i=1}^{n} X_i S_i / p_i S$ reflecting the revenue generating potential for a marginal change in the tax on $X_i$ due to the direct and indirect effects on consumption for all goods. Further simplify the notation by defining the income effect on the environment as $\theta_k = \pi_k \sum_{i=1}^{n} X_i \frac{\partial \bar{X}_z}{\partial Y}$. We can thus rearrange terms and simplify so that the optimal tax expressions can then be written as

\[
\frac{t_j}{(1 + t_j)} = \frac{(\mu - \alpha + \theta_k)}{\mu} R \quad \forall j \neq z \tag{A9}
\]

and

\[
\frac{t_j}{(1 + t_j)} = \frac{(\mu - \alpha + \theta_k)}{\mu} R + \frac{\pi_k}{\mu (1 + t_j)} \quad \forall j = z \tag{A10}
\]

These implicit solutions are difficult to interpret by inspection, in part because of the lack of transparency in interpreting $R$. Moreover, although the environmental component of the tax in [A10] appears to be separable from the standard formula, the independence is illusory both because of the denominator $(1 + t_z)$ is endogenous and because the actual level of the externality depends on the actual equilibrium and hence the optimal tax rates; the same is true in the other direction (Auerbach 1985).

The results differ from the expressions obtained by Sandmo involving uncompensated demands. Sandmo concluded that the environmental damages of $X_z$ “does not enter the tax formulas for the other commodities, regardless of the pattern of complementarity and substitutability” (1975, p. 92). In this alternative derivation, we see that the numerator in the first term on the right-hand side includes $\theta$, a term involving $\pi_k$, indicating that the presence of an externality raises the optimal tax on all goods due to their income effect: by reducing real income, all taxes discourage consumption of the externality-producing good to some extent, and

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these optimal tax rates will be higher as a result. These two versions of the optimal tax results are
not in conflict: in the model involving ordinary demands, the income effects are implicit.

**Amenity externality case**

In the amenity case, labor productivity, \( h \), is constant, so that aggregate output is defined
as \( mh(1-X_0) = \sum mX_i + mG \), and the \( E \) enters the utility function directly. Each household’s
maximization problem can be stated as

\[
\text{Max } \quad u(x_0, x_1, \ldots, x_n, E)
\]

\[
s.t. \quad h(1 - X_0) = \sum_{j=1}^{n} (1 + t_j)X_j
\]

The Lagrangian expression for each household taking \( E \) and \( G \) as given is thus

\[
L = u(x_0, x_1, \ldots, x_n, E) + \lambda \left[ h(1 - X_0) + G - \sum_{j=1}^{n} (1 + t_j)X_j \right]
\]

for \( j = 1, \ldots, z, \ldots, n \). \[A11\]

In this case the social planners problem is

\[
\text{Max } m \left[ u(x_0, x_1, \ldots, x_n, E) \right] \quad \text{s.t.} \quad h(1 - X_0) + G = \sum_{j=1}^{n} (1 + t_j)X_j
\]

\[
s.t. \quad m \sum_{j=1}^{n} (1 + t_j)X_j = G
\]

\[
E = e(mX_2)
\]

The household’s indirect utility function is \( V(p_0, p_1, \ldots, p_n, E) = u(x(p_0, p_1, \ldots, p_n), E) \), so we can
state the social optimization problem as the Lagrangian equation

\[
L = mu(v(p_0, p_1, \ldots, p_n, E) + \mu \left[ m \sum_{j=1}^{n} t_jX_j - G \right]
\]

The planner’s first-order conditions are
so that the marginal damage denoted as \( \pi_a \) for the amenity case, is defined as

\[
\pi_a = m \left[ \frac{\partial U}{\partial E} \frac{de}{d(mX_z)} + \mu \sum_i t_i \frac{\partial X_i}{\partial E} \frac{de}{d(mX_z)} \right].
\]  

We see that in this case as well, the marginal social damage includes both the direct loss of income to households and the loss of revenues due to changes in consumption. \(^9\)

At this point we can see that the expressions in \([A13]\) and \([A14]\) are parallel to those in \([A4]\) and \([A5]\). Thus, we forego presenting steps \([A6]\) through \([A8]\) for the amenity case, and simply write the final expressions for these optimal environmental taxes:

\[
\frac{t_j}{1 + t_j} = \frac{\mu - \alpha + \theta_z}{\mu} R \quad \forall j \neq z \tag{A15}
\]

and

\[
\frac{t_j}{1 + t_j} = \frac{\mu - \alpha + \theta_z}{\mu} R + \frac{\pi_k}{\mu (1 + t_j)} \quad \forall j = z \tag{A16}
\]

Thus, the optimal tax expressions for the production externality and environmental amenity cases differ only in terms of \( \pi_k \) and \( \pi_a \), the expression for marginal social damages in utility units.

---

\(^9\) Sandmo does not explicitly consider the effect of changes in environmental quality on demands for other goods, so the second term in brackets on the right-hand side of \([A14]\) is omitted in his analysis. However, given the highly stylized representation of an environmental externality, one may assume that Sandmo has either assumed the effects to be zero, or incorporated them as indirect components of the cross-price effects with respect to the polluting good.
Appendix B: Equivalence of results for alternative normalization

To demonstrate the equivalence between the results in (17), (18) and (19) with those found in Bovenberg and Goulder (1996), note that an income tax normalization amounts to multiplying the household budget constraint by (1-tL) where tL is the income tax rate and where (1+tC)=(1/(1-tL)). This step implies that the household budget constraint now represents units of net income, and the shadow value of net income, λN, will differ from the shadow value of gross income, λ, such that λ = λN(1-tL). In the Bovenberg and Goulder model, household income is expressed in units of net income whereas the government revenue constraint continues to be expressed in units of gross income, in which case the shadow value on the revenue constraint, µ, reflects gross income units. To be consistent, this shadow value can also be expressed in net income units, µN, where µ = µN(1-tL).

Bovenberg and Goulder’s model is precisely the one identified above as a special case, where ∂V/∂E = m∂U/∂E. Letting MDN denote m(∂U/∂E)/λN for this special case, and noting the identity a N MSDN = λNMDN, the optimal environmental tax expression from Bovenberg and Goulder can be represented several ways:

\[ t^*_D = (MD_a) \frac{\lambda_N}{\mu} = (MD_a) \frac{\lambda_N}{\mu^N (1-tL)} = (MD_a) \frac{\lambda_N (1+tC)}{\mu^N} \]  \[ \text{[B1]} \]

Taking the partial derivative of \( t^*_D \) with respect to MDa we have

\[ \frac{\partial t^*_D}{\partial (MD_a)} = \frac{\lambda_N}{\mu^N (1-tL)} = \frac{\lambda_N (1+tC)}{\mu^N} \]  \[ \text{[B2]} \]
which is equivalent to (19) except that units are expressed in terms of net income. Substituting the identity \( a^N MSD^N_a = \lambda^N MD^N_a \), [B1] can be rearranged (canceling \( \lambda^N/\lambda^N \) and multiplying by \( \alpha^N/\alpha^N \) where \( \alpha = \alpha^N(1-t_l) \)), to arrive at

\[
[B3] \quad t^* _d = \frac{\alpha^N (1+t_c) MSD^N}{\mu^N}.
\]

Comparing [B3] and (17), we see two differences. First there is no revenue-raising term in [B3] since the labor tax accounts for the revenue-raising tax. Second, there are differences in units due to the normalization. Otherwise, the two expressions are identical.
REFERENCES


Table 1. Second-best optimal environmental taxation with a productivity externality

Social optimization model

Household utility function: \( U = C^{0.5}D^{0.5} + 7L^{0.5} \)
Household budget constraint \( (1+t_C)C + (1+t_D)D = h(T-L)P_L + R \)
Environmental damage \( h = 1/(1+0.1D) \)
Budget constraint \( R = nt_CC + nt_DD \)
Revenue requirement \( R = 400 \)
Time endowment \( T = 1000 \)
Population \( n = 1 \)

Optimal solution:
\[
U = 448.1555 \\
C = 413.771 \\
D = 312.210 \\
L = 160.689 \\
E = 1.156 \\
t_C = 0.361 \\
t_D = 0.803 \\
\alpha = 0.446 \\
\mu = 0.477 \\
\]

Marginal social damages = 0.3477
Pollution tax differential \( (t_D - t_C) \) = 0.442
Ratio: pollution tax to MSD = 1.27

Household optimization:

Optimal solution same as above
Private marginal utility of income, \( \lambda \) = 0.319
Marginal damage in utility units = 0.1002
Marginal private damage = 0.314
Ratio: optimal tax/MPD = 1.407
Table 2. Second-best optimal environmental taxation with an amenity externalities

**Social optimization model**

<table>
<thead>
<tr>
<th>Household utility function:</th>
<th>U = C^{0.5}D^{0.5} + 7L^{0.5} - 0.1E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Household budget constraint</td>
<td>(1+t_c)C + (1+t_d)D = h(T-L)P_L + R</td>
</tr>
<tr>
<td>Budget constraint</td>
<td>R = nt_cC + ntp_D</td>
</tr>
<tr>
<td>Environmental damage</td>
<td>E = 0.1D</td>
</tr>
<tr>
<td>Revenue requirement</td>
<td>R=400</td>
</tr>
<tr>
<td>Time endowment</td>
<td>T=1000</td>
</tr>
<tr>
<td>Population</td>
<td>n=1</td>
</tr>
</tbody>
</table>

Social optimum:

- U = 476.88
- C = 490.645
- D = 405.960
- L = 103.395
- E = 40.596
- t_c = 0.321
- t_d = 0.597
- α = 0.461
- μ = 0.479

Marginal social damages (0.1/α) = 0.217
Pollution tax differential (t_d – t_c) = 0.276
Ratio: pollution tax to MSD = 1.27

**Household optimization:**

Optimal solution: same as above

- Private marginal utility of income, λ = 0.344
- Marginal private damages, 0.1/λ = 0.291
- Ratio: pollution tax to MPD = 0.95
Figure 1. Optimal environmental tax and two definitions of marginal environmental damage: productivity externality

Figure 2. Optimal environmental tax and two definitions of marginal environmental damage: amenity externality